REASONS FOR FOCUSING ON THE PREDICTION OF THE VERY EXTREME SEA STATES

Requirements from a design point of view (Norwegian Continental Shelf Practise)

Bad behaving response problems

Prediction of events corresponding to prescribed annual exceedance probabilities

Environmental contour lines

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Åsgard C January 2006: $H_s = 12-15m$
Sea state severity approaching a level where structural integrity can be at risk
Requirements from a design point of view
(Norwegian Continental Shelf Practise)

- **Norwegian Rules and Regulations** require that offshore structures are controlled at two levels regarding overload – ULS and ALS.

- **Ultimate limit state (ULS) requirement:**

  \[ \gamma_f x_c \leq \frac{y_c}{\gamma_m} \]

  - Safety factor, e.g. 1.3
  - Elastic capacity
  - Material factor, e.g. 1.15
  - 10^{-2} probability response
  - we should be concerned with response corresponding to very low annual exceedance probabilities.

- **Accidental limit state (ALS) requirement:**

  As above with \( \gamma_f = \gamma_m = 1.0, x_c = 10^{-4} \) annual probability load, and \( y_c = \) plastic system capacity
Why be concerned with $10^{-4}$ environmental load - Bad behaving problems

➔ If the response – annual exceedance probability relation changes abruptly in a worsening direction ULS requirement will not ensure a sufficiently low annual failure probability
Consistent prediction of q-probability loads

Assuming that the sea state is characterized by $H_s$ and $T_p$ and denoting a 3-hour maximum response quantity by $X_{3h}$, the long term distribution for $X_{3h}$ is given by:

$$F_{X_{3h}}(x) = \int \int F_{X_{3h} \mid H_s, T_p}(x \mid h, t) f_{H_s T_p}(h, t) \, dt \, dh$$

As the long term distribution is known, the q-probability value is found by solving:

$$1 - F_{X_{3h}}(x_{3h,q}) = q / m_{3h}$$

▶ We need to account for both the long term weather variability and the short term response variability for obtaining consistent q-probability values.

Expected number of 3-hour events above threshold per year
Approximate method for consistent long term extremes

i) Determine the q-probability contour of $H_s$ and $T_p$

ii) Determine the worst sea state along contour for selected response.

iii) Estimate the distribution of 3-hour maximum response for this sea state.

iv) The $\alpha$-percentile of this distribution is a good estimate for the q-probability response, $\alpha$ is typically around 90.

=> A good estimate is a percentile well above the most probable value.
Predicting q-probability response in a storm climate

• Full long term analysis:
  * Joint distribution of $H_s$ and $T_p$ for all 3-hour events exceeding storm threshold.
  * Conditional distribution of 3-hour maximum response given $H_s$ and $T_p$.

  ➔ This estimate will be considered as the “true” value.

• Environmental contour method:
  * q-probability contour for storm peak characteristics.
  * Conditional distribution for the most unfavourable sea states along the contour line.

  ➔ This is an approximate estimate, it will be demonstrated how good it is.
Storm peak contour lines
Response example

The conditional distribution of 3-hour maximum response given the sea state is modelled by a Gumbel model:

\[
F_{X_{3h}|H_{sp}T_{pp}}(x|h,t) = \exp\left\{ - \exp\left[ - \left( \frac{x - \alpha(h,t)}{\beta(h,t)} \right) \right] \right\}
\]

Distribution parameters:

\[
\beta(h,t) = 0.1h^2 \left[ 1 + \cos^{40} \left( \frac{2\pi(t - 11.5)}{80} \right) \right]
\]

\[
\alpha(h,t) = \beta(h,t) \ln \left( \frac{10800}{0.75t} \right)
\]

[Scale parameter of Eq. (11) diagram]

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Results using contour line method

Table 3: Various quantiles for the worst range of the 0.01-probability contour line

<table>
<thead>
<tr>
<th>0.01 – probability contour sea state</th>
<th>Selected quantiles (%)</th>
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<tbody>
<tr>
<td></td>
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<td>10.95</td>
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<td>11.36</td>
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<td>12.15</td>
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<td>12.51</td>
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<td>12.89</td>
<td>195.0</td>
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<td><strong>Full long term analysis</strong></td>
<td>266</td>
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</table>

Table 5: Various quantiles for the worst range of the 0.0001-probability contour line

<table>
<thead>
<tr>
<th>0.0001 – probability contour sea state</th>
<th>Selected quantiles (%)</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>12.75</td>
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<td>14.11</td>
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<td>14.53</td>
<td>279.1</td>
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<td>14.96</td>
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<td>17.06</td>
<td>206.5</td>
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<td><strong>Full long term analysis</strong></td>
<td>393</td>
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</tbody>
</table>
Concluding remarks

Why focus on the very extreme weather events?

• Annual probability of structural failure should be smaller than say $10^{-4}$

  Loads on this probability level are defined by weather conditions corresponding to annual exceedance probabilities of $10^{-5} - 10^{-3}$.

• The adequacy of $10^{-2}$ - response and safety factor (ULS design control) is affected by whether or not the structural system is well-behaving.

• It is important to account for both the weather randomness and the conditional randomness of the short term response extreme value given the weather.
THE CHALLENGE: ROBUSTNESS AGAINST THE UNEXPECTED

KNOWN THREAT
Dangerous – but can be controlled
Wave-parallel:
$10^{-2}$ – probability sea states

UNKNOWN THREAT
Difficult to control in a rational way.
Wave-parallel:
How will $10^{-4}$ probability sea states look? How bad can it become?
Example wave climate:
Storms exceeding 8m significant wave height in the Northern North Sea

- Long term climate model: Truncated version of model proposed by Haver and Nyhus (1986)

- Contour line: Joint model for $H_{sp}$ and $T_{pp}$ fitted to storm peaks exceeding 8m during the period 1973 – 2006 (159 storms).
Results of full long term analysis

\[
F_{X_{3h}}(x) = \int \int F_{X_{3h} \mid H_s, T_p}(x \mid h, t) f_{H_s, T_p}(h, t) \, dt \, dh
\]

\[1 - F_{X_{3h}}(x_{3h,q}) = \frac{q}{m_{3h}},\] where \( m_{3h\_Haver\&Nyhus}(>8m) = 19.56 \)

<table>
<thead>
<tr>
<th>Annual exceedance probability, q</th>
<th>Response, (x_q)</th>
<th>Ratio: (x_{0.01}/x_q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.63 (1-year)</td>
<td>155</td>
<td>1.72</td>
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<td>0.10 (10-year)</td>
<td>209</td>
<td>1.27</td>
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<tr>
<td>0.01 (100-year)</td>
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<tr>
<td>0.001 (1000-year)</td>
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<td>0.0001 (10000-year)</td>
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<td>0.68</td>
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