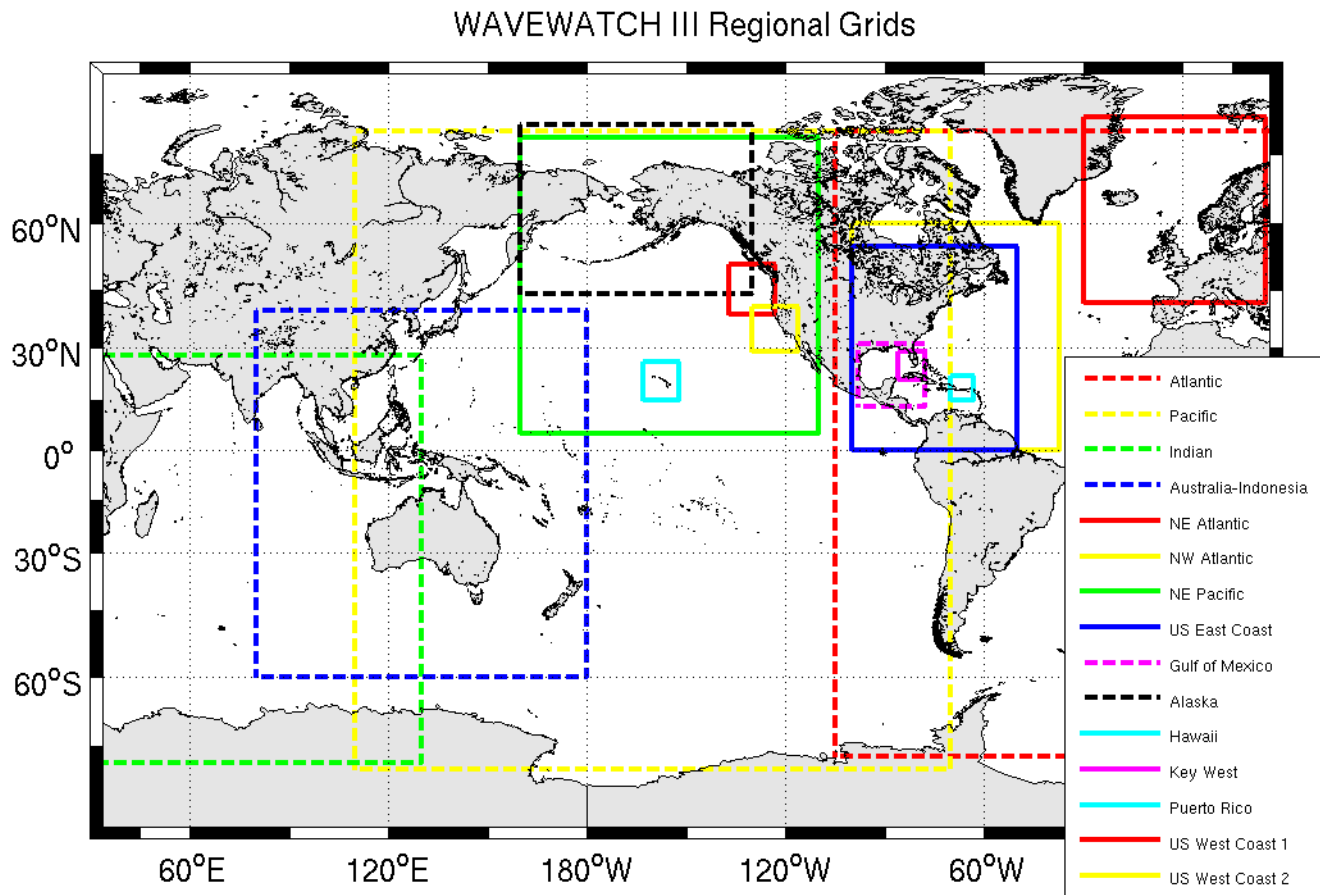


# Unified Global and Regional Wave Model on a Multi-Resolution Grid

Jian-Guo Li & Andy Saulter      28 October 2013

# Motivation for a unified model



Could we merge all these grids into a single grid ?



# Conclusions

**“Yes, we can!”**

How?

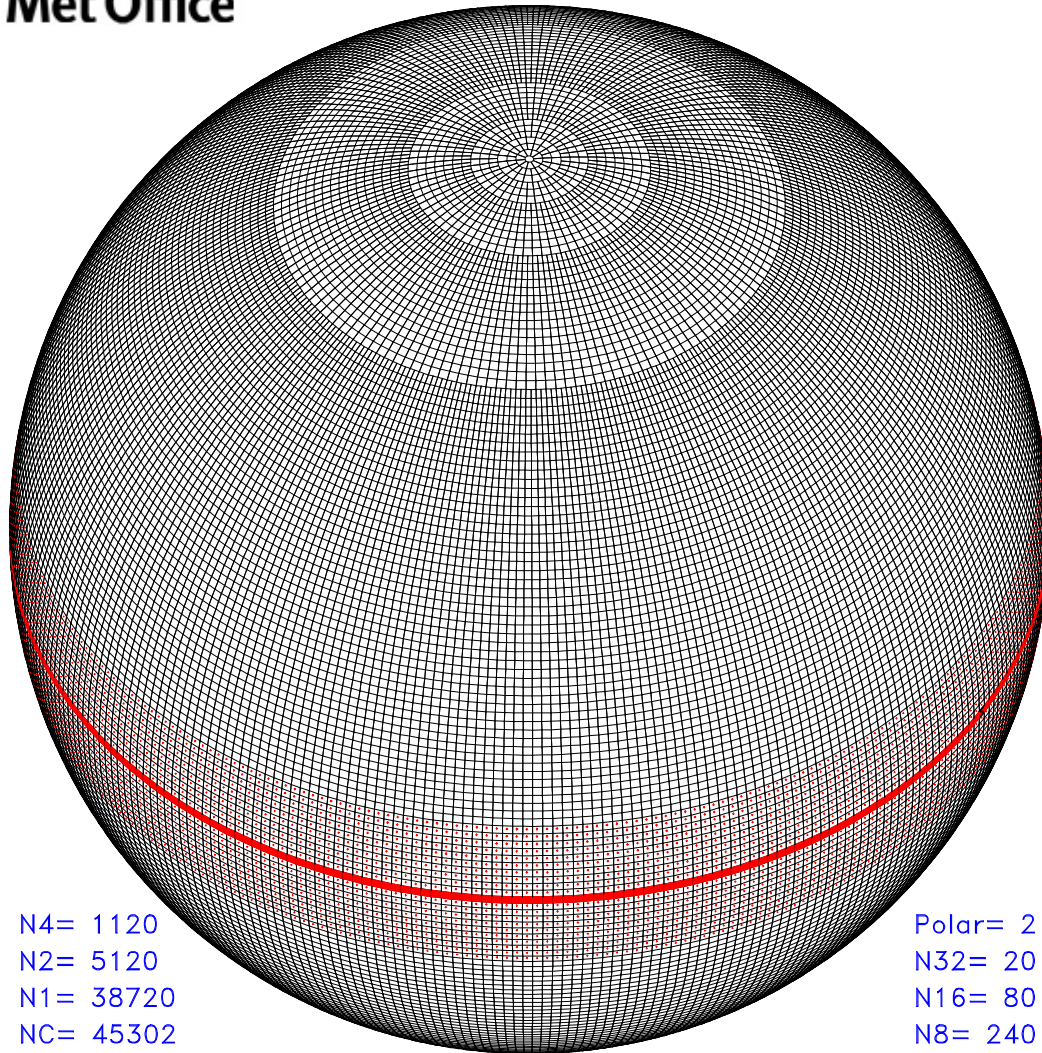
Multi-resolution SMC grid



# 1. Methodology

- Generating a spherical multiple-cell (SMC) grid.
- Propagation on a global SMC grid.
- Application in a wave model

# Spherical Multiple-Cell grid



N4= 1120  
 N2= 5120  
 N1= 38720  
 NC= 45302

Polar= 2  
 N32= 20  
 N16= 80  
 N8= 240

SMC 1° Grid Projection (60.0°W, 45.0°N) Rotation Pole (180.0°E, 0.0°N)

- Merged cells at high latitudes to relax CFL limit on time step.
- Introduce round polar cells with integral equation to avoid polar blocking and singularity.

$$\frac{\partial}{\partial t} \iint_A \psi dA = - \oint_{C_A} \psi \mathbf{v} \cdot d\mathbf{s}$$

$$\psi_P^{n+1} - \psi_P^n = \pm \frac{\Delta t}{A_P} \sum_{i=1}^m \psi_i^* v_i \Delta s_i$$

More details please see:  
 Li, J.G. 2011: *Mon. Wea. Rev.*, **139**, 1536-1555.



# Surface wave energy balance equation on SMC grid

- Ocean surface wave spectral energy balance equation in spherical coordinate system:

$$\frac{\partial \psi}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial (F_y \cos \varphi)}{\cos \varphi \partial y} + \frac{\partial (k \psi)}{\partial k} + \frac{\partial (\dot{\theta} \psi)}{\partial \theta} = S$$

$$F_x \equiv (u + U) \psi - D_x \partial \psi / \partial x$$

$$F_y \equiv (v + V) \psi - D_y \partial \psi / \partial y$$

Advection and diffusion terms are merged.

Great-circle turning is added to refraction.



# Upstream Non-Oscillatory 2<sup>nd</sup> Order (UNO2) Advection Scheme

$$\psi_j^{n+1} = \psi_j^n + \left( u_{j-1/2} \psi_{j-1/2}^{MF} - u_{j+1/2} \psi_{j+1/2}^{MF} \right) \Delta t / \Delta x_j$$

$$\psi_{j+1/2}^{MF} = \psi_C^n + (x_{MF} - x_C) G_C$$

$$x_{MF} - x_C = 0.5 \text{sign}(u_{j+1/2}) (\Delta x_C - |u_{j+1/2}| \Delta t)$$

$$G_C = \text{Sign}(G_{DC}) \min(|G_{DC}|, |G_{CU}|)$$

$$G_{AB} \equiv (\psi_A - \psi_B) / (x_A - x_B)$$

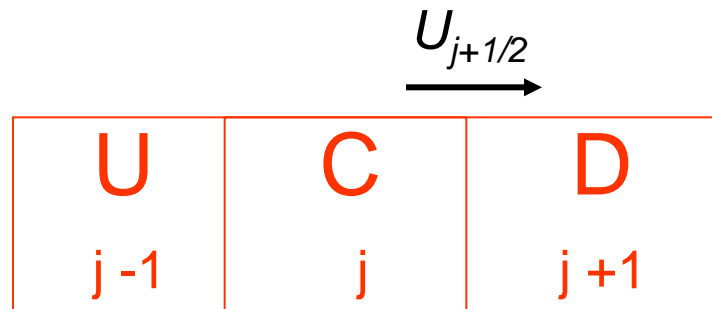
Details see:

Li, J.G. (2008)  
*Mon. Wea. Rev.*,  
136, 4709-4729.

## UNO2

You Know Too

Upstream,  
Central and  
Downstream  
cells relative  
to velocity  $u$ .





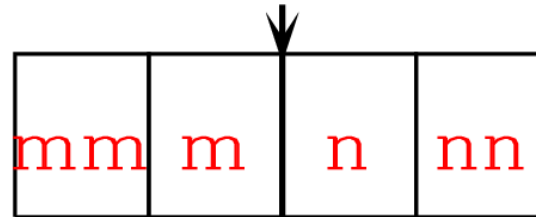
# Unstructured SMC grid flux face pointer array

Internal and boundary faces are treated alike in 1-D array. No boundary for global model.

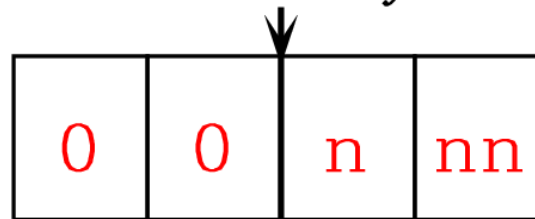
Single point island is extended by 0-cells, allowing single-island blocking.

Two-D spherical surface advection is done by 4 loops: u- and v-face flux loops and 2 cell update loops.

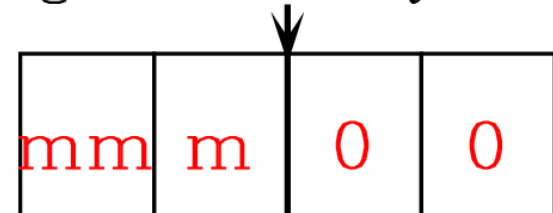
Internal face



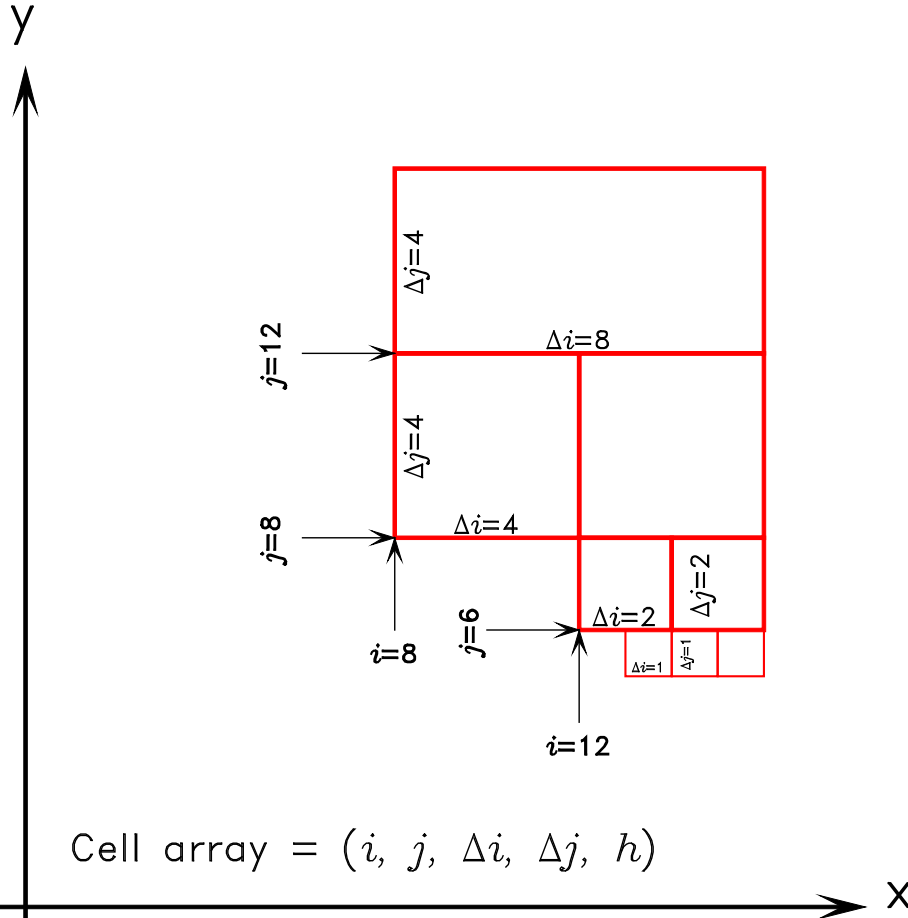
Left boundary face



Right boundary face



# How to create a SMC grid



- Use regular lat-lon grid bathymetry data at finest resolution.
- Make base resolution cells as much as possible and refined ones up to the finest resolution near coastlines.
- Merge cells at high latitudes once grid length is halved.
- Sort cells in ascending size order.

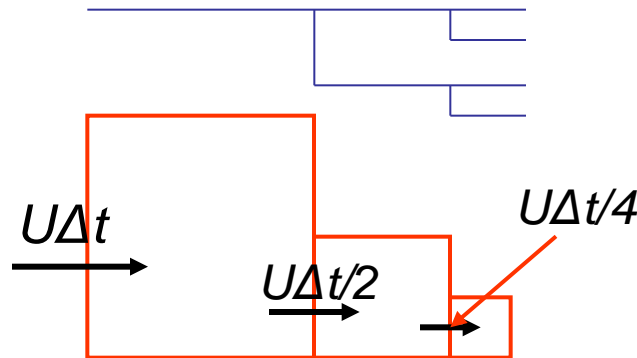
More details please see:  
 Li, J.G. 2012: *J. Comput. Phys.*, **231**, 8262-8277.



# Multi-Resolution SMC grid

- Sub flux loop for refined faces on sub time steps.
- Temporary net flux variable to hold flux into each cell between space and time levels.
- Sub-cell loop to update refined cells and reset net flux variable.
- Resolution levels defined by user.

Halved sub  
time step for  
halved faces



# Refraction by rotation and depth gradient limiter

Ocean surface wave energy travels at depth dependent group speed and refraction occurs in shallow waters.

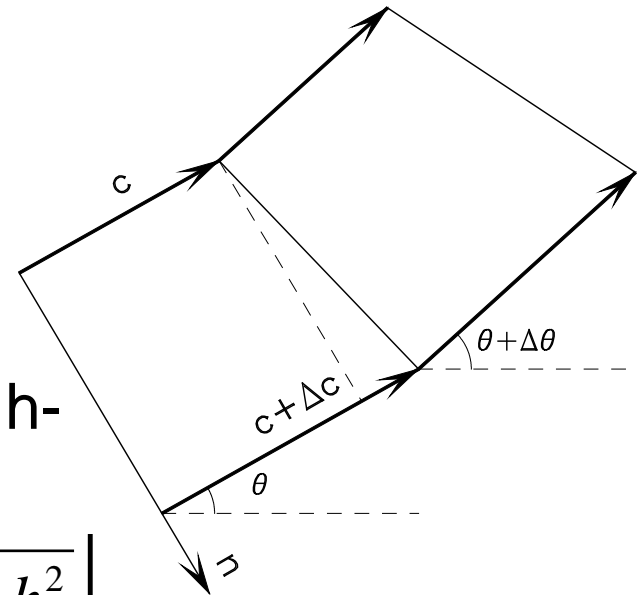
$$\dot{\theta}_{rfr} = -\xi \mathbf{n} \cdot \nabla h - \mathbf{n} \cdot \nabla U_k$$

$$\xi = \omega / \sinh(2kh)$$

Refraction by rotation and limited to h-gradient direction.

$$\alpha = \cos^{-1} \left[ - (h_x \cos \theta + h_y \sin \theta) / \sqrt{h_x^2 + h_y^2} \right]$$

$$\Delta \theta_{mxrfr} = \eta \min(\alpha, \pi - \alpha)$$





# Wave travels along great circle rather than fixed local direction

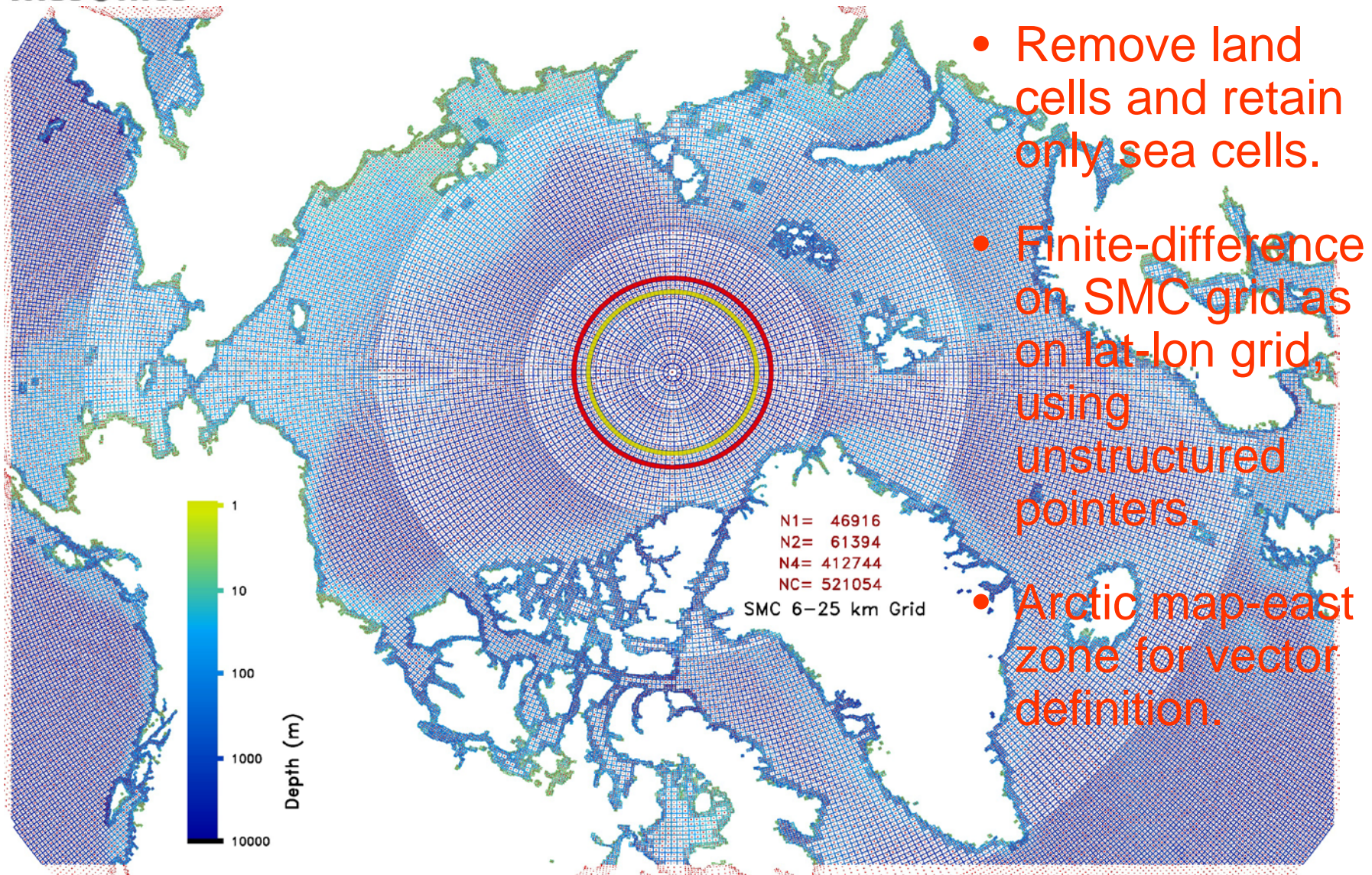
- Great circle distance is the shortest distance on a sphere, hence the nature likes it.
- Wave spectral component is defined in relative to its local east so it deviates from its direction in transport. The direction changing rate is

$$\dot{\theta}_{gct} = -\dot{\gamma} \cos \theta \tan \varphi, \quad \dot{\gamma} \equiv c_g / r_{Earth}$$

Refraction and GC turning may be treated as a single rotation, rather than directional advection.

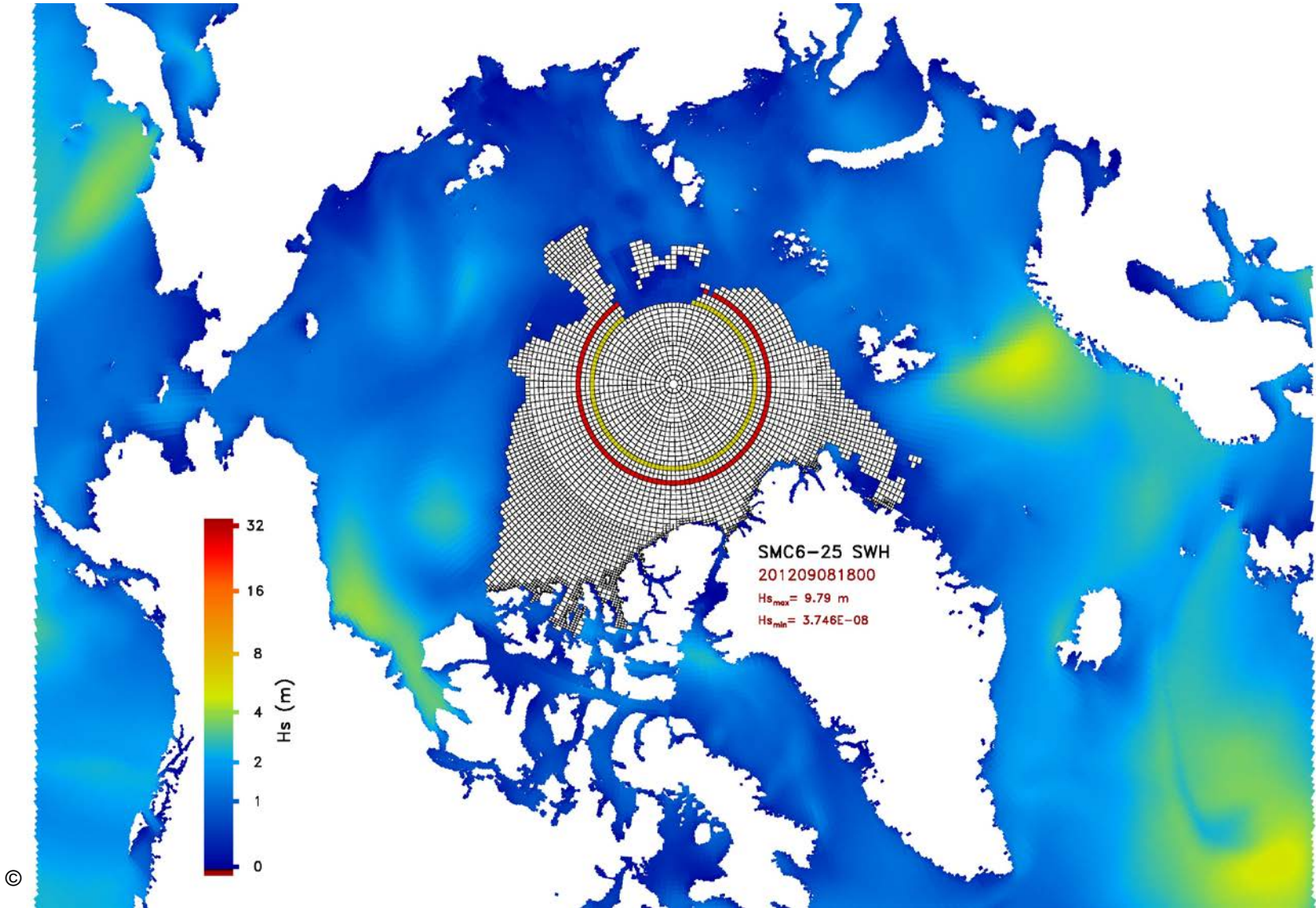
$$\Delta \theta = \left( \dot{\theta}_{gct} + \dot{\theta}_{refr} \right) \Delta t$$

# SMC6-25 grid Arctic region



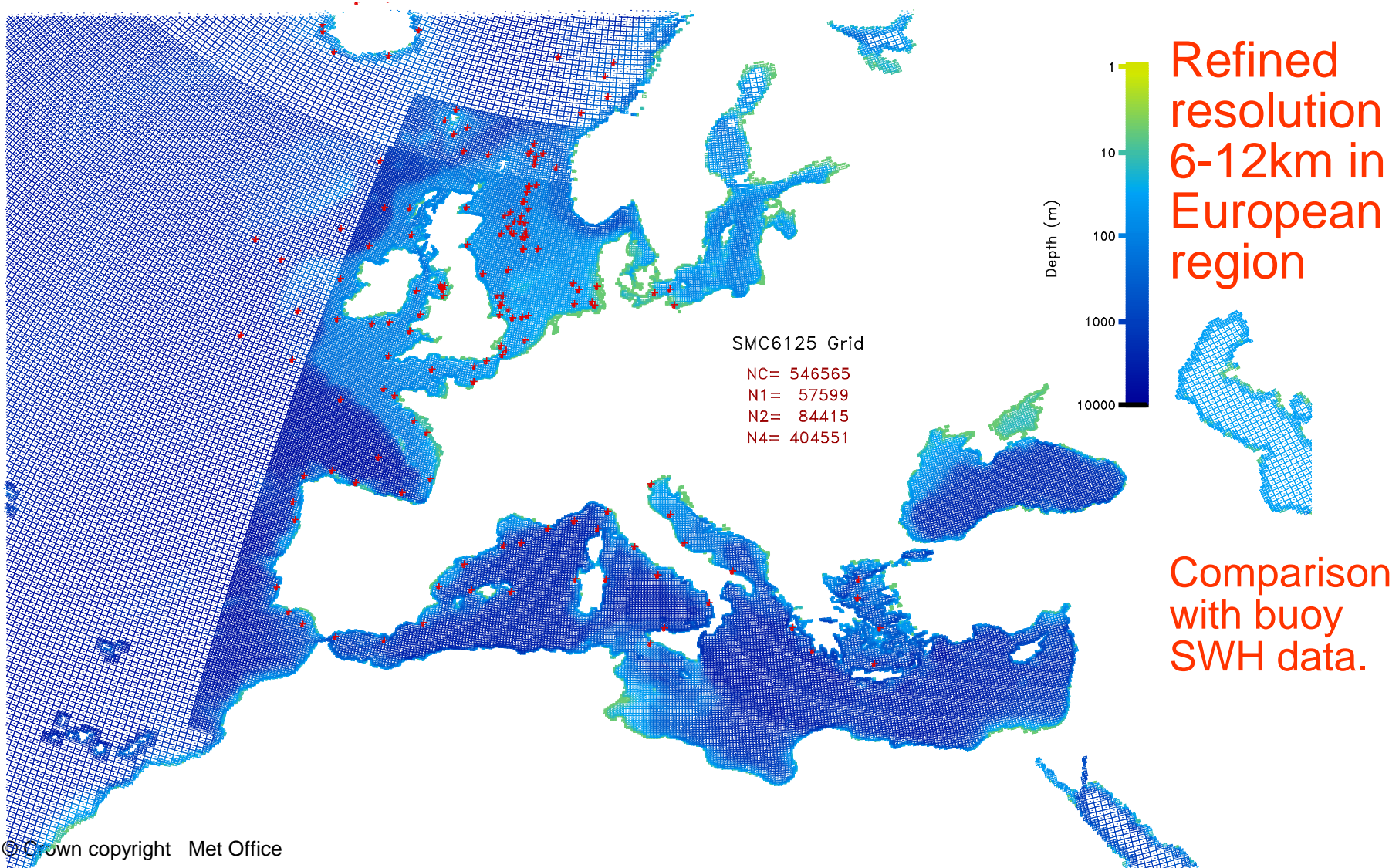
- Remove land cells and retain only sea cells.
- Finite-difference on SMC grid as on lat-lon grid, using unstructured pointers.
- Arctic map-east zone for vector definition.

# SMC6-25 SWH Arctic region



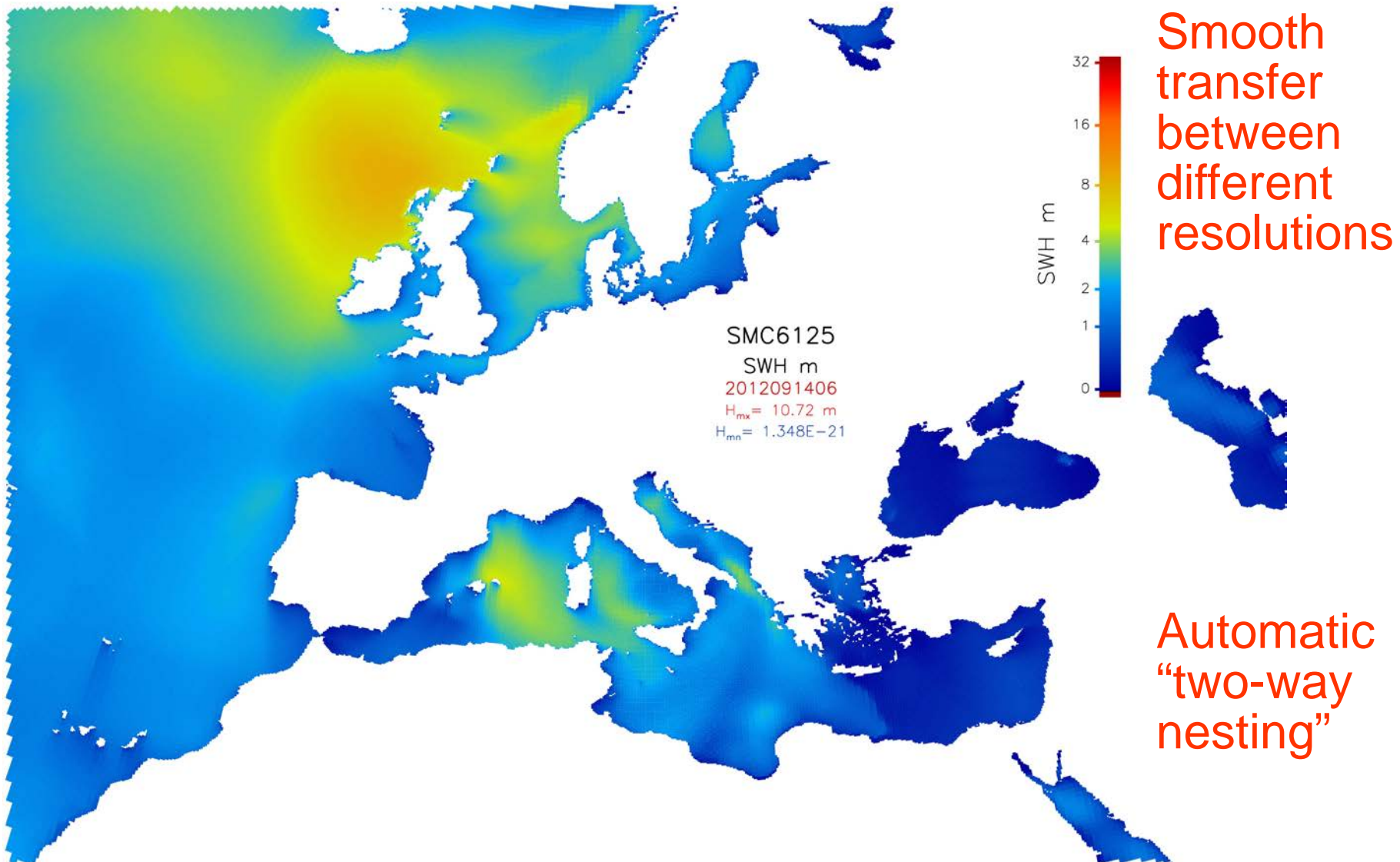


# SMC6-25 grid European region





# SMC6125 SWH European region





# SMC6-25 grid Pacific region

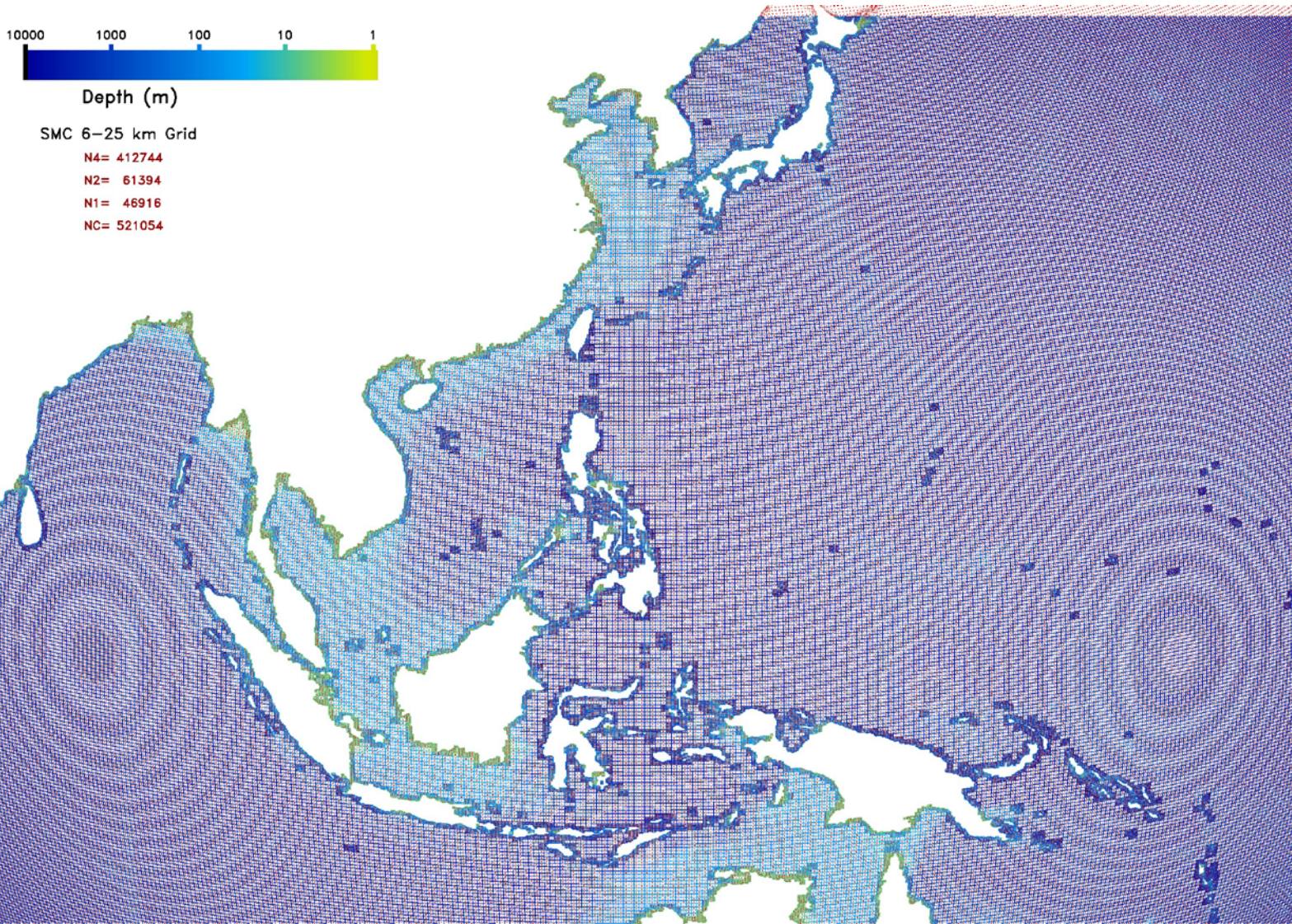
Pacific region  
including  
islands:

Diaoyu  
(Senkaku),

Huangyang  
(Scarborough  
Shoal),

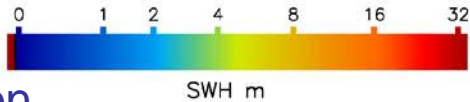
Xisha  
(Paracel)

Nansha  
(Spratly).





# SMC6-25 SWH in Pacific region



SMC625  
2012093000  
 $H_{mx} = 12.26$  m

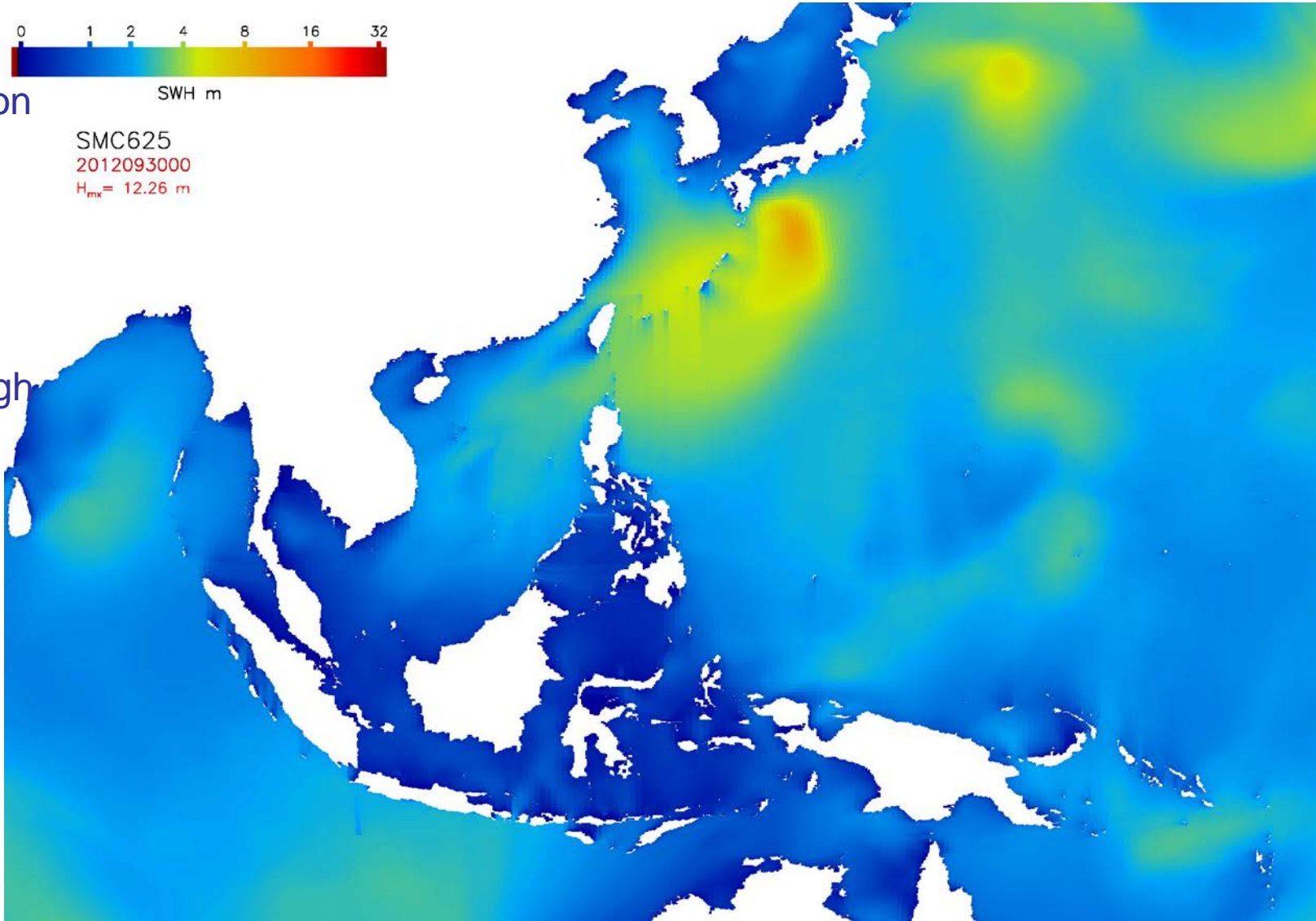
Pacific region  
including  
islands:

Diaoyu  
(Senkaku),

Huangyang  
(Scarborough  
Shoal),

Xisha  
(Paracel)

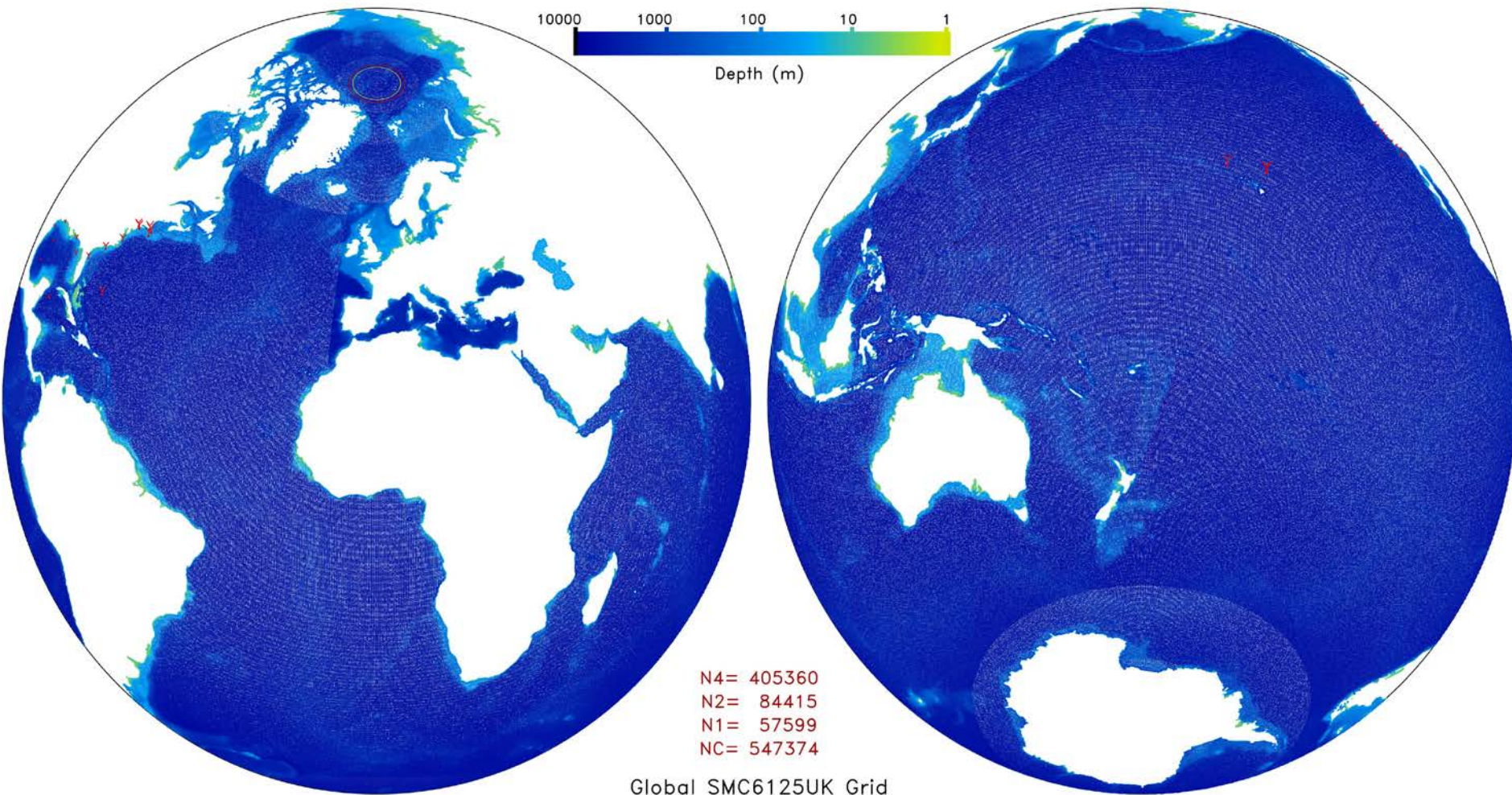
Nansha  
(Spratly).





# The 6-12-25km SMC grid

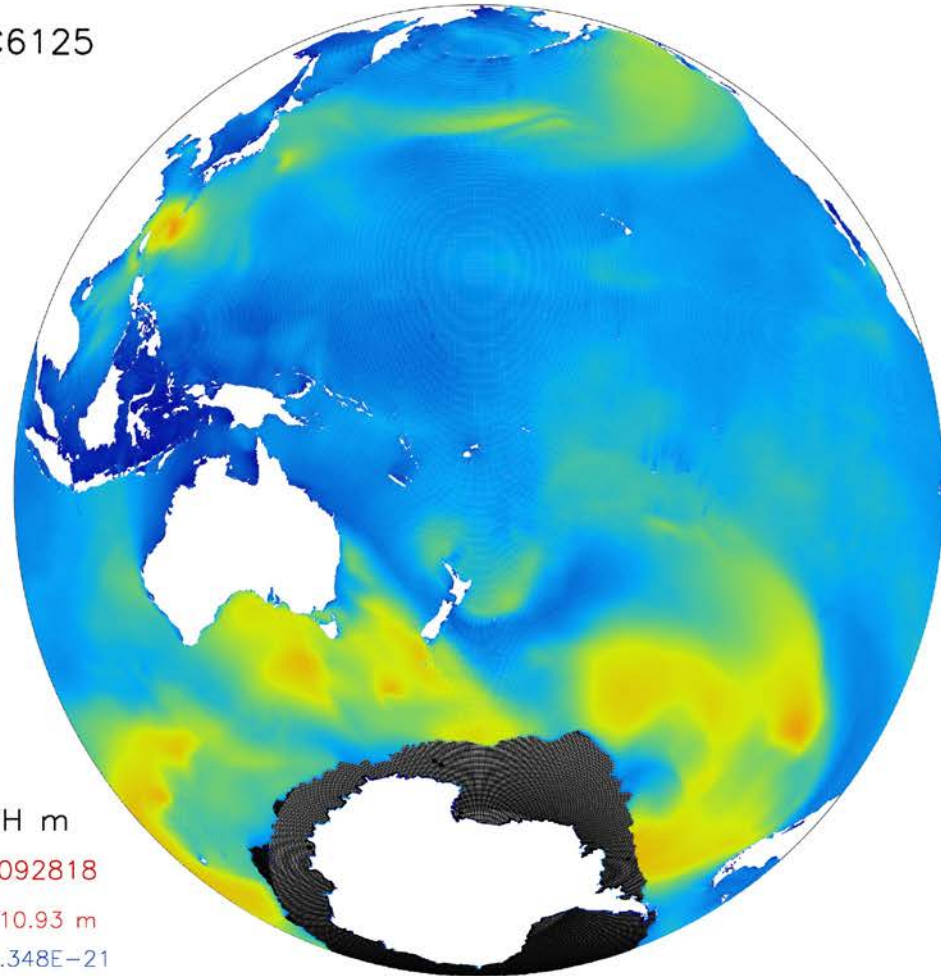
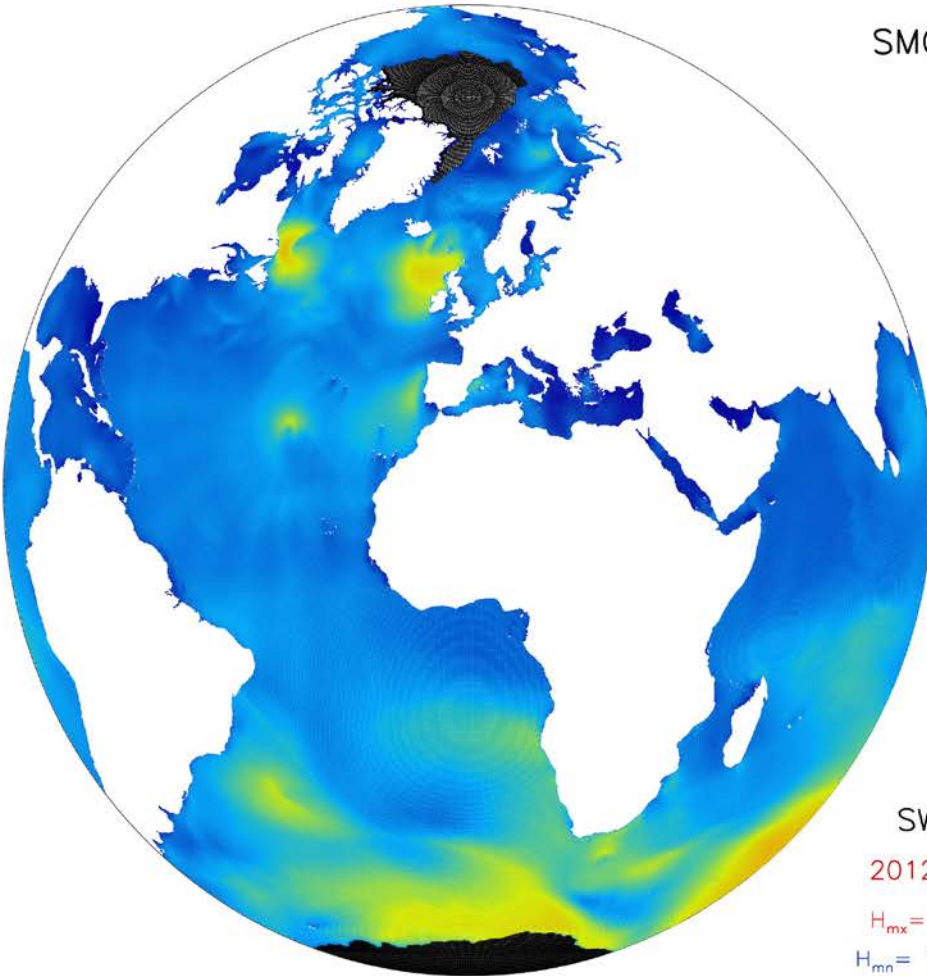
Total cell 547,374 ~ 70% of 25km lat-lon grid 1024x768





# SMC6125 SWH global view

SMC6125



SWH m  
2012092818  
 $H_{mx} = 10.93$  m  
 $H_{mn} = 1.348E-21$



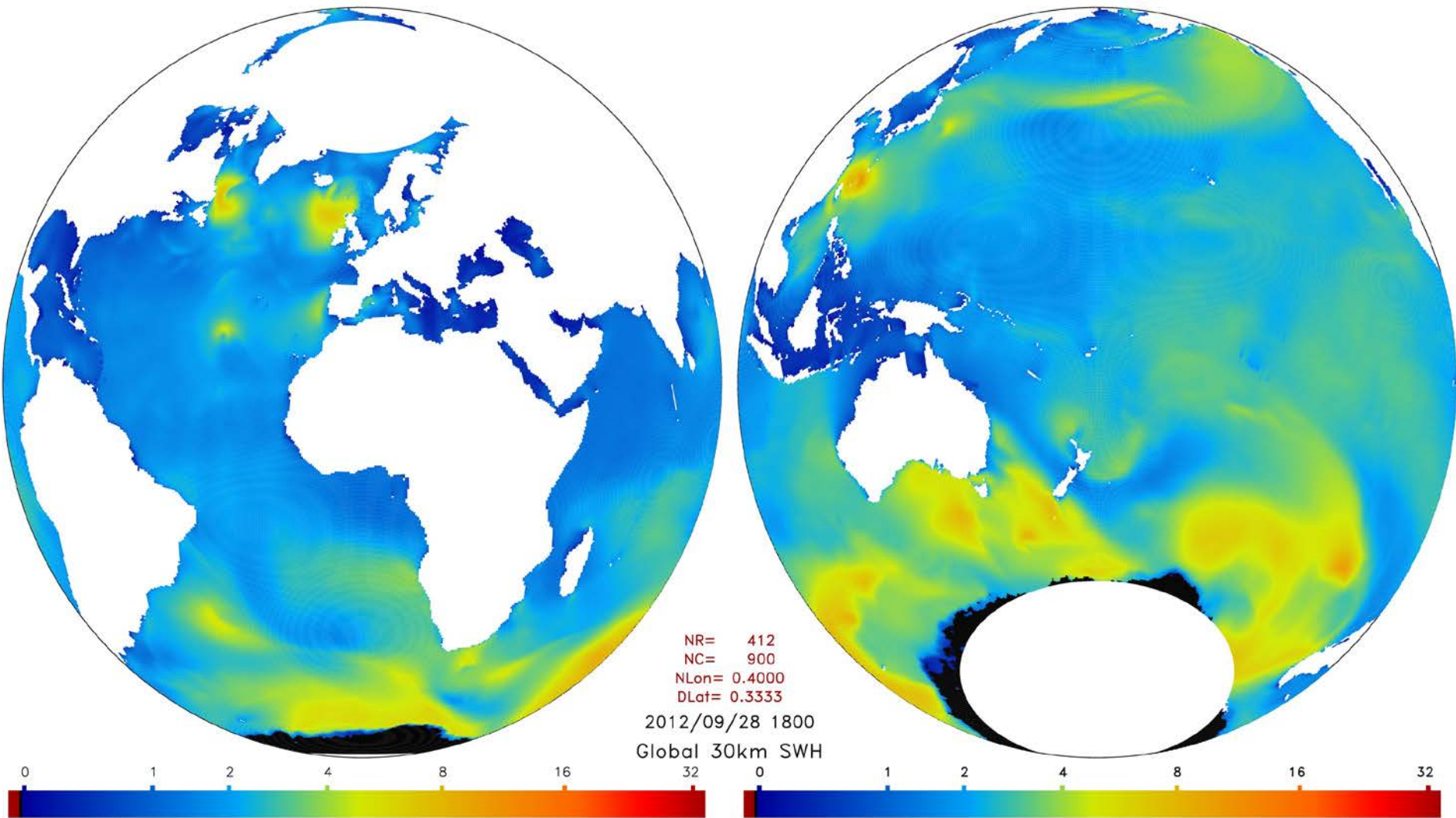


## 2. Compare SMC6125 with global and regional models

- Met Office global 30km operational model.
- Global 25km model.
- European 8 km regional model.
- UK 4 km model.
- Validation with buoy wave spectra.

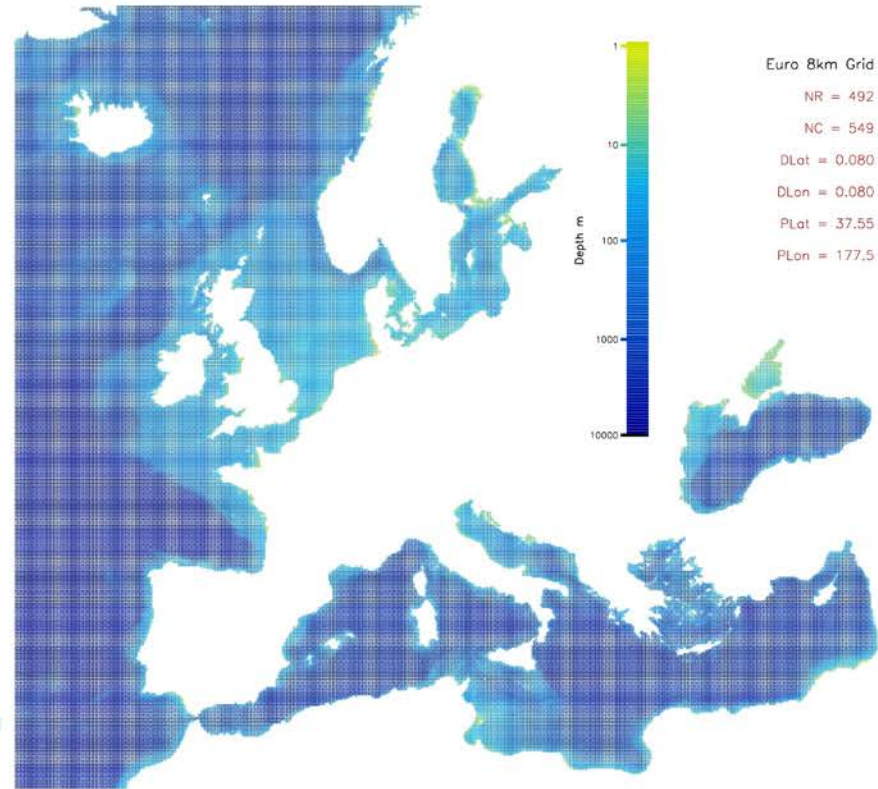
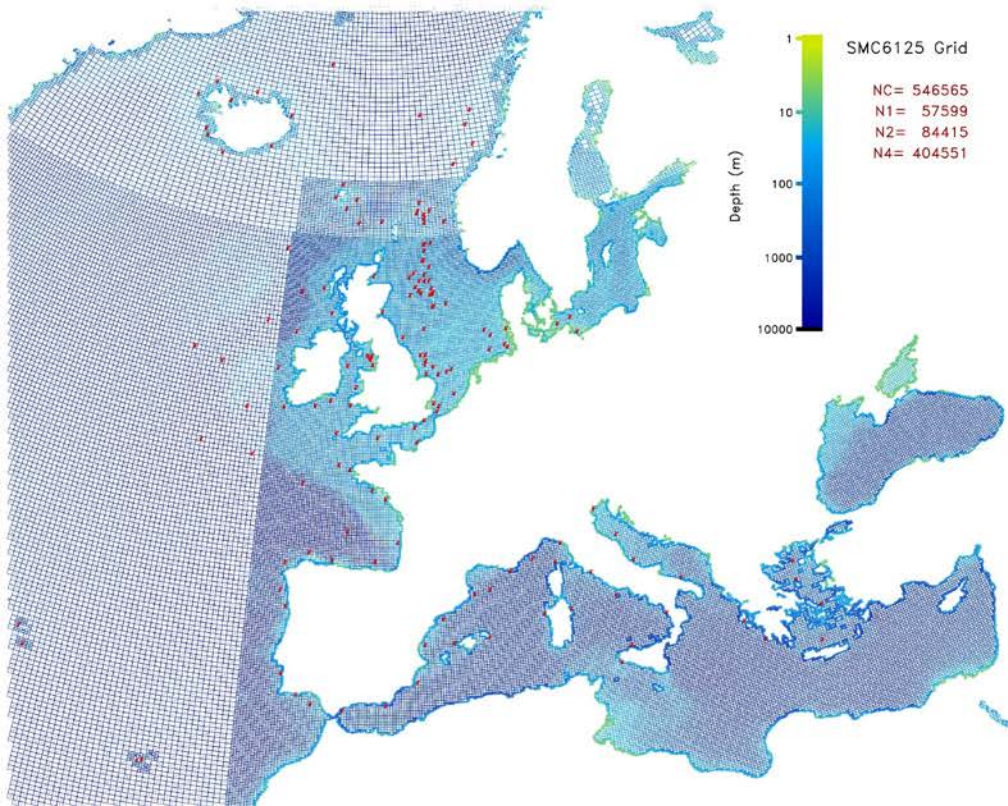


# Global 30km (mid-section) SWH



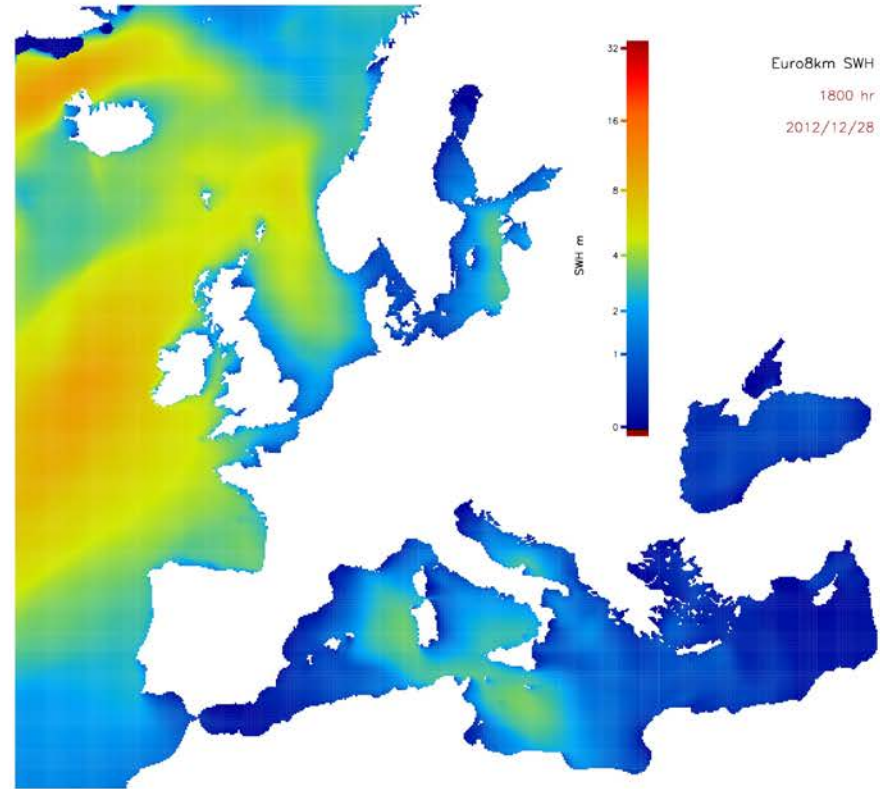
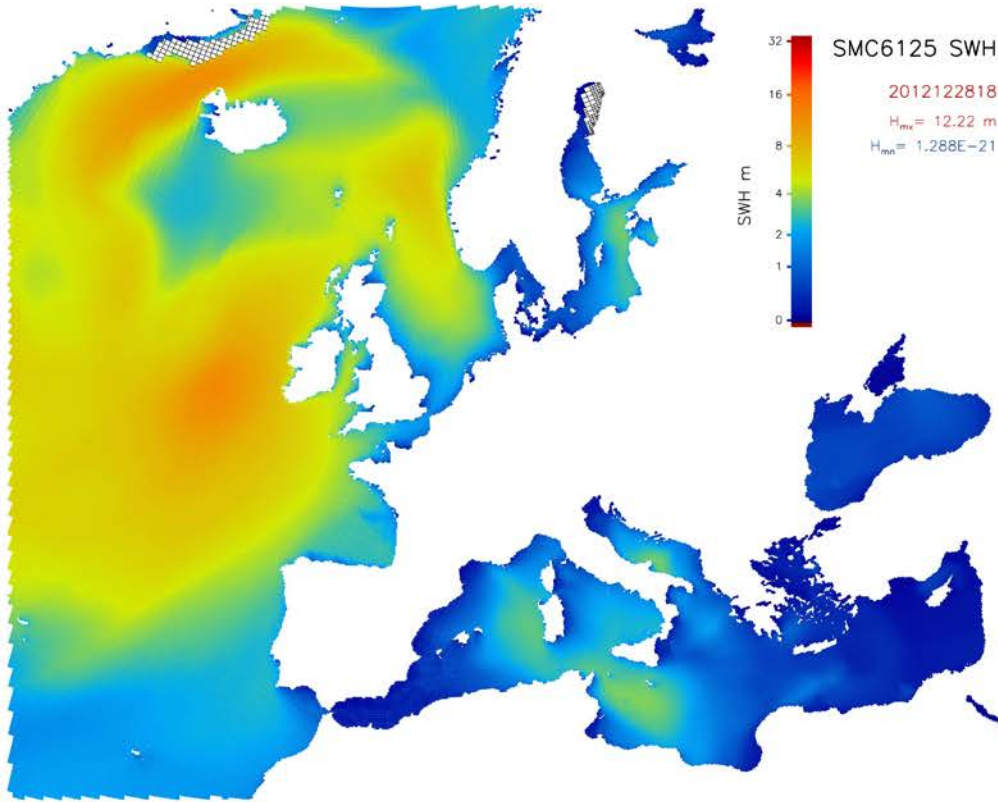


# Euro 8 km and SMC6125 grids





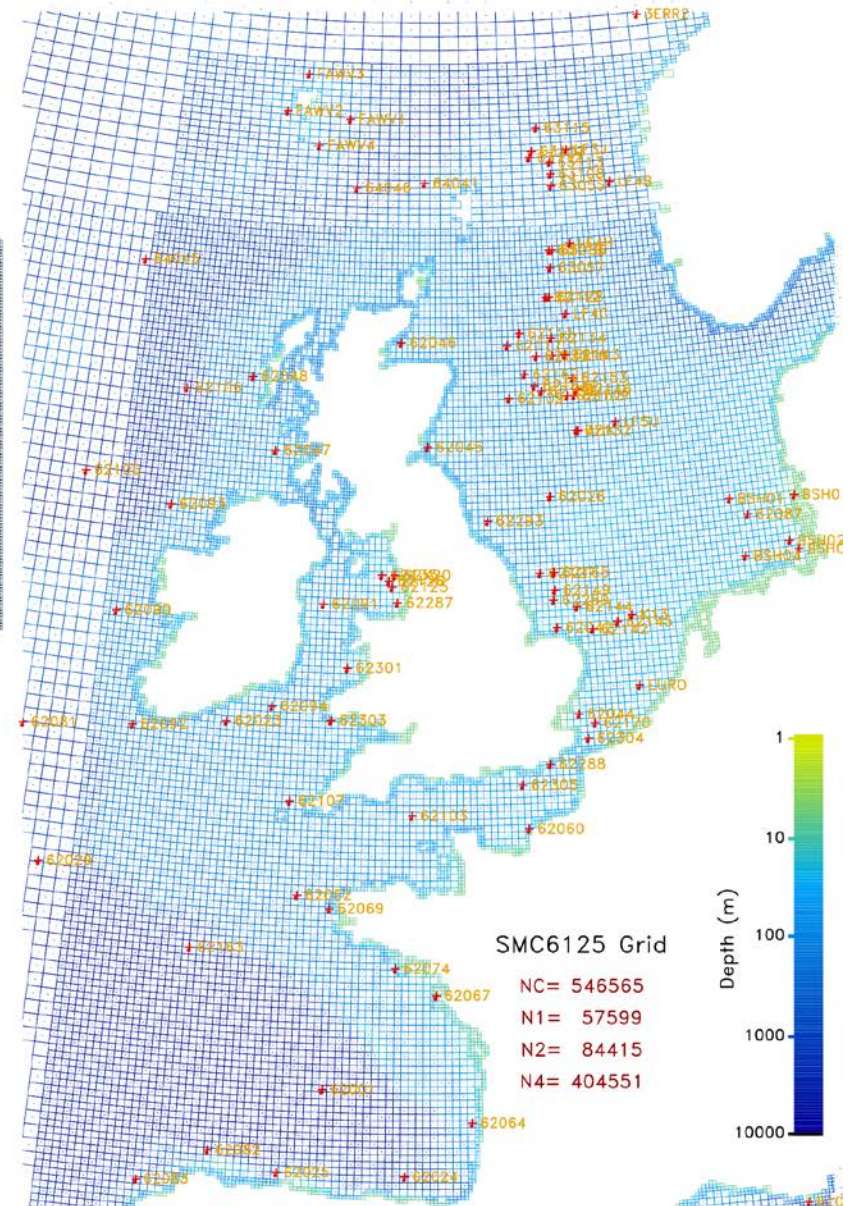
# Euro 8 km and SMC6125 grids





Met

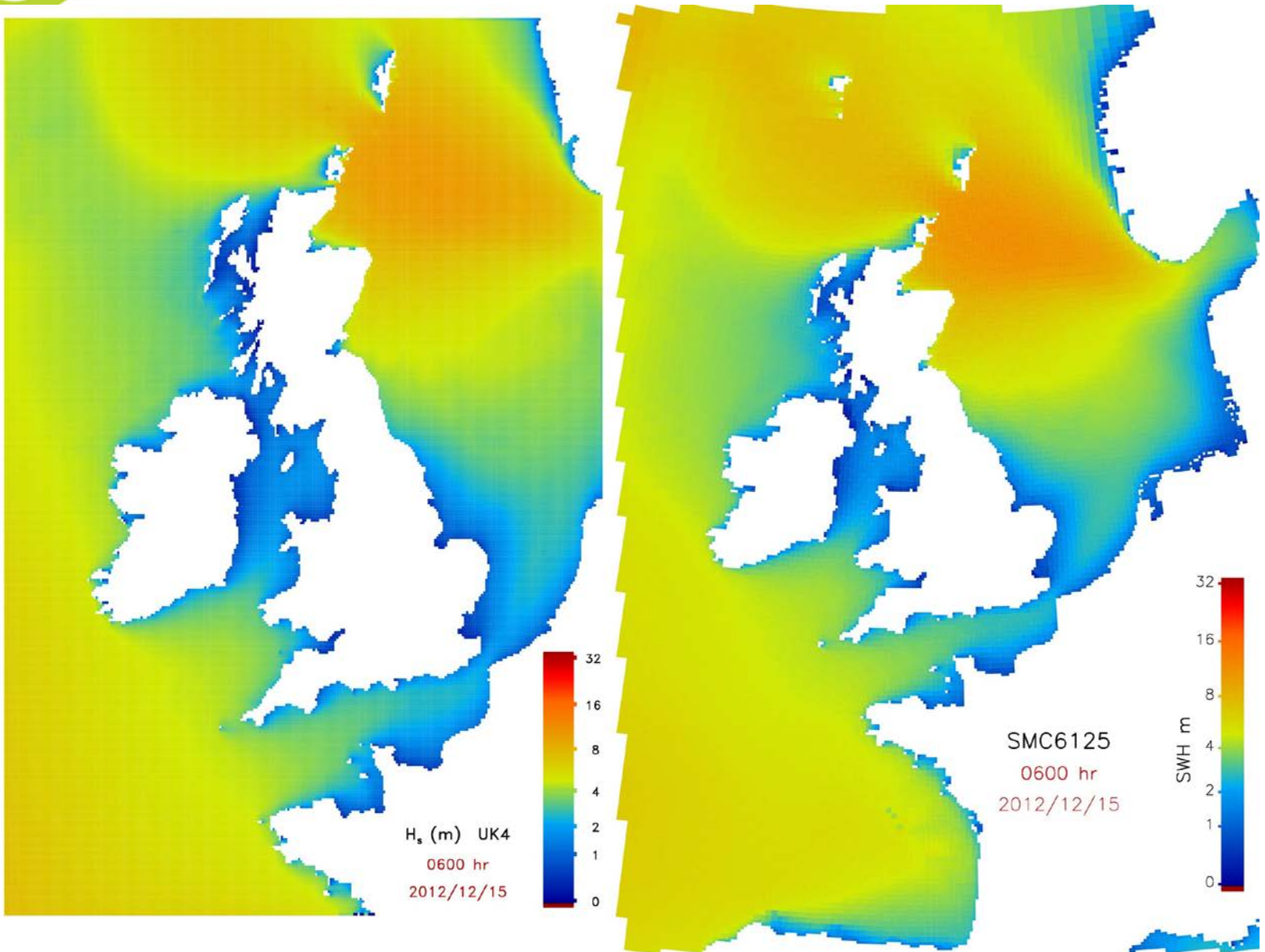
# UK4 km and SMC6125 grids





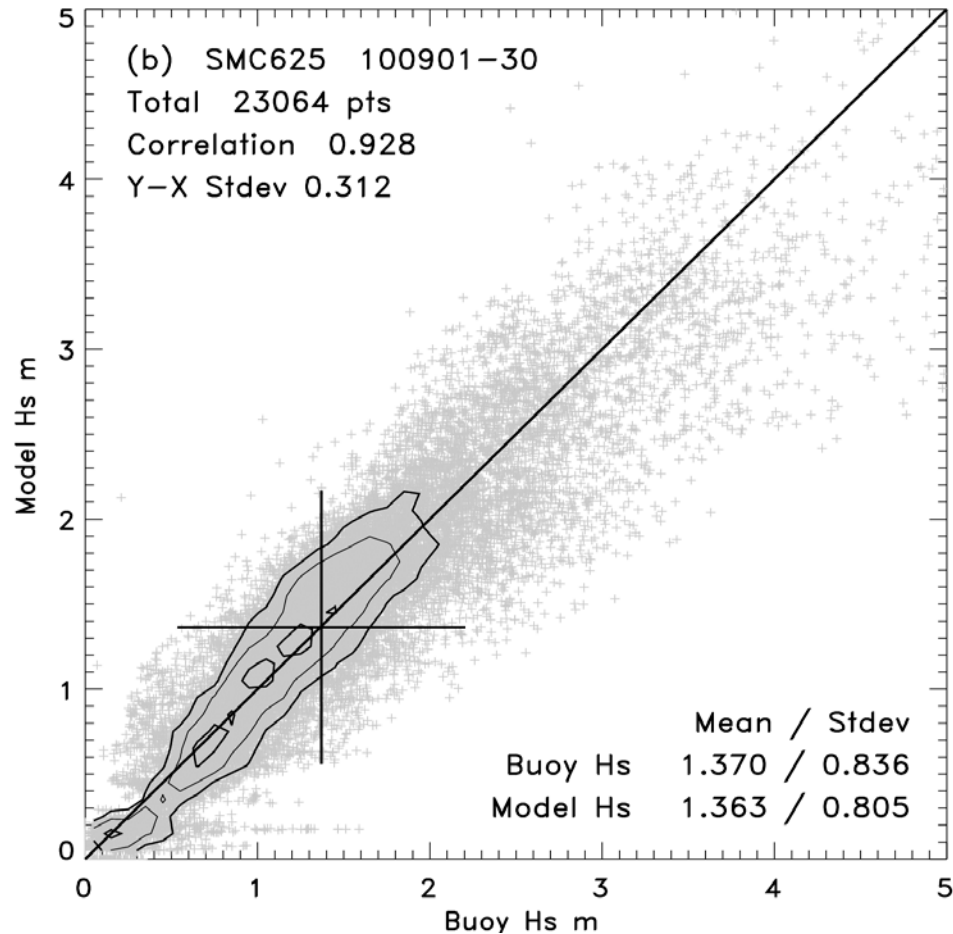
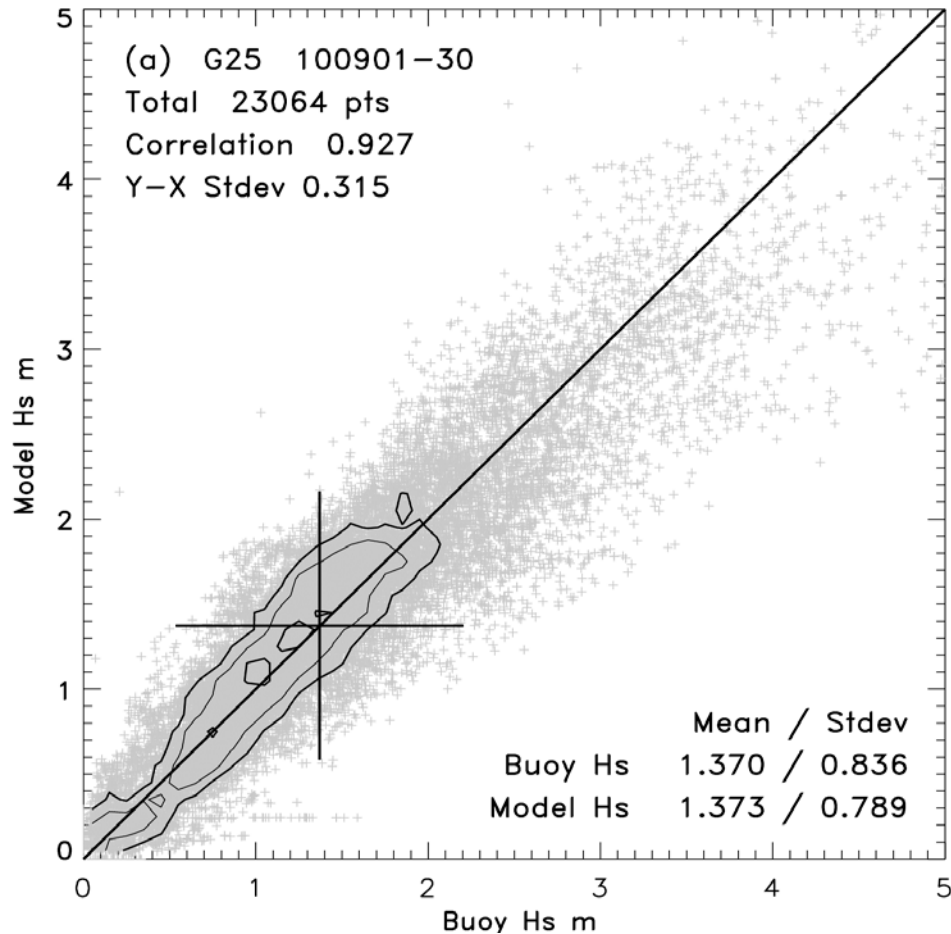
Met

# UK4 km and SMC6125 grids





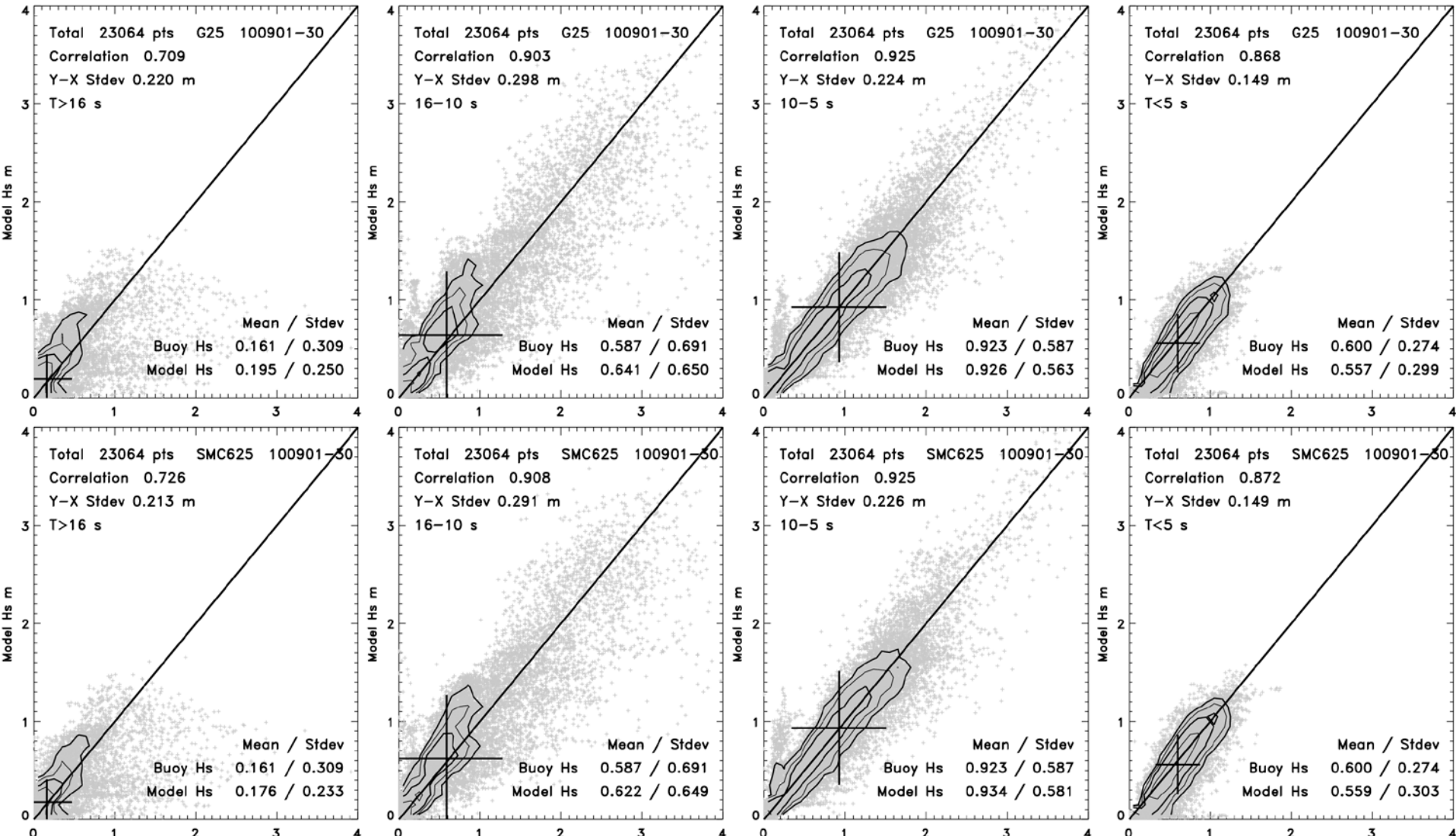
# Comparison G25 - SMC6-25 via Buoy SWH Sept 2010



For more see: [Li, J.G. 2012: J. Comput. Phys., 231, 8262-8277.](#)



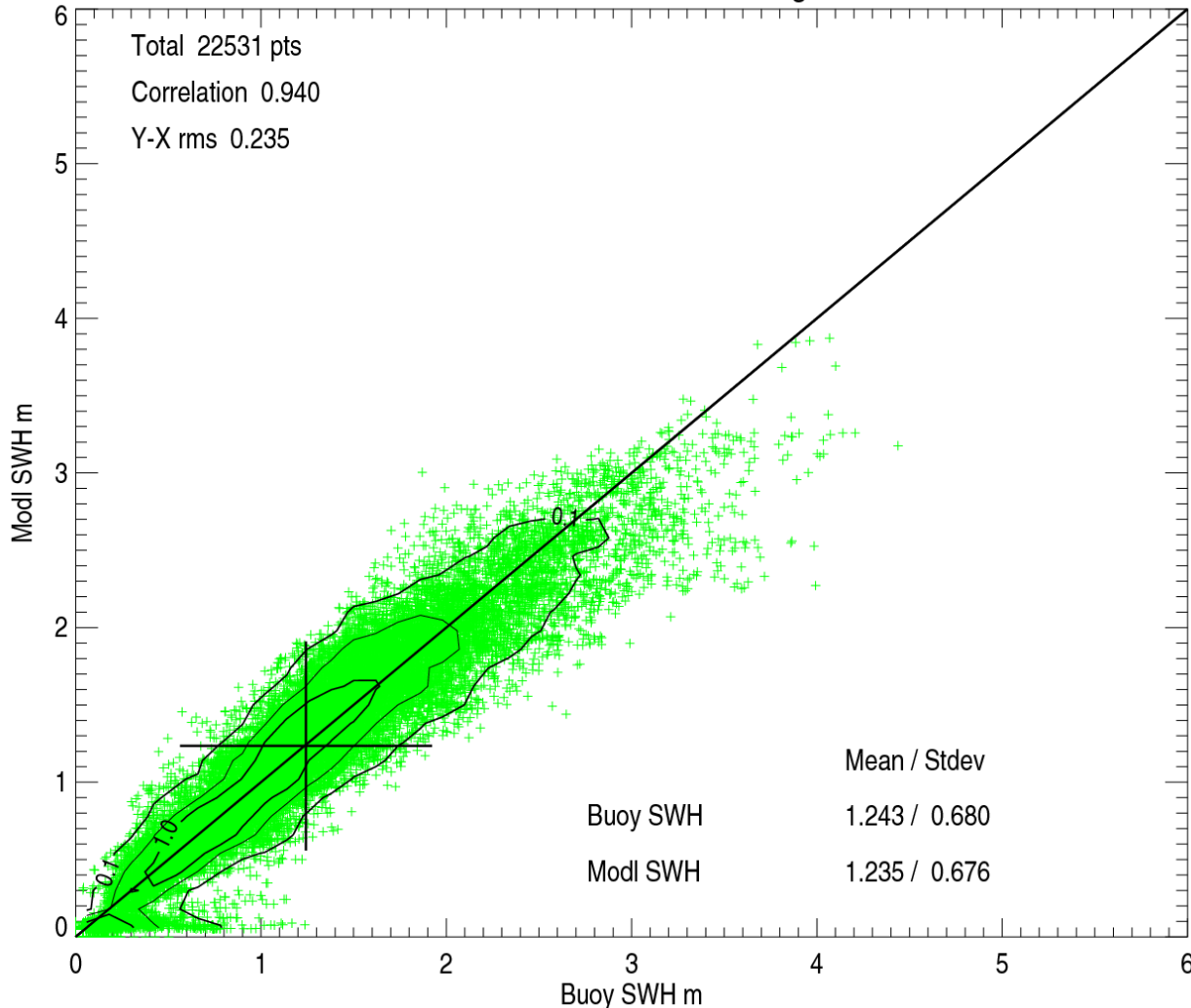
# Comparison G25 - SMC6-25 via Buoy 4-bin SRWH





# Comparison SMC6125-ST3 via Buoy SWH Sept 2012

SMC6125V314f model via A32Bs during 201209WAM

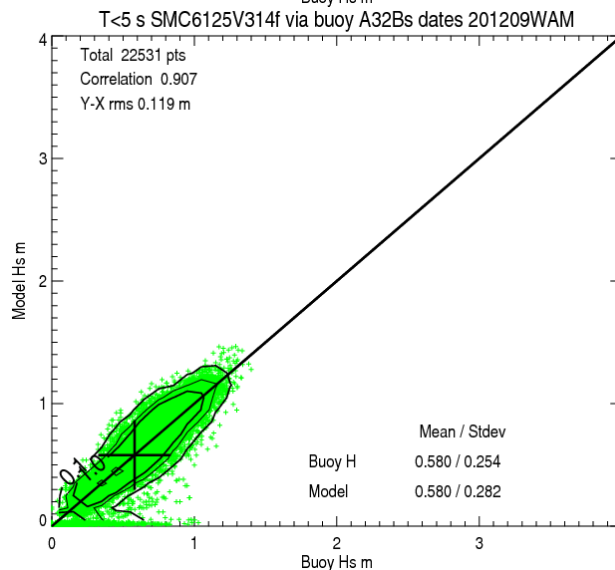
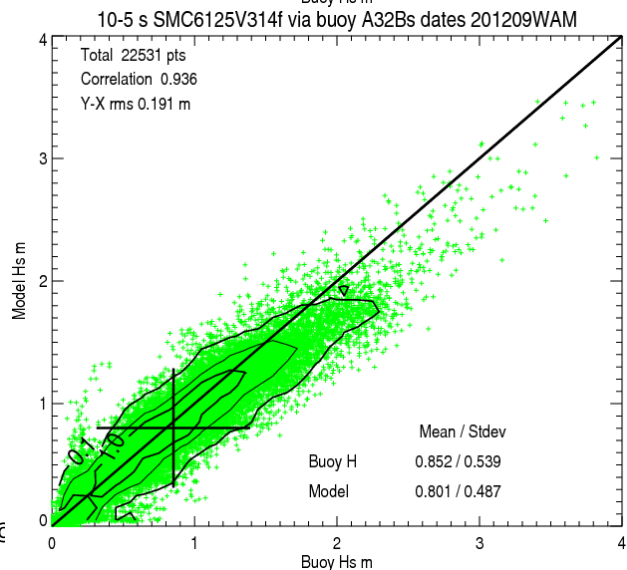
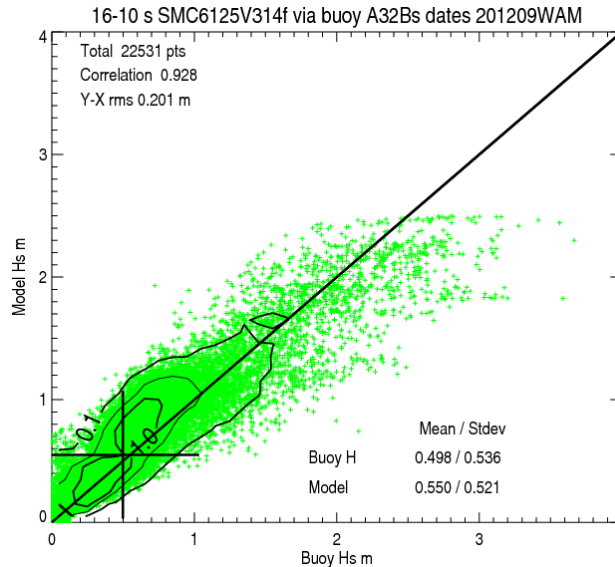
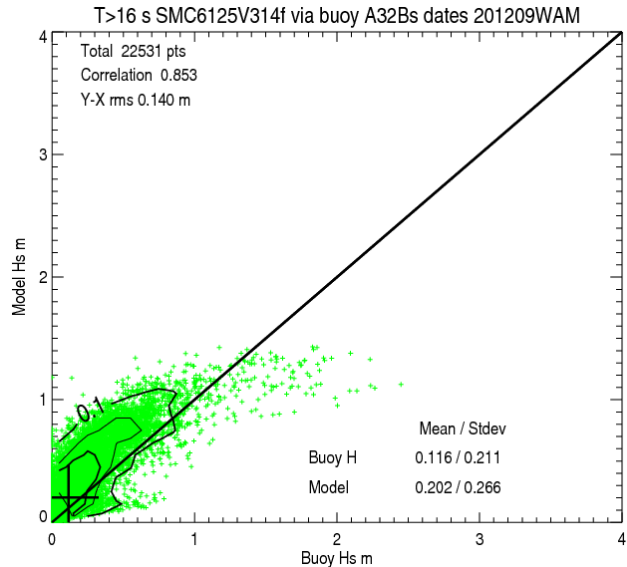


WW3 V3.14  
retuned ST3  
source term

32 spectral  
buoys from  
NDBC.



# Comparison SMC6125-ST3 with Buoy 4-bin SRWH



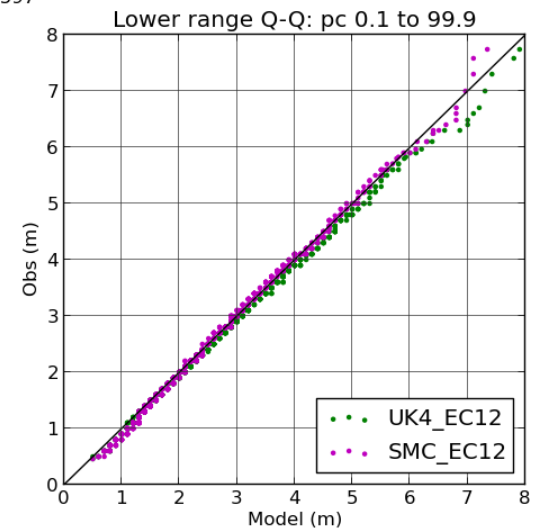
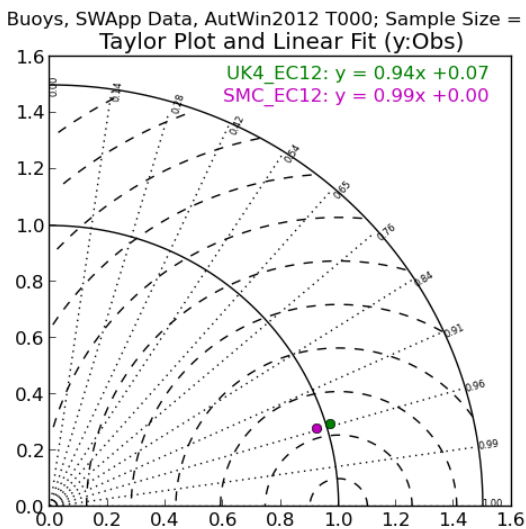
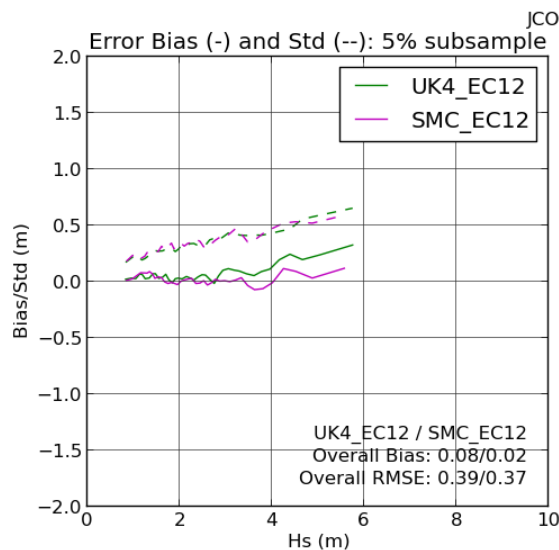
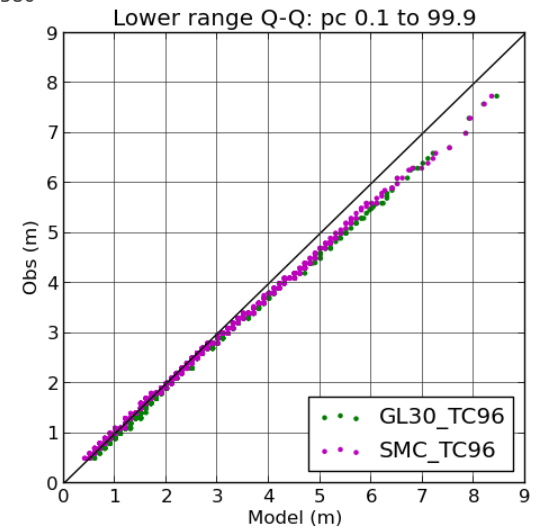
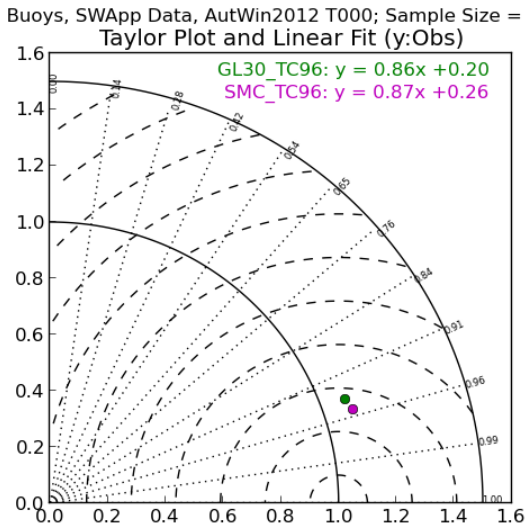
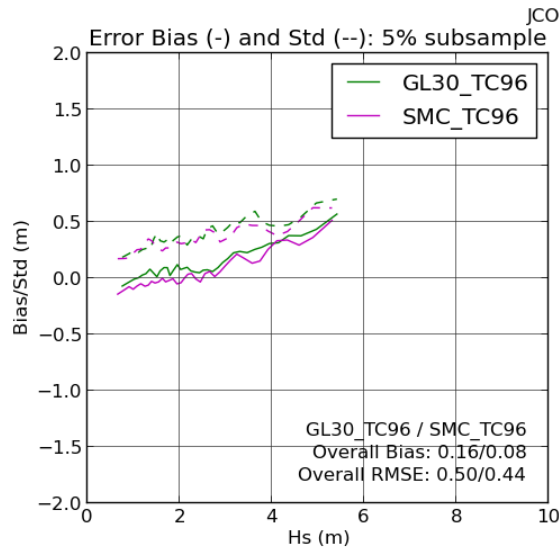
First two bins for swell, model is higher than buoy;

Last two bins for windsea, model is lower;

So retuned ST3 source term is our best in agreement with buoy spectra, using Met Office 25km wind.

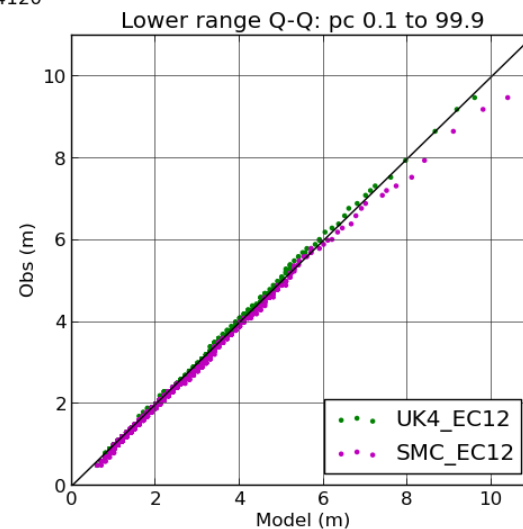
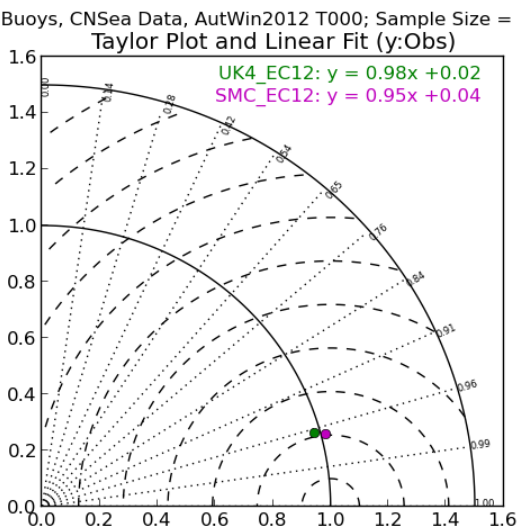
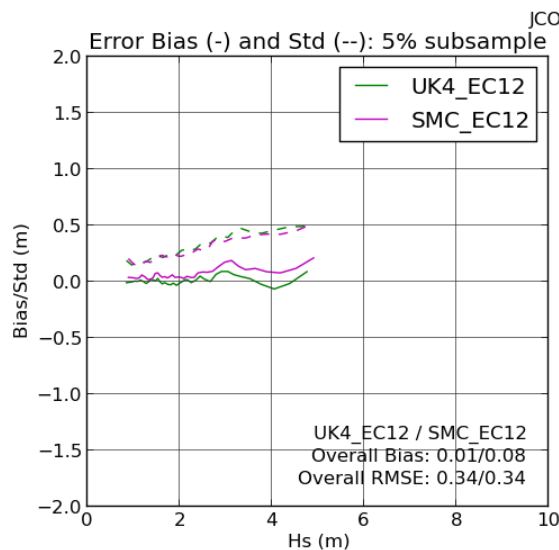
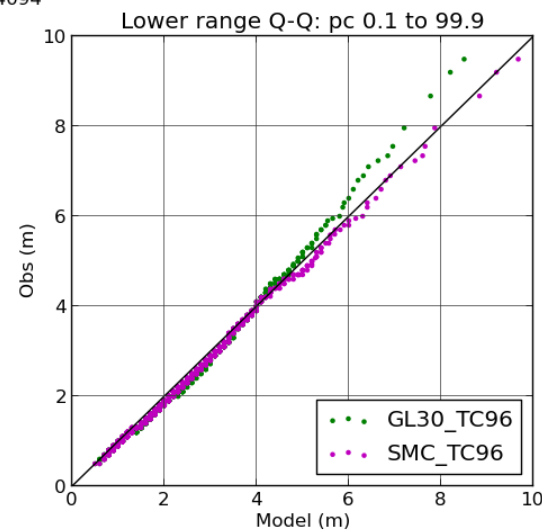
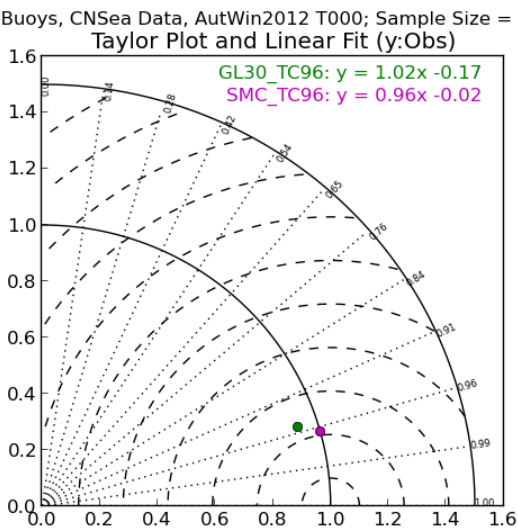
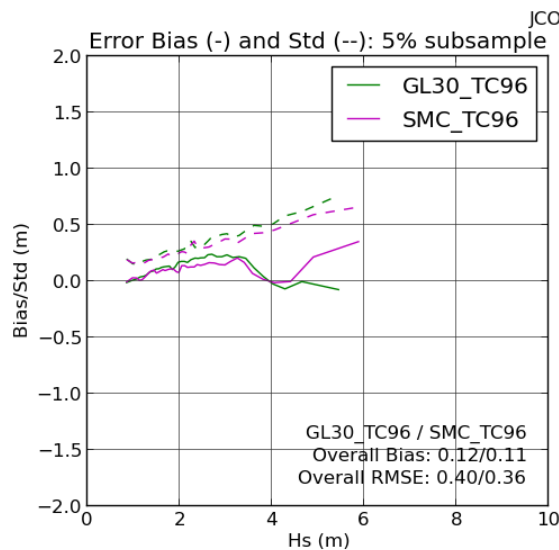


# Comparison SMC6125, G30 and UK4 with buoy SWH – inc. swell

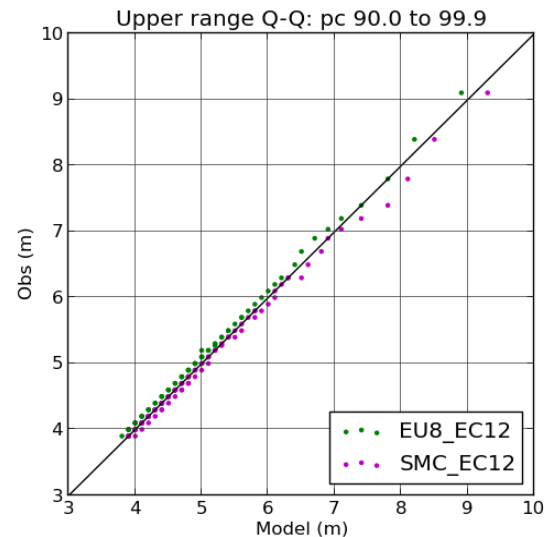
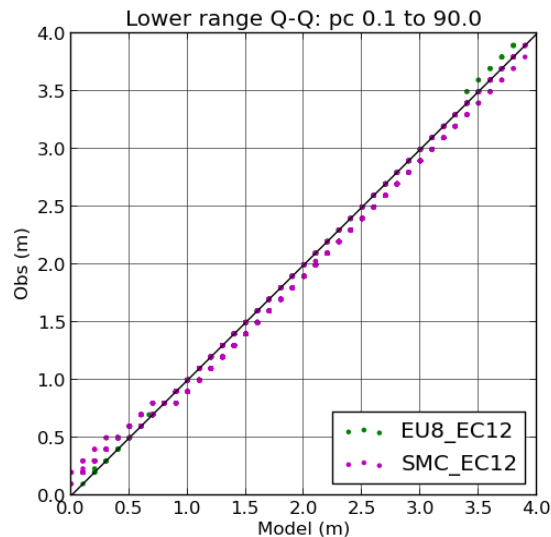
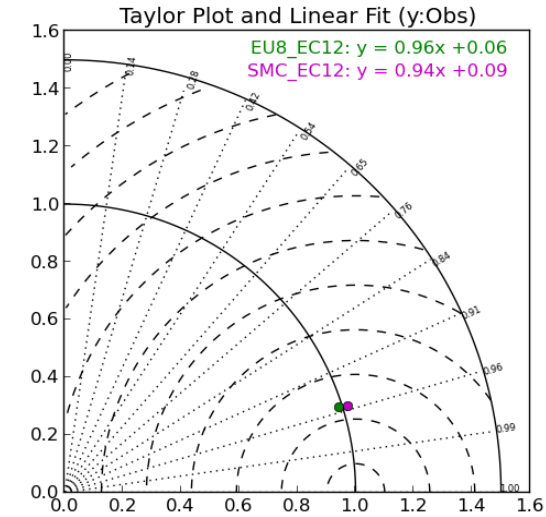
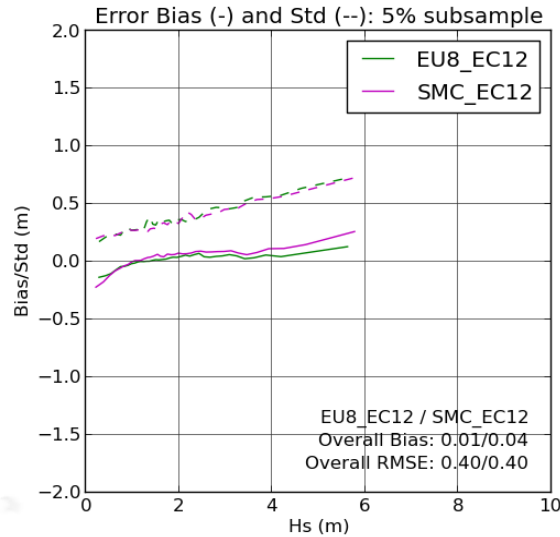




# Comparison SMC6125, G30 and UK4 with buoy SWH – wind sea



# Comparison SMC6125 & Euro8 with buoy SWH – all buoys.





# Summary and conclusions

- Ocean surface wave propagation on variable resolution spherical multiple-cell (SMC) grid is presented.
- A multi-resolution 6-12-25 km SMC grid WW3 model is compared with G30, G25, Euro8 and UK4 models.
- **The unified wave model is capable of replacing the present UK Met Office global and nested regional operational suite.**
- Other applications of the SMC grid for regional models can also benefit from the unstructured, variable resolution grid.
- Improvement may be possible if regional high resolution wind forcing could be blended with global one in the unified model.



## 3. Other applications

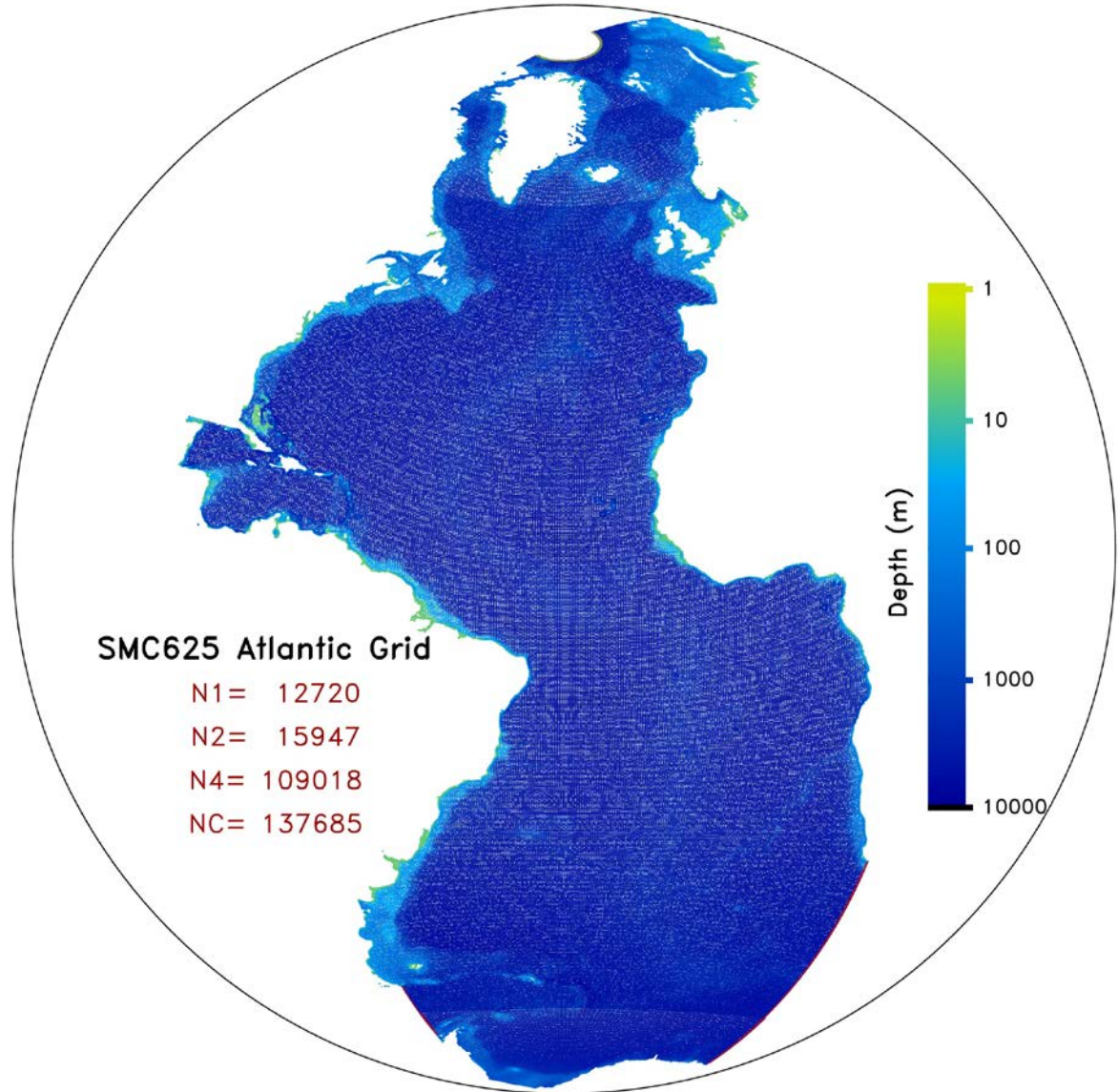
- Global SMC50km hindcast (Met Office).
- Atlantic SMC6-25 ensemble (Met Office).
- Great Lakes SMC 0.5-2km (NOAA NCEP)
- Global SMC 0.25–1° (Ocean University China).

# SMC6-25 grid Atlantic model

Regional grid for the Atlantic Ocean.

Boundary condition is minimised by extending to coastlines. Only the west section in SO requires input boundary condition.

The eastern section is mainly outward waves so no boundary input is required.



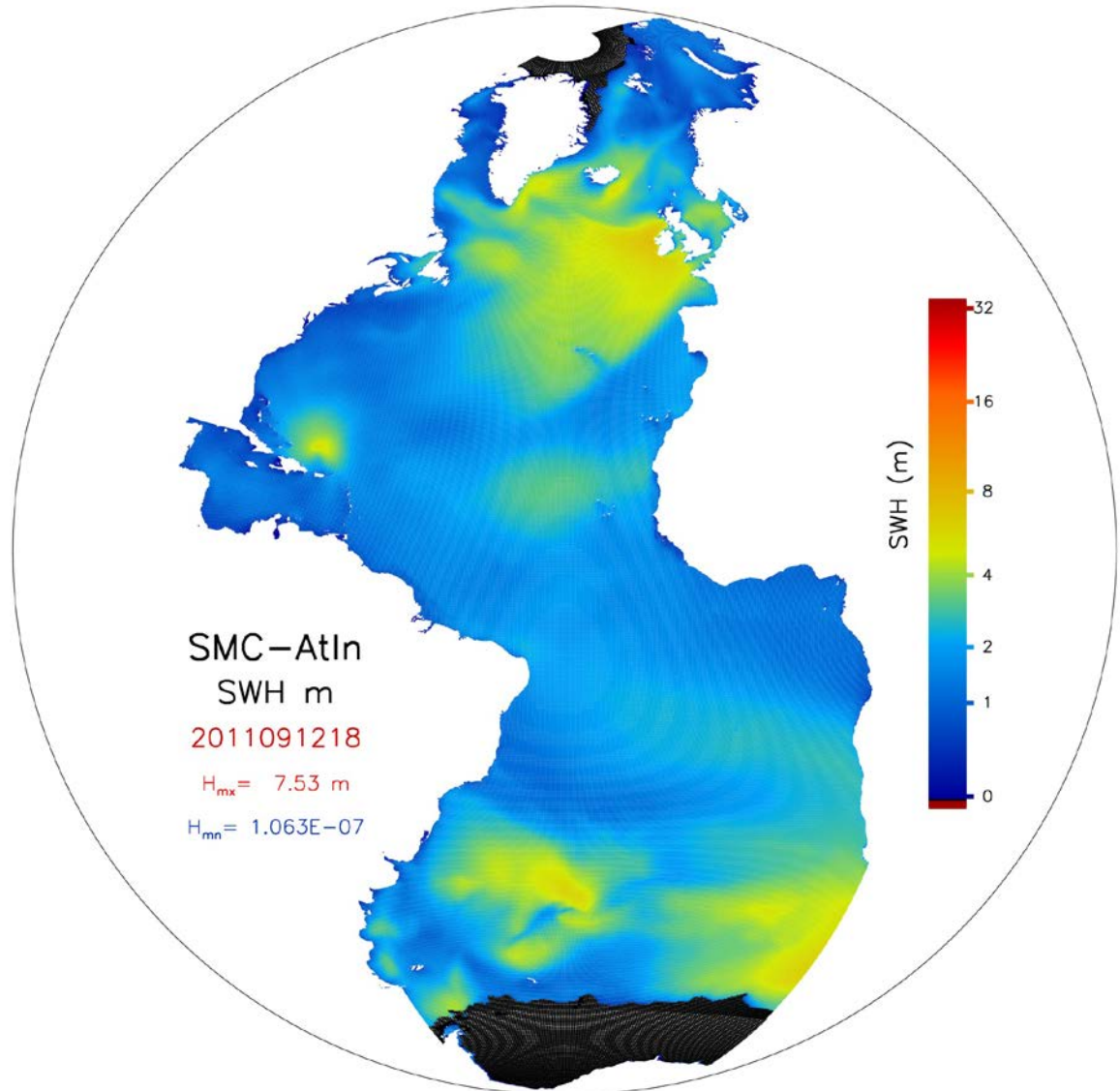
# SWH in SMC6-25 Atlantic model

North boundary sealed by Arctic sea-ice;

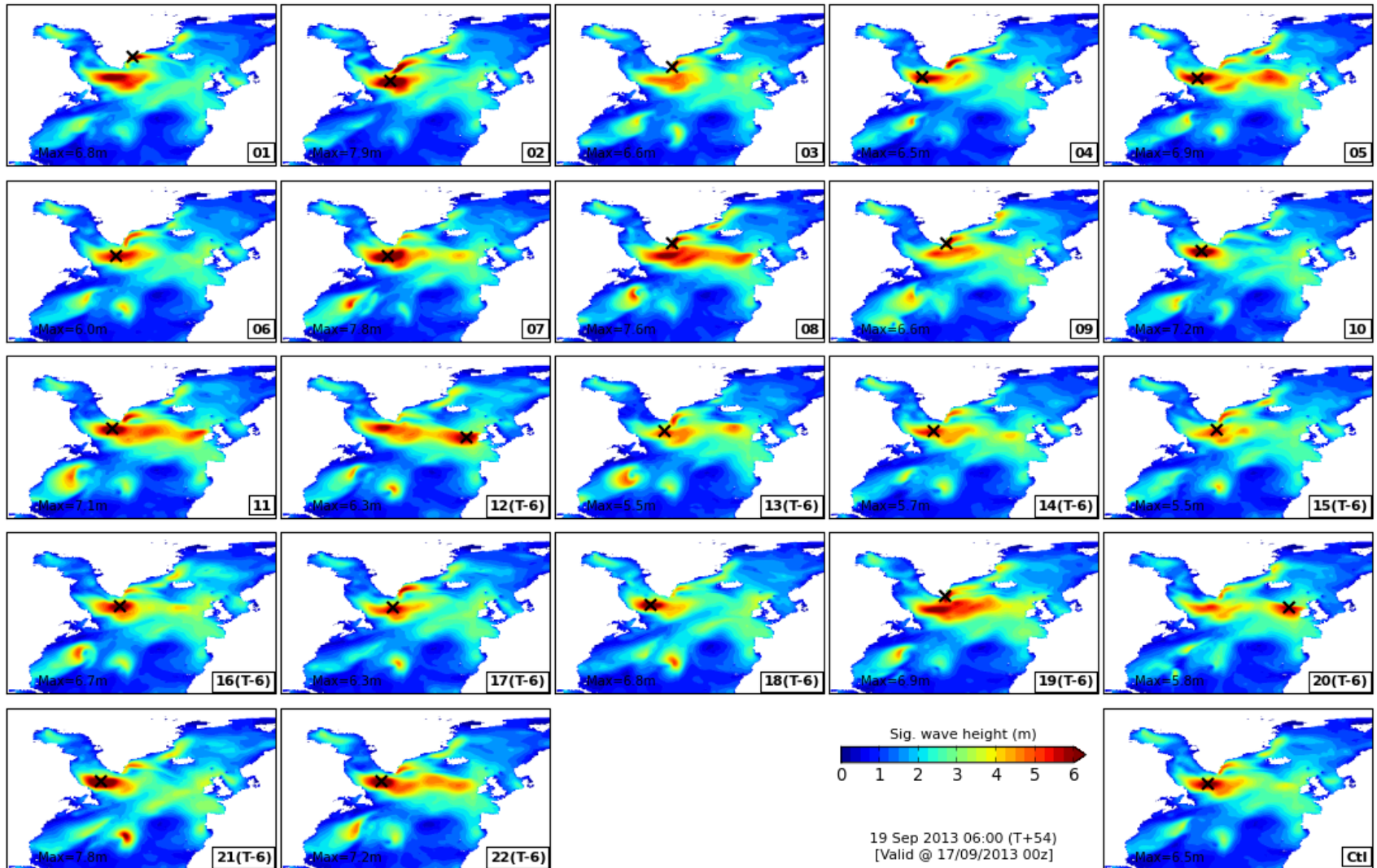
Southern boundary is defined by sea ice or Antarctic coastline;

Influence of SO west boundary is limited and has no effect on the N Atlantic.

Sheltered waters (Gulf of Mexico, Hudson Bay and Mediterranean Sea) are negligible to N Atlantic.

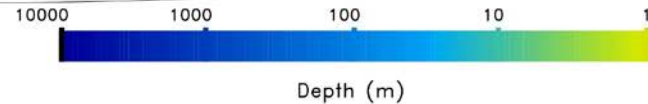


# SMC6-25 Atlantic ensemble





# SMC0512 grid for Great Lakes



Gt Lakes SMC0512 Grid

NX/NY =	3072	2048
Lon0/Lat0=	267.28000	41.16000
dLon/dLat=	5.600000E-03	4.000000E-03

SMC resolution at 0.5, 1 and 2km.

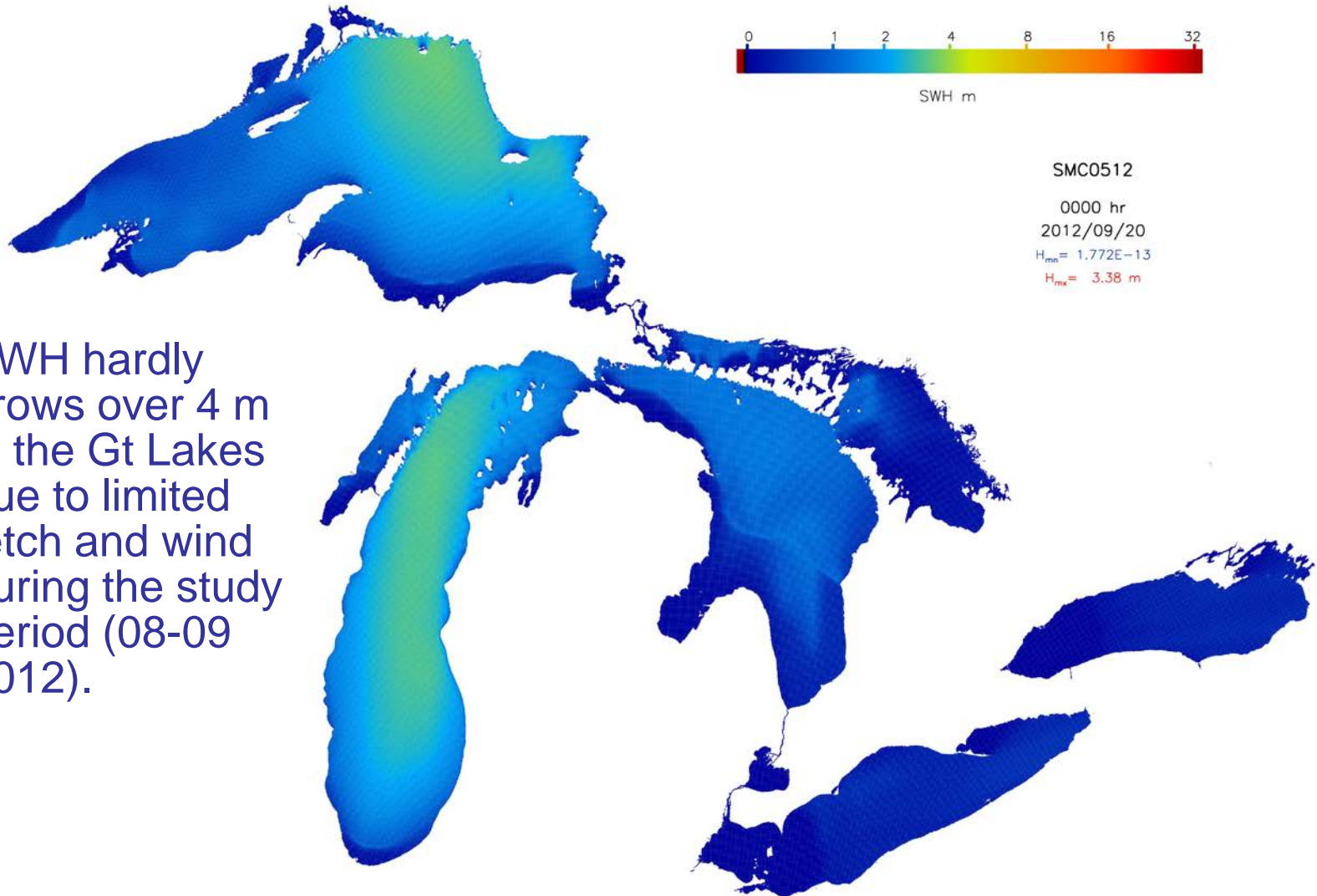
Total cell number 124,578 is about 32% of 2km grid (768x512).

- NC= 124578
- N1= 21732
- N2= 33854
- N4= 68992

Gt Lakes SMC0512 Grid



# SWH of SMC0512 GtLakes model



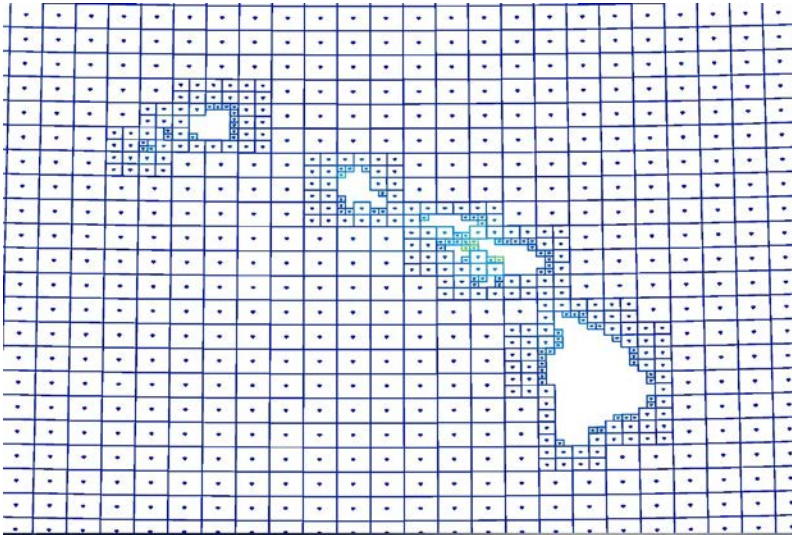
SWH hardly grows over 4 m in the Gt Lakes due to limited fetch and wind during the study period (08-09 2012).



# Summary and conclusions

- Ocean surface wave propagation on variable resolution spherical multiple-cell (SMC) grid is presented.
- A multi-resolution 6-12-25 km SMC grid WW3 model is compared with G30, G25, Euro8 and UK4 models.
- **The unified wave model is capable of replacing the present UK Met Office global and nested regional operational suite.**
- Other applications of the SMC grid for regional models can also benefit from the unstructured, variable resolution grid.
- Improvement may be possible if regional high resolution wind forcing could be blended with global one in the unified model.

# Refined 6-25km SMC grid

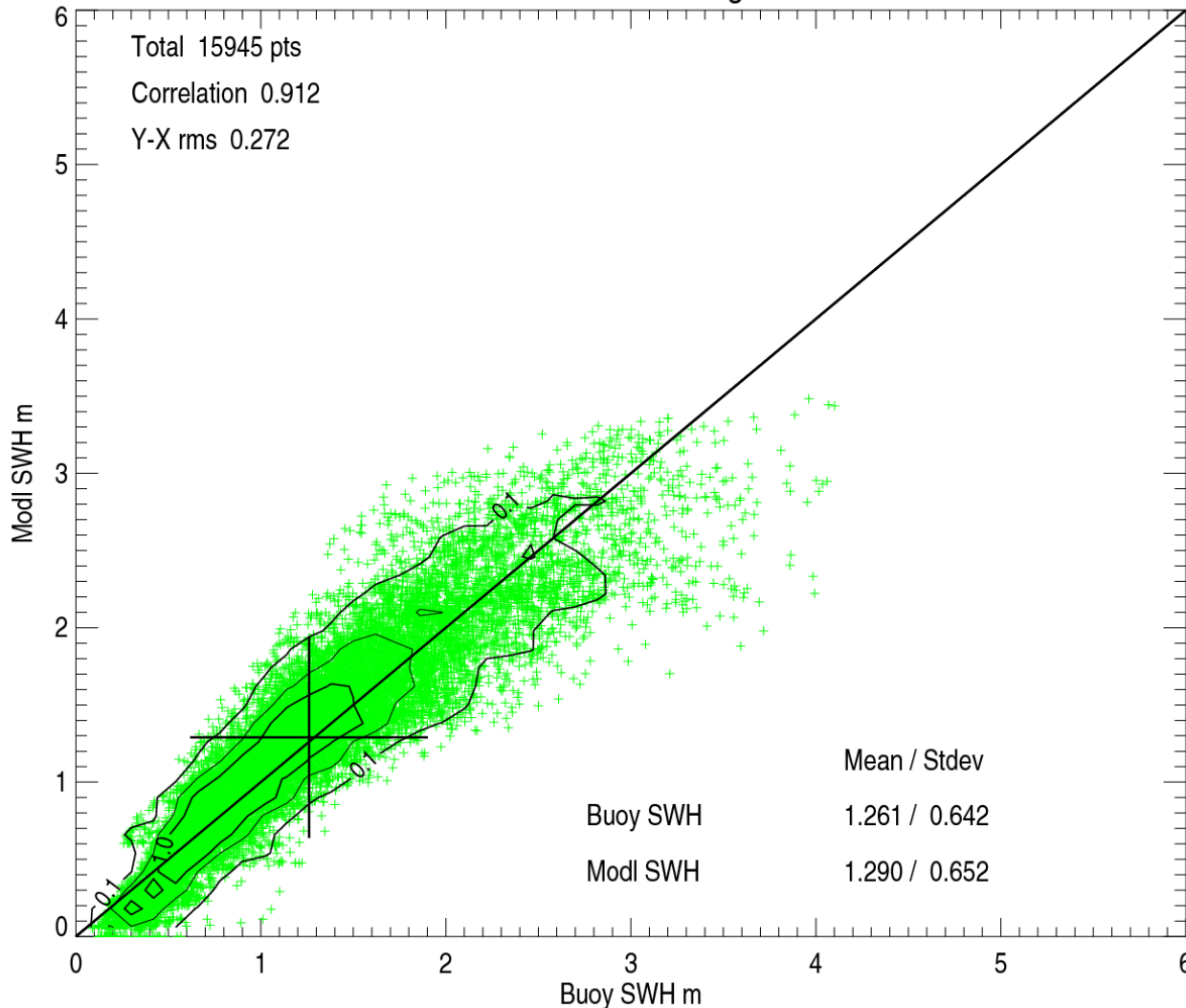


- Refined resolution up to 6km near coastlines, resolving small islands.
- Number of cells (520615) still smaller than lat-lon grid (1024x768=786 432) 66%.
- Could make regional models redundant.
- Multi-time-steps to handle refined cells.



# Comparison SMC6125-TC96 via Buoy SWH Sep-Dec 2012

SMCTCs model via A30Bs during 20120901-1231

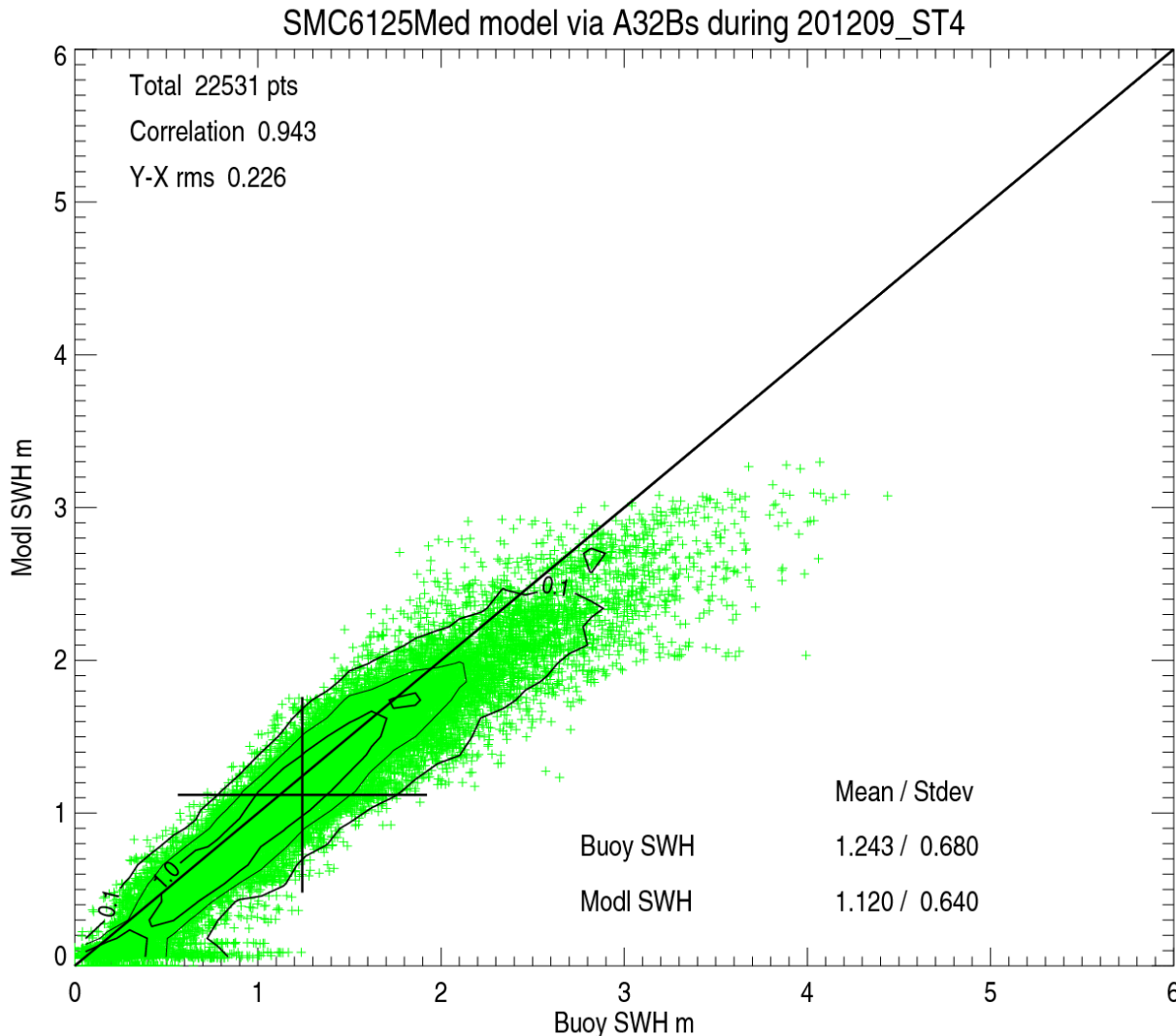


WW3 V4.08  
with TC96  
source term

32 spectral  
buoys from  
NDBC.



# Comparison SMC6125-ST4 via Buoy SWH Sept 2012

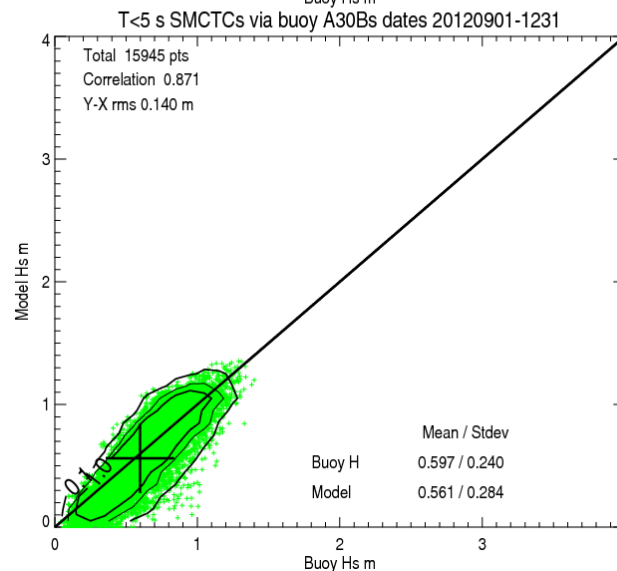
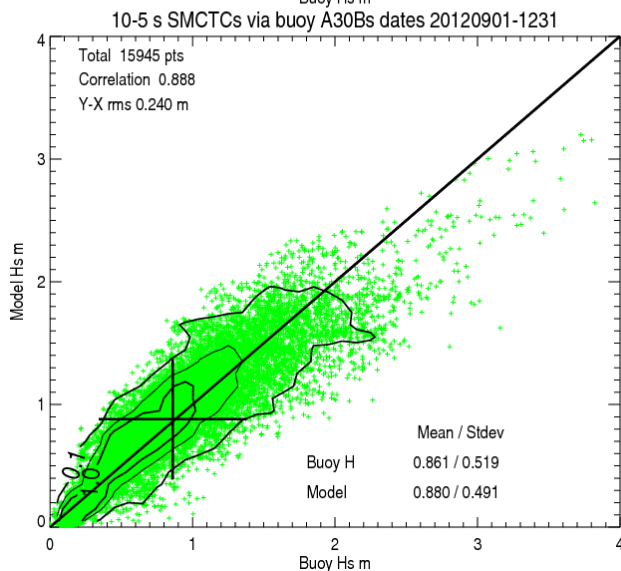
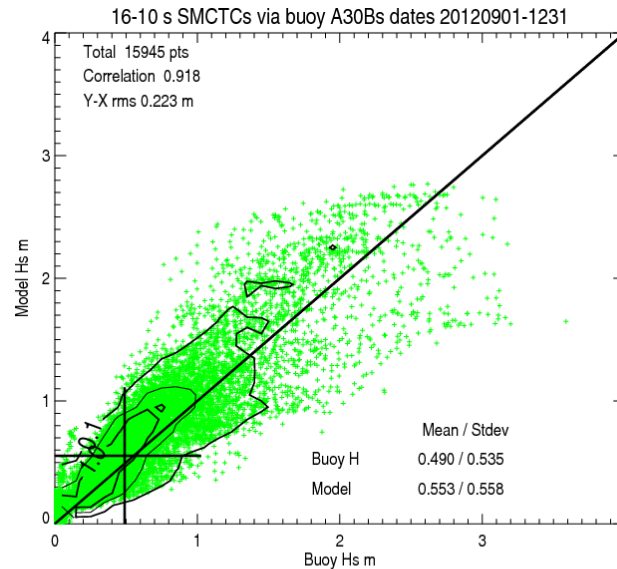
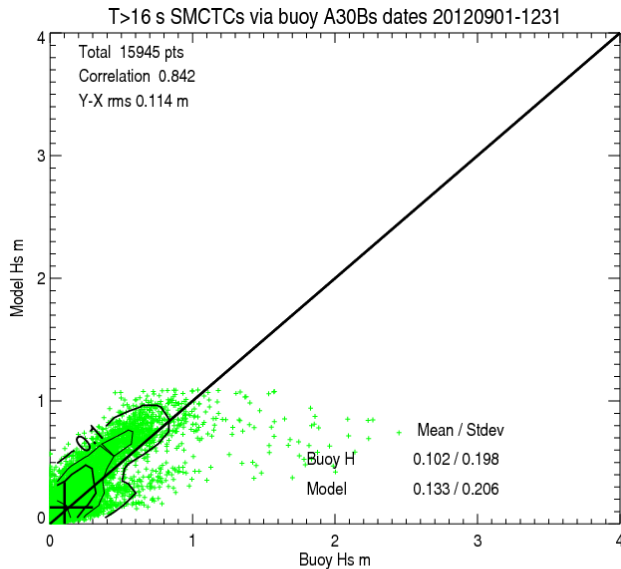


Latest WW3  
V4.11 with  
ST4 source  
term

32 spectral  
buoys from  
NDBC.



# Comparison SMC6125-TC96 with Buoy 4-bin SRWH



First two bins for swell, model is higher than buoy;

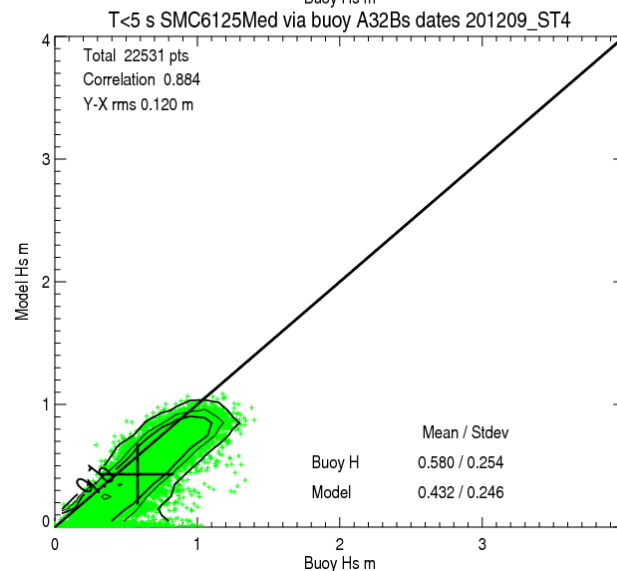
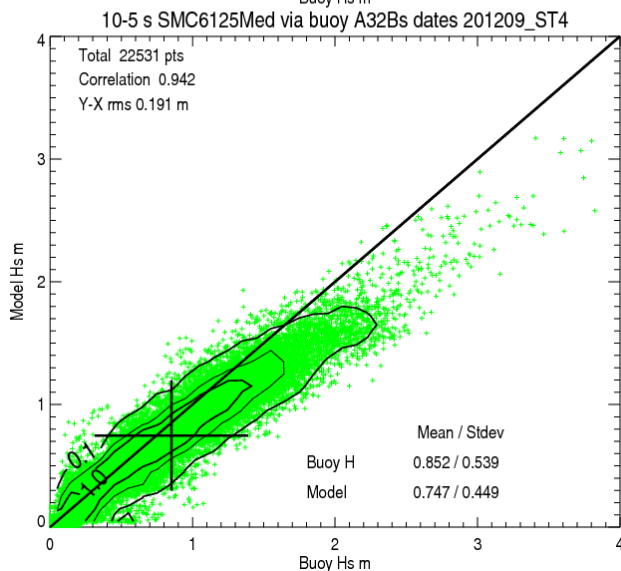
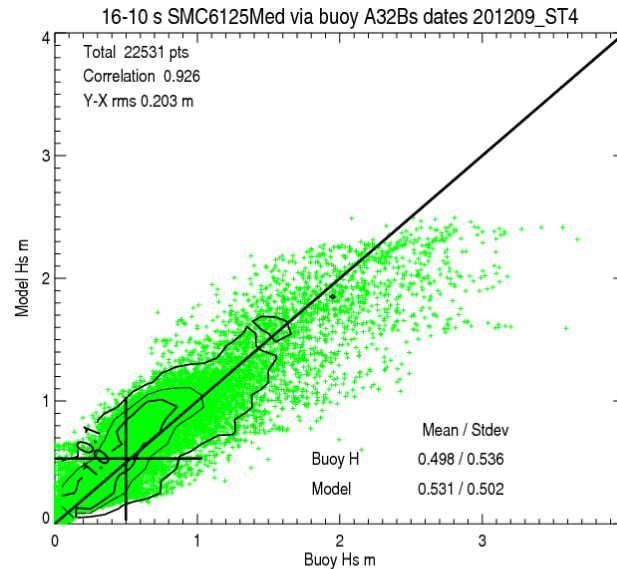
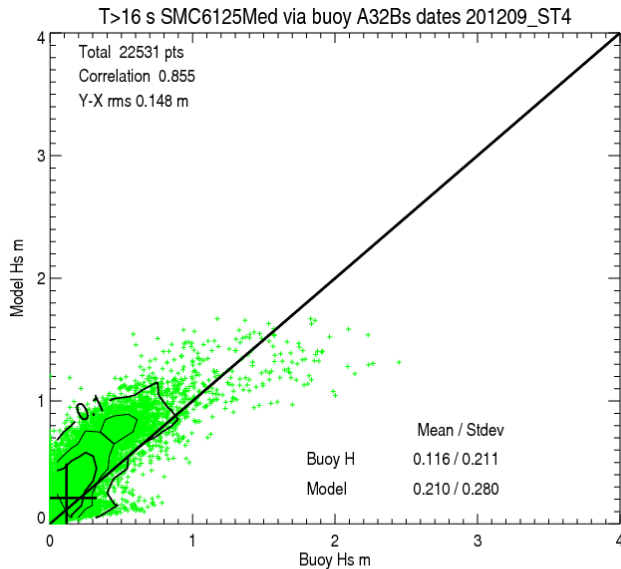
Last two bins for windsea, model is lower;

TC96 source term is fine for global buoys.

But wind sea grow becomes too slow for short fetch regions.



# Comparison SMC6125-ST4 with Buoy 4-bin SRWH



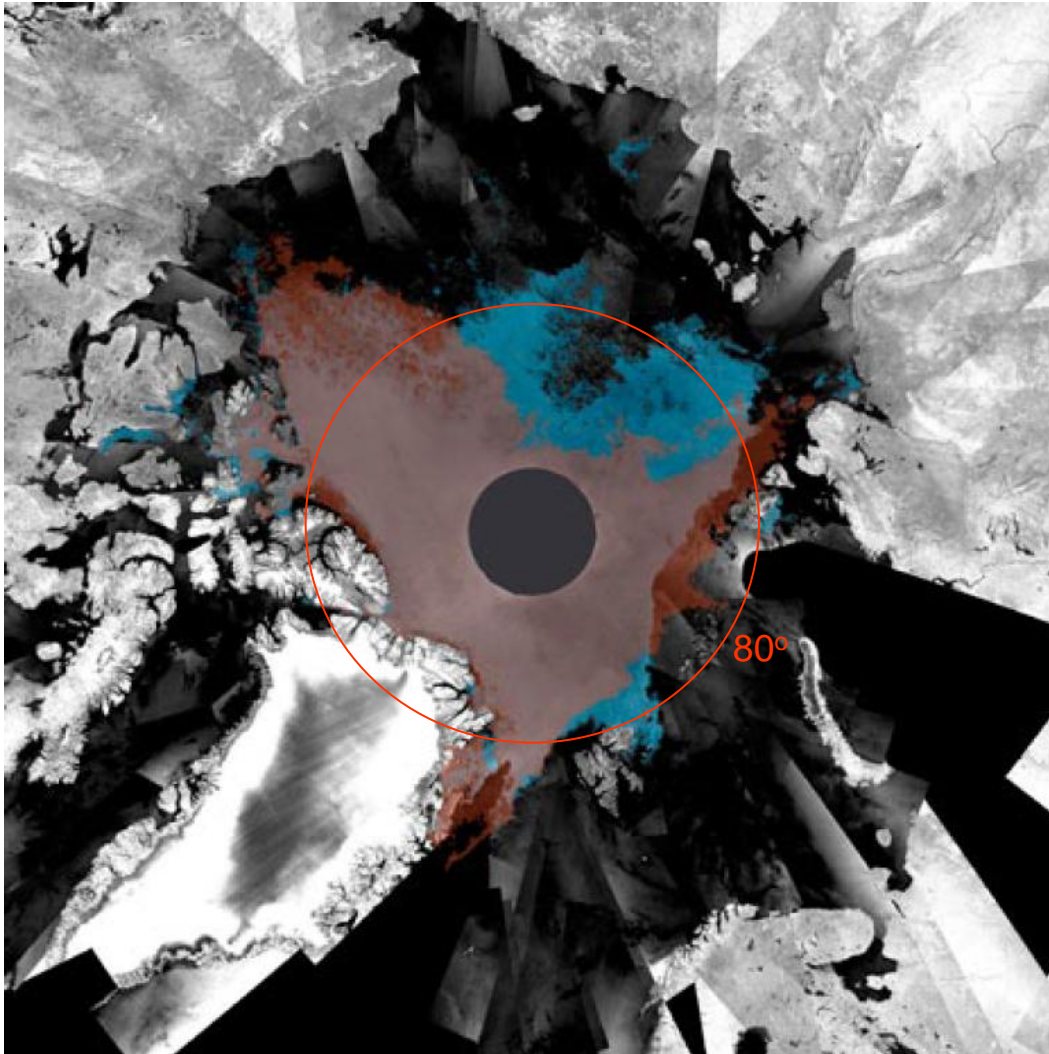
First two bins for swell, model is higher than buoy;

Last two bins for windsea, model is lower;

So the ST4 source term is still not enough for wind sea generation

But the weak dissipation term compensates a bit so swell is increased.

# Arctic ice by ESA Envisat ASAR in September 2007 and 2008



Arctic ice is retreating.

Global wave model needs extension to cover the Arctic.

Light brown: ice appear in both Septembers.

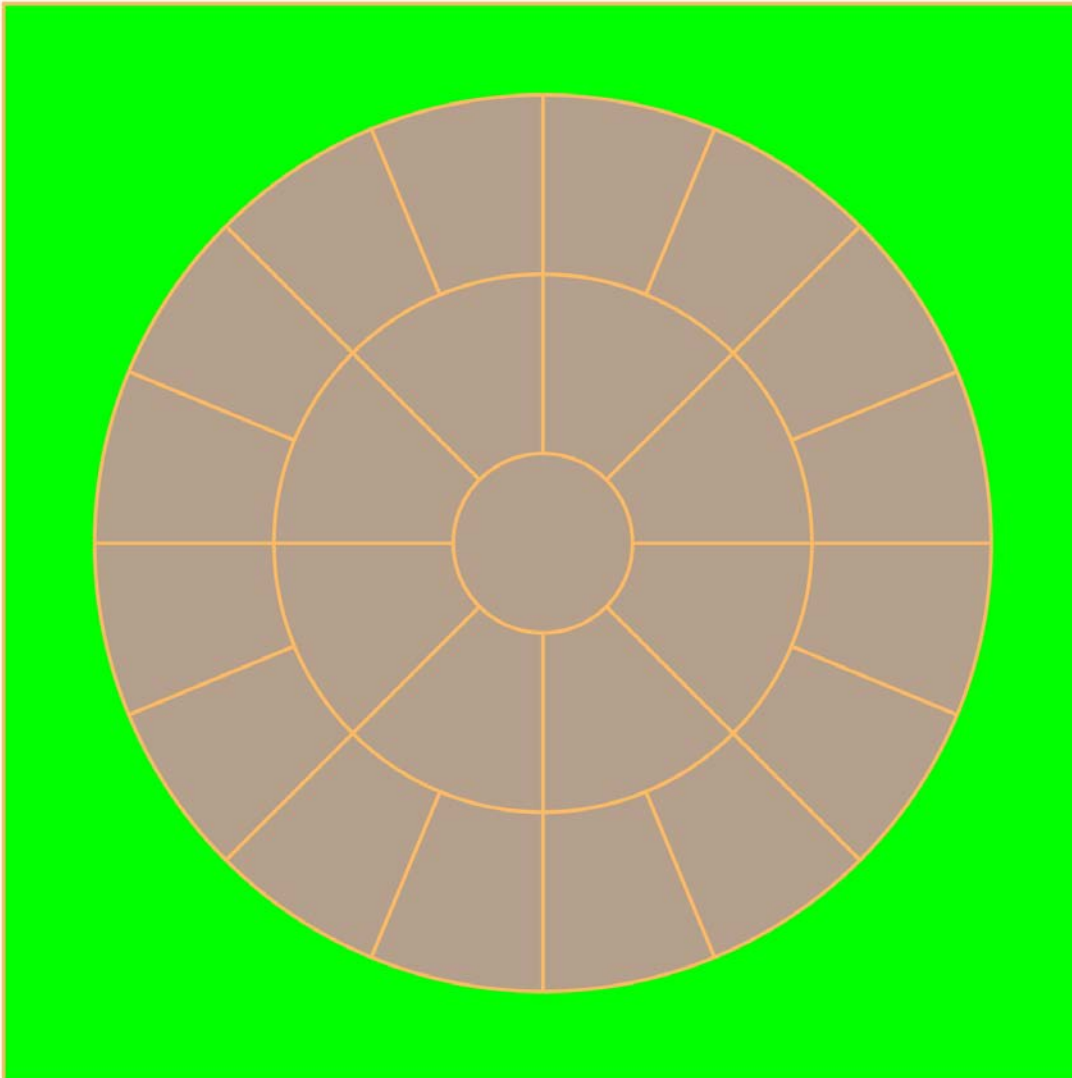
Dark brown: ice free in Sept 2008 but covered in Sept 2007.

Blue: ice free in Sept 2007 but covered in 2008.

Highest ice-free latitude  $86^{\circ}$ , present model  $82^{\circ}$ .

Worst scenario: Arctic ice free in summer 2013 (?)

# Lunar circle perfect 25 blocks



- Polar block radius  $r$
- First and second circle height  $2r$
- Each block area exactly  $\pi r^2$
- $(3r)^2 = (8+1)r^2$
- $(5r)^2 = (16+8+1)r^2$
- May find one in a garden centre.



# Comparison of G25 and SMC models

	G25 Lat-lon	SMC25	SMC6-25
Wave model grids	1024x688	429722 (61%)	520823 (74%)
Min adv time step (s)	180 s	600 (333%)	600/300/150
One-day hindcast on 1-node 32 PEs (64 co)	330 s/task	210 (64%)	336 (102%)
5.5-day forecast on 4-nodes 128 PEs	1800 s/task	1240 (69%)	1860 (103%)
RA2 rms / correlation	0.542/ 0.933	0.553 / 0.931	0.546 / 0.932
Buoy rms / correlation	0.315/ 0.927	0.335 / 0.917	0.312 / 0.928