ACKNOWLEDGEMENTS

The organizers are indebted to many people who provided valuable assistance in the planning and conduct of the workshop. We would especially like to thank the sponsors, the federal Panel on Energy Research and Development and the Atmospheric Environment Service of Environment Canada, who provided the financial assistance for the workshop. The Canadian Meteorological and Oceanographic Society and the American Meteorological Society also assisted by carrying notices of the workshop in their Bulletins. The committee would also like to thank all those who submitted papers for the workshop program, and those who served as chairmen and rapporteurs for the various sessions. Special thanks are due to Krystyna Czaja who assisted in the assembly and production of this preprint volume, and to Hao Viet Le and Catrin Doe who produced the cover photograph.

Cover: Photograph of the menu screen of an interactive graphical wind and wave analysis and forecasting system for the north Atlantic Ocean. The system, which has also been implemented in the north Pacific Ocean, is described in paper F-4 in this volume.

V.R. Swail, Chairman, Workshop Organizing Committee
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Maximum Likelihood Method Techniques for Directional Analysis of Heave-Pitch-Roll data

F.P. Brissette and I.K. Tsanis

Department of Civil Engineering, McMaster University
Hamilton, Ontario, L8S 4L7

SUMMARY

Pitch-roll-heave buoys are the most commonly used devices to obtain measurements of the wave directional spectrum, as they provide a relatively inexpensive and versatile way of obtaining an estimate of the angular energy distribution from the pitch-roll-heave signal. Since Direct Fourier transforms have been shown to produce smeared estimates of the true spectrum, one has to resort to use a different estimate, such as given by the Maximum Likelihood Method (MLM). The MLM estimate, while adequate for many engineering applications, has been shown to constantly overpredict the angular spreading of wave fields. In order to circumvent this problem, modifications to the MLM have been proposed such as the Iterative MLM (IMLM) and Eigen-Vector MLM (EVMLM) estimates. This paper compares the different MLM estimates with a new Normalized MLM estimate (NMLM). The normalization takes advantage of the MLM estimate characteristic of consistently overpredicting spreading by a constant factor independently of direction and frequency. The NMLM estimate is computed from the MLM estimate by normalizing the energy in every direction bin using a fixed ratio of sech 2 distributions. The new estimate is shown to outperform all of the other estimates in cases of unimodal and bimodal distributions. Field data from the Surface Wave Dynamics Experiment (SWADE) is also presented to illustrate features outlined in test cases.

1. INTRODUCTION

The energy distribution of a sea state has been traditionally described as a function of frequency. It is fairly easy to obtain the one dimensional (frequency) spectrum since the record of the water surface at one point is sufficient. A more accurate representation of a sea state also requires information on the directional spreading of the spectrum. This information is important to most ocean and coastal engineering applications such as: wave forecasting, satellite surveillance, shore protection, upper mixed layer dynamics, environmental hazards and design of marine structures and vehicles. On-site directional information can be obtained using measurements from either a wave gauge array or a mixed-instruments array. Although high resolution directional spectrum estimates can be obtained from an array of wave staffs (Brissette and Tsanis, 1991, Tsanis, Brissette and Donelan, 1992), estimates of the directional wave spectrum are
generally extracted from data obtained by a heave-pitch-roll buoy, as they provide a cheaper and more versatile way of obtaining directional information. Heave-pitch-roll buoys have also been the traditional way of measuring the directional spectrum since Longuet-Higgins, Cartwright and Smith (1963) introduced a direct Fourier transform method to extract a directional spectrum estimate from their signal. Obtaining directional information is a two-fold problem. The first part of the problem deals with collecting and processing the data while the second part is concerned with extracting a directional estimate from the corrected data. Many spectral estimators are now available and it is of practical importance to assess their characteristics as one estimator might be good in certain conditions and inappropriate in others. In a recent review of some of these methods, Brissette and Tsanis (1992) showed that the Maximum Likelihood Method (MLM), despite giving estimates constantly overpredicting angular spreading, was relatively insensitive to extraneous factors such as, presence of background noise, wavenumber dependence, and that it was a relatively easy to implement, efficient and robust estimator. These characteristics make the MLM attractive if the induced artificial spreading can be accounted for. Modifications to the MLM scheme have been proposed by Pawka (1983) and Oltman-Shay and Guza (1984) with the Iterative MLM (IMLM), Marsden and Juszko (1987) with the Eigen-Vector MLM (EVMLM) and more recently by Brissette and Tsanis (1992) with a normalized form of the MLM (NMLM).

2. THEORY

The original derivation of the MLM (Capon, 1969) was intended for wave gauge arrays. This derivation of the MLM was extended to mixed instruments arrays by Isobe et al. (1984). They found that a general form of the MLM was given by:

\[ \hat{S}(\theta_i, \omega_k) = \frac{\varepsilon}{\sum_{m} \sum_{n} C^{-1}_{mn}(\omega_k) H^*_m(\theta_i, \omega_k) H_n(\theta_i, \omega_k) e^{i\mathbf{\vec{k}} \cdot (\mathbf{\vec{x}}_m - \mathbf{\vec{x}}_n)}} \]  

(1)

where \( H_j(\theta_i, \omega_k) \) is the transfer function which linearly relates any mixed array measurement to the water elevation, \( C^{-1}_{mn}(\omega_k) \) is the inverse of the cross-power-spectral-density matrix, \( \omega \) is the angular frequency, \( \mathbf{\vec{k}} \) is the wavenumber, \( \mathbf{\vec{x}} \) is a vector of spatial coordinates and \( \varepsilon \) is a scaling factor which equates the total energy at frequency \( \omega_k \) to the power in the point spectrum. In the case of an array of wave gauges, all the transfer functions \( H_j \) are equal to 1 and Eq. (1) reverts to its original form. Eq. (1) can be rewritten as:
\[
\hat{S}(\theta_i, \omega_k) = \frac{\varepsilon}{\Xi^H(\theta_i, \omega_k) C^{-1}(\omega_k) \Xi(\theta_i, \omega_k)}
\] (2)

For a heave-pitch-roll signal:

\[
\Xi = \begin{bmatrix} 1 \\ ik \cos \theta \\ ik \sin \theta \end{bmatrix}
\] (3)

where \( k \) is the wavenumber which can be obtained from the linear theory, or more appropriately directly from the Cross-Power-Spectral-Density (CPSD) estimates:

\[
k = \left( \frac{C_{22} + C_{33}}{C_{11}} \right)^{0.5}
\] (4)

where the subscripts 11, 22 and 33 respectively represent the heave, pitch and roll signals.

The IMLM stems from the inability of the MLM to give an estimate which is perfectly consistent with the data. In an attempt to correct for the inconsistency, Pawka (1983) and Oltman-Shay and Guza (1984) used an iterative scheme to force the MLM estimate to converge toward a true solution of the spectrum. One simple iterative scheme is the one of Krogstad et al. (1988):

\[
S_{(n+1)} = S_n + \nu \left[ S_{MLM} - S_{MLM}^R \right]
\] (5)

where \( S_{MLM} \) is the MLM estimate and \( S_{MLM}^R \) is the MLM estimate obtained from the reconstructed CPSD matrix, \( C^R(\omega_k) \) using \( S_n(S_0=S_{MLM}) \). \( S \) denotes the directional spectrum \( S(\Theta, \omega_k) \).

The EVMLM has its origin in acoustic wave detection. The method assumes that the CPSD matrix can be partitioned into noise \( \langle \tilde{N} \rangle \) and signal \( \langle \tilde{R} \rangle \) components, according to the CPSD matrix eigenvalues \( (V) \) and corresponding eigenvectors \( (D) \). such that the values of the cross-spectra can be expressed as:
The energy estimate is then obtained by minimizing the estimate of the system noise (see Marsden and Juszko, 1987 for details).

Recently, using an hyperbolic secant squared (sech\(^2\)) spreading function \( S(\theta) \)
\[
S(\theta) = \frac{1}{2} \beta \sech^2 \left( \beta(\theta) \right)
\] 

(Brissette and Tsanis, 1992) investigated the response of the MLM toward distributions of variable spreading with values of the spreading parameter \( \beta \) well encompassing the normal range of 1.24 to 2.62 found in wind waves, as defined by Donelan, Hamilton and Hui (1985). They found that the MLM overprediction of the spreading was constant over the entire range of values of the spreading parameter \( \beta \). This prompted them to account for the artificially induced spreading of the MLM by forcing the estimate to a narrower form. This was done by normalizing the energy in each direction according to a factor \( \zeta_i \) defined as:

\[
\zeta_i = \alpha \frac{\sech^2(\alpha \beta \Theta_i)}{\sech^2(\beta \Theta_i)}
\]

TABLE 1. Model Tests

<table>
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<th>TEST #</th>
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<th>( \theta )</th>
<th>( f_p )</th>
<th>( W )</th>
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<td>unimodal sea</td>
<td>( \sech^2 )</td>
<td>DHH</td>
<td>180</td>
<td>0.15</td>
<td>1.2</td>
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<tr>
<td>2</td>
<td>bimodal sea</td>
<td>( \sech^2 )</td>
<td>DHH</td>
<td>100,240</td>
<td>0.11,0.2</td>
<td>1.2,1.0</td>
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\( f_p \) is the frequency at the spectral peak and \( W \) is the ratio of wave celerity at the spectral peak to the wind speed component in the direction of travel of the peak waves (\( \theta \)).

where \( \beta \) is the spreading parameter of the \( \sech^2 \) distribution function fitted to the MLM estimate and \( \alpha \) is a "constriction" factor whose value is fixed at 0.82 (Brissette and Tsanis, 1992). Essentially, the normalization factor \( \zeta_i \) simply rescales the energy in every direction according to a ratio of \( \sech^2 \) spreading functions chosen as to exactly account for the MLM induced angular spreading.
3. TEST CASES

The two presented test cases attempt to model realistic sea conditions using a Donelan, Hamilton and Hui (DHH) frequency spectrum with $\text{sech}^2$ spreading functions. Details on each cases are presented in Table 1. For each test, the Maximum Entropy Method (Lygre and Krogstad, 1986) estimate (MLM) is also presented for comparison purposes. Cross-Power-Spectral-Density matrices are reconstructed from the models and spectral estimates calculated. A minimum amount of noise (0.25%) is added to avoid a bimodal solution of the MEM (Brissette and Tsanis, 1992).

Test 1 models a unimodal sea. Figure 1 illustrates the results in the form of 3D plots. Figures 2 and 3 summarize the results by presenting a least square fit of the spreading parameter $\beta$ and the root mean square spreading (RMS spread) for each technique. As previously discussed the MLM technique consistently overpredicts the spreading over the entire range of frequencies while the IMLM overpredicts at low frequencies but behaves quite nicely in the upper half of the frequency range. The MEM and EVMLM exhibit a similar behavior in the low frequency end as they both predict the RMS spread very accurately despite underpredicting the spreading around the peak frequency where the spread is narrower. At higher frequencies and wider spreading, the MEM becomes more consistent while the EVMLM estimate loses progressively its accuracy. For unimodal distributions, the NMLM estimate significantly outperforms the others.

The second attempts to recreate a realistic case of a mixed sea, and at the same time allows us to examine the response of the various spectral estimates to various bimodal distributions at many different frequencies. Figure 4 presents the results in the form of 3-D plots while Figure 5 presents the results at a frequency of 0.24 Hz. The three-dimensional plots presented in Figure 4, allow a rapid visual assessment of each method. Clearly, the MEM, IMLM and EVMLM overestimate the energy, especially around the spectral peak, the MLM underestimates the same energy while the NMLM estimate is close to target. Figure 5 presents typical results and while not being perfectly on target, the NMLM outperforms the other methods.
Fig. 1  Surface plots of directional spectrum estimates (test # 1). DHH input spectrum, 
$f_p=0.15 \text{ Hz}$, $W=1.2$. 
4. FIELD DATA

The field data case presented is from the Surface Wave Dynamics Experiment (SWADE) which took place from October 1990 to April 1991 (Weller et al., 1991). The analyzed SWADE data comes from the National Data Buoy Center (NDBC) SWADE Discus-N buoy. The Discus-N buoy (NDBC buoy 44001) was located at 73°48.9′W longitude and 38°11.6′N latitude, at the edge of the continental shelf off-shore of Virginia. The water depth at these coordinates is 120 m.

The Discus-N buoy is a heave-pitch-roll NDBC/SWADE 3 meters discus directional buoy. The directional spectra presented represent a one-hour average taken on October 21, 1990 at 1200 GMT. Results are presented in the form of 3D plots in Figure 6. This case shows a swell at approximately 0.1 Hz and the trace of an old sea at 0.22 Hz. Original results presented on the left-hand side of Fig. 6 indicate that the NMLM is the only method that detects a bimodality in the swell. In order to make sure that this observation was not spurious, the NMLM estimate was used as a target and a CPSD matrix was reconstructed, from which new spectral estimates were computed. Essentially, if the NMLM estimate is close to the real underlying spectrum, the reconstructed estimate should all be close to their original form. The results presented on the right hand side of Fig. 6 show that all the reconstructed estimates, despite being slightly...
narrower, keep the same original features. This indicates that indeed the NMLM estimate is a good representation of the true spectrum.
Fig. 4 Surface plots of directional estimates for a bimodal sea (test # 2). Swell: DHH spectrum, $f_p = 0.11 \text{ Hz}, W = 1.2, \theta = 100$. Wind sea: DHH spectrum, $f_p = 0.2 \text{ Hz}, W = 1, \theta = 240$. 
5. DISCUSSION AND CONCLUSION

For comparison purposes, Table 2 presents values of two error parameters Err1 and Err2 calculated for each test and each spectral estimate.

\[
\begin{align*}
\text{Err1} &= \frac{\int_0^{2\pi} \int_0^{2\pi} |S(f,\theta) - \hat{S}(f,\theta)| \, d\theta \, df}{\int_0^{2\pi} \int_0^{2\pi} S(f,\theta) \, d\theta \, df} \\
\text{Err2} &= \left( \frac{\int_0^{2\pi} \int_0^{2\pi} \left[ S(f,\theta) - \hat{S}(f,\theta) \right]^2 \, d\theta \, df}{\int_0^{2\pi} \int_0^{2\pi} S(f,\theta) \, d\theta \, df} \right)^{0.5}
\end{align*}
\]

The first parameter Err1 is essentially the ratio of the volume (area) of the summation of the errors over the total energy. Some estimates have the tendency of being relatively accurate over most of the distribution except at the peak. For this reason, the second parameter Err2 should provide a better measure of this tendency, as its value will be more sensitive to over and underpredictions around the spectral peak, where accurate measurements must be made. Data in Table 2 indicate that for both test cases the NMLM performs the best, followed by the MEM and IMLM. The performance of the NMLM is particularly evident in both test cases. The error parameter Err2 is more favourable to the IMLM estimate, reflecting the tendency of the
MEM and EVMLM estimates to overestimate more severely the energy at the peak of the distribution.
Fig. 6  Surface plots of directional estimates for a SWADE field data case. Results are presented on the left-hand side. The right-hand side spectra are reconstructed spectra using the NMLM estimate as a model test.
The normalization technique, which is the basis of the NMLM, is a simple empirically based corrective scheme, and this must be kept in mind when interpreting results, although the estimate has stable properties which make it attractive from this point of view. One drawback of the NMLM is that the estimate requires more computational time than the other methods. The main reason being that it requires the calculation of other estimates, in addition to least square fits to each identified distributions in order to compute the normalization factors $\zeta_i$. This makes the NMLM slightly more expensive from a computational viewpoint, than even the IMLM which requires iterations and the calculation of multiple MLM estimates. Even though the larger part of the computing cost lies within the sectors of quality control, data correction and transformation in the frequency domain, an increase in the cost of computing spectral estimates could be significant, especially when dealing with routine analysis of large data sets. With the ongoing advent of faster computers and workstations, this constraint will be relaxed and more complex methods can be envisioned such as an Iterative Normalized scheme, or a knowledge based system making optimum use of all available spectral estimates in order to achieve a best-possible estimate.

REFERENCES


Methods for Directional Spectra Measurements by Small Arrays
I.K. Tsanis and F.P. Brissette
Department of Civil Engineering, McMaster University
Hamilton, Ontario, L8S 4L7

SUMMARY
An accurate description of a sea state is essential to many ocean and engineering applications such as: wave forecasting, satellite surveillance, shore protection, upper mixed layer dynamics, environmental hazards and design of marine structures and vehicles. Arrays of wave gauges have been shown to produce estimates of the wave directional spectrum with good resolution and accuracy. Since a number of different directional spectra methods are available it is of primary importance to select the one that will give the best estimate. Some considerations are resolution, accuracy, reliability and efficiency. Direct Fourier Transform methods, the Maximum Entropy Method (MEM and Maximum Likelihood Method (MLM) and variants, including a Normalization MLM (NMLM) are compared in cases of unimodal and bimodal distributions. Results obtained from a symmetrical six wave gauge array at the National Water Research Institute’s Waves Tower on Lake Ontario, are discussed to further outline the different methods characteristics. Results indicate that most methods perform reasonably well but that the NMLM is the best estimate overall.

1. INTRODUCTION
The energy distribution of a sea state is usually expressed as a function of wave frequency and wave propagation direction. It is easy to obtain the frequency spectrum, because the record of the water surface at one point is sufficient, but in order to obtain detailed directional information, wave records from a large number of points are needed. In practice, only several simultaneous wave records are possible and it is important to obtain them in such a way so as to be able to make the most accurate estimate. Wave gauge arrays (Oakley and Lozow 1977) are often used for estimation of wave directional spectra. The purpose of this paper is to address the above question by using different methods for directional spectra measured by small wave gauge arrays. Comparison of the results is accomplished by using unimodal and bimodal directional distributions. Field data from Lake Ontario will also be examined to illustrate the potential of the best performing methods.

The results of this study will be important to offshore activities, e.g., exploration and use of offshore resources, for petroleum production. It will also be a useful and practical tool for engineers
and scientists involved in offshore petroleum activities and concerned with engineering design, operational efficiency, safety and protection of the environment. In addition scientists and limnologists will also benefit because the results of this study will lead to improved hind/forecasting methods.

2. WAVE DIRECTIONAL METHODS

The wave directional spectrum (energy distribution in frequency and direction) is typically expressed as:

\[ S(f, \Theta) = F(f) D(f, \Theta) \]  

(1)

where \( f \) is the frequency, \( \Theta \) the direction, \( F(f) \) is the frequency spectrum and \( D(f, \Theta) \) is the normalized directional spreading function.

Five directional spectra techniques are described below. One direct Fourier transform method, three maximum likelihood methods (MLM) and one maximum entropy method (MEM).

The Pitch–Roll Buoy (PRB) method is the first direct Fourier transform method developed by Longuet-Higgins et al. (1963). The directional spectra can be determined by the information yielded by the motion of a buoy that measures the vertical displacement and angles of pitching and rolling. In the case of an array of wave gauges the slope components \( \partial \eta / \partial x \) and \( \partial \eta / \partial y \) in x and y directions, respectively, have to be approximated using the water elevation \( \eta \) from at least two wave gauges. The co–spectra \( C_{ij} \) and quad–spectra \( Q_{ij} \) of any pair of elevation and slopes are directly related to the first five Fourier coefficients \( a_n \) and \( b_n \) of the directional spectrum expressed as a Fourier sum.

The Maximum Likelihood Method (MLM) was developed by Capon (1969) and was applied to an array of sensors for determining the properties of propagating waves. Jefferys et al. (1981) used the MLM to estimate the directional spectra from wave height measurements obtained by a wave gauge array. The derivation of the MLM is similar to Lacoss (1971). If a sea state can be represented by the summation of a number of monochromatic waves of power \( S(f, \Theta_i) \) coming from directions \( \Theta_i \), with \( i=1, N \), then the true cross spectral density matrix is given in terms of \( S(f, \Theta_i) \) in the frequency band near \( f \) by:

\[ C_{jk}(f) = \sum_{i=1}^{N} X_{j}(f, \Theta_i) X_{k}^{*}(f, \Theta_i) S(f, \Theta_i) \]  

(2)

where \( N \) is the number of considered directions, \( X_{j}(f, \Theta_i) \) is the complex phase lag between the jth sensor and the origin for a wave of frequency \( f \) approaching from direction \( \Theta_i \). The energy incident from direction \( \Theta_i \) is evaluated by minimizing the influence from all the
other components. The minimization uses Lagrange multiplier theory and leads to an estimate of the energy in the plane wave:

\[ S_{\text{mlm}}(f, \theta) = \frac{\varepsilon}{\chi^T(f, \theta) C^{-1}(f) \chi(f, \theta)} \]  

where \( \varepsilon \) is a normalization factor, and \( C^{-1}(f) \) is the inverse of the cross spectral density matrix.

The Iterative Maximum Likelihood Method (IMLM) was developed by Pawka (1983) and applied to determine the island shadows in wave directional spectra. Oltman-Shay and Guza (1984) used the IMLM for point measurement systems such as the pitch and roll buoy and slope array. Krogstad et al. (1988) used the IMLM to obtain high-resolution directional spectra from horizontally mounted acoustic doppler current meters. Generally the cross-spectral density matrix reconstructed from the MLM estimate using Eq.(2) will not be equal to the observed cross-spectral density matrix. Krogstad et al. (1988) used a simple iterative scheme to resolve this inconsistency

\[ S_{(n+1)} = S_n + \omega [(S_{\text{MLM}} - M(S_n))] \text{ with } S_0 = S_{\text{MLM}} \]  

where \( S_{\text{MLM}} \) is the MLM estimate, \( M(S_n) \) is the MLM estimate obtained from the reconstructed cross-spectral density matrix using \( S_n \), \( \omega \) is a relaxation parameter slightly above 1 (a value of \( \omega = 1.2 \) appears to yield convergence in about five iterations) and \( n \) is the iteration number.

The Normalized Maximum Likelihood Method (NMLM) was first developed for heave-pitch-roll buoys (Brissette and Tsanis, 1992a,b) in an attempt to correct for the artificially induced spreading of the MLM by forcing the estimate to a narrower form. This was done by normalizing the energy in each direction according to a factor \( \zeta_i \) defined as:

\[ \zeta_i = \alpha \frac{\text{sech}^2(\alpha \beta \theta_i)}{\text{sech}^2(\beta \theta_i)} \]  

where \( \beta \) is the spreading parameter of the \( \text{sech}^2 \) distribution function fitted to the MLM estimate and \( \alpha \) is a "constriction" factor whose value was found to be 0.82 for heave-pitch-roll data. Essentially, the normalization factor \( \zeta_i \) simply rescales the energy in every direction according to a ratio of \( \text{sech}^2 \) spreading functions chosen as to exactly account for the MLM induced angular spreading. The extension to wave gauge data simply consists in recalculating the parameter \( \alpha \) for a given array geometry.

The MEM estimate is obtained by maximizing the entropy of the function \( f \) given by the partial Fourier series of the angular distribution of
energy (first two terms in complex form, $c_1$ and $c_2$). According to its spectral definition, the function to maximize is given by:

$$H(f) = -\int_{-\pi}^{\pi} \log (f(\theta)) d\theta$$

which yields an estimate

$$S(\theta) = \frac{1}{2\pi} \left( 1 - \phi_1 c_1^* - \phi_2 c_2^* \right)/\left| 1 - \phi_1 e^{-i\theta} - \phi_2 e^{-2i\theta} \right|^2$$

with

$$\phi_1 = (c_1 - c_2 c_1^*)/(1-|c_1|^2), \quad \phi_2 = c_2 - c_1 \phi_1$$

See Lygre and Krogstad (1986) for details.

3. TEST CASES

Array geometry and number of wave gauges are of primary concern. The spacing between the wave gauges should be chosen in order to avoid spatial aliasing and to match the waves period of interest. The number of wave gauges depends on technical and economical factors and on the needed resolution. Essentially the best wave directional method should give reasonably accurate estimates with a minimum number of wave gauges.

Figure 1 displays four array configurations that were tested and Figure 2 shows typical results for the MLM. Every method gives estimates that will improve with the number of wave gauges but the behavior of the different methods is only very weakly dependent on the same number, provided that the array is approximately symmetrical. This means that if a method overpredicts the spreading for a three wave gauges array, it will also overpredict it for any array, although by a lesser amount if the number of wave gauges is increased. For this reason this paper will focus on the six wave gauge array presented in Fig. 1. It can be seen from Figure 2 that amount of improvement does not necessarily justify the use of an extra wave gauge, at least for unimodal distributions. On the other hand, the extra wave gauge will allow better resolution of bimodal seas. It is also interesting to note that the four wave gauge array gives the same estimate as the three wave gauge one for unimodal distributions. This outlines the
need of defining what the array can resolve and the importance of testing different geometries before settling on a particular one.

To investigate the response of the different methods to variable spreading, two hyperbolic secant squared spreading functions (Donelan et al., 1985) given by:

\[ D(\theta) = \frac{1}{2} \beta \text{sech}^2 \left( \beta(\theta) \right) \]  

were used with values of the spreading parameter $\beta$ of 1.24 and 2.62. Results are presented in Figures 3a and 3b. It can be seen that the MEM severely overestimates the peak energy, the MLM induced artificial spreading causes it to underestimate it, while the NMLM and IMLM are close to the target spectrum. Figure 3b outlines a problem of the IMLM which tends to settle toward a bimodal estimate of the spectrum at low frequencies. This seems to be the result of a non-convergent iterative scheme at low frequencies. These observations are consistent with results obtained by Brissette and Tsanis (1992a) for heave-pitch-roll data. This is not surprising since the heave-pitch-roll signal is essentially equivalent to the one obtained from a three wave gauge array. Figure 4 shows the MLM and NMLM estimates for a $\cos^2(\theta)$, $S=10$, spreading function to outline that the MLM induced spreading is only weakly dependent on the shape of the spreading function.
Fig. 1  Three, four, five and six wave gauge arrays tested in this study.

Fig. 2  MLM estimates for arrays in Fig. 1 ($f = 0.2Hz$).

Fig. 3a  Directional estimates for $sech^2$ distribution function ($f = 0.5 Hz$, $\beta = 1.24$).

Fig. 3b  Directional estimates for $sech^2$ distribution function ($f = 0.2 Hz$, $\beta = 2.62$).
In order to investigate the response of the methods to bimodal distributions, a case of mixed sea was simulated. Two DHH spectrum (Donelan, Hamilton and Hui, 1985) centered at 100° and 240° with peak frequency of 0.11 and 0.2 Hz were used for this purpose. Figures 5a to 5f present 3-D plots for the target spectrum and all estimates. The number in parenthesis is a figure of merit of the estimate given by:

\[
\frac{\int \int |S(f, \theta) - \tilde{S}(f, \theta)| \, d\theta \, df}{\int \int S(f, \theta) \, d\theta \, df}
\]  

All estimates are reasonably accurate except the direct Fourier transform method (PRB) which is unable to resolve the bimodality of the spectrum. The MEM and the IMLM (at lower frequencies) overpredict the energy while the MLM underpredicts. The NMLM is closest to target. Figures 6a and 6b show the results at two distinct frequencies. In Fig 6a the IMLM is again unstable but is shown to perform the best for bimodal distribution in Fig. 6b. In Fig. 6b the IMLM and NMLM outperform all the other methods.

4. FIELD DATA

The field data were collected from the National Water Research Institute’s (NWRI’s) tower on Lake Ontario during a three year period.
(1985–1987) for the "Deep Water Wave Breaking and Wave–Turbulence Interaction" project (Tsanis & Donelan, 1987). The tower is in 12 m of water, 1.1 km off the beach at the west end of Lake Ontario, see Fig. 7. In the tower’s location the shoreline is straight and the bottom slope is gentle. The tides, seiches and wind set-up can change the water level by an amount less than 0.1 m and the wind-induced currents are typically less than 10 cm/s. The location of the tower makes possible fetches from 1.1 km for the prevailing west winds up to 300 km for east winds. Every year, Lake Ontario sees several episodes of higher than 10 m/s wind speed. Six wave gauges were arranged in a pentagon with one at the center as shown in Fig. 2. Sample data 31 minutes long were FFT averaged to a frequency bandwidth of 0.039 Hz, giving spectral estimates with over 100 degrees of freedom.

**TABLE 1. Storm Case**

<table>
<thead>
<tr>
<th>Run</th>
<th>segment</th>
<th>Julian date</th>
<th>GMT time</th>
<th>length (min)</th>
<th>Wind dir deg</th>
<th>$U_{12}$ (m/s)</th>
<th>$T_a$ (°C)</th>
<th>$H_s$ (m)</th>
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<tbody>
<tr>
<td>87185</td>
<td>a,b,c</td>
<td>349</td>
<td>11.54</td>
<td>93.0</td>
<td>84</td>
<td>14.5</td>
<td>0.33</td>
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<td>87186</td>
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<td>349</td>
<td>13.31</td>
<td>29.0</td>
<td>82</td>
<td>15.8</td>
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<tr>
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<td>349</td>
<td>14.02</td>
<td>48.5</td>
<td>89</td>
<td>15.2</td>
<td>0.61</td>
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<tr>
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<td>a,b</td>
<td>349</td>
<td>14.53</td>
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<td>14.1</td>
<td>0.86</td>
<td>2.37</td>
</tr>
<tr>
<td>87189</td>
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<td>13.1</td>
<td>1.34</td>
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<td></td>
<td>349</td>
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<td>9.5</td>
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<td>1.93</td>
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<td>320.0</td>
<td>235</td>
<td>11.1</td>
<td>1.58</td>
<td>0.30</td>
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</table>

*Directional spectra are shown for segments 87188a, 87191a and 87192.

$U_{12}$ - wind speed at 12 m, $T_a$ - air temperature, $H_s$ - sign. wave height*

Three runs taken from one storm case during the fall period of 1987 were analyzed using the NMLM for estimating wave directional spectra. Table 1 gives detailed information on the physical parameters during the storm such as wind speed and direction, air and water temperature and significant wave height (defined as four times the rms of the water surface elevation). The first run, R87188 is taken as the storm (with strong winds from the east) is nearing its end. Figures 8a,b show 3D and contour plots, which indicate that the peak waves come from about 75°. Run R87191a is taken a short time after the wind shifted from about 80° to 240°. The swell (Figures 8c,d) comes from 65° and the wind sea is not noticeable yet. Finally Figures 8e,f (run R87192) show the dying swell now coming from 55° with the wind sea now clearly visible. This sequence of plots, coupled with the high
resolution of the spectral estimate clearly indicate that in cases of strong easterly winds, the waves are forced and are not reaching the west end of the lake at about 60° (the long fetch direction) as previously thought. As soon as the wind starts dying, the swell starts to realign itself with the longer fetch (Donelan et al., 1985).

5. DISCUSSION AND CONCLUSIONS

When analyzing wave data to extract an estimate of the directional spectrum, the choice of the appropriate technique can be a difficult one. That choice may be influenced by such factors as: instrumentation used, directional resolution needed and frequency of interest. A modeler looking for the main direction of wave propagation does not need the same spectral representation needed by design engineers for accurate calculation of wave loads on offshore structures. For these reasons it is difficult to draw general conclusions that will apply to all cases. Nevertheless, certain points can be raised.

The direct Fourier transform method is simple to implement and does not require any lengthy computations such as matrix inversions or iterative calculations common to other methods. But for applications needing good directional resolution, or in the presence of bimodal distributions, the method is inadequate.

The maximum entropy method gives an estimate which tends to overpredict the energy at the spectral peak but is able to resolve mixed seas. The results presented in this paper are mainly based on a six wave gauge array which puts the method at a disadvantage since it cannot use all of the available information. The calculation of the slopes makes use of all wave gauges but the new signals are in fact a linear combination of these and as such, do not contain all the information. If the slope calculations are reasonably accurate, the MEM estimate should not be dependent on the number of wave gauges. The relative performance of the method (when compared to maximum likelihood methods) should increase as the number of wave gauges (and the accuracy of the MLM) decrease.

The three maximum likelihood methods presented in this paper all give good directional spectra estimates. At low wavenumbers, the IMLM tends to be unstable due to convergence problems in the iterative scheme but gives the best estimate at higher frequencies for bimodal distribution. The MLM tends to underpredict the wave energy at the spectral peak but does well in every test case. The NMLM estimate does not have any major drawback and overall it performs best. The NMLM and IMLM are more computationally expensive than all other methods, especially the IMLM when the number of wave gauges is higher than 4. The computation time of the NMLM is not dependent on the number of wave gauges. The use of the NMLM to analyze field cases in Lake Ontario indicates that high resolution directional spectral estimates
can be obtained which allow fine details of the evolution of wave fields to be examined.
Fig. 5 Surface plots of directional estimates for a bimodal sea. Swell: DHH spectrum, $f_p = 0.11 \text{ Hz}, W = 1.2, \theta = 120$. Wind sea: DHH spectrum, $f_p = 0.2 \text{ Hz}, W = 1, \theta = 240$. 

TARGET

NMLM ($0.037$)

MLM ($0.0071$)

IMLM ($0.0102$)

MEM ($0.0084$)

PRB ($0.0313$)
Fig. 6a  Directional estimates of Fig. 5 at $f = 0.11 \text{ Hz}$.

Fig. 6b  Directional estimates of Fig. 5 at $f = 0.17 \text{ Hz}$.
Fig. 7  NWRI research tower in Lake Ontario.
Fig. 8  Surface and contour plots for field data.
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Wave Directional Spectra and Current Interaction in Lake St.Clair

F.P. Brissette and J. Wu

Department of Civil Engineering, McMaster University
Hamilton, Ontario, L8S 4L7

SUMMARY

Analysis of wave data collected during a 1985 field study on Lake St.Clair revealed important differences in certain cases between the wind and wave directions. In cases of westerly wind, peak waves were found to propagate at an angle of up to 60° to the wind direction. Detailed study of the wave directional spectra obtained from three arrays of three wave staffs each indicates that the difference between the wind and wave direction increases with increasing westerly wind and that the difference decreases when higher wavenumbers are considered. Both observations are consistent with the fact that these differences may be due to the interaction between the waves and the currents in the St.Clair/Detroit river system flowing through Lake St.Clair. In order to establish the water current structure, a two-dimensional hydrodynamic model was used to compute the hydraulic and wind-driven circulation in Lake St.Clair. The circulation patterns predict strong north currents needed to reconcile observations and theory. The wave directional analysis indicates that the wave-current interaction in Lake St.Clair is a complex phenomenon.

1. INTRODUCTION

Lake St.Clair (Figure 1), located between Lake Huron and Lake Erie, has a surface area of 1200 km² and is characterized by its shallowness, its maximum depth being just about 6 m, and its average depth of 4 m. The St.Clair River carries an outflow of about 6000 m³ from the upper three Great Lakes into Lake St.Clair. This high volume combined with the shallowness of Lake St.Clair (residence time of 9 days) indicate that hydraulic and wind-driven circulation are likely to form strong currents (Schwab et al., 1989). During the fall of 1985, an experiment conducted by the National Water Research Institute (NWRI) and the NOAA Great Lakes Environmental Research Laboratory (GLERL) allowed data to be gathered during a two and a half months period at different locations on Lake St.Clair (Venkatesh et al., 1987). Directional information was obtained at three locations on the lake, using a Direct Fourier Transform method (Longuet-Higgins et al., 1963). Figure 2 presents a plot of the peak wave direction compared to the wind direction at the central location on the lake (C3). The plot shows systematic deviations from the wind direction that are in excess of those that would be expected due to the gradient in fetch.
with direction (Donelan, 1980). The large deviations seem compatible with the presence of a strong hydraulic current aligned roughly north to south which would tend to deflect the waves toward the south (Figure 3). There are data that are not consistent with this simple idea, so that, in addition to a detailed analysis of the current structure, high resolution directional estimates are warranted.

2. DIRECTIONAL SPECTRUM ESTIMATION AND RESULTS

In order to obtain wave directional information during the 1985 experiment, arrays of wave staffs were mounted on three towers, C1, C2 and C3, see Figure 1. The three identical arrays consisted of three wave staffs each arranged in the vertices of a isosceles (0.25 m) right triangle (Figure 4). Water surface elevation was sampled at 4 Hz for 17 minutes every two hours. Samples were FFT averaged to a frequency bandwidth of 0.03 Hz, giving spectral estimates with 64 degrees of freedom. Cross-Power-Spectral-Density (CPSD) matrices were computed using the water elevations at each wave staff ($\eta$) but also on the heave ($\eta$), pitch ($\partial \eta / \partial x$) and roll ($\partial \eta / \partial \gamma$) signals using:

$$\eta = \eta^2$$  \hspace{1cm} (1a)  

$$\frac{\partial \eta}{\partial x} = 4 (\eta_1 - \eta_2)$$  \hspace{1cm} (1b)
where the subscripts refer to a particular wavestaff. Using both CPSD, a directional analysis was performed and directional spectrum estimates were obtained using a direct Fourier Transform method (Longuet-Higgins et al., 1963), Maximum Likelihood Methods (Capon, 1969; Jefferys et al. 1981; Isobe et al., 1984) and Maximum Entropy Method (Lygre and Krogstad, 1986). The following discussion will focus on the Normalized form of the MLM (Brissette and Tsanis, 1992a,b).

3. HYDRODYNAMIC MODEL OF LAKE ST.CLAIR

In order to further investigate the wave-current interaction in Lake St.Claire, the water current structure needs to be established. A two-dimensional nearly horizontal flow model is used to simulate the hydraulic and wind-induced circulation in Lake St.Claire. Using standard assumptions for a two-dimensional approximation (Blaisdell et al., 1991), the equations of motion in the $\chi$ and $\gamma$ direction and the continuity equation can be reduced to:

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -g \frac{\partial \eta}{\partial x} + 2V \Omega \sin \phi + \frac{k}{h} \frac{1}{W_x} - \frac{C_b}{h} \frac{U \sqrt{U^2 + V^2}}{h}$$

(2)

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -g \frac{\partial \eta}{\partial y} + 2U \Omega \sin \phi + \frac{k}{h} \frac{1}{W_y} - \frac{C_b}{h} \frac{V \sqrt{U^2 + V^2}}{h}$$

(3)
\[
\frac{\partial \eta}{\partial t} + \frac{\partial (U h)}{\partial x} + \frac{\partial (V h)}{\partial y} = q
\]  

(4)

where \( U \) and \( V \) are the depth-averaged velocities in the \( x \) and \( y \) directions respectively, \( g \) is the gravitational acceleration, \( \eta \) the water free surface elevation relative to the still water level, \( d \) is the water depth, \( \Omega \) the angular rotation of the earth, \( \phi \) the latitude, \( h \) the total water depth \((h=d+\eta)\), \( \bar{W} \) the wind speed at a 10 meter elevation, \( q \) the specific discharge of a source or a sink, \( k \) a surface friction coefficient and \( C_b \) is a dimensionless bottom friction coefficient defined as:

\[
C_b = \left( \frac{n^2 g}{h^{1/3}} \right)
\]  

(5)

\( f_p \) is the frequency at the spectral peak. Wave direction is for peak wave as obtained following Longuet-Higgins et al. (1963). * Towers C1 and C2 are in shallow water and shoaling effects may be important.

### TABLE 1. Field Data

<table>
<thead>
<tr>
<th>RUN #</th>
<th>Julian Day</th>
<th>Wind Dir</th>
<th>Wind Sp. (m/sec)</th>
<th>Wave Dir. C1 sup *</th>
<th>Wave Dir. C2 sup *</th>
<th>Wave Dir. C3</th>
<th>( f_p ) (Hz) C1</th>
<th>( f_p ) (Hz) C2</th>
<th>( f_p ) (Hz) C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>85270.22</td>
<td>270</td>
<td>326</td>
<td>4.5</td>
<td>329</td>
<td>331</td>
<td>353</td>
<td>0.31</td>
<td>0.31</td>
<td>0.32</td>
</tr>
<tr>
<td>85311.20</td>
<td>311</td>
<td>275</td>
<td>11.2</td>
<td>325</td>
<td>316</td>
<td>324</td>
<td>0.26</td>
<td>0.27</td>
<td>0.31</td>
</tr>
<tr>
<td>85320.06</td>
<td>320</td>
<td>87</td>
<td>12.3</td>
<td>31</td>
<td>47</td>
<td>63</td>
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<tr>
<td>85320.16</td>
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<td>11.7</td>
<td>109</td>
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<td>82</td>
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<td>0.34</td>
<td>0.26</td>
</tr>
<tr>
<td>85321.08</td>
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<td>9.3</td>
<td>316</td>
<td>305</td>
<td>287</td>
<td>0.25</td>
<td>0.26</td>
<td>0.28</td>
</tr>
</tbody>
</table>

4. RESULTS

As shown in Table 1, five different cases will be briefly discussed to illustrate some of the characteristics of wave directional spectra in Lake St. Clair. These cases have all in common that the peak wave direction is significantly different than the wind direction. The first case illustrates a NNW wind and, as shown in Table 1, the wave direction at C3 is almost straight from the north while results from
C1 and C2 are more consistent with the wind direction (Table 1). The water current structure for this case is shown in Figure 5 (for a 1.2 km grid), where the strong N-S current is thought to cause refraction of the wave field. In the east end of the lake, where currents are weak, wind and wave direction are consistent.

The second case is for a strong (11.2 m/sec) westerly wind. The circulation pattern (Figure 6) predicts a strong alongshore current on the NE boundary of the lake. Data in Table 1 indicates that the peak waves are coming from the NNW, a difference of up to 60° from the wind direction. Figure 7a presents a 3-D plot of the directional spectrum (NMLM estimate) at C3 and Figure 7b presents the same spectrum multiplied by the frequency to the fourth power in order to outline features at higher frequencies. Fig. 7b indicate that the spectrum is bimodal, one mode is travelling with the wind direction (270°), and the other mode is probably being refracted by the strong currents. The presence of a bimodal spectrum in such a relatively small lake is surprising and outlines the complexity of the wave-current interaction in Lake St.Clair.

The third and fourth cases (85320.06, 85320.16) are 10 hours apart, and the main difference between the two is a 23° wind shift to the ESE from the east, which causes a 60° difference in the wave direction at C1. Figures 8a and 8b show circulation plots for both cases. It can be seen that the circulation patterns are different despite the small difference in wind direction. In Fig. 8a, a strong alongshore current seems to control the wave direction whereas in Fig. 8b, strong currents are absent and the wave direction coincides with the wind direction.

The last case is for a 230° wind, which is essentially opposite to a strong North-East South-West current as shown in Fig. 9. Surface and contour plots of the NMLM directional spectrum estimate (at Tower C3) are presented in Figures 10 and 11. The spectrum is again bimodal, but surprisingly, neither modes are travelling with the wind, one coming from around 290° and the other from about 185° and gradually turning to 150° at higher frequencies. At this stage of the work, it is too early to speculate on the physics of the phenomenon and more cases of southwesterly are presently being analyzed.

5. CONCLUSIONS

A 2-D hydrodynamic model was used to investigate possible wave-current interaction in Lake St.Clair. The shallowness and large surface area of Lake St.Clair, coupled with the strong inflow from the St.Clair River can result in strong hydraulic and wind-induced currents. Wave
direction seems to be very dependent on the water–current structure, complicating the task of forecasting wave direction. Significantly different hydraulic and wind-induced circulation patterns can result from small differences in wind conditions, making it even harder to accurately forecast wave direction. The wave–current interaction in Lake St.Clair is a complex phenomenon. More detailed work needs to be done and more field data need to be analyzed, especially in cases of steady wind growth and relaxation in turning winds before a clearer picture emerges.
Fig. 5  Circulation pattern for case R85270.22 (scale = 10 cm/sec)

Fig. 6  Circulation pattern for case R85311.20 (scale = 10 cm/sec)

Fig. 7a  NMLM estimate for case R85311.20.

Fig. 7b  NMLM estimate for case R85311.20 multiplied by $f^4$. 

Fig. 8a  Circulation pattern for case R85320.06 (scale = 10 cm/sec)

Fig. 8b  Circulation pattern for case R85320.16 (scale = 10 cm/sec)

Fig. 9  Circulation pattern for case R85321.08 (scale = 10 cm/sec)

Fig. 10  NMLM estimate for case R85321.08
ACKNOWLEDGEMENTS

We thank M. Donelan (NWRI), M. Skafel (NWRI), S. Venkatesh (Atmospheric Environment Service) P. Liu and D. Schwab (GLERL) for providing the field data. The field program was funded in part by the Panel on Energy Research and Development. The present work was supported by NSERC Grant No. URFBS142.

REFERENCES


Fig. 11a NMLM estimate for case R85321.08 multiplied by $f^4$.

Fig. 11b Contour plot of 11a.


DEVELOPMENT OF A STATISTICAL WAVE CLIMATE DESCRIPTION BASED ON 10-PARAMETER SPECTRA

1Barbara-Ann Juszko and 2Ross Graham

1Juszko Scientific Services
127 Cliff Drive, RR 4, Victoria, B.C., V9B 5T8

2Defence Research Establishment Atlantic
P.O. Box 1012, Dartmouth, N.S, B2Y 3Z7 Canada

ABSTRACT

A 10-parameter model representation of directional wave spectra is described which can provide a concise description of individual records and allows for both a statistical wave climate description and the development of design spectra. The usefulness of this approach is demonstrated with three years of ODGP hindcast directional spectra from the Grand Banks. The 10-parameter spectra are fit to the hindcast data by means of a non-linear, least-squares, variance conserving procedure which resulted in acceptable fits for 92% of the records. Probability analysis on the fit parameters then allows for the development of design spectra associated with a given significant wave height or spectral group. The fit parameters can also be used in a statistical wave climate analysis. The applicability of the results to actual field conditions is discussed in light of existing ODGP model assessments.

1. INTRODUCTION

Many scientific and engineering applications use established relationships based on wave climate variables such as significant wave height, peak period and peak direction in order to examine the structural response of offshore and shoreline structures and for ship operability studies. The statistical distribution of these variables allows for estimates of the probability of occurrence of specific wave conditions, risk assessment of extreme conditions and establishment of design constraints. The wave climate variables are limited in the information they supply as they do not provide an understanding of the spectral shape which is very important for frequency and/or direction dependent response analyses. To remedy this, parametric models (e.g. Pierson-Moskowitz, Pierson and Moskowitz 1964; JONSWAP, Hasselmann et al. 1973), incorporating selected wave statistics, have been used to regenerate the amplitude frequency spectrum. These parametric models were designed to reproduce single sea peaks assuming self-similarity in spectral development. Many applications, however, require understanding of multiple peak spectra (e.g. swell and sea) and the associated wave direction and directional spread. Therefore, more complex parametric models are required. Ochi and Hubble (1976) suggest
a six-parameter model comprised of two three-parameter expressions which can separately model a maximum of two wave components. They show that by performing a statistical analysis on the individual parameter distributions, one could generate sets of design spectra, with a known probability of occurrence, which encompass the range of observed spectral shapes. A similar analysis was performed by Juszko (1990) on a large number of field spectra recorded off Canada’s West Coast. Hogben and Cobb (1986) suggest expanding the Ochi and Hubble model to include direction information by adding a \( \cos^2 \) term to each component providing a 10-parameter directional model. This expression was used successfully by Juszko (1989a,b) in order to parameterize directional wave spectra obtained from both a WAVEC buoy and a hindcast model. Application of the Ochi and Hubble statistical analysis to the 10-parameter directional model is an obvious next step.

In this paper, a 10-parameter directional model will be fit to three years of ODGP (Offshore Data Gathering Program) hindcast spectra generated for the Hibernia region of the Grand Banks. Hindcast spectra are used as they provide a sufficiently large sample for the statistical analysis. These data, as well as the numerical and statistical methodology, will be described in Section 2. In Section 3, the ‘goodness-of-fit’ of the 10-parameter model will be discussed and examples of the predicted design spectra will illustrate the ability of the analysis to reproduce the wide range of spectral shapes encountered when records are grouped according to significant wave height and/or spectral type. The paper will conclude with a discussion in Section 4 addressing the accuracy of the results in representing field conditions in light of existing hindcast model assessments.

2. METHODOLOGY

2.1 Data Sources

Hindcast directional wave spectra, produced by the ODGP deep water spectral wave model, were supplied by the Marine Environmental Data Service (MEDS). The data cover the period from October 1983 through September 1986, and were calculated for grid point 1106 (46 deg. 15 min. N, 48 deg. 45 min. W) on the Grand Banks. The data set consisted of a total of 4382 directional spectra, sampled every six hours, and described using 15 frequencies and 24 directions. The frequency resolution was variable with nominal frequencies at: 0.2545, 0.2792, 0.3142, 0.3491, 0.3840, 0.4189, 0.4538, 0.4952, 0.5760, 0.6458, 0.7331, 0.8377, 0.9948, 1.309 and 1.9373 radians per second (rps). The direction resolution was constant at 15 degrees.

2.2 Parameterization

The parameterization of the directional spectra and the details concerning the non-linear fit procedure, first guess selection, and
stop criteria are described in Juszko (1989b and 1991). Briefly, the analysis consisted of a four step process. The first step involved scanning the directional spectra for the location of peaks, ranking these peaks according to their spectral density value, assigning a two-digit code (ICODE) to the record to denote "spectral type" and storing the features of the two major peaks (e.g., direction, frequency, significant wave height) for possible use as first guess values for the 10-parameter model fit. The first digit of ICODE was assigned the total number of scanned peaks and the second took on a value based on the direction and peak period separation of the two largest peaks, given by:

<table>
<thead>
<tr>
<th>2nd digit</th>
<th>Spectral Peak Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>single peak</td>
</tr>
<tr>
<td>1</td>
<td>direction separation &gt; 45°; period separation &gt; 2 seconds</td>
</tr>
<tr>
<td>2</td>
<td>direction separation &gt; 45°; period separation &lt; 2 seconds</td>
</tr>
<tr>
<td>3</td>
<td>direction separation &lt; 45°; period separation &gt; 2 seconds</td>
</tr>
<tr>
<td>4</td>
<td>direction separation &lt; 45°; period separation &lt; 2 seconds</td>
</tr>
</tbody>
</table>

These code values will allow for later grouping of records according to spectral type to aid in both the assessment of the parametric fit and in establishing design spectra specific to spectral type. It was found that close to 60% of the records were multiple peaked, justifying the use of a more complex parametric model.

In the second step, an initial fit of the six-parameter Ochi and Hubble (OH) spectra, given by,

$$S(\omega) = \frac{1}{4} \sum_{i=1}^{2} \frac{\left(\frac{4\lambda_i+1}{4} \omega_{mi}^4\right)^{\lambda_i}}{\Gamma(\lambda_i)} \frac{\delta_i^{2\lambda_i+1}}{\omega^{4\lambda_i+1}} e^{-\left[\frac{(4\lambda_i+1)}{4}(\omega_{mi}/\omega)^4\right]}$$

was performed to the surface displacement spectra. Here \(S(.)\) represents the OH spectrum, \(\delta_i\), \(\omega_{mi}\), and \(\lambda_i\) are parameters describing the significant wave height, modal frequency and shape, respectively, of the \(i\)th component of the spectrum, and \(\Gamma(.)\) denotes the gamma function.

Each component three parameter expression is identical to the Wallops spectrum (Huang et al., 1981) given deep water, sinusoidal
wave assumptions (Juszko, 1990). The fit procedure uses the
Levenberg-Marguardt non-linear iterative process which is a
combination of a steepest-descent and Newton’s method. In this
application a weighted fit was performed in order to reflect the
different frequency bandwidths of the hindcast spectra. A variance
conserving procedure was also incorporated. The significant waveheight
of the spectra over the frequency range covered by the input spectrum,
h_{1/3,1}, was calculated from:

$$h_{1/3,1} = 4\sqrt{\sum_{i=1}^{N} E(\omega_i)DF(\omega_i)}$$  \hspace{1cm} (2)

where $E(\omega_i)$ represents the input spectral density at the frequency $\omega_i$,
$DF(\omega_i)$ is the variable frequency resolution and the sum is performed
over N frequencies. This value is less than $h_{1/3,2}$ obtained from the
two significant wave height parameters via:

$$h_{1/3,2} = \sqrt{\delta_1^2 + \delta_2^2}$$  \hspace{1cm} (3)

which is associated with the theoretical variance of the expanded
spectrum having a frequency range extending from 0 to $\infty$. In order to
conservate variance, the fit procedure included an algorithm which
forced the total variance under the model OH spectrum over the
appropriate frequency range to equal that of the input ODGP spectrum.

In the third step, the 10-parameter directional model given by:

$$M(\omega, \theta) = \frac{1}{4} \sum_{i=1}^{2} \left( \frac{4\lambda_i+1}{\Gamma(\lambda_i)} \right)^{\lambda_i} \frac{\delta_i^2}{\omega^{4\lambda_i+1}} A(P_i) \cos^2 P_i \left( \frac{\theta - \theta_{mi}}{2} \right) e^{-\left[(4\lambda_i+1)/4(\omega_{mi}/\omega)^4\right]}$$  \hspace{1cm} (4)

was fit to the hindcast spectra. Here, $P_i$ are directional spread
parameters, $\theta_{mi}$ are the modal directions and $A(P)$ is a normalization
factor for the area under a $\cos^{2P}$ curve given by:

$$A(P) = \frac{2^{2P-1}\Gamma^2(P + 1)}{\pi\Gamma(2P + 1)}$$  \hspace{1cm} (5)

The six OH parameters were used as first guesses for the equivalent
non-directional parameters while the directional properties associated
with the two modelled peaks were used as first guesses for $\theta_{m1}$ and $P_i$ (see Juszko, 1991). The variance conserving, non-linear fit procedure was similar to that used for the six-parameter model. The “goodness-of-fit” was assessed by calculating the percent residual error, RESD, given by

$$\text{RESD} = \frac{\sum_{k=1}^{N} \sum_{j=1}^{M} [S_D(\omega_k, \theta_j) - S_M(\omega_k, \theta_j)]^2 WT_i^2}{\sum_{k=1}^{N} \sum_{j=1}^{M} [S_D(\omega_k, \theta_j)]^2 WT_i^2}$$

where $S_D$ represents the input spectra, $S_M$ the 10-parameter fit spectra, $WT$ is a frequency weighting based on bandwidth and the sum is performed over $N$ frequencies and $M$ directions.

In the fourth step, the 10-parameter fit was re-performed on selected records, to see if the fit could be improved, by using a set of first guesses for the parameters obtained from the peak scanning procedure of step 1 with constant values of 2.0 and 1.5 for $\lambda_1$ and $\lambda_2$, respectively. The records that were re-processed included single peak spectra (ICODE=10) whose RESD values were greater than 10% (implying a poor first guess using the OH model parameters) or if ICODE indicated a multiple peak spectra, RESD>7% and, at the same time, the difference between the two modal directions and the scanned peak directions was greater than 45 degrees (implying that a peak was missed).

### 2.3 Statistical Analysis

The statistical analysis followed closely the procedure described by Ochi and Hubble (1976) for the six-parameter spectra. In this application, the statistics were calculated for the following eight parameters: $\arctan(\delta_1/\delta_2), \omega_{m1}, \omega_{m2}, \ln(\lambda_1), \ln(\lambda_2), \ln(P_1)\ln(P_2)$ and $|\theta_{m1} - \theta_{m2}|$. The ratio $\delta_1/\delta_2$ was used in order to reduce the number of parameters and taking the arctangent simplified the fit of a Gaussian form to the probability distribution. The natural logarithms were taken for the shape and spread parameters in order to limit the range in these values. The difference in modal direction, forced to lie between 0 and 180 degrees, was used as the relative direction between the two peaks was more important than the absolute peak directions. The data were divided into nine groups according to significant wave height. Records with RESD values greater or equal to 20% were excluded. An occurrence probability histogram was generated for each of the eight parameters and waveheight group. Table 1 lists the group number, number of records per group and mean significant wave height of the group which will be used in later regression analyses. A two-sided bounded Gaussian distribution was then fit to the probability histograms after outliers were removed and upper and lower
bounds were determined. An example of the fit for \( h_{1/3} \) group 3-4 m is shown in Figure 1. A family of probability spectra was developed for each parameter and waveheight group by locating the mode or median (50% probability) and the 80, 90 and 95% confidence limit parameter values. (The mode and median were often very close in value except for the direction parameter and for some of the higher waveheight groups where there were few data points.) These values were then treated as target parameters and the data were scanned for records whose associated parameters fell within 5% about the target value. The remaining seven parameters were then individually summed and averaged. This procedure results in 24 sets of eight parameters for each wave height group, eight of which are associated with the median (or modal) value. The eight median (or modal) sets can be averaged together to generate one average spectra for each group. The functional relationship

\[
Y = aX^b + cX + d
\] (7)

was fit to the results to allow for prediction of any parameter value \( (Y) \) from a known significant wave height \( (X) \).

![Figure 1: Probability histograms and fitted bounded Gaussian distribution (solid line) for fit parameters associated with records having significant wave heights between 3 and 4 m.](image-url)
Table 1: Number of records and average significant wave height per wave height group

<table>
<thead>
<tr>
<th>GROUP</th>
<th>RANGE</th>
<th>NPTS</th>
<th>$h_{1/3}$ average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-1m</td>
<td>61</td>
<td>0.88</td>
</tr>
<tr>
<td>2</td>
<td>1-2</td>
<td>843</td>
<td>1.60</td>
</tr>
<tr>
<td>3</td>
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<td>1230</td>
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</tr>
<tr>
<td>4</td>
<td>3-4</td>
<td>914</td>
<td>3.47</td>
</tr>
<tr>
<td>5</td>
<td>4-5</td>
<td>489</td>
<td>4.43</td>
</tr>
<tr>
<td>6</td>
<td>5-6</td>
<td>250</td>
<td>5.43</td>
</tr>
<tr>
<td>7</td>
<td>6-7</td>
<td>97</td>
<td>6.49</td>
</tr>
<tr>
<td>8</td>
<td>7-8</td>
<td>34</td>
<td>7.38</td>
</tr>
<tr>
<td>9</td>
<td>&gt;8</td>
<td>24</td>
<td>8.98</td>
</tr>
</tbody>
</table>

The probability analysis was also performed for records grouped according to spectral types (ICODE) and significant waveheight. This allowed for further understanding of the range in spectral shape that may be encountered as single and multiple peak spectra are examined separately and not "averaged" together as in the previous analysis.

Table 2: Percent joint occurrence of % RESD and $h_{1/3}(m)$

<table>
<thead>
<tr>
<th></th>
<th>0-1m</th>
<th>1-2</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>&gt;10</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10%</td>
<td>0.57</td>
<td>12.03</td>
<td>22.02</td>
<td>17.98</td>
<td>10.34</td>
<td>5.29</td>
<td>2.17</td>
<td>0.80</td>
<td>0.37</td>
<td>0.07</td>
<td>0.09</td>
<td>71.73</td>
</tr>
<tr>
<td>10-20%</td>
<td>0.80</td>
<td>7.51</td>
<td>6.55</td>
<td>3.29</td>
<td>1.07</td>
<td>0.66</td>
<td>0.09</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>19.99</td>
</tr>
<tr>
<td>20-30%</td>
<td>0.68</td>
<td>2.51</td>
<td>1.48</td>
<td>0.34</td>
<td>0.05</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
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<td>0.00</td>
<td>5.13</td>
</tr>
<tr>
<td>30-40%</td>
<td>0.46</td>
<td>0.91</td>
<td>0.52</td>
<td>0.11</td>
<td>0.05</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>2.08</td>
</tr>
<tr>
<td>40-50%</td>
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<td>0.07</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.75</td>
</tr>
<tr>
<td>50-60%</td>
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<td>0.11</td>
<td>0.05</td>
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<td>0.00</td>
<td>0.00</td>
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</tr>
<tr>
<td>60-70%</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>&gt;70%</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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</tr>
<tr>
<td>TOTAL</td>
<td>2.76</td>
<td>23.64</td>
<td>30.69</td>
<td>21.75</td>
<td>11.50</td>
<td>6.02</td>
<td>2.26</td>
<td>0.80</td>
<td>0.41</td>
<td>0.07</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>
3. RESULTS

3.1 Fit Assessment

The ability of the 10-parameter model to represent hindcast spectra is illustrated by the distribution of RESD (Equation 6) with significant wave height, shown in Table 2, and with ICODE in Table 3. These results indicate that approximately 91.7% of the records had acceptable fits (RESD ≤ 20%). The larger errors tend to be associated with multiple peak spectra (8% of the 8.3% unacceptable fits were scanned as multiple peaks) reflecting limitations of the model functional form and/or low energies (5.5% had $h_{1/3} < 2m$) when small errors in the fit can result in large relative errors. Table 4 contains the distribution (i.e. mean average deviation, standard deviation, coefficient of skewness and kurtosis) and comparison statistics (i.e. mean error, RMS error, scatter index and correlation coefficient) for spectral properties calculated on the hindcast and regenerated parametric spectra. The spectral properties included in the comparison were significant wave height, the peak period TP (the period associated with the spectral peak of the surface displacement spectrum), the peak direction PDIR (the direction associated with the maximum of the directional spectrum $S(\omega, \Theta)$), TDIR (the period associated with PDIR), and the vector mean direction VMD. Note that in this table we are comparing $h_{1/3,1}$ with $h_{1/3,2}$ (described in Section 2, Equations 2 and 3) since total variance is being conserved and, as expected, $h_{1/3,2} > h_{1/3,1}$ (negative mean error). The high correlations further support the accuracy of the parametric representation.
### Table 3: Percent joint occurrence of ICODE and % RESD

<table>
<thead>
<tr>
<th></th>
<th>0-10%</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>&gt;70</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>39.18</td>
<td>1.37</td>
<td>0.18</td>
<td>0.05</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>40.80</td>
</tr>
<tr>
<td>21</td>
<td>10.13</td>
<td>4.34</td>
<td>0.71</td>
<td>0.27</td>
<td>0.05</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>15.54</td>
</tr>
<tr>
<td>22</td>
<td>8.90</td>
<td>3.99</td>
<td>0.94</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>14.10</td>
</tr>
<tr>
<td>23</td>
<td>1.57</td>
<td>0.55</td>
<td>0.07</td>
<td>0.05</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.26</td>
</tr>
<tr>
<td>24</td>
<td>3.88</td>
<td>0.94</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>4.88</td>
</tr>
<tr>
<td>31</td>
<td>3.15</td>
<td>2.74</td>
<td>0.64</td>
<td>0.25</td>
<td>0.14</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>6.96</td>
</tr>
<tr>
<td>32</td>
<td>1.48</td>
<td>1.87</td>
<td>0.78</td>
<td>0.39</td>
<td>0.18</td>
<td>0.02</td>
<td>0.02</td>
<td>0.00</td>
<td>4.75</td>
</tr>
<tr>
<td>33</td>
<td>1.03</td>
<td>1.05</td>
<td>0.41</td>
<td>0.11</td>
<td>0.02</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>2.65</td>
</tr>
<tr>
<td>34</td>
<td>1.16</td>
<td>0.94</td>
<td>0.25</td>
<td>0.07</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.42</td>
</tr>
<tr>
<td>41</td>
<td>0.43</td>
<td>0.75</td>
<td>0.32</td>
<td>0.21</td>
<td>0.09</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>1.83</td>
</tr>
<tr>
<td>42</td>
<td>0.23</td>
<td>0.48</td>
<td>0.21</td>
<td>0.23</td>
<td>0.09</td>
<td>0.02</td>
<td>0.02</td>
<td>0.00</td>
<td>1.28</td>
</tr>
<tr>
<td>43</td>
<td>0.21</td>
<td>0.37</td>
<td>0.07</td>
<td>0.05</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.71</td>
</tr>
<tr>
<td>44</td>
<td>0.27</td>
<td>0.39</td>
<td>0.18</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.89</td>
</tr>
<tr>
<td>&gt;50</td>
<td>0.09</td>
<td>0.23</td>
<td>0.32</td>
<td>0.16</td>
<td>0.09</td>
<td>0.02</td>
<td>0.02</td>
<td>0.00</td>
<td>0.94</td>
</tr>
<tr>
<td>TOTAL</td>
<td>71.73</td>
<td>19.99</td>
<td>5.13</td>
<td>2.08</td>
<td>0.75</td>
<td>0.25</td>
<td>0.07</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
3.2 Families of Probability Spectra

Although families of probability spectra were developed for a variety of probability levels, only the results for the modal, median and 95% limits will be discussed. Further information can be found in Juszko (1991). The parameter values for the average median and modal spectra in each $h_{1/3}$ class are plotted in Figure 2. Examples of the 95% confidence parameter values, as a function of $h_{1/3}$, are shown in Figure 3 for the target parameter $\arctan (\delta_1/\delta_2)$. Results of the regression analyses (using Equation 7) are shown as the solid line in Figures 2 and 3. The regression had various degrees of success. The dependence of the shape and spread parameters on significant wave height was poorly represented except when they were the target parameters. It can be seen in Figures 2 and 3 that there were distinct trends in the parameter values with significant wave height. The decrease in modal frequency (1 and 2) and direction separation with increasing energy is consistent with sea peak development during storms. The behavior of the other parameters reflect the relative importance of swell in the spectra as well as describing how the 10-parameter model divides the spectral energy between the two components in cases of single peaks. For example, in Figure 3, the

Table 4: Summary statistics for spectral properties of the hindcast model (M) and corresponding 10-parameter (F) spectra (4382 pts)

<table>
<thead>
<tr>
<th></th>
<th>$h_{1/3}$</th>
<th>TP</th>
<th>PDIR</th>
<th>TDIR</th>
<th>VMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>2.999</td>
<td>9.243</td>
<td>1.574</td>
<td>9.955</td>
<td>1.437</td>
</tr>
<tr>
<td>F</td>
<td>3.120</td>
<td>9.335</td>
<td>1.556</td>
<td>9.688</td>
<td>1.447</td>
</tr>
<tr>
<td>ADEV M</td>
<td>1.116</td>
<td>1.427</td>
<td>1.023</td>
<td>1.593</td>
<td>0.947</td>
</tr>
<tr>
<td>F</td>
<td>1.141</td>
<td>1.555</td>
<td>1.019</td>
<td>1.578</td>
<td>0.958</td>
</tr>
<tr>
<td>SDEV M</td>
<td>1.429</td>
<td>1.834</td>
<td>1.257</td>
<td>1.990</td>
<td>1.220</td>
</tr>
<tr>
<td>F</td>
<td>1.497</td>
<td>1.919</td>
<td>1.259</td>
<td>2.005</td>
<td>1.230</td>
</tr>
<tr>
<td>SKEW M</td>
<td>1.081</td>
<td>0.141</td>
<td>0.502</td>
<td>0.274</td>
<td>0.409</td>
</tr>
<tr>
<td>F</td>
<td>1.359</td>
<td>0.286</td>
<td>0.597</td>
<td>0.240</td>
<td>0.427</td>
</tr>
<tr>
<td>KURT M</td>
<td>1.759</td>
<td>0.189</td>
<td>-0.389</td>
<td>-0.089</td>
<td>0.089</td>
</tr>
<tr>
<td>F</td>
<td>3.396</td>
<td>0.006</td>
<td>-0.314</td>
<td>-0.081</td>
<td>0.047</td>
</tr>
<tr>
<td>ME</td>
<td>-0.121</td>
<td>-0.939</td>
<td>0.008</td>
<td>0.267</td>
<td>-0.004</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.506</td>
<td>0.627</td>
<td>0.379</td>
<td>0.881</td>
<td>0.157</td>
</tr>
<tr>
<td>% SCAT</td>
<td>16.549</td>
<td>6.754</td>
<td>24.227</td>
<td>8.972</td>
<td>10.918</td>
</tr>
<tr>
<td>CC</td>
<td>0.945</td>
<td>0.946</td>
<td>0.895</td>
<td>0.912</td>
<td>0.935</td>
</tr>
</tbody>
</table>
shape parameters ($\lambda_1$) show a distinct dip for $h_{1/3}$ between 2 and 6 m. At low energies, swell signals, which are narrow-banded, can provide a relatively large contribution to the spectra and these are being modelled by the first set of components. At high $h_{1/3}$ values, the 10-parameter model has a tendency to fit the sharp sea peak with the first set of components (i.e. large $\lambda_1$ value) while the second set models the flat high frequency "tail" of the single peak. At mid-energy levels, the separation between sea and swell is not as distinct as at lower energies and the developing sea peaks are "flatter" than at higher energies, both effects leading to a reduction in $\lambda_1$.

Figure 2: Average median (star) and modal (diamond) parameter values as a function of significant wave height. The median regressions of Table 5 are shown as solid lines.
Table 5: Regression coefficients for the overall median spectra

<table>
<thead>
<tr>
<th>Median Parameter</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{arctan}(\delta_1/\delta_2) )</td>
<td>0.278</td>
<td>0.767</td>
<td>-0.140</td>
<td>0.476</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>-0.554</td>
<td>0.293</td>
<td>0.016</td>
<td>1.335</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>-1.085</td>
<td>0.353</td>
<td>0.081</td>
<td>2.174</td>
</tr>
<tr>
<td>( \ln(\lambda_1) )</td>
<td>0.004</td>
<td>2.436</td>
<td>0.022</td>
<td>0.650</td>
</tr>
<tr>
<td>( \ln(\lambda_2) )</td>
<td>-3.472</td>
<td>0.185</td>
<td>0.229</td>
<td>3.679</td>
</tr>
<tr>
<td>( \ln(P_1) )</td>
<td>-0.461</td>
<td>0.609</td>
<td>0.116</td>
<td>3.364</td>
</tr>
<tr>
<td>( \ln(P_2) )</td>
<td>-0.425</td>
<td>0.779</td>
<td>0.153</td>
<td>2.370</td>
</tr>
<tr>
<td>(</td>
<td>\theta_{m1} - \theta_{m2}</td>
<td>)</td>
<td>-1.741</td>
<td>0.259</td>
</tr>
</tbody>
</table>

Figure 3: 95% confidence parameter values as a function of significant wave height for the target parameter \( \text{arctan}(\delta_1/\delta_2) \). Squares: lower limit; circles: median; triangles: upper limit. The parameter regressions of Table 6 are shown as solid lines.
Table 6: Regression coefficients for 95% confidence limits spectra. L: Lower; U: Upper.

| 1: $\arctan{(\delta_1/\delta_2)}$ | 2: $\omega_{m1}$ | 3: $\omega_{m2}$ | 4: $\ln(\lambda_1)$ | 5: $\ln(\lambda_2)$ | 6: $\ln(P_1)$ | 7: $\ln(P_2)$ | 8: $|\theta_m - \hat{\theta}_{m2}|$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>L 1</td>
<td>0.268</td>
<td>0.650</td>
<td>0.151</td>
<td>0.319</td>
<td>-0.113</td>
<td>0.791</td>
<td>0.045</td>
</tr>
<tr>
<td>U 1</td>
<td>0.585</td>
<td>0.627</td>
<td>-0.229</td>
<td>0.739</td>
<td>-0.133</td>
<td>1.013</td>
<td>0.094</td>
</tr>
<tr>
<td>L 2</td>
<td>0.199</td>
<td>0.465</td>
<td>0.004</td>
<td>0.171</td>
<td>0.018</td>
<td>0.806</td>
<td>-0.023</td>
</tr>
<tr>
<td>U 2</td>
<td>0.179</td>
<td>0.534</td>
<td>-0.040</td>
<td>0.618</td>
<td>-0.502</td>
<td>0.659</td>
<td>0.130</td>
</tr>
<tr>
<td>L 3</td>
<td>-0.264</td>
<td>0.722</td>
<td>0.127</td>
<td>0.790</td>
<td>-0.001</td>
<td>1.812</td>
<td>-0.015</td>
</tr>
<tr>
<td>U 3</td>
<td>1.395</td>
<td>0.398</td>
<td>-0.214</td>
<td>-0.584</td>
<td>-0.164</td>
<td>0.805</td>
<td>0.051</td>
</tr>
<tr>
<td>L 4</td>
<td>0.711</td>
<td>0.757</td>
<td>-0.329</td>
<td>0.117</td>
<td>-0.057</td>
<td>0.883</td>
<td>0.007</td>
</tr>
<tr>
<td>U 4</td>
<td>0.685</td>
<td>0.684</td>
<td>-0.281</td>
<td>-0.184</td>
<td>0.502</td>
<td>0.366</td>
<td>-0.092</td>
</tr>
<tr>
<td>L 5</td>
<td>1.212</td>
<td>0.846</td>
<td>-0.779</td>
<td>0.056</td>
<td>-0.224</td>
<td>0.849</td>
<td>0.117</td>
</tr>
<tr>
<td>U 5</td>
<td>0.280</td>
<td>0.585</td>
<td>-0.076</td>
<td>0.456</td>
<td>-0.675</td>
<td>0.289</td>
<td>0.014</td>
</tr>
<tr>
<td>L 6</td>
<td>0.415</td>
<td>0.643</td>
<td>-0.150</td>
<td>0.425</td>
<td>-0.170</td>
<td>0.719</td>
<td>0.036</td>
</tr>
<tr>
<td>U 6</td>
<td>-0.326</td>
<td>0.645</td>
<td>0.143</td>
<td>0.678</td>
<td>-0.382</td>
<td>0.446</td>
<td>0.043</td>
</tr>
<tr>
<td>L 7</td>
<td>0.002</td>
<td>-37.786</td>
<td>0.030</td>
<td>0.696</td>
<td>-0.128</td>
<td>0.289</td>
<td>-0.053</td>
</tr>
<tr>
<td>U 7</td>
<td>-3.368</td>
<td>-0.164</td>
<td>-0.126</td>
<td>4.147</td>
<td>0.290</td>
<td>0.270</td>
<td>-0.059</td>
</tr>
<tr>
<td>L 8</td>
<td>-0.277</td>
<td>0.683</td>
<td>0.116</td>
<td>0.895</td>
<td>-0.473</td>
<td>0.419</td>
<td>0.036</td>
</tr>
<tr>
<td>U 8</td>
<td>-0.170</td>
<td>0.605</td>
<td>0.127</td>
<td>0.461</td>
<td>-0.389</td>
<td>0.363</td>
<td>0.026</td>
</tr>
</tbody>
</table>

The regression coefficient values for the median and 95% confidence limit spectra are provided in Tables 5 and 6, respectively. Table 6 may require some explanation. For each target parameter, for
example arctan \((\delta_1/\delta_2)\), upper and lower confidence limits are determined for the eight parameters, arctan \((\delta_1/\delta_2)\), \(\omega_{m1}\), \(\omega_{m2}\), \(\ln(\lambda_1)\), \(\ln(\lambda_2)\) \(\ln(P_1)\) \(\ln(P_2)\) and \(|\Theta_m - \Theta_m'|\), which are labeled as parameters 1 to 8 in the table. For example, the columns under arctan \((\delta_1/\delta_2)\) give the regression coefficients for parameters 1 to 8, which define, with Equation 7, the dependence of these parameters on significant wave height. Plots of the resulting fits for this target parameter are shown in Figure 3.

An understanding of the ability of the statistical approach to provide design spectra representing a wide range of spectral shapes, can best be obtained through visual examination of contoured directional spectra. The average median directional spectra are illustrated in Figure 4. They show the presence of both swell and sea peaks, which are young in terms of spectral development, at lower energies (1 to 4) with increasing loss of swell signals and greater directional symmetry at higher energies. Figure 5 contains the lower confidence spectra for the target parameter \(\omega_{m1}\). The lower 95% spectra represent occurrences of multiple peaks at all energy levels while, conversely, the upper 95% spectra (not shown) are single peaked. The median spectra associated with the different spectral type groupings are provided in Figure 6. In this case, fit parameters for single and multiple peaked spectra are not being averaged together so that the median spectra are consistent with their designation. Figures 4, 5 and 6, though only a subset of the available information, clearly illustrate the variability in spectral shape that can be modelled and predicted.
Figure 4: Contoured average median spectra by significant wave height group as described in Table 1. Contour levels set at: 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0, 15.0, 20.0 and 30.0 m²/rps-rad.

4. DISCUSSION

The use of a parametric representation of directional wave spectra and subsequent statistical analysis of the probability distribution of the parameters, was shown to provide design spectra which reflect the wide range in energy distribution that is observed. The probability analysis based on significant wave height groups allowed for the development of prediction relations for the model parameters within the significant wave height range examined. As there may be both single and double peak spectra with the same significant height, whose parameters are therefore averaged together, the results based on spectral type groupings may be more appropriate for some applications. Unfortunately, there were insufficient data to establish similar prediction relations. All of the results must be considered in light of the basic limitation of the parametric model to describe only two spectral peaks.
Design spectra, such as presented in this paper, will continue to be an important tool for engineers as they provide a basis for model and design testing. On the one hand, hindcast spectra are widely used for this purpose due to the absence of long records of field measured directional wave spectra necessary for the statistical analyses. On the other hand, hindcast models have only limited success in reproducing field conditions. Conclusions based on their design spectra must be qualified due to the following considerations. There are intrinsic differences between model and field spectra. For example, the directional resolution of field spectra, obtained using existing analysis techniques, is limited to detecting the presence of a maximum of two directional peaks per frequency (which are under-resolved). The resolution limits of the model spectra are determined solely by the direction grid. There is no isotropic noise included in the model spectra which is present in the field. Beyond these basic differences, Juszko and Graham (1992), through a coherence analysis, show that a major limitation of the ODGP model was an inability to reproduce variability in the wave field on time scales less than about 30 hours (for overall spectral energy) and that low frequency energy (i.e. swells) were not represented, in a statistically acceptable manner, on any time scale. The latter does not mean that swell signals are never modelled but rather that their occurrences do not correspond well with the field data. These observations imply that the underlying distribution of the parameters may be quite different between hindcast and field data fitted with the 10-parameter model (i.e. expect greater variability in the field values as well as possible differences in skewness and kurtosis) and further, that design spectra which indicate the presence of swell/sea peaks should be used cautiously.
Figure 5: Contoured lower 95% confidence spectra by significant wave height group as described in Table 1 for target parameter $\omega_m$. Contour levels as in Figure 4.
5. ACKNOWLEDGEMENTS

This work was supported by a Department of Supply and Services Contract No. W7707-0-1301/01-OSC. The authors would like to thank Dr. Ron Wilson, as well as his support staff, of the Marine Environmental Data Service, Department of Fisheries and Oceans, for supplying the hindcast data.

6. REFERENCES


WIND ESTIMATES FROM OBSERVED DIRECTIONAL WAVE SPECTRA

William Perrie¹, Bechara Toulany¹ and Zhengya Zhu²

¹ Physical and Chemical Sciences
Scotia-Fundy Region
Department of Fisheries and Oceans
Bedford Institute of Oceanography
Dartmouth, Nova Scotia

² Dept. of Oceanography
Dalhousie U.
Halifax, Nova Scotia

1. INTRODUCTION

Marine wind measurements are not made at all locations where waves are measured and this motivates us to search for wind information from the wave observations. Furthermore, wind measurements can have biases. In situ wind measurements are usually obtained from buoy-mounted anemometers or ship-mounted anemometers. The former are believed to give reliable estimates of surface wind speed by Pierson (1991) and are the basis for wind observations in the Surface Wave Dynamics Experiment (SWADE), described by Weller et al (1990), Marsden and Juszko (1989) suggest that wave slopes computed from the time series collected by directional wave buoys may also be used to calculate surface wind direction and speed.

Remotely sensed data from scatterometers or altimeters estimate the winds with algorithms derived by comparing in situ data to the remotely sensed data. It has recently been suggested by Glazman and Pilorz (1990) that wind speed inferred from altimeter measurements may have systematic errors that decrease with increasing wave age. Therefore observations about sea-state may be necessary for an accurate determination of the winds. We expect the same may be true regarding the scatterometer measurements.

Marsden and Juszko (1989) looked at time series collected from directional wave buoys in open ocean conditions on the Grand Banks of Newfoundland. They found high correlations between sea surface slope and surface wind speed and direction. The present analysis extends Marsden and Juszko (1989) by considering the spectral data directly recorded by directional wave buoys rather than the time series, and by considering offshore winds associated with fetch-limited growing waves. We first consider wind directions in Section 2. Section 3 looks at calibrations of onshore wind speeds in terms of wave slopes. Offshore wind speeds are calibrated in terms of wave slopes in Section 4. Finally, Section 5 considers wind speed as function of fetch when winds are offshore.

2. Wind Direction

In separating swell from wind sea during their analysis of pitch/roll buoy data from the CASP observation period, Dobson et al
(1989) found that the region of the wave spectrum above about twice the spectral peak follows the shifting direction of the wind very closely. The CASP array is shown in Figure 1.

Figure 1: The CASP wave buoy array. Shoreline wind measurements were recorded near point 'R'.

Well-behaved wave direction which is closely associated with the wind direction, may be computed by constructing the mean wave direction $\overline{\theta}$ for the upper equilibrium range (the region of the spectrum above 0.4 Hz or $1.5f_m$ : whichever is higher, where $f_m$ is the peak frequency of the windsea spectrum)
where $\Theta$ is the wave direction at frequency $f$ determined from the quad-spectra between the heave signal and the north-south wave slope $Q_{12}$, and the heave signal and the east-west slope $Q_{13}$.

\[ \bar{\Theta} = \frac{1}{\min[3.0f_{w}, f_{N}]} - \frac{\max[0.4Hz, 1.5f_{w}]}{\max[0.4Hz, 1.5f_{m}]} \int_{\max[0.4Hz, 1.5f_{m}]}^{\min[3.0f_{w}, f_{N}]} \theta(f) \, df \] (2.1)

\[ \theta = \arctan\left(\frac{Q_{13}}{Q_{12}}\right) \] (2.2)

corrected for the magnetic declination. The nyquist frequency (0.64 Hz) is denoted by $f_{N}$. Figure 2 compares the wind direction at the meteorological buoy (MINIMET) with the wave direction as calculated from equation (2.1) for the outermost CASP pitch/roll buoy when wind speeds less than 5 m/s are not included in the analysis.

**Figure 2:** Wind Direction measured at the MINIMET as a function of the high frequency wave direction defined by equation (2.1) measured at WAVEC WC31, in degrees true. Perfect agreement is shown by the line -- -- --. The agreement is almost perfect for all the pitch/roll buoys in the array. Correlation coefficients, for this and other CASP buoys, are above 0.98 with rms errors less than 14.7°. Marsden and Juszko
(1989)’s analysis yields a correlation coefficient of 0.99 and rms error 8.0°. The direction of the high frequency equilibrium region of the spectrum provides a good measure of the observed wind direction without the 180° ambiguity encountered by Marsden and Juszko (1989).

3. Onshore Winds

When the wind is onshore, there is no roughness change such as occurs at the land-water boundary in the offshore case in the sense that no boundary layer height adjustment occurs over water. This is the case of open ocean long-fetch waves. In principle the fetch is unlimited. In practice, the fetch is limited depending upon where the waves were generated in the open ocean. Certainly the corresponding wind sea spectrum used in the calibration of observed winds tends to be different from the fetch-limited windsea arising from offshore winds. Conditions are equivalent to Marsden and Juszko (1989)’s directional buoy data from the Grand Banks or to that of any other observation point in the open ocean.

Our analysis considers situations where the measured winds at the MINIMET are within a window of ±60° to orthogonal from the coast to eliminate contamination due to winds interacting with the irregular coastline. We calibrate the wind speed $U$ measured at the MINIMET as a function of the total north-south spectral wave slope energy ($E_2$), at the outermost pitch/roll of the array.

$$E_2 = \int_{f_s}^{f_N} C_{22}(f) \, df$$

(3.1)

where $f_s$ is the boundary between swell and windsea and $C_{22}$ is the north-south wave slope spectrum. Comparison is made between wind speed and the total wave slope energy integrated over all frequencies of the wind sea spectrum. Parameterizations in terms of the expression

$$U = A_0 + A_1 s + A_2 s^2$$

(3.2)

where $s$ represents the spectral wave slope are consistent among themselves and verify the time series analysis of Marsden and Juszko (1989). In fact, our parameterization of total spectral wave slope energy $E_2+E_3$, where $E_3$ is the east-west wave slope energy, gives slightly better correlation coefficients and lower rms errors than the Marsden and Juszko (1989) parameterizations of total wave slope variance.

4. Offshore Winds: Winds as a Function of Wave Slope
When the wind is offshore, there is a change in roughness at the land-water boundary. In fact, before the winds pass over the water, they pass over forest, swamp and low hills, according to the trajectory that they follow at a given instant, and these factors contribute to the effective boundary layer roughness which shapes the wind field that finally reaches the water, as discussed in Dobson et al (1989). At the land-water boundary, the roughness changes once more and an associated boundary layer height adjustment occurs. Fetch-limited wind-generated waves are therefore highly variable for differing slanting fetch situations in the offshore case.

To correlate wind speed and wave slope we restrict our attention to situations where the winds are within ±30° of orthogonal to the coastline to eliminate contamination from slanting fetch cases. Figure 3 and Table 1 present the wind speed measured at the MINIMET as a function of the north-south wave slope (E2),

Figure 3: Wind Speed as a function of wave slope E2 (north-south) for offshore winds within ±30° measured by the MINIMET at WC33. Shoreline wind directions are required to be within ±10° of high frequency wave directions. Least squares parameterization as given in Table 1 is shown by -----.
Table 1: Quadratic parameterizations given by equation (3.2), correlation coefficients $R$ and rms errors for wind speed as a function of wave slope in the north-south direction E2 for offshore winds within $\pm 10^\circ$ and $\pm 15^\circ$ to the high frequency wave directions for WC33.

<table>
<thead>
<tr>
<th></th>
<th>$\pm 10^\circ$</th>
<th>$\pm 15^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>2.08</td>
<td>2.09</td>
</tr>
<tr>
<td>$A_1$</td>
<td>1129.5</td>
<td>1172.3</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$1.6 \times 10^4$</td>
<td>$1.4 \times 10^4$</td>
</tr>
<tr>
<td>$R$</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>rms</td>
<td>1.73</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Wind direction as measured at the shoreline is within $\pm 10^\circ$ (and also within $\pm 15^\circ$ of the high frequency wave direction determined by equation (2.1)). Knowing the wave slopes at the other 2 pitch/roll buoys, this calibration allows estimation of the wind speeds at these buoys during offshore scenarios.

5. Wind speed as a function of fetch

There are very few measurements of the variation of surface wind speed with fetch for offshore winds. Smith and MacPherson (1987) achieved a set of aircraft observations at 50 m height and, assuming a logarithmic profile, these may be reduced to winds at 10 m height as presented in Figure 4. Although the Smith and MacPherson (1987) data is unique, the error bars are large. It is therefore important that further observations be analyzed to give insight into the fetch dependence of the wind speed. We were therefore motivated to try to infer the wind speed dependence on fetch from the spectral wave slope energy.
Figure 4: Offshore winds as a function of fetch from the Smith and MacPherson (1987) measured aircraft winds at 50-m elevation, showing measured ratios and standard deviations winds at a 50-km fetch to those at fetch, are denoted * with error bars. The parameterization derived from spectral wave slopes is shown as o.-.-.-o with estimated error bars.

To select wave slope data that are ‘relaxed’ to the associated wind forcing, we required that wind direction at the shoreline agree with the high frequency wave direction at the buoys WC31, WC32 and WC33, as calculated from equation (2.1), to ±10°. To eliminate contamination due to slanting fetch situations, only offshore winds within the ±15° window to orthogonal to the coastline are considered. Using the relation between north-south wave slope (E2) at WC33 and wind speed measured at the MINIMET as shown in Figure 3 and parameterized in Table 1, we inferred the corresponding wind speeds at WC32 and WC31. Results are compared in Figure 4 to the measurements of Smith and MacPherson (1987).

6. Conclusions

A wave direction can be computed from the high frequency region of the spectrum, well within the equilibrium range. This wave direction compares well and is statistically identical with in situ measurements of wind direction from a nearby MINIMET buoy,
For onshore winds, we derived parameterizations of wind speed as a function of spectral wave slopes in good agreement with those found earlier by Marsden and Juszko (1989) from time series. This differs from the correlation of wind to wave slope for offshore winds. In the latter case the waves are wind-generated and (short) fetch-limited. Because of the change in roughness at the land–water boundary, the wind speed also changes with fetch as observed by Smith and MacPherson (1987).

We derived the variation of wind speed with spectral wave slope energy for offshore winds at the outermost WAVEC of the array shown in Figure 1. From these results we inferred the wind speed at the other two directional buoys. In deriving these other winds, we assumed that the results of Figure 3 and Table 1 could be applied to the two inner directional buoys. This was necessary because wind measurements were only available at the outermost WAVEC where the meteorological buoy was located. Thus we assumed that it is a good approximation to regard the parameterization of wind speed in terms of spectral wave slope as independent of fetch. We were successful in obtaining resultant winds as a function of fetch that agree qualitatively with Smith and MacPherson (1987)’s measurements of offshore winds. Our scatter is comparable to their error bars.

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REFERENCES


ON THE STRUCTURE OF THE MARINE SURFACE WIND FIELD IN EXTRATROPICAL STORMS

Vincent J. Cardone
Oceanweather Inc
Cos Cob, CT

1. INTRODUCTION

Interest in surface marine winds has been stimulated in recent years by the demands of increasingly accurate (i.e. well calibrated) ocean response models. For example, the third generation wave model (WAMDI, 1988) provides extremely accurate specifications of integrated sea state properties and of the wave spectrum in tropical and extratropical storms provided that surface winds are specified with negligible systematic (bias in wind speed and/or direction) or spatially correlated errors, and possess random errors of less than about 2 m/sec in wind speed and 20 degrees in direction. Unfortunately, surface wind analyses and forecasts produced at operational numerical weather prediction (NWP) centers rarely achieve such low errors (e.g. Gemmill et al., 1988) resulting in fairly large errors in operational sea state predictions (e.g. Eid et al., 1986).

The desire for accurate design criteria for offshore structures (namely, peak loads associated with surface winds, waves and currents) has led to increasing reliance on the hindcast method (Cardone et al., 1989). For basins in which the peak load is associated with tropical cyclones, wind fields are developed mainly through the application of numerical models (Cardone et al. 1976). In most mid- and high-latitude basins, peak loads are associated with extratropical cyclones, and the main work of a hindcast study is to describe the time and space evolution of the wind field in severe historical storms, working mainly from conventional historical meteorological data.

Cardone et al. (1980) reviewed alternate surface wind field analysis methods and concluded that intensive subjective reanalysis of pressure charts, combined with selective application of manual kinematic analysis techniques were needed to provide unbiased hindcasts of storm peaks in mid-latitude Northern Hemisphere basins. These methods depend upon conventional data, mainly historical ship reports. Where such data are too sparse (e.g. very high latitudes, most Southern Hemisphere basins) no amount of reanalysis can reduce the wind errors to tolerable levels. For example, in the Labrador Sea Extreme Waves Experiment (LEWEX) so much of the wave energy observed in the measurement area originated far north and south of the mid-latitude North Atlantic shipping lanes, that when nine different wave models were driven by a common wind field, derived in a post-analysis (Cardone, 1991) and the hindcasts compared to
directional wave measurements, common errors pervaded all models, suggesting that wind field errors masked the effect of any deficiencies in wave model physics.

As the data density decreases, the analyst is forced to rely increasingly on preconceived patterns of the structure of surface pressure fields (which are then transformed to surface wind fields using marine planetary boundary layer models such as Cardone, 1969) and of the surface wind field. For extratropical cyclones, these patterns tend to conform to the classical Norwegian frontal-cyclone model, which describes the evolution of a marine cyclone from the initial amplification of a wave on a preexisting front, through a deepening stage accompanied by frontal occlusion. This conceptual model continues to dominate operational surface marine analysis (which are drawn at most centers either by hand, or in a man-machine interactive process), and, of course, analyses preserved in archives of weather maps of the past half century, which form the starting point for most hindcast studies. The Norwegian model of cyclogenesis has been questioned in recent years not from observational evidence, but from numerical simulations using high-resolution NWP models (e.g. Hoskins and West, 1979). The development of this new conceptual model of cyclogenesis has been reviewed by Shapiro et al. (1991). The key surface level elements of the new model include (1) the development of a pronounced “bent-back warm front” from the developing cyclone into the polar air-stream westward from the developing cyclone forming a “T-bone” frontal structure with the cold front; (2) the migration of the intensifying cyclone along the bent-back front as the T-bone fractures from the storm center; (3) the development with time of a warm-core cyclone, or “warmsector seclusion” as polar air encircles the center at low levels. The theoretical basis for the new conceptual model continues to be supported in numerical model studies. For example, Kuo et al. (1991) used a synoptic/mesoscale NWP model to simulate the details of the evolution of the Ocean Ranger storm. The resulting integration exhibited the main features of the storm, and several ERICA storms have also exhibited the structural detail of the new conceptual model (Neiman et al., 1991). The above conceptual model joins several other modern “anomalous” modalities of extratropical storm formation such as polar lows, cyclogenesis in polar air-streams, and re-intensification of mature cyclones.

The main purpose of this paper is to begin to explore the implications of some of these new concepts for surface marine wind modelling of extratropical storms. In particular, we ultimately seek answers to the questions: what is the nature of the anomalies of the surface wind (relative to the simpler classical picture) and are these anomalies important enough to affect ocean-response? If so, has failure to resolve these anomalies in historical hindcast studies seriously biased extreme design criteria? This pilot study begins to
address these issues with the description of surface wind field “anomalies” in several recent severe east coast extratropical cyclones, revealed through intensive kinematic analysis of greatly enhanced (relative to typical historical coverage) surface marine observation networks.

2. DATA BASE AND CASES SELECTED

There has been steady improvement in coverage of high quality surface marine observations off the east coast of North America within the past couple of years because of the growing deployment of moored data buoys by NOAA and AES. In addition, during the winters of 1988–1989, the Experiment on Rapidly Intensifying Cyclones over the Atlantic (ERICA) (Hadlock and Kreitzberg, 1991) contributed an extensive network of drifting buoys, and during selected periods (so-called IOP’s) low-flying research aircraft sampled the boundary layer wind flow in the vicinity of explosively developing cyclones. During the late fall and winter of 1990–1991, the buoy network off the east coast of the US was further augmented by additional moored buoys deployed for the Surface Wave Dynamics Experiment (SWADE) (Weller et al., 1991).

Operational surface pressure and wind field products have apparently not improved significantly during these periods, despite the availability of most of these data in real time. Sanders (1990) used reanalyses of surface pressure fields derived through intensive subjective hand analysis to document the evolution of ERICA storms and to study errors in operational analyses and conventional observations. His comparison of the manual and automated analyses showed “serious deficiencies in the latter” in storm central pressure and position. Graber et al. (1991) used objective analysis winds from the U.S. Navy Fleet Numerical Oceanography Center (FNOC), the NOAA National Meteorological Center (NMC), the NASA Goddard Space Flight Center (GSFC) and ECMWF, to hindcast the severe SWADE storm of October, 1990 with the WAM model. All of those wind fields provided hindcasts which significantly underpredicted storm wave peaks in the SWADE array, and wave height scatter indices (ratio of the standard deviation to mean of measurement sample) about a factor of 2 larger than usually achieved with WAM when research quality wind fields are used. Greatly improved agreement was found when this same storm was rehindcast by WAM Using winds developed by hand kinematic analysis (Cardone et al, in preparation).

For this study we draw examples of anomalous surface wind field structures from complete kinematic wind field analyses produced for the following four scenarios:

Cases 1, 2 and 4 are characterized by explosive cyclogenesis, (24-hour deepening exceeding 24 mb) while in Case 3, maximum 24-hour deepening observed was about 18 mb. Wind fields for Case 1 were developed at part of a model study of surface wave modulation of momentum flux (Chin et al. 1991). Case 2 winds were prepared for this study because this storm has drawn intense interest from ERICA atmospheric modelers. Case 3 wind fields were prepared in support of SWADE. Case 4 winds were prepared mainly to add an additional wave hindcast to the population of historical storms hindcast for the PERD sponsored Canadian East Coast extreme wave climate study (Swail et al., 1989).

3. WIND FIELD ANALYSIS METHOD

The wind field analysis method adopted is basically the same as that described and recommended by Cardone et al. (1980). While no fundamental changes in the hindcast method were necessary to account for the greatly enhanced surface data in the western North Atlantic, wind fields could be derived at temporal (three-hourly intervals) and spatial scales (0.5 degree grid spacing) hardly justified for analysis of ship report data in open ocean regions. The method consists of the following steps:

1. Assemble conventional surface weather maps (mainly NOAA NMC Final Analysis series and NGM initial analyses), six-hourly ship report collections (including late reports and punched logs in NOAA NCDC TDF-11 archive), US and Canadian buoy and C-MAN data, including where available, consecutive 10-minute average winds, surface observations from drifting buoys (ERICA), aircraft winds reduced to surface (ERICA).

2. Reduce measured winds to effective neutral 20-m height hourly averages. For ship reports, only the height and stability adjustment is possible for measured winds and Beaufort estimates, following Cardone et al. (1990). For buoy and C-MAN stations without continuous wind sampling, hourly-average winds at three-hourly intervals are synthesized by averaging three consecutive hourly winds (speed and direction) with weights of 1/4, 1/2, 1/4. Measured air-sea temperature difference (three-hourly) and anemometer height are used to adjust the average winds to equivalent 20-m neutral, using a stability-dependent surface layer model.

3. On computer plotted base maps, carry out a detailed hand-analysis of surface pressure and surface air-temperature at three-hourly intervals, and sea surface temperature once per day, carefully maintaining continuity of centers of action and frontal boundaries.

4. Digitize each analyzed map on a x-y digitizing table, including digitization of isopleths, locations and values of maxima and minima.
5. Calculate gridded pressures and temperatures by fitting a paraboloid to seven or more digitized points located within a small circle about each point.

6. Calculate effective neutral wind fields over the whole domain of the analysis from the gridded pressures and temperatures using the marine planetary boundary layer model (MPBL) of Cardone (1969). That model links the following external factors governing the near surface-flow in a steady-state, horizontally homogeneous boundary layer: latitude (Coriolis parameter, \( f \)), surface roughness parameter \( (Zo) \), air-sea temperature difference \( (Ta-Ts) \), gradient wind vector, \( (Vgr) \) and horizontal air temperature gradient, or thermal wind vector \( (Vth) \). Compare the modelled (hence PRESTO) and measured winds at each buoy location and compute standard measures of difference.

7. Over a more limited domain, carry out a subjective kinematic analysis of the adjusted wind observations (the MPBL winds are available to the analyst as an underlay to the analysis base map), digitize the wind speeds and directions at grid points, enter these (hence KINEMA) winds to a disk file, and replace the PRESTO winds with the KINEMA winds inside the domain of the kinematic analysis.

8. Interpolate the final wind fields to the measurements sites, compare time histories of measured and modelled winds, and compute standard statistical measures of difference.

4. SURFACE WIND FIELD ANOMALIES

Case 1. ERICA IOP-2. The complex evolution of the sea-level pressure field is described in detail by Sanders (1990), and our own pressure field reanalyses follow his closely. To illustrate this complexity we show only one of the fields (selected from 3-hourly fields derived over a 10-day period) in Fig. 1a-d, which are valid 1800 UT 13 December, 1988. This figure shows, in order, the NOAA NMC North American surface analysis, PRESTO winds calculated from our reanalyzed pressure and temperature analyses, the manual kinematic analysis, and the final KINEMA wind field. Two weak depressions are resolved in the NMC analysis, the southern low depicted as a partially occluded wave on a preexisting front, the northern system in a trough shown extending to the north-northwest from the southern low. The PRESTO winds more precisely place the circulation centers of these two systems. The KINEMA field, however, resolves a third distinct circulation center. Continuity analysis indicates that the southernmost system itself had evolved over the previous 24-hours, the northernmost system has formed within the previous 12-hours, and the central system is newly formed. These three separate circulation centers could be tracked eastward and northward as indicted in Fig. 1d for at least the following 12 hours, with each deepening rapidly,
until all three centers apparently coalesced into one intense center by about 0900 UT 14 December. By 1800 UT, 14 December the central pressure of the single center had lowered to 957 mb, which is 43 mb lower than the lowest of the central pressures 24 hours earlier.

Comparison of Fig. 1b and 1d (difference plots shown at the workshop) reveal large spatially coherent differences in wind speed and direction, especially in the area between the coast and the northern low, and, as expected, in the region surrounding the central low. Since the PRESTO winds were derived from reanalyzed pressure and temperature analyses, we suspect that differences between KINEMA and operational wind products, or PRESTO winds derived from conventional pressure charts, would be even larger. Table 1 shows standard scalar wind differences between the analyses (over the whole PRESTO and KINEMA periods modelled) and the adjusted measured winds at all NOAA buoys within the domain of the analysis. The errors for PRESTO are typical of the lowest errors reported (e.g. Cardone, 1991) for model diagnosed surface marine winds, and yet these errors evidently mask large systematic errors relative to KINEMA. In general, ocean response models are much more tolerant of random errors than systematic wind errors.

The differences between the KINEMA and buoy winds are about one-half those of PRESTO. The buoy winds were, of course, used in the kinematic analysis, but since the analysis process strives to resolve only synoptic scales (albeit small-synoptic scales) of motion (unlike some objective analysis schemes which can assimilate and match measured data almost perfectly, but at the expense of adding noise to the analysis), we suspect that these differences probably characterize the surface wind fields in the domain of the kinematic analysis.

**Case 2. ERICA IOP-4.** This storm was among the most intense extratropical cyclones observed to move off the mid-Atlantic coast this century. Between 00 UT 4 January, 1989 and 00 UT 5 January, central pressure decreased from 996 mb to 936 mb. Surface and aircraft data revealed (Neiman et al. 1991) the evolution of the frontal structure and the warm-sector seclusion during the deepening stage, and several mesoscale cyclones which apparently propagated westward along the bent-back front. We followed Neiman et al. (1991) pressure analyses closely in specification of PRESTO winds (after adding air and sea temperature analyses) and used all surface and aircraft measured winds to develop three hourly wind fields during the period of rapid intensification. It is very doubtful that the deepening rate and extreme intensity of this system would have been detected from analysis of typical historical data, since as soon as the cyclone central pressure reached its minimum of 936 mb, central pressure began to rise gradually reaching about 946 mb when the center crossed Newfoundland at 12 UT 5 January.
Fig. 2 compares the PRESTO (Fig. 2a) and KINEMA (Fig. 2b) wind fields at 06 UT 4 January, during the explosive deepening. Data from an aircraft probe of the center helped to define the frontal locations, the field of motion on both sides of major frontal boundaries, and snapshots of two mesoscale cyclones propagating rapidly along the bent-back warm front. The latter are not well resolved on the 1/2 degree wind grid, but KINEMA clearly shows greater spatial variation in the surface wind field within about 300 miles of the “center”. The PRESTO wind field is reminiscent of the depiction of a more classical frontal cyclone, while the KINEMA field indicates considerably more shear in speed and direction along the warm front to the east, and the deformation of the wind field to the west of the center nearly as far westward as the coast, associated with the bent-back warm front, and the more complicated and highly irregular wind field near the “center”. Fig. 3 compares PRESTO (Fig. 3a) and KINEMA (Fig. 3b) winds with averaged measured winds at buoy 44004 (38.5N, 70.7W). This comparison highlights the tendency for the PRESTO winds to exceed KINEMA winds within the cyclonic part of the circulation. The positive bias in PRESTO wind speeds (relative to KINEMA) is probably related to the steady-state assumption in the MPBL which implies instantaneous adjustment of the wind field to the pressure-field (gradient wind). Fig. 3 also gives wind speed scattergrams for PRESTO (Fig. 3a) and KINEMA (Fig. 3b) winds based upon comparisons at ten NOAA buoys off the east coast. Table 1 gives the difference statistics. The bias and scatter in PRESTO wind speed and direction reflect not only random errors but the large spatially coherent systematic differences between the PRESTO and KINEMA fields. The rather elongated circulation structure associated with the bent-back warm front west of the center and the occluding front east-northeastward of the center persisted throughout the explosive deepening phase.

**Case 3. SWADE Special IOP.** This storm occurred at the beginning of SWADE and has become one of the most interesting SWADE IOP’s because of the excellent measured wave database acquired in a fairly intense, though sub-bomb, case of cyclogensis. The center and most intense part of the storm passed directly over the SWADE array, thereby allowing the wind field to be resolved with an accuracy rarely achieved in an open deep water region. Fig. 4a shows the NOAA NMC Surface analysis of the developing storm. The System is represented as an open wave on a preexisting front, with warm front to the northeast and cold front trailing southward from the “center”. The three-hourly manual kinematic analyses (e.g. Fig. 4b) indicated a far more complex structure, as several small scale cyclonic centers propagated along an elongated shear zone which extended eastward from North
Carolina to just east of Cape Hatteras then northeastward nearly 1000 miles to near Cape Race, Newfoundland. This frontal structure greatly "linearized" the surface wind field about the "center" of the storm through most of the development stage, as exemplified in the KINEMA field shown in Fig 4c. The nearly linear fetch of northeasterly winds from Nova Scotia to Cape Hatteras favored generation of significant wave heights (HS) of about 8m just offshore North Carolina and Virginia. The KINEMA windfields fit the buoy wind series very closely (see Fig. 5 and Table 1) and when they were used to drive the WAM model, the resulting hindcasts were so skillful, that residual differences between the wave hindcasts and the measurements probably do not arise in wind errors but instead probably will allow further refinement of the wave model physics.

Case 4. Grand Banks Storm. This case was studied mainly to provide winds for a hindcast of waves on the Canadian east coast. The hindcast was prompted after three Canadian buoys moored just south of the Grand Banks measured peak HS each almost exactly equal (range 13 m to 14 m) to the 100-year return period maximum HS specified at these buoy locations in the recently completed PERD study (Swail et al, 1989). Interest in this storm was further stimulated when hindcasts using operational wind fields and the same wave model as used in the PERD study, failed to approach these peak wave heights. Indeed, a casual examination of the intensity and track of the extratropical cyclone associated with this event would not have ranked the Storm as one capable of such extreme wave generation, using the storm selection methods applied in the PERD study. This paradox was resolved when KINEMA winds were developed for this storm from the enhanced surface data base provided by the SWADE array (the beginning of its IOP-2) and the recently deployed Canadian buoys and were used to drive the PERD model. The resulting hindcast specified peak HS within 0.5 meters of measured at each buoy location.

The kinematic analysis revealed that a rapidly propagating transient feature of the wind field in the peripheral circulation of this storm greatly enhanced its wave generation potential, and was mainly responsible for the extreme wave heights observed south of the Grand Banks. Fig. 6a shows the KINEMA wind field at 00 UT 10 January, 1991, about the newly formed storm centered (minimum pressure 1012 mb) east of New Jersey. Fig. 6b shows the KINEMA field 24 hours later. The center has deepened to 980 mb and peak winds are barely 25 m/s. The shape of this storm, elongated northwest-southeast throughout this period, combined with its northeastward movement do not favor strong wave generation in the right quadrants (where the peak waves were measured). However, by 00 UT 11 January, an important feature of the wind field appears in the northern Gulf of St. Lawrence as an area of 30 m/s northwesterly winds. This "jet-streak" like feature
propagated southeastward through Cabot Strait and offshore (toward the Canadian buoy network) at nearly 15 m/s as peak surface winds in the core of the streak increased to 38 m/s south of NF by 12 UT 11 January, 1991. This “streak” feature is almost completely missing in the PRESTO winds, which were derived from tediously hand-reanalyzed pressure analyses, as exemplified in Fig 6d, which shows the PRESTO wind field for the same time as for the KINEMA field shown in Fig. 6b. This suggests that this feature is strongly ageostrophic and may be linked to an upper-air jet-stream streak which entered the upper short wave trough associated with this storm, further energizing the storm as it moved over the Grand Banks, where by 00 UT 12 January minimum pressure decreased to 955 mb.

Table 1 includes wind difference statistics for this storm, formed again from NOAA data buoys. Since these buoys are outside the area of strongest cyclogenesis and surface wind, they reflect differences between the PRESTO and KINEMA winds for the precursor and early stages of this storm.

5. PRELIMINARY CONCLUSIONS

(1) The network of moored data buoys off the east coast of North America allows much more accurate analysis of surface pressure and wind fields in marine extratropical storms than ever before possible, especially during the winters in which the network was augmented for the ERICA and SWADE experiments.

(2) The potential of this enhanced surface data network has evidently not yet been realized in operational surface analyses produced by objective analysis systems at major centers (FNOC, NMC, NASA, ECMWF). Most likely, this is because the analysis systems (such as SCM, OI) which work well for over land surface and upper air observation networks, have not been optimized for the buoy coverage in this limited ocean region.

(3) Detailed kinematic analysis of a few selected cases of intense east coast extratropical cyclogenesis from the enhanced data has in each case revealed complex synoptic scale and mesoscale surface wind field structures which we have termed “anomalous” because they are not resolved typically in wind fields developed from historical analyses and ship reports. Some of these “anomalies” merely reflect complex frontal structures associated with a new emerging conceptual model of extratropical cyclogenesis, but most other anomalies defy generalization at this time. Analysis of a much larger sample of cases (ERICA and SWADE provide the opportunity to study dozens of cases) might help discriminate what is “typical” from what is “anomalous”.

(4) This study suggests at least two areas for further research beyond the simple extension of study to a larger sample of cases.
(a) Much more efficient (than hand-analysis) surface wind analysis schemes are required. Fully automated objective analysis schemes could be improved, especially systems which incorporate an “expert system” approach to learn the positive contributions of a skilled analyst. But we suspect the full potential of the enhanced data will be realized only in a man-machine interactive system, such as the prototype systems being explored in AES (INGRED) and NOAA NMC for surface pressure and frontal analysis. Development of these systems should be extended to surface wind analysis.

(b) Sensitivity studies with calibrated ocean response models should be carried out in order to assess the impact of failure to resolve surface wind field “anomalies” on surface wave and current hindcasts and extreme wave and surface current climate descriptions developed from such hindcasts. We suspect, for example, that features such as mesocyclones propagating rapidly along the “bent-back warm front”, as observed in ERICA IOP-4, though fascinating, have little impact on storm peak wave generation, while large deformations of the surface wind field north and south of the front do have significant impact. Systematic studies with a larger storm sample, however, will be required to verify these speculations.

ACKNOWLEDGEMENTS

This study was suggested by Val Swail, AES, who is acknowledged also for his interest and steadfast support of improved marine surface wind field analysis methods. The preparation of this paper and development of wind fields for Cases 2 and 4 was supported under AES Contract KM 170-1-8549. The development of wind fields for Case 3 was supported under AES Contract KM 191-0-8092. We thank Dr. Madhav Khandekar, AES, for his timely support of the SWADE IOP wind analysis. Wind fields were developed for Case I (ERICA IOP-2) as part of a research program supported by U. S. Navy office of Naval Research under ONR Grant #NOO1490-F-0093. Finally, we thank oceanweather meteorologists B. T. Callahan, A. T. Cox and M. J. Parsons for their patient and skillful synoptic analysis work.

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Table 1

Scalar wind speed and direction differences between analyzed (PRESTO or KINEMA) wind fields and averaged winds measured (adjusted to equivalent 20m neutral) by NOAA East Coast buoys

<table>
<thead>
<tr>
<th>Case Type</th>
<th>Number of Pairs</th>
<th>Wind Speed Mean diff. (m/s)</th>
<th>Wind Speed RMS diff. (m/s)</th>
<th>Wind Direction Mean diff. (deg)</th>
<th>Wind Direction RMS diff. (deg)</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 PRESTO</td>
<td>351</td>
<td>.22</td>
<td>3.10</td>
<td>7.3</td>
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<td>135</td>
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<td>1.58</td>
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<td>20.7</td>
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<td>4.37</td>
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<td>.05</td>
<td>.82</td>
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</tr>
<tr>
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<td>2.87</td>
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<td>.12</td>
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<td>3.0</td>
<td>18.9</td>
</tr>
<tr>
<td>4 PRESTO</td>
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<td>-1.15</td>
<td>3.33</td>
<td>5.3</td>
<td>36.4</td>
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<tr>
<td>KINEMA</td>
<td>598</td>
<td>.30</td>
<td>1.79</td>
<td>.4</td>
<td>21.6</td>
</tr>
</tbody>
</table>

Composite of previous studies for MPBL winds (Cardone, 1991)
PRESTO vs buoys	-.3	2.9
Fig. 1. Analyses at 1800 UT, 13 December, 1988 of ERICA IOP-2 storm:
(a) NOAA NMC Final Analysis; (b) PRESTO wind from reanalyzed
pressure field; (c) kinematic analysis; (d) final KINEMA winds
Fig. 2. Analyzed wind fields at 0600 UT, 4 January, 1989 of ERICA Top-4 storm: (a) PIREPO winds; (b) KINEMAT winds.
Fig. 3 Comparison of modelled and measured winds in ERICA IOP-4: (a) PRESTO winds at 44004; (b) KINEMA winds at 44004; wind speed scattergrams: (c) PRESTO vs all buoys; (d) KINEMA vs all buoys.
Fig. 4. Analyses at 0000 UT 26 October, 1990 of SWADE IOP storm: (a) NOAA NMC NA analysis; (b) kinematic analysis; (c) final KINEMA winds.

Fig. 5. Scattergram of KINEMA wind speed vs buoy measured winds in SWADE IOP
Fig. 6. Selected wind fields in Grand Banks storm of 10–15 January, 1991: 
(a) KINEMA winds at 0000 UT 10 January; (b) KINEMA winds at 
0000 UT 11 January; (c) KINEMA winds at 1200 UT 11 January; 
(d) PRESTO winds at 0000 UT 11 January.
A COMPUTER-BASED KINEMATIC ANALYSIS ROUTINE
AND ITS APPLICATION TO STORMS IN THE CANADIAN ATLANTIC REGION

D.T. Resio¹ and V.R. Swail²

¹Florida Institute of Technology
Melbourne, Florida

²Atmospheric Environment service
Downsview, Ontario

1. INTRODUCTION

The objective of this paper is to describe progress toward the development of an improved computer-based wind estimation methodology. During the last 15 years or so, agencies around the world have developed extensive wind and wave climatologies on an ocean-basin and even global scale. Methodologies used to produce these data sets have been based on objective computer algorithms, such as geostrophic wind approximations and planetary boundary layer (PBL) models. In some cases available observations have been blended into the pressure-based surface wind fields in an attempt to improve the overall accuracy of the estimated winds. Such blending algorithms have been based purely on the basis of a spatial weighting function centered around the observation point. Wind estimation methodologies such as these are referred to as machine-only methodologies.

Studies during the 1980’s (Cardone et al., 1989; Resio, 1982) have shown that machine-only methods, even those which have observations included within them, tend to underpredict maximum winds in storms. In wave hindcast studies, objective wind field underpredictions produce a consistent bias toward low waves in hindcast wave extremes. A solution employed by many wave modelers has been to calibrate their wave models to minimize this bias; however, this procedure distorts the overall distribution of waves and can lead to serious problems in extrapolating to long-term extreme values.

As a consequence of the apparent inability of machine-only methods to produce adequate wind fields within storms, a second methodology has become an accepted practice in the derivation of "best-possible" wind fields. In this approach, termed a kinematic analysis, an experienced meteorologist analyzes each weather map based on concepts of continuity and past observations. Typically, the meteorologist uses machine-only winds as a baseline consideration and combines these into his own winds. This combination of objective and kinematic analyses has been termed a man-machine mix. Although this methodology is more costly than the machine-only winds, its success in improving hindcast results and in direct comparisons in storm areas has led to its application in recent studies (Cardone et al., 1989).
The approach attempted here is to develop a computer-based kinematic analysis (CKA) methodology capable of recognizing and quantifying natural organizations in synoptic-scale wind fields. As shown in classic texts such as Petterssen (1940), the categorization and understanding of streamline patterns is considered to be an essential component of an analyst’s kinematic analysis; consequently an important component of this work examines machine-based constructions of streamlines and an interpretation of a “natural” coordinate system for interpolating wind-field structure.

2. BASIC ASSUMPTIONS

Ships’ observations have been shown to contain significant bias and random errors within them; consequently, they do not, by themselves, constitute a first-rate data set for accuracy assessments. Due to this, we shall choose here to accept the available kinematically analyzed (man-machine mix) winds as “correct.” The goal of our computer-based methods then will be directed toward techniques which can reproduce the man-machine winds. A second point that should be made here is that we are seeking to develop a tool that not only can learn to emulate past patterns but also one which can continue to learn as new data becomes available. In other words, this system should be able to gain experience as additional wind fields are analyzed. Toward this end, we will be developing data-adaptive analyses which can function at different levels of human intervention, ranging from zero-intervention to the ability to incorporate secondary inputs from analyses. In this paper, the emphasis will be on the zero-intervention limit of these analyses.

3. DATA PROCESSING

The data sets available for this study are:
1. pressure fields and derived objective winds,
2. kinematic winds (man-machine mix), and
3. ship and buoy observations.

These data are available for 69 storms over the interval 1959 through 1989. When one is dealing with techniques that incorporate actual observations within them, it is sometimes difficult to evaluate errors in an independent manner. For example, in a kinematic analysis all available data is considered in formulating a given windfield; consequently, attempts to quantify the error in this approach are impeded by the lack of independent observations to be used for comparisons. To avoid this situation, we will keep all storms that occurred after 1980 out of the diagnostic analyses. Storms after 1979 will then be used as an independent check on the relationships derived from the set of storms from 1959 through 1979.
The spatial grid for these data is shown in Figure 1. The southern limit of the grid is at 25 degrees North latitude and the northern limit is at 75 degrees North latitude. The western limit is at 80 degrees West longitude and the eastern limit is at 20 degrees West longitude. The grid spacing is 1.25 degrees latitude and 2.5 degrees longitude. No data were saved at on-land locations; thus, all analyses were developed to recognize the existence of a discontinuity at the land-sea interface. Another potential problem in terms of proceeding with a "straight forward" analysis is the existence of two grid scales, as can be seen in Figure 1. In order to facilitate data processing, the data set was expanded, via bilinear interpolation in the area outside the fine-mesh region, to a single fine-mesh grid with 69 rows and 49 columns.

3.1 Pressure Fields and Derived Objective winds

Once pressures fields were established from interpolations, geostrophic-level winds were estimated and surface winds were derived via the program MKWIND, supplied by AES. This wind model was expected to be equivalent to that used by Oceanweather in their own objective analyses. Since no fields of air temperature and/or sea temperature were available, synthetic values based on wind direction and grid location were used to estimate these quantities during actual model runs. For consistency with the available kinematic winds, the
reference level for predicted surface winds was taken at 19, 5 metres. Since no pressures were included for land points in the archived data, the objective winds cannot be specified except at locations greater than one grid cell away from a land point or a boundary.

3.2 Kinematic Winds

Kinematic winds were obtained via a procedure analogous to the interpolation method for pressure fields. Directions were interpolated independently from winds via interpolating x-y vector components and taking the arctangent based on these interpolated values as arguments. These winds represent careful kinematic analysis results produced by Dr. Cardone of Oceanweather and, thus, should form a consistent data set for analysis.

Kinematic analyses were only performed for the fine mesh area shown in Figure 1 and were performed only for selected time intervals, not for the duration of each entire storm. Four storms (620306, 631114, 680104, and 770205) in the interval 1959-1979 contained no kinematic analyses.

3.3 Ship and Buoy Observations

Ship and buoy observations were extracted from tape archives on a storm by storm basis. Latitude-longitude values were converted to equivalent (rounded-off not truncated) i-j locations within the grid for comparisons which will be presented in section 4. No ships’ observations were available for three storms in the interval 1959-1979 (670427, 690209, and 691226).

4. BASELINE ERROR EVALUATIONS

Primary error assessments here are, in terms of the deviations between kinematic winds and objective wind speeds; however, during the course of this study it became apparent that a second useful error evaluation could be made by comparing both sets of wind fields to the ship observations. This second analysis provides some interesting information relative to the potential for the ship observations alone to reduce errors between the machine-only and kinematic winds. Only storms in the diagnostic interval (1959-1979) will be used in this baseline error evaluation in order to retain storms occurring within the 1980’s as an independent data set for later use.

It was initially assumed that the objective analysis routine provided to us would be capable of exactly replicating the archived machine-only winds (i.e. the parts of the windfields outside the fine-mesh region and the parts which were outside the time limits analyses). This did not prove to be true. Comparisons between the objective analysis performed here to the earlier results indicate that
there exists some small differences between the two analyses. Hence, the CKA approach developed and applied in this study will be removing two sources of deviations which indirectly, at least, should show the flexibility of this tool.

Figure 2.

![Histogram of Mean Errors for Storms in 1959-1979 Period](image)

Figure 3.

Comparisons between kinematic winds and objective winds come from a sample of 40 storms (the original 44 from 1959-1979 minus the 4 for
which no kinematic analyses were conducted). Figure 2 shows the histogram of mean deviations (bias) between the kinematic and machine-only winds for all storms. Figure 3 shows a similar presentation for the distribution of error variances. As can be seen from those analyses, significant differences apparently exist between the two data sets. Errors in these comparisons appear to have a strong spatial correlation. Cardone et al. (1989) has pointed out that this spatial correlation is responsible for some of the significant problems encountered in using objective winds in wave hindcast studies.

The transformation equation used here is given via the relationship

$$W_t = 2.16 W^{7/9}$$

where \( W_t \) is the transformed wind speed and \( W \) is the original wind speed. Surprisingly, deviations between the kinematic winds (which incorporate information from ships’ observations) and ships’ observations did not seem to be significantly smaller than deviations between objective winds (which do not incorporate information from ships’ observations) and ships’ observations.

5. DEVELOPMENT OF DIAGNOSTIC CKA METHODOLOGY

5.1 Introduction

In this report we concentrate on the development of a method which requires no human intervention. This will serve as a baseline guide to possible future improvements and provide an idea of the utility of this approach. Another limitation in this first phaser of development is the consideration of information only from one weather map at a time. This means that no concepts of map-to-map “continuity” will be considered here.

A robust CKA must be able to deal with analyses of wind fields for a wide range of conditions, including cases with limited data beyond a specified pressure field. Therefore, it must be capable of reasonable decision-making relative to the appropriate usage of available data. For this reason, the approach here includes three different levels of CKA analysis, based on the data availability beyond the gridded pressure field:

1. level 1 - (little or no data available) base wind field modifications only on persistent patterns of deviations observed between kinematic and objective winds;
2. level 2 - (moderate information available) base wind field modifications on combination of level 1 patterns and consideration of information from ships’ observations; and
3. level 3 - (considerable information available) base wind field modifications on streamline analysis of additional wind information.
Each of these levels of the CKA are introduced in this paper; however, as will be seen in subsequent sections, primary consideration this year is given to the level 1 development.

The evaluation of the performance of these different levels will be judged by the following criteria:

1. reduction in mean difference (bias);
2. reduction in variance of deviations (random errors);
3. reduction in correlated errors over large areas; and
4. reduction in errors in estimated maximum conditions.

5.2 CKA – Level 1

5.2.1 Fundamental considerations

Three general types of CKA transformations are investigated here. First, it is possible that, independent of all other factors, an analyst might reduce high values, raise low values, or perform some other consistent operation which he has determined appropriate from past experience. In this case, a "universal" transformation for all grid points and times should exist. Second, it is possible that an analyst determined that winds in certain geographic areas were consistently incorrect. In this case, a geographically-based transformation would provide an appropriate correction for all wind fields. Third, it is possible (and should be anticipated from the actual procedures of most analysts) that the CKA must consider transformations based on concepts of inherent natural organization in the wind fields. In this case, the computer must be "taught" to recognize certain synoptic situations and adapt its own analysis to this basis. For an analyst, such recognition typically comes in two stages. First, he must recognize the elements that shape the map. Second, he must use concepts of continuity and streamline analysis to develop a kinematic context for these elements. For practical purposes, the primary elements recognized by an analyst consist of the following:

1. low pressure centers,
2. high pressure centers,
3. frontal boundaries, and
4. regions influenced by the above three elements.

It should be noted that these four elements have three different topological characteristics. The first two are points on a weather map. The third represents lines on a weather map. And, the last represents regions on a weather map. Consequently, our CKA methodology must be able to deal with these differences in a consistent fashion. Of these four elements, three (elements 1, 2, and 4) are definable from information readily available in the archived data. Thus, for our
zero-intervention, initial stage of CKA development, we will concentrate on these three elements.

5.2.2 Locating low-pressure centers

A two-stage algorithm for finding discrete low-pressure centers is used here to locate low pressure centers in a fashion consistent with an analysts’s identification of the same feature. First, each point in the grid and is considered to be a low center if that point contains a minimum pressure within a centered region of 7 grid cells. The second stage of the analysis removes extra points created by two nearby minima which have equal pressure values. This part of the algorithm examines each minimum found by criterion 1 relative to each other point found by this criterion. If any two of these points are within 5 grid cells of each other, the point with the lowest pressure is retained as a valid low center.

5.2.3 Locating high-pressure centers

The algorithm for finding high-pressure centers is analogous to that used in finding low-pressure centers.

5.2.4 Determining domains of influence of lows and highs

Several approaches to defining domains of influence based on pressure gradients and pressures were tried and found not to provide results consistent with an analysts’s interpretation. It turned out that a relatively consistent estimate of the primary domain of influence for a low could be found by starting at a low-pressure center (as determined above) and marching outward along radials in four cardinal directions, until a pressure greater than 1012 was reached or until the radial pressure gradient fell beneath a prescribed threshold (1.4 millibars per grid increment). The distance along these radials was found to be reasonably consistent with what might be termed a closed circulation pattern. An equivalent methodology was formulated for defining the domain of high pressures areas.

5.2.5 Streamline analysis

As part of this phase-1 effort a computer-based program for constructing streamlines was undertaken. Unfortunately, development has not progressed to the point where it is sufficiently general for arbitrary wind fields, Completion of this program will is now a priority item in phase 2.

5.2.6 Definition of persistent wind speed deviations for incorporation into CKA - level 1

In the simplest type of transformation, one might hope to apply a transformation that was independent of geographic position and/or
position relative to circulation centers. In this context, a “universal” regression equation might then be applied to transform objective wind speeds and directions into kinematic wind speeds and directions. However, after examining only the first six storm intervals, this approach was found to be very ineffective in removing deviations between kinematic and objective winds (consistently removing less than 5% of the total variance and contributing no reduction in mean deviation). No evaluations of this method’s performance relative to reducing large areas of correlated errors or errors in maximum conditions were made, due to its poor performance relative to bias and variance reduction. This approach can probably be ignored.

For the first six storm intervals, tests of methods based on transforms with only a geographic dependence consistently removed less than 7 percent of the total error variance and had little or no apparent effects on the mean deviations. Hence, this too can probably be ignored.

In attempting to replicate the processes that a human analyst undertakes in constructing kinematic wind fields, one must recognize that a key element to all manual analyses is the recognition of the natural organization of the wind fields. Many past studies have attempted to neglect this aspect of wind field estimation and have attempted to determine transformations which were in a sense equivalent to those attempted above.

Given that we now have a computer algorithm which can identify natural features in an arbitrary wind field, we need to develop a method for evaluating error characteristics within the context of this structure. After examining several storm intervals, a persistent error pattern relative to the lows and highs became somewhat apparent. This pattern was such that a negative bias persisted in the objective winds for the half-plane of directions centered approximately around a southeasterly vector, and a positive bias persisted in the objective winds for the opposite half-plane of directions. Figure 4 shows the average error characteristics in terms of contours of nondimensional multipliers around within a unit radius.

The pattern of deviation observed in Figure 4 appears to isolate a potentially important source of bias between the kinematic and objective winds. In particular, in the region of highest winds (usually the western, northwestern, or southwestern quadrants of a storm), wind speeds were found to be consistently underestimated. This possibly explains the tendency of hindcasts based on objective winds alone to underestimate extreme wave heights. Conversely, any comparisons at locations in the opposite side of the storm suggest that the model was overpredicts wind and wave conditions there. The
determination of spatial correlation in the error field seems to provide a basis for removing a significant part of the total correlated error by applying a transformation to the objective wind of the form

$$W_{cka} = W_o \ L(r, \Theta)$$

where $W_{cka}$ is the CKA-transformed wind speed, $W_o$ is the initial objective wind speed, $r$ is a dimensionless radial coordinate measured from the storm center, and $\Theta$ is the relative angle of the radial measured counterclockwise from northeast. The dimensionless radial distance is defined by the ratio

$$r = R/R_{max}$$

where $R$ is the actual distance along the radial and $R_{max}$ is the limit of the low's domain along that radial.

Figure 4. Contours of mean errors within low pressure areas for storm beginning 590206.

For consistency, it was decided to limit the error evaluation only to regions determined to fall within the low pressure domain. For our first approach, we will use the diagnostic information from these transacts through the low pressure domains to form the basis for the prognostic CKA model. In this case equation 1 can be used to estimate the revised CKA wind directly from the objective winds, given that $r$ and $\Theta$ are defined.
The form of equation 1 suggests that an analyst “climatological” transformations of certain storm regions (based presumably on past experience) is scale dependent, i.e. it inherently modifies an objective value at a particular $r, \Theta$ location by a fixed percentage. Besides strictly multiplicative operations such as this, it is possible that an additive operation can be inherent in a manual analysis. For example, if an analyst limits minimum velocities to 10 knots within low pressure systems, a purely multiplicative operation cannot represent this effect very well. In this case, a more general (albeit still linear) form of the CKA transformation could be given in terms of a standard regression equation

$$\text{2. } W_{cka} = L_1(r, \Theta) + L_2(r, \Theta) W_o$$

where coefficients $L_1(r, \Theta)$ and $L_2(r, \Theta) W_0$ can be determined by least squares fits to stratified samples for various discretized values of $r$ and $\Theta$. When this was attempted, the regression coefficients appeared to be relatively invariant with regard to categories of $r$, while still maintaining a strong dependence on $\Theta$. Due to inaccuracies in defining $r$ and variabilities in locations of strong maximum conditions along $r$, this perhaps should have been expected. Consequently, it was found that a simplified form for equation 2 which actually removed more error variance and mean error in test applications could be written as

$$\text{3. } W_{cka} = L_1(\Theta) + L_2(\Theta) W_o$$

where $L_1(\Theta)$ and $L_2(\Theta)$ are regression coefficients determined from samples taken within 45° direction bands.

5.2.7 Definition of wind direction error characteristics

Due to length restrictions this is omitted here,

5.3 CKA - Level 2

At the CKA-2 approach, the concept is to use available wind information to modify the basic patterns of differences determined in the CKA-1. Since this first level analysis appears to be effective at removing substantial portions of the deviations between objective and kinematic winds, it is desirable to base additional levels of improved CKA estimates on modifications to the CKA-1 results. Furthermore, since ships’ observations occur only as discrete samples in time and space, we must consider two facets of this problem separately. First, we must isolate the amount of additional (independent) information actually contained in the ships’ observations over and above that represented by the CKA-1 transformations. This information is applicable to CKA-2 transformations for all points at which observations actually exist. Second, a method of interpolating this information for the entire grid must be adapted, This interpolation
function should be able to recognize the natural organization patterns in various regions of the grid and take this organization into account in forming the CKA-2 estimates over the grid.

To determine the apparent information content of the ships’ observations, two hybrid variables were formed as follows:

4. \( \Delta W_1(i,j)_k = W_{cka}(i,j)_k - S_k \)

and

5. \( \Delta W_2(i,j)_k = W_{cka}(i,j)_k - W_{kin}(i,j)_k \)

where \( \Delta W_1(i,j)_k \) is the deviation between the estimated CKA-1 wind speed at the \( i,j \) location closest to the \( k^{th} \) ships’ observation and \( S_k \), the wind speed of the \( k^{th} \) ships’ observation. Similarly \( \Delta W_2(i,j)_k \) is the deviation between the CKA-1 wind speed and the kinematic wind speed at the \( i,j \) location closest to the \( k^{th} \) ships’ observation. If \( \Delta W_1 \) and \( \Delta W_2 \) are uncorrelated, no information from the ships’ observations would contribute toward a net reduction of the deviations between the CKA-1 and kinematic wind speeds. Based on the results from section 4, we do not expect any “exact” relationship between the kinematic wind speeds and ships’ wind speeds. However, when we examined correlations between \( \Delta W_1 \) and \( \Delta W_2 \) we found consistently significant correlations, implying that the kinematic analysis did take a substantial amount of information on wind speeds from the ships. Overall, the regression based on all data up through 1979 gave the following result

6. \( \Delta W_2 = 0.25 + 0.248 W_1 \)

with a correlation coefficient for this total sample of 0.394.

Using the \( r,\Theta \) coordinate system developed in section 5, we can form an interpolation function based on aspects of storm organization. Unfortunately, the low density of observations makes a direct interpolation, even in this structured context, difficult. Consequently, for this first phase effort a modified interpolation function was adopted which limits the distance over which interpolation and extrapolation is performed. In this approach, a linear interpolation is used in the \( \Theta \)-direction between two observational points falling within 90° of each other and in the redirection between two observational points falling along the same radial. It should be noted that this did not happen often; therefore, an alternate estimate is used close to 100% of the time. In this method, the \( r,\Theta \) concept is still used to estimate the size of the domain of influence, and the form adopted for the interpolation/extrapolation function is as follows:
7. \[ W_{cka}(i,j)_2 = W_{cka}(i,j)_1 + \sum_{k=1}^{N} \delta_k \]

where the sum in equation 7 is taken over all ships’ observations falling within a particular low pressure area and is defined as follows:

8. \[ \delta_k = \delta_{k0} \exp[-(\lambda_1 \delta r + \lambda_2 \delta \Theta)] \]

where \( \delta_{k0} \) is the deviation defined from equation 6 at the location of the \( k \)th ships’ observation \( \delta r \) is the distance between a given \( i,j \) location and the location of the \( k \)th ships’ observation in normalized \( r \) coordinate units, \( \delta \Theta \) is the distance between a given \( i,j \) location and the location of the \( k \)th ships’ observation in degrees, and \( \lambda_1 \), and \( \lambda_2 \) are empirical coefficients. The empirical coefficients are estimated here purely from concepts of the size of storm velocity fields and not from any optimal estimation methods. For this first year report, they are taken to be \( \lambda_1 \) equal to 0.3 and \( \lambda_2 \) equal to 30°. Based on the use of equation 8 in test applications of the diagnostic CKA-2 methodology, it was found that the performance of CKA-2 was in general superior to the performance of CKA-1. As expected, there are some cases in which the CKA-2 actually produced a slight degradation in the results. As an observational note, the cases in which CKA-2 results degrade wind estimates appear to be cases in which CKA-1 is already close to the kinematic winds.

6. PROGNOSTIC APPLICATIONS OF CKA METHODOLOGY

At this stage of development, we will be concerned with two aspects of prognosis:

1. CKA predictions achieved without using additional observed wind data (CKA-1 prognostic performance), and
2. CKA predictions achieved with the inclusion of observed winds (CKA-2 prognostic performance).

6.1 CKA-1 Prognostic Performance

Table 1 lists the baseline wind speed errors (before CKA-1 application) for the 1980 storms (our independent data set for testing). Table 2 lists the wind speed errors after CKA-1 application. The CKA-1 approach reduced the mean errors by more than 50% and reduced the error variance by about 40%. Moreover, as can be seen in Table 3, the CKA-1 method appears to reduce the underestimate of the objective wind speeds in the storm sector behind the cold front by about 80%, even though our methodology was not derived specifically to accomplish this.
6.2 CKA-2 Prognostic Performance

Unlike the injection of observational data with only a mathematical context of location, we now have a concept data assimilation within a natural coordinate system. Using the methods discussed in section 5, we were able to obtain a general improvement over the CKA-1 methodology as seen in Table 3.

Table 1

<table>
<thead>
<tr>
<th>Date</th>
<th>Mean Error</th>
<th>Error Variance</th>
</tr>
</thead>
<tbody>
<tr>
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7. SUMMARY AND CONCLUSIONS

The CKA-1 methodology appears to reduce deviations between the objective and kinematic wind fields by about 50%.

Table 2

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2. The optimal CKA uses both the CKA-1 methodology and the injection of observed wind speeds.
3. The characteristic deviations determined in the CKA diagnostic application appears to be consistent with the documented tendency of objective winds to underpredict wind extremes and hindcast wave extremes. In particular, the CKA appears to be able to reduce the spatially correlated error magnitude along with the underestimation of the extremes in the objective wind speeds. Both of these attributes of the CKA could be important in future applications.
4. The application of an ongoing diagnostic mode of the CKA during prognostic applications should allow the CKA to continue to "learn" during future applications.

It should be noted that, although a foundation for the CKA methodology is established here, this stage of development is still quite rudimentary.

8. REFERENCES


WAVE GROUPS IN COASTAL WATERS

Peter Chandler and Diane Masson

Institute of Ocean Sciences
Sidney, B.C.

Abstract

Wave groups are identified in wave data collected off the coast of Nova Scotia during the 1986 CASP project. Group activity is parameterized by the mean number of waves, $j$ in a sequence of discrete wave heights larger than a given threshold. This parameter is also computed from wave fields numerically simulated, having a typical JONSWAP shape. Several theoretical approaches to wave group analysis are presented. First, Kimura’s theory is described which relates $j$ to the correlation between successive wave heights, $\gamma_h$, by treating the sequence of wave heights as a Markov chain. Also included is an extension to this theory where spectral properties are used to estimate $\gamma_h$. The second theory relates the length of a high run, defined in terms of a wave envelope, to the spectral width. Comparisons are made between the average group lengths determined from theory and those measured from real and simulated wave data. The relationship between $j$ and the correlation coefficient between successive wave heights is shown to agree well with Kimura’s theory. On the other hand, the data do not support the relationships between group statistics of discrete wave heights and spectral properties proposed by the two spectral approaches.

1 Introduction

A sequence of high waves is known as a wave group and is evident in both visual observations of the sea and measured wave data. Waves occurring in groups represent a more severe design condition than ungrouped waves with a similar height distribution (e.g. Medina et al., 1990). Wave groups also introduce a time scale into the motion of the sea (on the order of one minute) that is longer than the five to 20 second periods of the gravity wave spectrum (Longuet-Higgins and Stewart, 1964). This longer time scale may be important in phenomena such as resonance with the natural oscillations of coastal embayments or moored vessels, and generation of edge waves.

A measure of the wave groupiness can be determined by averaging the number of waves in the groups of successive waves exceeding a
height threshold, $H_c$. By definition, the average group length $\bar{j}$ equals or exceeds 1, and increases with the groupiness of the wave field. Whereas $\bar{j}$ is readily calculated from a sequence of wave heights derived from a time series of the sea surface displacement, most of the wave information collected is commonly available as an energy spectrum. Thus the problem becomes one of identifying a robust-relationship between some groupiness statistics computed in the time domain, such as $\bar{j}$, and a parameter that can be derived from the wave spectrum.

The first part of this paper presents the theoretical basis of some commonly used approaches to wave group analysis: Kimura’s theory (Kimura, 1980) which considers the sequence of wave heights as a Markov chain process, an extension to this theory proposed by Battjes and Van Vledder (1984) who added a new spectral wave groupiness parameter, and finally the one based on the wave envelope function (e.g. Longuet-Higgins, 1984). Following is a comparison of the mean number of waves in a group, $\bar{j}$, calculated from both simulated and measured wave data, with grouping parameters derived from the various theories. Finally, a conclusion has been included for the convenience of those who want to avoid reading the whole paper.

2 Kimura’s theory

Kimura (1980) introduced the correlation between successive wave heights into a model for the mean group length. The joint probability density function (pdf) of successive wave heights $H_1$ and $H_2$ is given by Kimura as the two-dimensional Rayleigh distribution:

$$p(H_1, H_2) = \frac{\pi^2}{4} \frac{H_1 H_2}{H_m^4 (1 - \kappa^2)} \exp \left( -\frac{\pi H_1^2 + H_2^2}{4 H_m^2} \frac{1}{(1 - \kappa^2)} \right) I_0 \left( \frac{\pi}{2 (1 - \kappa^2)} \frac{H_1 H_2}{H_m^2} \right)$$

(1)

where $\kappa$ is a correlation parameter, $H_m$ is the mean wave height, and $I_0$, the modified Bessel function of zeroth order.

The correlation coefficient between successive wave heights, $\gamma_h$, is determined from a series of $N$ wave heights by

$$\gamma_h = \frac{1}{\sigma^2(H)} \frac{1}{N-1} \sum_{i=1}^{N-1} (H_i - H_m)(H_{i+1} - H_m),$$

(2)

with $\sigma(H)$ the standard deviation of the wave heights. The correlation coefficient, $\gamma_h$, and the correlation parameter $\kappa$, are related through (e.g. Longuet-Higgins, 1984)
\[ \gamma_h = \frac{E(\kappa) - \frac{1}{2}(1 - \kappa^2)K(\kappa) - \frac{\pi}{4}}{1 - \frac{\pi}{4}}, \quad (3) \]

where \( E \) and \( K \) are complete elliptic integrals of the first and second kind, respectively.

To compute the probability of a sequence of high waves, Kimura used the conditional probability that a wave height exceeds the threshold value, \( H_c \), given that the previous wave also exceeds \( H_c \), \( p_{22} \), computed from the joint pdf \( p(H_1, H_2) \)

\[ p_{22} = \frac{\int_{H_c}^{\infty} \int_{H_c}^{\infty} p(H_1, H_2) dH_1 dH_2}{\int_{H_c}^{\infty} \int_{0}^{\infty} p(H_1, H_2) dH_1 dH_2} \quad (4) \]

the probability that a group is comprised of \( j \) waves can be written

\[ p(j) = (1 - p_{22}) p_{22}^{(j-1)} \quad (5) \]

giving an average group length

\[ \bar{l} = \frac{1}{1 - p_{22}} \quad (6) \]

The present theory allows the mean number of waves per group to be estimated from the correlation coefficient between consecutive wave heights. Goda (1983) found \( \gamma_h \) to adequately describe the run lengths from an analysis of long-travelled swell. However, a shortcoming of Kimura’s formulation is that the groupiness is not defined in terms of the energy spectrum. Along this line, Battjes and Van Vledder (1984) proposed a modification to the theory in which a new spectral wave groupiness parameter is introduced. Based on the work of Arhan and Ezraty (1978), they proposed to use, instead of \( \gamma_h \), a new spectral parameter \( \gamma_s \), which can be determined from the frequency spectrum \( E(\omega) \), via Eqn. (3) and a new correlation parameter \( \kappa_s \),

\[ \kappa_s = \frac{\sqrt{X^2 + Y^2}}{m_0} \quad (7) \]

where
\[ X = \int_0^\infty E(\omega) \cos(\omega T_m) \, d\omega, \]
\[ Y = \int_0^\infty E(\omega) \sin(\omega T_m) \, d\omega, \]  

and \( T_m \) is the average period between zero up-crossings obtained from the spectrum.

### 3 Wave envelope theory

The wave envelope theory was developed by Rice (1944, 1945) to study noise in electrical circuits, and applied to groups of surface gravity waves by Longuet-Higgins (1957).

Considering the sea surface elevation, \( \eta(t) \), as a random Gaussian process, the envelope function can always be defined. The signal \( \eta(t) \) can be expressed as a linear combination of sinusoids with radian frequency \( \omega_n \),

\[
\eta(t) = \Re\left\{ \sum_n c_n \, e^{i(\omega_n t + \epsilon_n)} \right\}, \tag{9}
\]

where the random phase, \( \epsilon_n \), is uniformly distributed over the range \([0, 2\pi]\), and the fixed amplitude \( c_n = \sqrt{2E(\omega)\Delta \omega} \). By choosing a carrier wave frequency, as a representative midband frequency, \( \eta(t) \) may be reformulated as

\[
\eta(t) = \Re\left\{ e^{i\bar{\omega}t} \sum_n c_n \, e^{i[(\omega_n - \bar{\omega})t + \epsilon_n]} \right\} \tag{10}
\]

\[
= \Re\{ R(t)e^{i\bar{\omega}t} \}. \tag{11}
\]

A complex wave envelope function can now be defined as

\[
R(t) = \sum_n c_n \, e^{i[(\omega_n - \bar{\omega})t + \epsilon_n]} \equiv a(t)e^{i\phi(t)}, \tag{12}
\]

where \( a(t) \) is the amplitude of the envelope function and \( \phi(t) \) the phase. For a narrow spectrum, the variation of \( a \) with time is slow in comparison to \( \bar{\omega} \), and the wave crests (troughs) closely follow the envelope function.
From the known probability density of the envelope function, an estimate of the average number of waves in a group, $H'$, can be derived by dividing the average length of the episodes for which the wave envelope exceeds a given level by the mean zero upcrossing period, $T_m$. Longuet-Higgins (1984) expressed $H$ in terms of one single spectral parameter, the spectral width, $\nu$

$$H' = \sqrt{\frac{2m_0}{\pi} \frac{\sqrt{1 + \nu^2}}{\nu} \frac{1}{H_c}}$$  

(13)

The spectral width parameter, is defined as

$$\nu = \sqrt{\frac{m_2m_0}{m_1^2} - 1}$$  

(14)

with the spectral moment $m_r = \int_0^\infty \omega^r E(\omega)d\omega$. When $\nu^2 \ll 1$ the spectrum is considered to be narrow. In coastal waters, $\nu$ typically ranges from .10 to .50. The wave envelope function $a(t)$ can be determined in practice using either of two related techniques; the Hilbert transform and complex demodulation.

4 Wave data analysis

The sea surface elevation data were collected during the CASP project carried out by the Atmospheric Environment Service and the Bedford Institute of Oceanography in 1986. The locations of the wave buoys are shown in Fig. 1, and the data represent a range of coastal wave conditions in water depths of 20 to 100 m (Dobson et al., 1989). As emphasized by Longuet-Higgins (1984), the concept of a wave group implies the neglect of wave components of frequencies significantly different from the peak frequency. It is then appropriate to bandpass the wave record around the peak frequency with the condition, however, that the total energy, $m_o$, should not be changed significantly by the filtering. Thus, any spectrum that has energy distributed in two or more widely separated frequency bands is not suitable for simple group analysis. As the CASP data set mainly comprises bimodal spectra from the typical Atlantic conditions of a local wind sea developing on an underlying swell, this requirement was a serious limitation to the selection of wave spectra used in our group analysis. In view of these conditions, the time series were lowpass filtered at a frequency of 1.5 times the peak frequency. A modified wave spectrum was then
computed from the filtered time series, and the wave record selected if the energy loss in the filtering process did not exceed 20%. In selecting the data, care was also taken to choose wave fields from a wide range of coastal conditions including growing and decaying seas, low and high energy levels, and finally spectra of various widths. Following these criteria, 98 time series were processed and used in the group analysis described below.

A spectral analysis of the original (unfiltered) 30 minute records of surface elevation sampled at 1.28 Hz provided spectral estimates with a resolution of 0.005 Hz and 18 degrees of freedom. Each selected time series was lowpass filtered, despiked, and detrended. The time series of discrete wave heights was then generated using the standard zero-upcrossing technique, the wave envelope was computed using the Hilbert transform, and a filtered spectrum determined from the filtered data.

To provide an additional source of data, time series of sea surface elevation were also generated numerically. The random waves are simulated using the random coefficient method in which the signal
\( \eta(t) \) consists of \( N \) values sampled at discrete times \( t_m \) with intervals \( \Delta t \), such that

\[
\eta(t_m) = \sum_{n=0}^{N/2} \{a_n \cos(\omega_n t_m) + b_n \sin(\omega_n t_m)\},
\]

(15)

where \( \omega_n = \frac{2\pi n}{N\Delta t} \). The random coefficients \( a_n \) and \( b_n \) are generated from a Gaussian distribution with variance \( E(\omega_n)\Delta\omega \). This method is preferred here to the commonly used random phase method as the latter was shown to be adequate only for sufficiently large values of \( N \) (Tucker et al., 1984). The wave spectrum, \( E(\omega) \), was chosen as the empirical JONSWAP spectrum characteristic of growing seas, and the time series sampling and duration periods similar to the ones of the measured data. A total of 150 time series were generated, and their group characteristics examined.

The wave height threshold used in the group analysis was selected as the mean wave height, \( H_m = \sqrt{2\pi m_c} \), as it is a familiar characteristic of the wave field, and allows a greater number of wave groups per record than a larger value such as the significant wave height. Kimura’s prediction of the mean length of wave groups, \( j \), in terms of the correlation coefficient between successive wave heights, \( \gamma_h \), as given by (1)-(6) is first examined. In Fig. 2, the \( j \) measured from the series of wave heights from both the simulated and real data are shown to tightly scatter around the Kimura’s relationship. It is also seen that, as the mean group length increases, the scatter of the data also increases because of a reduced number of wave groups detected in each record. In addition, data presented in Fig. 2 (as well as in the following figures) do not reveal any difference among group statistics extracted from real and simulated data. This supports the adequacy of the analytic linear Gaussian model for the surface elevation, Eqn. (15), to reproduce measured wave characteristics.

Battjes and van Vledder (1984) suggested to modify Kimura’s theory by relating \( j \) to a new correlation coefficient, \( \gamma_{s'} \), conveniently computed directly from the spectrum (see (7)-(8)). However, as shown in Fig. 3, the modified approach consistently underpredicts the measured wave group length. This is due to the fact that the spectral correlation coefficient proposed, \( \gamma_{s'} \), is also consistently smaller than the measured coefficient, \( \gamma_h \), extracted from the time series of discrete wave heights.
The group characteristics of the wave field were then analysed in terms of the wave envelope theory. For each time series, the predicted average number of waves in a group, \( \bar{H} \), was computed from the measured spectral width of the filtered spectrum as in Eqn. (13). The results were compared with another estimate of the mean number of waves in a group computed directly from the wave envelope, \( \bar{j}_{en} \), by averaging the duration of the time intervals for which the amplitude of the envelope function, \( a(t) \), exceeds the mean wave amplitude, \( H_m/2 \), and multiplying this value by the mean zero-crossing frequency. Results of the comparison are given in Fig. 4 which shows a good agreement between the theory's predictions and the wave envelope groupiness characteristics (\( r^2 = 0.83 \)).

![Figure 2: The average group length \( \bar{j} \) as a function of the correlation coefficient between discrete wave heights \( \gamma_H \) for real (•) and simulated (+) data. The solid curve represents Kimura's theoretical relationship.](image)

It is to note, however, the success of this spectral approach in predicting \( \bar{j}_{en} \) does not guarantee its usefulness in predicting wave group characteristics of the discrete wave heights. In fact, the wave envelope theory is strictly valid when \( V^2 \ll 1 \), in which case the wave crests follow the wave envelope. As the spectral width increases, the discrepancy between the wave crests and the envelope increases, and
the wave group theory becomes inadequate. This is well illustrated in Fig. 5 where the mean number of waves in groups of discrete waves, \( \bar{j} \), is compared to the mean number of waves derived from the wave envelope, \( \bar{j}_{en} \). The envelope parameter, \( \bar{j}_{en} \), is, over the present range of spectral width, always smaller than \( \bar{j} \). Also evident in the figure, is the divergence of the two estimates as the spectrum widens, or \( \bar{j} \) decreases, as expected from the limitations of the theory.

5 Conclusion

Several commonly used approaches to the study of wave groups are first described: Kimura’s theory relating the mean number of waves in a group, \( \bar{j} \), to the correlation coefficient between successive waves, \( \gamma_h \), an extension of the latter in which \( \gamma_h \) is conveniently replaced by a spectral correlation parameter, \( \gamma_s \), and finally the wave envelope theory in which \( \bar{j} \) is estimated from the amplitude of the wave envelope function. The predictions

Figure 3: The average group length \( \bar{j} \) as a function of the correlation coefficient \( \gamma_s \) for real (•) and simulated (+) data. The solid curve represents Kimura’s theoretical relationship.

of these three different methods of wave group analysis are then compared with real and numerically simulated wave data.
One striking but expected feature of the results presented here is the similarity between the wave group characteristics extracted from the real and the simulated data. This gives support to the commonly used linear Gaussian model of the sea surface to examine statistical properties of ocean waves.

In terms of wave group predictions, Kimura’s theory provides a very good model for the data set analysed here, whereas the two spectral approaches do not. The modified Kimura’s theory significantly underpredicts the mean group length due to the fact that the spectral $\gamma_s$, is not equal but consistently smaller than the measured $\gamma_H$. Although the wave envelope theory successfully predicts a mean number of waves per group defined by the amplitude of the wave envelope function $\bar{j}_{en}$, it does not adequately model $\bar{j}$ measured from the time series of wave heights. The difficulty with the spectral approaches to wave groups comes from the fact that the interpretations in the spectral domain are quite different from the ones derived from the discrete waves identified with the standard zero-upcrossing method. The practical need of a robust relationship between groupiness characteristics computed in the time domain and the wave spectrum still remains.

Figure 4: The average run length derived from the amplitude of the wave envelope $\bar{j}_{en}$ as a function of $\bar{H}$ for real (*) and simulated (+) data.
Figure 5: The average run length derived from the amplitude of the wave envelope $\tilde{j}_{en}$ as a function of $\tilde{j}$ for real (•) and simulated (+) data.

Acknowledgements. Access to the CASP wave data was achieved through the good offices of Fred Dobson and Rick Marsden.

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Longuet-Higgins, M.S., 1984. Statistical properties of wave groups in a


simulation of a random sea: a common error and its effect upon
LOW-FREQUENCY RESPONSE OF A CIRCULAR BASIN TO A TIME-DEPENDENT SPATIALLY UNIFORM WIND

Urmas Raudsepp
Institute of Ecology and Marine Research
1 Paldiski Str.
200031 Tallinn, Estonia

ABSTRACT

The low-frequency response of a circular basin to a time-dependent but spatially uniform wind (this assumption is valid for a basin smaller than the size of the wind pattern) is investigated analytically by a barotropic intermediate model of constant depth and Coriolis parameter. Dissipation effects are neglected. Whereas near-inertial and super-inertial frequencies are filtered out, it is expected that resonant forcing takes place at sub-inertial frequencies.

Wind stress is presented as a sum of linear trends of east-west and north-south components and clockwise-anticlockwise rotating components of different amplitudes and initial phases at discrete frequencies. Wind stress is imposed at t=0 to the water basin initially at rest, which leads to initial value problem.

A fundamental solution is obtained using Laplace transformation techniques, which leads to a final solution in terms of convolution integrals and initial wind stress. Uniform wind stress excites the first natural basin mode only, with radial dependence in the form of a first order Bessel function.

Basin response to linear wind stress consists of static surface elevation with linearly increasing amplitude and two anticlockwise propagating waves with different amplitudes and phase speed equal to the first free basin mode and phase shift π/2.

The response to a rotary forcing component with frequency different from the first natural mode consists of two propagating waves. The anticlockwise rotating wind stress component generates a wave propagating in the anticlockwise direction with phase speed half the sum of forcing and the first natural mode frequency with slowly changing amplitude. A clockwise rotating wind stress component with frequency less (greater) than first natural mode generates a wave propagating anticlockwise (clockwise). The phase speed is small, while amplitude changes are more rapid.

The limit of forcing frequency to be equal to the first natural mode gives a resonance condition, which leads to an anti-clockwise rotating first natural mode of linearly increasing amplitude and small amplitude standing wave response.
The complete response is the superposition of these waves and has a complicated pattern.
NUMERICAL EXPERIMENTS OF AN OCEAN WAVE MODEL 
FOR THE SOUTH ATLANTIC OCEAN 

Valdir Innocentini and Dirce Maria Franco Pellegatti

Instituto Nacional de Pesquisas Espaciais
Sao Jose dos Campos, Brazil

ABSTRACT

A spectral limited-area ocean wave model has been developed for the South Atlantic ocean. It is a 2nd generation type with adjustment to the Kreuseman spectra. A numerical artifice is implemented in order to avoid excessive redistribution of energy in cases of changing wind direction. The energy generation-dissipation tuning is obtained by the duration-limited growth curve. The semi-Lagrangian scheme is applied in the advection. The model behaviour is discussed in meteorological situations of frontal systems and an artificially generated hurricane.
CHANGES IN THE SPECTRUM OF WIND-WAVES BY THE OPPOSING SWELL

Zhan Cheng

National Research Center for Marine Environment Forecast
No. 8 Da Hui Si, Hai Dian District, Beijing, China

1. INTRODUCTION

Since the theoretical study of Longuet-Higgins & Stewart (1960), it is well known that when gravity waves of short wavelength ride on the surface of much longer waves then the amplitude of a short irrotational wave of small slope is greatest at the long-wave crest and least at the trough, partly as a result of the geometrical convergence at the crest and partly as a result of the working by radiation stresses. For finite water depth, the short-wave amplitude is increased by a factor

\[ a'/a = 1 + AK[(1/4)\tanh Kd + (3/4)\coth Kd]\cos\psi \]

(1)

where \( a \) denotes the amplitude of the short wave, \( A \) the amplitude, \( \psi \) the phase and \( 2\pi/K \) the wavelength of the long-wave; and \( d \) denotes the mean water depth.

Besides this change in the amplitude, the frequency of the short-wave is also increased by a factor

\[ \sigma'/\sigma = 1 + AK[(1/2)\coth Kd - (1/2)\tanh Kd]\cos\psi \]

(2)

Phillips (1981) pointed out that when the swell has a small steepness \( AK \), as the water depth approaches infinite (relative to long-wave), the second term on the RHS of equ. (2) vanishes and the frequency of the short-wave varies with respect to the phase of the swell (the long-wave) by a factor of \( (AK)^2 \), the ‘Doppler effect’ associated with the convection of the short-wave by the orbital velocities of the long ones dominates the variation in the frequency of the short-wave. However, if the water depth keeps finite for the long-waves, both effects described above contribute to the variation in the frequency of the short-wave.

The suppression of short wind-generated waves by a train of longer, mechanically generated waves of same direction as wind was apparently observed by Mitsuyasu (1966) in laboratory. As the slope of the long waves increased, the total energy and the spectral density of the short waves decreased progressively. More recently, Phillips & Banner (1974) obtained the same result in laboratory and established a dynamical model to explain the phenomenon as the increase of the wind drift at the crests of a train of long waves limiting the amplitude of superimposed wind-generated waves.

The reduction in the energy density of wind-waves by the swell propagating in the direction of wind conceals the changes in the
spectrum of wind-waves due to the swell, which were expected by the theory of Longuet-Higgins & Stewart (1960), and that of Phillips (1981). When the swell propagates against the wind, the case is likely to be different. Mizuno (1976), Young & Sobey (1985), Tsuruya (1988) and Mitsuyasu & Yoshida (1989) studied the interaction between wind and the opposing swell under the action of wind. Mitsuyasu & Yoshida (1989) paid some attention on the wind-waves and found that the energy of the wind-waves was not reduced by the opposing swell.

In present study the changes in the spectrum of the wind-waves by the opposing swell are discussed in detail. In addition to what revealed by Mitsuyasu & Yoshida (1989), the expansion of the wind-wave spectrum by the opposing swell was found in the experiment. After extending the theory of Longuet-Higgins & Steward (1960) and that of Phillips (1981) into the spectral space, the author believes the sources of the expansion have been discovered.

2. EXPERIMENTAL CONDITIONS AND TECHNIQUES

2.1 The wind-wave flume

The experiment was conducted in the wind-wave flume of Kyushu University in Japan. Figure 1 show a schematic of the flume, whose interior dimensions are 0.8m high, 0.6m wide and with a usual test-section of 15m long. The mean water depth was kept at 0.353m in the experiment. A beach for absorbing swell energy, a centrifugal fan for blowing wind through the flume and a transition plate for thicking the air boundary layer situated upwind side of the test-section (the fan is outside left of Fig. 1). At the downwind side, a filter made of vinylon net was installed across the water section for absorbing the downwind-propagating wind-waves. A flap-type wave generator at the downwind side was used for generating the swell (regular oscillatory waves).

Figure 1: Schematic diagram of experimental arrangements (units in cm)
Wind speed $U_r$ in the flume was changed stepwise as $U_r = 3, 5$ and 7m/s, and monitored with a Pitot-tube installed above the transition plate. Here, the wind speed $U_r$ corresponds roughly to a cross-sectional mean speed after the correction of a small change in the cross-sectional area of the flume. Vertical wind profiles over the water surface were measured with another Pitot-tube at a fetch of $F=8m$, where $F$ was measured from the tip of the transition plate. At a definite fetch the wind-waves generated by the low-speed wind are not likely to break and of a high peak frequency, so it is easier to distinguish the behavior of wind-waves and that of the swell. This is a reason why the maximum wind speed is limited to 7m/s. Another consideration is that in the utilization of the theory of Longuet-Higgins & Stewart (1960) to discuss the experimental results, large differences between short waves and long ones are also needed.

The period of the swell in the flume was kept at $T=1.024s$, and the swell steepness under no wind action was changed stepwise as $H_o/L=0.01, 0.02, 0.03$ and 0.04. For each wind speed the wave steepness was changed successively as 0 (corresponds to pure wind-waves), 0.01, 0.02, 0.03 and 0.04. The waves were measured simultaneously at three stations ($F=6, 8$ and 10m) with capacitance-type wave gauges. The reason for the particular selection of the wave period $T=1.024s$ is as follows. Firstly the change of the swell of this period is very small along the fetch even under the wind action, which makes the analysis of the data easier. Secondly if we digitize the wave record with a sampling frequency of 100Hz or 200Hz, $2^n$ data, which is used for FFT analysis, give an integral multiple number of the regular waves. The latter reduces the leakage effect in the spectral analysis.

2.2 The experimental procedure

Calibration of r.p.m. of the wind blower versus the reference wind speed $U_r$ and the stroke of the wave generator versus the wave height were done before the experiment. After the calibration, experimental conditions, such as the reference wind speed, the period and wave height of the regular water waves, could be controlled by a microcomputer.

In each run of the experiment the wind blower was started immediately after the start of the wave generator. The measurement of the wind profile over the water surface was done 5 minutes after the start of the wind blower to wait for a stationary state of the wave system. The waves were measured independently after the measurement of the wind profile, because the wave gauges would disturb the wind field.

3. WAVE DATA ANALYSIS

The wave records of each run were digitized at a sampling frequency of 200Hz. From the wave records of 11 minutes for each run the wave characteristics were analyzed.
we obtained 32 samples of the wave data, each of which contained 4096 data points. Power spectra of waves were computed through a fast-Fourier-transform procedure for each sample of the wave data. The sample mean of 32 spectra was used for further analysis. Due to the procedure described in Section 2, the leakage effect of the spectral components of the regular waves was negligibly small. The frequency resolution of the wave spectra was Δf=4.88X10⁻²Hz..

4. RESULTS AND DISCUSSION

4.1 Wind profile over the water surface

Vertical wind profiles over the water surface U(z) were measured for pure wind-waves and for the co-existing system of wind-waves and the swell propagating against the wind. The lower parts of all profiles show logarithmic distributions. The friction velocity, u*, and the roughness parameter of the water surface, z₀, were determined from the wind profile U(z) near the water surface by applying the logarithmic distribution,

\[ U(z) = \frac{u*}{\kappa} \ln\left(\frac{z}{z_0}\right) \]  

where \( \kappa \) is the Karman constant (≈0.4). The friction velocity so determined is used for further analysis.

4.2. The drag coefficient of the water surface

The wind speed at the height z=10m, U₁₀, was determined by using the data of u* and z₀, and extrapolating the logarithmic wind profile (3). By the definition of the drag coefficient C_D,

\[ C_D = \tau_*/(\rho U_{10}^2) = \left(\frac{u*}{U_{10}}\right)^2 \]  

\( C_D \) can be calculated from the measured values of u* and U₁₀, where \( \tau_* = \rho u_*^2 \) is the wind stress acting on the water surface and the density of the air. It can be seen from Fig. 2 that the drag coefficient of the water surface \( C_D \) increases clearly with the wind speed \( U_r \) but is not so much affected by the swell propagating against the wind except for the case of the lowest wind speed \( U_r = 3 \)m/s. For the wind speed \( U_r = 3 \)m/s, the drag coefficient \( C_D \) increases clearly with the increase of the swell steepness. According to (3) and (4), \( C_D \) is uniquely determined by the roughness parameter of the water surface \( z_0 \). At low wind speed, the existence of the swell will affect the wind-waves overlapping on it and changes the roughness of the water surface and then \( C_D \).
4.3 The change in the opposing swell under the action of wind

Mitsuyasu & Yoshida (1989) studied the attenuation of the opposing swell under the action of wind in detail. As the main purpose of present study is to investigate the effects of the opposing swell on the wind-waves, the wave height of the swell is just monitored. The spectra of the co-existing system of wind-waves and the swell propagating against the wind are shown in Fig. 3, where $U_r = 5\text{m/s}$, $F = 8\text{m}$. In order to divide the total energy into the energy of wind-waves and that of the swell, we first eliminated the fundamental spectral peak of the swell by eliminating nine spectral points at and near each spectral peak and applying a linear interpolation to each spectral gap of the eliminated spectral points. By this way we obtained the spectrum of wind-waves in the presence of swell. Then we determined the energy of the wind-waves $E_w$ by integrating the wind-wave spectrum. The energy of the swell $E_s$ is obtained by subtracting the wind-wave energy $E_w$ from the total wave energy $E_t$ of the co-existing system as

$$E_s = E_t - E_w$$

(5)

The present method for separating the energy of wind-waves and that of the swell is different from that used by Mitsuyasu & Yoshida (1989). This is due to the following reason: At low wind speed, spectral peaks corresponding to the higher harmonic of the swell, which appear in the frequency region of the wind-wave spectrum, contribute significant fraction to the spectrum of the co-existing system. From the value of $E_s$ determined above we calculated...
approximately the wave height \( H \) of the swell under the action of the wind by using the relation \( H = 8 \sqrt{E_w} \).

As previously mentioned, relatively long regular water wave (\( T=1.024s \) and \( L \approx 1.5m \)) was used in the experiment in order to reduce the change in the wave height of the swell propagating under the action of the wind. In fact, relative changes in the wave height of the swell due to the wind action were

\[
\frac{(H - H_0)}{H_0} < 5\% \tag{6}
\]

for all runs in the experiment except for the case of \( U_r = 7m/s \) and \( H_0/L = 0.01 \), for which the relative change was about 25%. In (6) \( H \) is the wave height of the swell under the action of the wind and \( H_0 \) the wave height of the swell without wind action. Therefore, the original wave steepness \( H_0/L \) is used as a parameter representing the swell steepness.

4.4 Wind-wave spectra

Fig. 4 shows the wind-wave spectra of the co-existing system of wind-waves and the opposing swell, where the spectrum of the swell is eliminated. In this figure 11 point triangular filter is also used to make the high frequency part of the spectrum a more clear picture. All spectra shown in Fig. 4 are those for \( U_r = 5m/s \) and \( F = 8m \). The swell
steepness changes stepwise from 0.01 to 0.04. The spectrum of the pure wind-waves at same wind speed and same fetch is also sited for comparison. In the presence of the opposing swell, the magnitude of the spectral peak of the wind-waves decreases, but the magnitudes of the spectrum in the other regions around the peak increase (compared with the case of the pure wind-waves), especially in the region where \( f<f_m \) for \( H_o/L=0.03 \) and 0.04. In other words, the spectrum of wind-waves is expanded by the opposing swell. This phenomenon is also observed for \( U_r=3\text{m/s} \) and 7m/s. Another feature shown in Fig. 4 is that the high frequency portions of the wind-wave spectrum, where \( f>10\text{Hz} \), are not obviously affected by the opposing swell.

![Power spectra](image)

Figure 4. Power spectra of the pure wind-waves and the wind-waves affected by the opposing swell. \( U_r=5\text{m/s}, F=8\text{m} \). --.--. The pure wind-waves; ______ the wind-waves affected by the opposing swell.

In order to demonstrate the expansion of the wind-wave spectrum quantatively, we introduce the conventional spectral width parameter \( \varepsilon^2 \) into discussion. The ratio \( \varepsilon^2/(\varepsilon^2)_o \) indicates the relative change in the spectral width of wind-waves under the action of the opposing swell, where \( (\varepsilon^2)_o \) is the spectral width parameter for pure wind-waves. Fig. 6 shows the dependence of \( \varepsilon^2/(\varepsilon^2)_o \) on the swell steepness \( H_o/L \) at different wind speed. There is no data locating in the region where \( \varepsilon^2/(\varepsilon^2)_o<1 \). In general, when the swell steepness in small \( (H_o/L<0.03) \), \( \varepsilon^2/(\varepsilon^2)_o \) increases with the swell steepness and achieves its maximum around \( H_o/L=0.03 \), and then decreases. For \( U_r=5\text{m/s} \), the case is a little different, in which there is no maximum achieved up to \( H_o/L=0.04 \).
The scaled wind-wave energy, $E_w/(E_w)_0$, is shown in Fig. 6, where $(E_w)_0$ denotes the energy of the pure wind-waves. The squares with a tolerance bar are data obtained by Mitsuyasu & Yoshida (1989). Taking the data as a whole, the present result is consistent with that of Mitsuyasu & Yoshida (1989), despite different methods were used in calculating the wind-wave energy. At the wind-speed $U_r=3\,\text{m/s}$, the effect of the swell on the wind-wave energy is more obvious, $E_w/(E_w)_0=6.7$ when $H_o/L=0.04$, which is out of the scope of the figure. When the swell propagates against the wind, which shows a quite different trend from that when the swell propagates in the direction of the wind (Mitsuyasu, 1966).

![Figure 5. The scaled spectral width parameter $\varepsilon^2/(\varepsilon^2)_0$ versus the swell steepness $H_o/L$. $U_r$ (m/s): • 3; □ 5; ▲ 7.](image1.png)

Figure 5. The scaled spectral width parameter $\varepsilon^2/(\varepsilon^2)_0$ versus the swell steepness $H_o/L$. $U_r$ (m/s): • 3; □ 5; ▲ 7.

![Figure 6. The scaled wind-wave energy $E_w/(E_w)_0$ versus the swell steepness $H_o/L$. $U_r$ (m/s): • 3; □ 5; ▲ 7. Data of Mitsuyasu and Yoshida (1989). --- - - Mitsuyasu (1966).](image2.png)

Figure 6. The scaled wind-wave energy $E_w/(E_w)_0$ versus the swell steepness $H_o/L$. $U_r$ (m/s): • 3; □ 5; ▲ 7. Data of Mitsuyasu and Yoshida (1989). --- - - Mitsuyasu (1966).
The scaled peak frequency of the wind-wave spectrum, $f_m/(f_m)_o$, versus the slope of the swell is shown in Fig. 7. When $U_r=3\text{m/s}$, $f_m/(f_m)_o$ decreases with the increase of $H_o/L$. As the wind speed increases the dependence of $f_m/(f_m)_o$ on $H_o/L$ is weakened. When we look back at Fig.2 for the drag coefficient of the water surface $C_D$, we find this result is physically reasonable. It is well known that at a definite fetch, as $u_*$ increases, the peak frequency of the wind-wave spectrum decreases. At $U_r=3\text{m/s}$, $C_D$ increases obviously with the slope of the swell, that is to say $u_*$ increases with the slope of the swell, so $f_m$ is like to decrease. As wind speed $U_r$ increases, $C_D$ tends to be independent of $H_o/L$, and so does $f_m$.

![Figure 7. The scaled peak frequency of the wind-wave spectrum $f_m/(f_m)_o$ versus the swell steepness $H_o/L$. $U_r(\text{m/s})$: $\bullet$ 3; $\square$ 5; $\triangle$ 7.](image)

As the short-wave pattern is swept over the long-waves, the amplitude and the frequency of the short-waves will change. Longuet-Higgins & Stewart (1960) discussed rigorously the problem by carrying out systematic evaluation of the wave motion by Stokes’ method of approximation, as far as the second order. More recently, Crapper (1984) formulated the same problem in an easier way. However, the previous discussions remained in the physical space and most of wind-wave phenomena are described in the spectral space. In this study effort is made to discuss the problem in the spectral space and to apply it to investigate the effect of the opposing swell on the wind-wave spectrum. Firstly, the previous results of Crapper (1984) is simply reviewed.

We suppose we have a long wavelength swell given by

$$\eta_L=A\cos(Kx_1-\Omega t) \quad (7)$$

on which is superimposed a short wavelength ’sea’ given by
\[ \eta_s = a_m \cos((k_m \cos \psi) x_1 + k_m \sin \psi) x_2 - \sigma_m t \]  \hspace{1cm} (8)

where \( \psi \) is the angle between \( K \) and \( k_m \).

Under the assumptions that 1) the short-waves are nevertheless long enough to be considered as pure gravity waves; 2) the changes in parameters of short-waves, such as \( da, d\sigma \) and so on, are of order of \( AK \); 3) \( C_m << C \) and 4) the water is deep enough for the short-waves to be considered as deep-water type, the changes in the intrinsic frequency and amplitude of the short-waves are

\[ \frac{d\sigma}{\sigma_m} = -C_1 \cos(Kx_1 - \Omega t) \]  \hspace{1cm} (9)

and

\[ \frac{da}{a_m} = C_2 \cos(kx_1 - \Omega t) \]  \hspace{1cm} (10)

where \( C_1 \) and \( C_2 \) are two nondimensional constants for the given waves. They can be expressed as

\[ C_1 = (\cos^2 \psi \coth Kd - \tanh Kd) AK/2 \]  \hspace{1cm} (11)

\[ c_2 = [\tanh Kd + (\cos^2 \psi + 2) \coth Kd] AK/4 \]  \hspace{1cm} (12)

where \( d \) is the water depth.

When the ‘Doppler effect’ associated with the convection of the short-waves by the orbital velocities of the long ones (Phillips, 1981) is taken into account, the frequency of the short-waves measured at a fixed position will be further changed. Under the assumption that \( \sigma_m AK/\Omega \) is the order of unit, the ‘Doppler effect’ by the orbital velocities of the long-waves dominates the change in the frequency of the short-waves. The frequency of short waves measured at a fixed position is

\[ \frac{d\sigma}{\sigma_m} = -C_3 \cos(Kx_1 - \Omega t) \]  \hspace{1cm} (13)

which is correct to the first order and to the opposing swell. Where

\[ C_3 = (k_m/\sigma_m) A \Omega \coth Kd \]  \hspace{1cm} (14)

The physical significance of Eqs. (9) to (14) can be explained as: When there is no long waves, if we measured the spectrum of the monochromatic short-waves at a fixed position, the measured spectrum is sharply peaked, with the peak located at \( \sigma = \sigma_m \). But in the presence of the long-waves, the spectrum of the short-waves is expanded, with the frequency interval from \( \sigma_m(1-C_3) \) to \( \sigma_m(1+C_3) \). The description is formulated as follows. For mathematical convenience, the amplitude-spectrum is used in discussion.

At a fixed position, Eqn. (8) becomes
\[ \eta_s = a_m \cos(\sigma_m t) \] 

(15)

the appropriate amplitude-spectrum function of it is

\[ a(\omega) = a_m / \sigma_m \delta(1-\omega / \sigma_m) \] 

(16)

where \( \delta() \) is the delta function.

\[ \eta_s = a_m \cos(\sigma_m t) \]

Figure 8. The theoretical prediction of the expansion of the amplitude-spectrum of the monochromatic short-wave by the opposing long wave. \( a(\omega) \) is the original spectrum of the monochromatic short wave; \( a'(\omega) \) the expanded spectrum by the opposing long wave.

In the presence of the long-waves, the spectrum of the short-waves recorded at the fixed position from \( t \) to \( t+dt \) is

\[ da'(\omega) = \{ (a_m / \sigma_m) [1+C_2 \cos(\Omega t)] \delta(1-C_3 \cos(\Omega t) - \omega / \sigma_m) \} dt \] 

(17)

The long-time recording of the short-wave spectrum is

\[ a'(\omega) = \frac{1}{T \sigma_m} \int_0^T dt \{ a_m (1+C_2 \cos(\Omega t) \delta(1-C_3 \cos(\Omega t) - \omega / \sigma_m) \} \] 

(18)

Using the integrating feature of the delta function, we obtain

\[ a'(\omega) = a_m / (\pi C_3 \sigma_m) [1-C_2 (\omega-\sigma_m) / (\sigma_m C_3)] [1-[(\omega-\sigma_m) / (\sigma_m C_3)]^2]^{-1/2} \] 

(19)

where \( \sigma_m (1-C_3) < \omega < \sigma_m (1+C_3) \).

Equation (19) is the new spectrum of the short-waves under the action of the opposing long wave, which is somewhat complicated. In order
make it a more clear picture, we draw a figure of \( a(\omega) \) \( a'(\omega) \) and for a particular set of waves. For example, let \( L=1.5m \), \( d=0.35m \), \( A/L=0.02 \) and \( \sigma_m=30\text{rad/s} \), then \( C_2=0.1331 \) and \( C_3=0.628 \). As the long-waves propagate against the wind, at the crest of the long-waves the frequency of the short-waves becomes least and the amplitude of the short-waves achieves maximum, but at the trough of the long-waves the case is reversed, so \( a'(\omega) \) is asymmetrical respect to \( \omega=\sigma_m \) (see Fig. 8). This same asymmetry can also be found in Fig. 4 for the wind-wave spectra affected by the opposing swell. the low-frequency part of the wind-wave spectrum is much plumper than its high-frequency part compared with the spectrum of the pure wind-waves. As \( \omega \) approaches to \( \sigma_m (1-C_3) \) or \( \sigma_m (1+C_3) \), \( a'(\omega) \) approaches infinite, which is due to the feature of delta the function. This does not happen to a continuous wind-wave spectrum.

5. CONCLUSION

The expansion of the wind-wave spectrum by the swell propagating against the wind was observed in the wind wave flume. This phenomenon is quite different from that observed when the swell propagated in the direction of the wind, i.e. the energy density of the wind-waves was reduced by increasing the slope of the swell. The new finding is qualitatively consistent with the theory of Longuet-Higgins & Stewart (1960) and that of Phillips (1981), when these theories are formulated in the spectral space. In the presence of the swell, whether it propagates in the direction of the wind or against the wind, it will change the energy of the wind-waves or redistribute the energy among the frequencies of the wind-wave spectrum. These effects must be included in the future ocean wave prediction model.

6. REFERENCES

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A GENERIC THIRD-GENERATION WAVE MODEL: AL

D. Resio,¹ W. Perrie,² S. Thurston,¹ and J. Hubertz³

¹Florida Institute of Technology
Melbourne, Florida

²Bedford Institute of Oceanography
Bedford, Nova Scotia

³U.S. Army Engr. Coastal Engineering Research Center
Vicksburg, Mississippi

1. INTRODUCTION

The purpose of this paper is twofold. Following a brief review of wave modeling, we will introduce a new "generic" deep-water, third-generation wave model. As part of this description, we will also introduce an accurate means of estimating nonlinear energy transfers within a spectrum, suitable for use in any third-generation wave model.

By the early 1960’s, a large body of evidence had accumulated which clearly demonstrated that waves in nature are better represented by a linearly superposed directional spectrum than by parametric wave trains. This motivated initial development of discrete spectral wave prediction models. In this type of model, individual discretized components of a directional wave spectrum are modeled independently. Due to the work of Hasselmann in the early 1960’s (Hasselmann 1962, 1963a, 1963b), a theoretical foundation for the estimation of nonlinear energy transfers among waves in a spectrum already existed. However, most researchers believed nonlinear interactions to be so weak that they were insignificant. Since no field evidence existed to contradict this belief and since methods were not available to evaluate the complete wave-wave interaction integral, the evolution of a wave spectrum was believed to be controlled only by direct wind input and wave breaking (Bunting, 1970). Under this assumption, the concept of an equilibrium range in a spectrum was formulated as an absolute limit to wave steepness, controlled only by wave breaking (Phillips, 1958). Consequently, early discrete-spectral wave prediction models were based on the concepts that direct wind input was the primary mechanism in wave generation and that all spectra had a universal value in their equilibrium range (Inoue, 1967; Bunting, 1970; Cardone et al., 1976) Models of this type have been termed first-generation wave models. Such models have been calibrated to predict a range of open-ocean wave conditions reasonably, but have been shown to have serious problems in their representation of spectral evolution in fetch-limited situations (Resio, 1981).

Evidence contradicting the direct-wind-input concept of wave generation began to appear in studies in the late 1960’s (Mitsuyasu;
1968a, 1968b); and in 1973 data from the JONSWAP experiment, along with a synthesis of several other data sets, Hasselmann et al. (1973) demonstrated fairly conclusively that energy levels in the equilibrium range varied systematically as a function wind speed and fetch. This evidence along with observations of “overshoot/undershoot” in energy levels near the spectral peak (Barnett and Wilkinson, 1967; Barnett and Sutherland, 1968) and the calculations of the form of net source terms for wave spectra propagating along a fetch (Mitsuyasu, 1968a; Hasselmann et al., 1973) were all consistent with the idea that nonlinear effects (wave-wave interactions) were a dominant source term in the wave generation process.

Due to these revised concepts of the physics governing wave generation, discrete-spectral models incorporating nonlinear wave-wave interactions were developed. Such models have been termed second-generation models. Early models of this type (Barnett, 1968; Ewing, 1971) still assumed that energy levels in the equilibrium range were controlled only by wave breaking; and hence, an absolute limit to wave steepness existed in the equilibrium range. Such models still produced spectral evolution in time and space similar to that of first generation models. Later second-generation models (Resio, 1981) recognized the importance of allowing energy levels in the equilibrium range to vary dynamically. Models of this type have been found to produce results consistent with observed patterns of temporal and spatial wave growth.

In second-generation models, the form of the wave spectrum is assumed to be governed by a dynamic balance between wind inputs into the equilibrium range and the nonlinear flux of energy out of this region of the spectrum via nonlinear wave-wave interactions. Hasselmann et al. (1976) argued that the strength of the shape-stabilizing effects inherent in this dynamic balance was so dominant in the spectral evolution equation that wave spectra in nature always stayed fairly close to a prescribed equilibrium form. In fact, Hasselmann et al. argued that the dominance of this dynamic balance was sufficient to allow the spectrum to be modeled by simple parametric methods comparable to those of Bretschneider (1952). Recently, however, Hasselmann et al. (1985a, 1985b, 1985c) have introduced a class of wave model in which it is assumed that the shape-stabilizing effects of wave-wave interactions are not sufficiently dominant to control spectral shape. Such wave models are termed third-generation models. In these models certain spectral constraints (up to frequencies about twice that of the spectral peak) are removed and energy levels throughout much of the spectrum are allowed to vary as a function of the actual estimated source terms.

Due to its importance in controlling both spectral shape and wave growth, the proper evaluation of the nonlinear source function is of
central importance to the implementation of a valid third-generation wave model and is critical to understanding the physics of wave generation. For such reasons, Hasselmann et al. (1985b) investigated four different means of obtaining estimates of the nonlinear source term. As will be shown subsequently in this paper, none of the four methods investigated in that earlier study have been shown to produce very satisfactory estimates of the nonlinear source term for a reasonable range of spectral shapes.

2. THIRD GENERATION WAVE MODELING

2.1 Historical Perspective

To date, all third generation wave modeling has been restricted to a single wave model (WAM) and modeling group. This has primarily been due to the existence of only a single representation of the nonlinear wave-wave interaction source term appropriate for third-generation modeling. As will be discussed in the following section, this representation has not been well tested and appears to suffer from some serious inaccuracies which could seriously affect model performance. It should be recognized here that, even though the physics inherent in a third-generation model may be superior to previous classes of models, if the model implementation of that physics is not accurate the model itself may not exhibit superior performance characteristics.

2.2 Model structure for a generic third-generation model: AL

Due to length restrictions here, we can only outline the overall model structure here. In essence, AL is partitioned into the following three parts:
1. input of options for model run,
2. initializations, and
3. time steps through the simulation interval.

In the time step part of the program the fundamental radiative transfer equation is solved for each grid point located in the water, i.e.

\[ \frac{\partial E(f, \theta)}{\partial t} = -C_g \nabla E(f, \theta) + \sum_{k} S_k(f, \theta) \]

where \( E(f, \theta) \) is the energy density at frequency \( f \) and propagation direction \( \theta \) at grid location \( ij \), \( C_g \) is the group velocity, and \( S_k(f, \theta) \) is the \( k \)th local source term. The solution of this inhomogeneous partial differential equation is usually obtained by first solving the homogeneous part of the equation (the advection term) and then adding the source term integration to this interim
solution. There are some problems with this somewhat simplistic approach; however, they will not be addressed here. We will adopt a simple explicit scheme for the numerical approximation in AL. As can be seen from the form of equation 1, we must solve for the net change in the energy at each spatial point in the grid, for each discretized frequency, direction and time step.

Three source terms are used in AL, as is the convention in WAM. These are $S_{in}$ (wind input), $S_{ds}$ (local wave breaking), and $S_{nl}$ (nonlinear wave-wave interactions). The forms for the wind input and wave breaking source terms are taken from Hasselmann et al. (1988)

3. PREVIOUS REPRESENTATIONS OF NONLINEAR ENERGY TRANSFERS IN A WAVE SPECTRUM

3.1 Representations of the complete interaction integral

Hasselmann’s (1962) representation of the rate of change of energy density at a given location within a wave spectrum involves four interacting waves and is of the form (Hasselmann and Hasselmann, 1981)

$$ n(k) = \int \int \int \int C(k_1, k_2, k_3, k_4) \delta(k_1 + k_2 - k_3 - k_4) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) $$

where $k_i$ is the wavenumber vector specifying the location of the $i_{th}$ interacting wave within the spectrum, $\omega_i$ is the radial frequency of the $i_{th}$ interacting wave, $n(k_i)$ is the action density of the $i_{th}$ interacting wave, $C(.)$ is the coupling coefficient which describes the strength of the interactions, and $\delta(.)$ is the Dirac delta function. It should be noted that the form of equation 1 ensures conservation of action, energy, and momentum. Webb (1978) provided a transformed version of equation 2 as

$$ \frac{\partial n(k_1)}{\partial t} = 2 \int \int T(k_1, k_3) dk_3 $$

where, $T(k_1, k_3)$ represents the transfer of action from $k_3$ to $k_1$ and is given by

$$ T(k_1, k_3) = \oint C(k_1, k_2, k_3, k_4) \left| \frac{\partial W}{\partial n} \right| \theta(k_1, k_3, k_4) ds $$
In equation 3 the interactions are now prescribed along an interaction locus, with \(\mathbf{s}\) and \(\mathbf{n}\) representing unit vectors along and across the locus, respectively and \(W = \omega_1 + \omega_2 - \omega_3 - \omega_4\). The function \(\Theta(k_1, k_3, k_4)\) is defined as

\[
\begin{align*}
\Theta(k_1, k_3, k_4) &= 1 \text{ when } |k_1 - k_3| \leq \times |k_1 - k_3| \\
&= 0 \text{ when } |k_1 - k_3| > |k_1 - k_4|
\end{align*}
\]

Webb (1978), Tracy and Resio (1982), and Resio and Perrie (1991) have all shown that equation 3 provides a stable form for evaluating nonlinear energy transfers within a spectrum. Hence, the numerical method described in Resio and Perrie (1991), based on this equation, will be used here to in comparisons of given approximations to the full Boltzmann integral.

An advantage to the form of equations 3 over equation 2 is inherent in the reduction of the integration over \(k_2\) and \(k_4\). Contributions along a specific interaction locus for each given combination of \(k_1\) and \(k_3\). In a numerical approximation of equation 3 \(k_1\) and \(k_3\) can be specified precisely as the centers of integration grid cells. The values of \(k_2\) and \(k_4\) are then fixed to fall along appropriate interaction loci. The error in the evaluation of the location of \(k_2\) and \(k_4\) (and the action densities at these locations) is limited only by the accuracy of the numerical solution of the locus equation and can be specified independent of the size of the integration grid cells.

In integration methods based on equation 2 interacting sets of wavenumbers are all specified only within the discretized accuracy of the integration grid. Any two of the wave numbers can be arbitrarily specified to coincide with points in the discretized integration grid, with no loss of generality. The other two wave numbers are also approximated by values at the center of their grid cells. Although the delta functions are formally removed from the integral, the value for the energy density in taken from the discretized location of the center of the integration grid cell. This allows an exploitation of certain symmetries in the interaction integral (Hasselmann and Hasselmann, 1981); however, this leads to differences between the locations of the actual position of wavenumbers 2 and 4 and the discretized location of these wavenumbers, resulting in instabilities in the integral and jaggedness in the results.

3.2 Representations of parameterizations of the interaction integral
To date, methods for estimating nonlinear source terms due to wave-wave interactions in a wave spectrum can be divided into four main categories:

1. direct parameterizations based on spectral energy content and shape;
2. parameterizations based on empirical orthogonal functions;
3. parameterizations based on local interaction approximations;
4. parameterizations based on selected integration domains of the total integral.

Direct Parameterizations

Barnett (1968) and Ewing (1971) both developed parameterizations of the nonlinear source term which depended explicitly on total wave energy, prescribed shape functions, and scaling frequencies related to the location of the mean frequency. In this form, the representation for the nonlinear source term, $S_{nl}$, is given by

$$4) \quad S_{nl} = \frac{\partial E(f, \Theta)}{\partial t} = \phi_1(f_0, \Theta_0) \phi_2(f/f_p) \phi_3(\Theta - \Theta_0)$$

where $E_0$ is the total energy in the wave spectrum, given by

$$5) \quad E_0 = \int \int d\int E(f, \Theta) df d\Theta$$

$E(f, \Theta)$ in equation 4 is the spectral energy density at frequency $f$ and propagation direction $\Theta$, and $f_0$ is a frequency scaling function of the form

$$6) \quad f_0 = \left[ \frac{1}{E_0} \int E(f) f^m df \right]^{1/m}$$

where $E(f)$ is the nondirectional spatial density, given by

$$7) \quad E(f) = \int E(f, \Theta) d\Theta$$

and $m$ is a positive integer (usually taken as 1 or 2 for use in equation 6). In the equations, $\phi_i$ represents different nondimensional shape functions.

Resio (1981) recognized certain exact similarity characteristics of equation 3 and chose to base his parameterization of $S_{nl}$ in the form

$$8) \quad S_{nl}(f, \Theta) = \lambda a^3 f_p^{-4} \phi_4(f/f_p) \phi_5(\Theta - \Theta_0)$$
where $\alpha$ is the equilibrium range coefficient for an $f^{-5}$ equilibrium range, and $f_p$ is the frequency of the spectral peak. Since, at that time, it was widely believed that the spectral equilibrium range did follow an $f^{-5}$ law (Phillips, 1958; Kitaigorodskii, 1961) and that spectral evolution along a fetch and through time followed a self-similar form (Mitsuyasu, 1968; Hasselmann; 1973), this parameterization appeared to provide a reasonable approximation to the wave-wave interaction source term for spectra undergoing active wave generation.

All parameterizations of this type are accurate only for a narrow class of spectral shapes (albeit spectral shapes which may be prevalent during most active wave generation scenarios). Also, all of these parameterizations were formulated with the understanding that side conditions (such as the allowable energy levels in the equilibrium range) were to be invoked whenever they were used in predictive schemes. Hence, none of these parameterizations can be considered either sufficiently general or sufficiently unencumbered with constraints to be incorporated into a third-generation wave model.

**Empirical Orthogonal Function (EOF) Representations**

The theoretical basis for EOF analyses shows that, for a given set of correlated variables, an EOF analysis provides an optimal basis for representing a data set in the sense that the maximum variance is explained in the smallest number of functions. Vincent and Resio (1977) showed that such an analysis for measured spectra at a site was capable of giving a good, efficient representation of nondirectional wave spectra in the absence of swell. However, in order to derive these functions one would have to have an a priori set of all possible spectra or $S_{nl}$ (or at least a very large set) in order to form the covariance matrix for the eigenfunction analysis.

Hasselmann et al. (1985b) formulated a set of EOF’s for a synthetic set of simulated spectra based on combinations of different nondirectional spectral shape parameters and angular spreading characteristics. Since an empirical parameterization can be no better than the data set on which it is based, it does not seem that too much is gained by using the EOF’s in this instance instead of a direct parameterization based on the spectral shape and angular spreading parameters themselves. This approach offers interesting possibilities; but is not likely to afford a viable approach anytime in the near future.

**Local Interaction Approximation**

If one takes the full interaction integral and assumes that contributions to this integral are dominated by interaction which are
close to \( k_1 \) (using the notation of equation 3, then a local expansion can be used to develop a diffusion operator for representing \( S_{n1} \) (Hasselmann and Hasselmann, 1981; Hasselmann et al., 1985b). However, as shown by Webb (1978), significant contributions to energy transfers come from wavenumbers quite removed from \( k_1 \). Thus, although this approximation does conserve action, energy, and momentum and seems to follow the general shape of the actual form for \( S_{n1} \) it cannot be considered as a general solution to the parameterization problem.

**Discrete Interaction Approximation (DIA)**

A final parameterization effort due to Hasselmann et al. (1985b) is based on the representation of the total integral by an integral over a reduced region of the interaction space. This method is used in the WAM model to evaluate \( S_{n1} \). Details can be found in Hasselmann et al. (1985b) and will not be repeated here. Figure 1 shows a comparison of the DIA parameterization of \( S_{n1} \) to the complete interaction integral for a JONSWAP spectrum from Hasselmann et al. (1985b). As can be seen from that figure, the agreement is not very good in the equilibrium range of the spectrum. Since a third-generation wave model’s purpose is to use the principle of detailed balance throughout the spectrum (up to at least 2.5 \( f_p \) or so), misrepresentations of \( S_{n1} \) in the equilibrium range can pose a serious problem. Figure 1 contains the only previously published comparison of the DIA to the full integral, known to the authors. Figure 2 shows a series of independent comparisons between the DIA and the total integral. As can be seen there, these results suggest that the DIA does not provide a reasonable approximation to the total integral in many cases.
Figure 1. Comparison of the exact one-dimensional distribution $S_{nl}$ with the discrete-interaction approximation for a JONSWAP spectrum. (From: Hasselmann et al., 1985).
Figure 2a.

\[ \text{DIA} \]
\[ \text{Exact Solution} \]

Figure 2b.
4. THEORETICAL FORMULATION OF A NEW APPROXIMATION FOR $S_{nl}$

It is clear that, for the purpose of representing $S_{nl}$ in a viable third-generation wave model, an accurate, generalized formulation is required. Otherwise, the detailed balance throughout the spectrum will be incorrect. In this case, it is likely that a third-generation model will be less accurate than a well-posed second-generation model. In this section, we will describe a representation for $S_{nl}$ which is accurate over a wide range of spectral shapes.

Let us consider a spectrum which is represented as the sum of two terms at each point within the spectrum, i.e.

$$n(k) = \overline{n(k)} + n'(k)$$

where the overbar denotes a broad-scale averaging and the prime denotes a local departure from the broad-scale structure. The action density term in equation 3 can now be represented in an expanded form as

![Comparison of DIA to Exact Solution](chart)

Figure 2c.
\[ D(k_1, k_2, k_3, k_4) = \]
\[
\frac{n(k_1) n(k_2) [n(k_3) - \bar{n}(k_3)] + n(k_2) n(k_3) [n(k_1) - \bar{n}(k_1)]}{[n(k_1) n'(k_1) + n'(k_1) \bar{n}(k_1)] [n(k_2) - \bar{n}(k_2)] + n'(k_1) - n'(k_2)} \\
+ \frac{n(k_1) n(k_3) [n'(k_2) - n'(k_2)] + n'(k_1) n'(k_2) [n(k_1) - \bar{n}(k_1)]}{[n(k_1) n'(k_1) + n'(k_1) \bar{n}(k_1)] [n(k_3) - \bar{n}(k_3)] + n'(k_1) - n'(k_3)} \\
+ \frac{n(k_2) n(k_3) [n'(k_2) - n'(k_2)] + n'(k_2) n'(k_3) [n(k_2) - \bar{n}(k_2)]}{[n(k_2) n'(k_2) + n'(k_2) \bar{n}(k_2)] [n(k_3) - \bar{n}(k_3)] + n'(k_2) - n'(k_3)} \\
+ \frac{n'(k_1) n'(k_3) [n'(k_1) n'(k_3)] + n'(k_2) n'(k_4) [n'(k_1) - n'(k_1)]}{[n(k_1) n'(k_1) + n'(k_1) \bar{n}(k_1)] [n(k_3) - \bar{n}(k_3)] + n'(k_1) - n'(k_1)}
\]

as before the overbar here denotes a broad-scale feature of the spectrum and the prime denotes a local perturbation. If we expand these terms and substitute them into separate integrals for \( S_{nl} \), we can sum the two integrals to give an estimate of an actual spectrum which contains variations at both scales and the effects of cross interactions due to their superposition. In this representation, we can assume that the perturbations will contribute to the total integral only at a local scale; consequently, we can write the representation for this interaction in terms of a local interaction approximation (including cross-interaction effects). Figure 3 shows the results of using this two-scale approximation (TSA) to the full Boltzmann integral for the same cases as tested with the DIA. As can be seen here, the TSA approximation clearly provides a more accurate representation for \( S_{nl} \).

5. TIME AND FETCH GROWTH IN AL

Figures 4 and 5 show the evolution of nondimensional energy versus nondimensional time and fetch. Also shown are the equivalent published relationships for the WAM model. Although AL’s results appear quite reasonable, it should be pointed out that this growth rate could be varied substantially by changing assumptions in the dynamic balance of the spectral region above 2.5 \( f_p \).
Figure 3a.

- DIA
- Exact Solution

Figure 3b.
Figure 3c.

Figure 4. Nondimensional duration-limited growth curves for the total energy.
6. DISCUSSION AND CONCLUSIONS

In this paper we have shown that AL contains a significantly more accurate method for estimating $S_{nl}$ than does the present version of WAM. It should be recognized that this accuracy is critical to a model that attempts to use the principle of detailed balance in place of spectral energy constraints. Since this has been a major stumbling block in creating additional third-generation wave models, it is hoped that this will enable other model developers to continue to investigate the role of third generation models in improving our understanding of the wave generation process and in wave model applications.

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PROPERTIES OF EVOLVING AND FULLY DEVELOPED WIND-GENERATED WAVES

W. Perrie and D. Resio

1Physical and Chemical Sciences, Scotia-Fundy Region
Department of Fisheries and Oceans
Bedford Institute of Oceanography
Dartmouth, Nova Scotia, Canada

2Department of Oceanography, Florida Institute of Technology
Melbourne Florida USA

ABSTRACT

The spectral energy balance equation for wind-generated waves is integrated in time for duration-limited growth. We use nonlinear transfer due to wave-wave interactions from Resio and Perrie (1991: JFM) and variations of the WAM formulation (Hasselmann et al: 1989) for energy input due to the wind, and energy removed due to dissipative breaking. We show that the spectrum evolves to fully developed state. We show the variation in time of total energy, peak frequency, peakedness and Phillips’ a constant. We also compute the action fluxes cascading to high and low frequencies within the spectrum and show their variation in time. Finally, we relate these fluxes to total energy and peak frequency and we show the relation between these fluxes and both the evolution of the forward face and the high frequency range within the spectrum.

1. INTRODUCTION

The spectral energy density for surface gravity waves in deep water $E(f, \Theta)$ evolves in space and time according to the relation

$$\frac{\partial E(f, \Theta)}{\partial t} + C \cdot \nabla E(f, \Theta) = S_{\text{in}} + S_{\text{nl}} + S_{\text{ds}} \quad (1.1)$$

where $S_{\text{in}}$ is the spectral energy input by the wind, $S_{\text{ds}}$ is the dissipation due to wave breaking and white-cap formation and $S_{\text{nl}}$ is the change in spectral energy due to nonlinear transfer resulting from wave-wave interactions.

Parameterizations for wind input energy $S_{\text{in}}$ are heavily motivated by the observations of Snyder et al (1981). The form is

$$S_{\text{in}} \equiv \beta E(f, \Theta) \quad (1.2)$$

where $\beta$, as specified by Hasselmann et al (1989), is given by

$$\beta = \max(0, 0.25 \frac{\rho_a}{\rho_w} \left(28 \frac{U_c}{c} \cos \Theta - 1\right)) \omega \quad (1.3)$$
air density is $\rho_A$, water density $\rho_W$, friction velocity in the wave direction is $U$, $\cos \Theta$ with $\Theta$ the direction of the wind relative to the wave propagation direction, phase velocity is $c = \omega/k$ and angular frequency $\omega$ is related to wavenumber $k$ through the deep water dispersion relation.

Dissipation due to wave breaking $\zeta_{ds}$ is assumed to have a simple form, motivated by Hasselmann (1974), as well as numerical experiments completed in Hasselmann et al (1989), and may be written,

$$\zeta_{in} \equiv C g k^{-4} \mathcal{F}(k^4 F(k))$$  \hspace{1cm} (1.4)

where $k=|k|$, $F(k)$ is the energy spectrum in vector wavenumber space $k$ and $\mathcal{F}$ is an appropriate functional. It is usually taken as

$$\zeta_{ds} = -2.33 \times 10^{-5} \left(\frac{\omega}{\hat{\omega}}\right)^2 \left(\frac{\omega}{\hat{\omega}_M}\right)^2 E(f, \theta)$$  \hspace{1cm} (1.5)

where

$$\hat{\omega} = \left[ E^{-1} \int E(f, \theta) \omega^{-1} df d\theta \right]^{-1}$$  \hspace{1cm} (1.6)

$$\hat{\omega}_M = E_0 g^{-2} = \int E(f, \theta) df d\theta$$  \hspace{1cm} (1.7)

and

$$\hat{\omega}_M = \frac{2}{3} E_0 g^{-2} \left[ E^{-1} \int E(f, \theta) \omega df d\theta \right]^4 \bigg|_{\text{Pierson-Moskowitz}}\equiv 0.003$$  \hspace{1cm} (1.8)

The complete Boltzman integral representation for nonlinear transfer due to wave-wave interactions $\zeta_{nl}$ can be evaluated in an efficient stable manner, using selected symmetries. Resio and Perrie (1991) transform the nonlinear transfer, represented in terms of a 6-fold Boltzmann integral in wavenumber space by Hasselmann (1961),

$$\zeta_{nl}(k_1) = \int \int \int \int \int e^{2(k_1 + k_2 + k_3 + k_4)} \mathcal{D}(k_1, k_2, k_3, k_4)$$

$$\delta(k_1 + \omega_1 - k_2 - \omega_2) \delta(k_3 + \omega_3 - k_4 - \omega_4) dk_1 dk_2 dk_3 dk_4 \cdot$$  \hspace{1cm} (1.9)
to the locus defined by the interaction resonance conditions, for example \(k_1 + k_2 - k_3 - k_4 = 0\), which reduces the 6-fold integral to a 3-fold integral in wavenumber space. Polar coordinates in wavenumber space then allow all terms to scale, except the action densities.

The reader is referred to Resio and Perrie (1991) for details on \(\varepsilon^2\), \(\mathcal{V}\) and the evaluation of equation (1.9) in terms of energy or action densities.

We make the assumption that

\[
\mathbb{C}_g \cdot \nabla E(f, \Theta) \ll \int \mathcal{I}_{\text{in}} + \int \mathcal{I}_{\text{n1}} + \int \mathcal{I}_{\text{ds}} \quad (1.10)
\]

which is valid for growing windsea spectra at large fetch.

2. SPECTRAL GROWTH TO THE FULLY DEVELOPED STATE

There are a number of parameters that could be computed and used as indicators of spectral evolution and growth to maturity. These include total energy \(E_o\), peak frequency \(f_p\), Phillips’ (1958) alpha coefficient \(\alpha\) and JONSWAP peakedness function \(\gamma\). Although field experiments have tried to determine the variation of these parameters in space and time, considerable uncertainty remains. As pointed out in Perrie and Toulany (1990), part of the problem lies in accounting for atmospheric stability and determination of the component of the wind affecting the wave growth.

(i) Total Energy \(E_o\) and peak frequency \(f_p\)
Figure 1. Variation of total energy $E_0$ with time. Balances of wind input, dissipation and nonlinear transfer are considered as presented in the text. Time is in hours, total energy, in m$^2$.

Five curves are shown. A simple integration of equation (1.1) has been completed using the formulations for wind input $\mathcal{F}_{in}$, nonlinear transfer $\mathcal{F}_{nl}$ and wave-breaking dissipation $\mathcal{F}_{ds}$ described above. The other four curves presented result from assuming wind input $\mathcal{F}_{in}$ that are 25% or 50% in excess of the formulation given in equation (1.3), or wave-breaking dissipation $\mathcal{F}_{ds}$ that is 25% or 50% below the formulation given in equation (1.7). Wind input nonlinear transfer and wave-breaking dissipation must achieve a balance if the system is to proceed to a state of full developments Equilibrium values for $E_0$ and $f_p$ can be varied by varying the balance between wind input $\mathcal{F}_{in}$ and wave-breaking dissipation $\mathcal{F}_{ds}$ and thus a match to the Pierson-Moskowitz level, for example

$$E_{PM} \approx 3.64 \times 10^{-3} U_{10}^4 g^{-2}$$

(2.1)

where $U_{10}$ is wind speed at 10 m, can always be achieved. Figure 1 implies 20 hr as a typical time for spectra to reach stationarity.

(ii) Phillips $\alpha$ coefficient

Assuming the JONSWAP parameterization of Hasselmann et al (1973) for one-dimensional wave spectra $E(f)$ we may write
\[ E(f) = \alpha \ g^2 (2\pi)^{-4} f^{-5} \exp(-1.25f^{-4}/f_p^{-4}) \gamma \exp(-(f-f_p)²/(2\delta f_p^2)) \] (2.2)

where \( \gamma \) is the spectral peakedness and \( \delta \) is the spectral spreading. We compute \( \alpha \), the high frequency coefficient proposed by Phillips (1958) for an \( f^{-5} \) spectral tail, as the hourly average of

\[ <\alpha> = \left< \int_{f_p}^{2.5f_p} \frac{df}{(2\pi)^4} f^5 \exp(1.25 \times f_p^4/f^4) \ E(f)/g^2 \right> \] (2.3)

where the integration covers the equilibrium range of the spectrum. Results corresponding to Figure 1 are shown in Figure 2.

Figure 2. As in Figure 1 for hourly averaged Phillips coefficient \( <\alpha> \).

\( \text{(iii)} \) Peakedness \( \gamma \)

The variation in the spectral peakedness \( \gamma \), as calculated from an inverted form of equation (2.2),
Figure 3. As in Figure 1 for hourly averaged peakedness $\langle \gamma \rangle$.

The two dominating factors which drive the variation shown for all the integrations of Figure 3 are the maximum spectral wave energy $E_{\text{max}}$ and the spectral peak frequency $f_p$. The former increases whereas the latter decreases in time. Thus, they compete with each other and the resultant behavior is shown in $\langle \gamma \rangle$, which first increases to a maximum and then decreases.

3. THE ROLE OF ENERGY FLUXES WITHIN THE SPECTRUM

For given wind input and dissipation, the spectrum grows with the nonlinear transfer driving the spectrum. The action flux past a reference frequency $\omega$ from high to low frequencies is

$$
\Gamma_\omega^- = \int \int \varepsilon^2 D \left| \frac{\partial W}{\partial n} \right|^{-1} ds \ H(|k|-k(\omega)) \ H(\omega-l_j) \ dk_j \ dk_l
$$

and similarly for the action flux from low to high frequencies, where $k(\omega)$ is the wavenumber corresponding to frequency $\omega$ through the
dispersion relation. The rate of change of action due to nonlinear transfer \( \mathcal{J} \) may be written as the 1-dimensional divergence,

\[
\mathcal{J}_{nl} = \frac{\partial}{\partial f} \left( \Gamma^+(f) + \Gamma^-(f) K \right)
\]  

(3.2)

In the absence of wind input and dissipation, the energy fluxes are shown in Figure 4 as functions of time.

![Graph](image)

Figure 4. Variation in total action flux, \( \Gamma^+ + \Gamma^- \), past a reference frequency, as given for example by equation (3.1), corresponding to a point high in the equilibrium range (2.25\( f_p \)), lower in the equilibrium range (1.5\( f_p \)) and the spectral peak \( f_p \). Wind input and dissipative wave-breaking are assumed absent.

At sufficiently high frequencies, energy fluxes through the equilibrium range are to high frequencies, as shown in Figure 4. Energy fluxes past the spectral peak \( f_p \) are always dominated by fluxes to low frequencies, although the net flux to low frequencies decreases monotonically with time. Convergence of energy fluxes past the equilibrium range and the spectral peak to zero, as shown after about 50 hr, coincides with the entire spectrum from spectral peak to equilibrium range reaching stationarity.

When the wind input \( \mathcal{J}_{in} \), nonlinear transfer \( \mathcal{J}_{nl} \) and wave breaking dissipation \( \mathcal{J}_{ds} \) are used in evaluating spectral evolution, energy fluxes through the equilibrium range and past the spectral peak are
always dominated by fluxes to lower frequencies. As shown in Figure 5, the fluxes past the equilibrium range initially begin with a comparatively small magnitude and thereafter rise monotonically with time, eventually achieving an equilibrium plateau after about 15–20 hours which they maintain.

Figure 5. Total energy fluxes past the equilibrium range as a function of time for the source term balances considered in Figure 1.

By contrast the fluxes past the spectral peak \( f_p \) to the forward face of the spectrum, shown in Figure 6, although also having a very small initial magnitude, quickly rise to maxima in magnitude, during the rapid development of the spectrum when the forward face of the one-dimensional energy spectrum is quickly migrating to lower frequencies. Thereafter, as the spectral development decelerates and the system moves to a state of full development, the energy fluxes to the forward face decrease in magnitude and after about 20 hours achieve an equilibrium plateau which they maintain for the remainder of the computation. In either case, the plateau achieved by energy fluxes past the equilibrium range or the spectral peak \( f_p \) has a much larger magnitude when wind input \( \gamma_{in} \) and wave breaking dissipation \( \gamma_{ds} \) are present then when they are absent, as shown in Figures 5–6. Moreover, energy fluxes past a reference point above the spectral peak \( f_p \), at for example \( 1.1f_p \) achieve maxima which are much less pronounced and the final plateaux to which the energy fluxes converge are higher.
Figure 6. Total energy fluxes past the spectral peak as in Figure 5.

In each case presented in Figure 6, the final equilibrium plateau of the energy fluxes to the forward face of the spectrum is noticeably lower than the equilibrium plateau achieved by the energy fluxes past the equilibrium range. The spectrum is now driven by the energy that is moved to the forward face of the spectrum, and which is not dissipated there by wave-breaking or white-capping. This is clearly a small factor compared to the large amount of energy that is transferred past the equilibrium range and essentially dissipated in the spectral region separating the equilibrium range from the spectral peak $f_p$. Energy is not conserved in its transfer from the equilibrium range to the forward face of the spectrum.

4. THE ROLE OF ENERGY CHANGES IN ENERGY CHANGE AND SPECTRAL GROWTH

Energy fluxes within the spectrum must be related to the evolution of spectral energy within the spectrum. For example, the rate of change of energy in the high frequency equilibrium range and the rate of change of energy on the spectral forward face and the region about the spectral peak $f_p$ must be related to the energy fluxes that connect these regions to the remainder of the spectrum. A directly related question concerns the partitioning of energy within the spectrum. How much is fluxed to the forward face, how much is retained in the mid-range frequencies and how much is fluxed past the equilibrium range and what is the time-dependence of this partitioning with time for an actively evolving spectrum?

To explore the relation between energy fluxes within a region of the spectrum and the rate of change of energy associated with that
region, for example the forward face (ff) and spectral peak region, we compute

\[ \mathcal{G}_{ff} = \sum_{i=1}^{3} \int_{0}^{1.2f_p} \mathcal{G}_i(f) \, df = \int_{0}^{1.2f_p} \frac{\partial E(f, \theta)}{\partial t} \, df \quad (4.1) \]

and similarly for the rate of change of energy in the high frequency region (hf) of the spectrum above the equilibrium range.

Figures 7-8 present the evolution of the rate of change of energy \( \frac{dE}{dt} \), as computed from equations (4.1), for the high frequency equilibrium range (hf) and the forward face (ff) regions as a function of the energy fluxes past these regions. As time increases, the high frequency equilibrium range experiences an increase in \( \frac{dE}{dt} \) and quickly reaches a stationary plateau long before the spectrum becomes fully developed. Thereafter it remains constant implying that energy increases linearly with time. By contrast, the forward face region experiences a decreasing \( \frac{dE}{dt} \) with time, as shown in Figure 8. Although the latter only achieves stationarity as the system nears full development it is 10\(^{-3}\) smaller than its initial value within a few hours and, as in Figure 8, long before the spectrum has reached full development. Thereafter the change in total energy within this region is small.
Figure 7. Total rate of change of energy in the equilibrium range as a function of total energy fluxes past the equilibrium range for the source term balances considered in Figure 1.
5. CONCLUSIONS

We have evaluated energy fluxes within the wave spectrums specifically within the spectral peak region, and also past the equilibrium range region. When the wind input $\mathcal{J}_{\text{in}}$ and wave-breaking $\mathcal{J}_{\text{ds}}$ source terms are present, energy fluxes to low frequencies dominate over energy fluxes to high frequencies. As the spectrum grows and evolves, energy fluxes past the equilibrium range increase monotonically until they reach a plateau after about 15–20 hours when the spectrum becomes fully developed. Energy fluxes past the spectral peak region first increase dramatically to a maximum, during the very rapid initial growth and development of the spectrum, and then decrease to a much lower value after about 15–20 hr at which time the spectrum is becoming fully developed.

The rate of change of total energy within the equilibrium range and the spectral peak region also exhibits two phases of development as the spectrum evolves and becomes fully developed. The rate of change of total energy in the equilibrium range increases monotonically until it reaches a plateau after about 5 hr. There is a high correlation between dimensionless energy fluxes past the equilibrium range and the rate of change of total energy within the equilibrium range. By contrast the rate of change of total energy in the spectral peak region...
region decreases monotonically for about 15-20 hr and then falls to zero.

ACKNOWLEDGEMENT

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REFERENCES


A SECOND-GENERATION WAVE MODEL
FOR THE NEW ZEALAND REGION

Andrew K. Laing
New Zealand Meteorological Service
Wellington, New Zealand

1. INTRODUCTION

The process of designing a wave model involves a great deal more than attempting to gain the most accurate description of the physics. Whilst the model must accurately represent wave growth, decay and propagation, and must be able to respond to extreme wind forcing, a balance must be reached between the level of sophistication in physical description, the computing resources available and the timeliness with which the model estimates are required.

Wave modelling is sufficiently well advanced for there to now be a number of good models available, all of which perform reasonably well in varied conditions. The state-of-the-art models such as the so-called 3rd-generation WAM model (the WAMDI group, 1988) gain much of their performance from an ability to quite accurately calculate the resonant energy exchanges resulting from weakly nonlinear interactions. This mechanism redistributes energy amongst the range of wave frequencies present (see, for example, Hasselmann, 1962) and hence determines the shape of the energy density spectrum. In fact it plays a dominant role in the evolution of the spectrum (see for example Komen et al, 1984).

Unfortunately, such models require high computing power. Although the computations required for a single timestep can be achieved with reasonably modest resources (a small workstation or PC is sufficient) the large number of timesteps required and the size of array over which the model is operated often make much more substantial demands of the computer. For modelers with modest resources a necessary compromise is to seek simple but versatile parameterisations of the nonlinear interactions which can be effectively implemented in wave models and executed with longer timesteps.

This paper describes a wave model which has been developed for New Zealand waters. It also describes the verification of the model during a study made covering a 5 month period in 1989.

2. THE WAVE MODEL

The waters around New Zealand can be regarded as deep. Even in the continental shelf margins depths of 80m are exceeded except for the few kilometres closest to the coast. In the present model a grid spacing of the order of 200km is used and waves approaching the coast
are little affected by the bottom until they are well within one gridlength of land. This enables us to safely base the wave model on a deep water simplification of the energy balance equation (i.e. the radiative transfer equation).

Thus, the evolution of the energy density spectrum \( E(f,\Theta;\chi,t) \), at frequency \( f \) and direction \( \Theta \), location \( \chi \) and time \( t \), can be described by

\[
\frac{dE(f,\theta;x,t)}{dt} = \frac{\partial E(f,\theta;x,t)}{\partial t} + c_g \cdot \nabla E(f,\theta;x,t) = S
\]

where \( c_g \) is the group velocity, \( \nabla \) is the vector gradient operator in the \( \chi \) plane and \( S \) represents the source term comprising contributions from direct wind input (\( S_{in} \)); dissipative loss due to white-capping and viscous dissipation (\( S_{dis} \)) and weakly nonlinear interactions between spectral components (\( S_{nl} \)).

2.1 Advection

The partial differential equation (1) is solved numerically in two steps. The first is the advection equation \( \frac{dE}{dt}=0 \). To achieve this we use a finite difference scheme based on the modified Lax-Wendroff method suggested by Gadd (1978). Similar schemes were employed in spectral ocean wave models by Golding (1983) and Laing (1983).

The scheme is fourth-order and gives very accurate translation of a field across the grid. A side effect of this accuracy is the so-called "sprinkler" effect whereby the discretisation of the spectrum in frequency (and hence propagation speed) and direction leads to a spatial separation of components over long traverses. Although a correction term (see Booij and Holthuijsen, 1987) can be applied it effectively doubles the number of quantities to be calculated at each timestep. Given the limited spatial extent of the grid and the reasonably dense discretisation in the present application the additional computation is not warranted. It should be noted that many model designs trade off numerical accuracy against the occurrence of this problem. Low order numerical schemes usually have considerable inherent numerical diffusion. This acts to smooth fields and mask the results of dispersion manifest in the "Sprinkler" effect.

Near boundaries or coastlines various adaptations of the scheme are necessary. Downstream land or grid boundaries are regarded as perfect sinks, and upstream differencing of lower order is required near such boundaries. Upstream grid boundaries are kept "open" by assuming the component values remain constant (at the boundary value) beyond the boundary. At upstream land boundaries the wave energy is forced to zero at a number of points appropriate to the scheme. The balance
between advection away from such boundaries and the source terms then ensures fetch limitations on growth.

2.2 Source terms

To complete the solution for equation (1) the source terms are applied by explicit forward differencing in time to solve the equation \( \partial E/\partial t = S \).

It is in the specification of these source terms where most of the originality in designing wave models is to be found. The wind input term \( S_{in} \) is reasonably consistently formulated by wave modelers in terms of the results of Snyder et al (1981). Following Komen et al (1984), Janssen and Komen (1985) and the WAMDI group (1988) a formulation based on the friction velocity \( u^* \) is adopted:

\[
S_{in}(f, \theta) = \begin{cases} 
0.25 \omega \frac{\rho_a}{\rho_w} [28 \frac{u^*}{c} \cos(\theta - \theta_w) - 1]E(f, \theta), & \text{if } |\theta - \theta_w| < 90^\circ \\
0, & \text{if } |\theta - \theta_w| \geq 90^\circ 
\end{cases}
\]  

(2)

where \( \rho_a \) and \( \rho_w \) are the densities of air and water respectively, \( \theta_w \) is the wind direction, \( \omega = 2\pi f \) and \( c \) is the phase speed \( (g/\omega) \).

This form does require some care when specifying the drag coefficient at high wind speeds. In its hindcast and forecast applications the present model uses friction velocities calculated from surface pressure fields via a diagnostic boundary layer model akin to Cardone (1969) and Laing (1983). In model tests a 10 metre wind is specified and translated to a friction velocity \( (u^* = U_{10}\sqrt{C_d}) \) using a drag coefficient as specified by Wu (1982), viz. \( C_d = 1.2875 \times 10^{-3} \) for \( U_{10} < 7.5 \text{m/s} \) or \( (0.8 + 0.065 U_{10}) \times 10^{-3} \) for \( U_{10} > 7.5 \text{m/s} \).

A dissipation term \( S_{dis} \), is also included, to represent white-capping and viscous dissipation. This takes the form specified by Komen et al (1984) and used by the WAMDI group (1988),

\[
S_{dis}(f, \theta) = -2.33 \times 10^{-5} \hat{\omega} (\omega / \hat{\omega})^2 (\hat{a} / \hat{a}_{PM})^2 E(f, \theta)
\]  

(3)

where \( \hat{\omega} \) denotes the inverse of the mean period, \( \hat{a} \) is a wave steepness parameter defined by \( \hat{a} = E_{tot} \hat{\omega}^4 / g^2 \) \( (E_{tot} \) is the total integrated energy) and \( \hat{a}_{PM} = 3.04 \times 10^{-3} \). There are some indications that it may be more realistic to formulate a dissipation term with greater dependence on the high frequency tail, and hence a higher power for
the ratio $\omega/\hat{\omega}$. However, given the simplified representation of the source terms in this spectral range, fine tuning this aspect would not serve any purpose here.

The weakly nonlinear interactions ($S_{nl}$) are critical to accurate spectral representation and a computationally cheap way of calculating these is necessary. This can be achieved both by a simple formulation, limiting the amount of computation per calculation, and by adopting a form which is robust to large integration timesteps.

The $S_{nl}$ function usually displays two major lobes, one positive below the spectral peak and one negative at mid-frequencies above the spectral peak. Young (1988) recognised that it is the magnitude and position of the positive forward lobe of this function which are most important in controlling spectral evolution. Further, the position of this lobe relative to the peak frequency of the spectrum and its spread are dependent on the spectral peak magnitude measured in terms of the so-called "peak enhancement factor", $\gamma$ (see JONSWAP spectrum, Hasselmann et al 1973). Young based his formulation for $S_{nl}$ on a triangular shaped positive lobe with the frequencies of the 3 vertices determined as functions of $\gamma$ and a peak frequency measure $f_m$, namely, $f_i = D_i(\gamma) f_m$, where the functions $D_i(\gamma)$ are deduced from the calculations of Hasselmann and Hasselmann. The magnitude of $S_{nl}$ is found by scaling from a reference function $S_{nl}^{ref}(\Theta)$. This scaling depends on the ratios of the spectral parameters $f_m$ and $\alpha$ (Phillips' parameter) to the reference values ($f_m^{ref}$ and $\alpha^{ref}$ respectively) and a scaling function $A(\gamma)$:

$$S_{nl}^{MAX} = S_{nl}^{ref}(\Theta) \left(\frac{\alpha}{\alpha^{ref}}\right)^3 \left(\frac{f_m}{f_m^{ref}}\right)^{-4} A(\gamma) \quad (4)$$

For robust estimates of $f_m$, $\alpha$ and $\gamma$, they are calculated as integral properties of the spectrum (for details see Young, 1988). $f_m$ is a high-order moment of the spectrum and closely approximates the peak frequency.

For the present application Young’s $S_{nl}$ representation is used with modifications to improve deep water spectral growth rates and multimodal spectral development and decay. Firstly we address the problem of a suitably long integration timestep for the model. Wind history, propagation and local wave evolution are recognized as having distinct timescales and are often treated with different timesteps. For the more accurate representations of $S_{nl}$ integration is often stabilised, for application over practicable timesteps, by employing "implicit" integration schemes. Even so, whilst 1 to 3 hours may be appropriate
for propagation and up to 6 hours for the wind history, a timestep of well under 1 hour may be required for the source terms. The WAM model (WAMDI, 1988) uses 15–20 mins.

One reason for poor results at longer timesteps is the inaccuracy introduced by calculating $S_{n1}$ from conditions at the beginning of a timestep. The spectral peak can change substantially in position and magnitude over a period of 1–3 hours and failing to recognise this in approximating $S_{n1}$ will retard the spectral evolution. Thus, in the present model the evolution of the peak frequency, $f_p$, as specified by Hasselmann et al (1976) (Equation 6.3) is used to estimate a new value $f_m^* = f_m + df_p$. Similarly the parameter $\alpha$ is modified to $\alpha^*$. These $\alpha^*$ and $f_m^*$ are used in calculating $S_{n1}$ as specified above. The effect of this modification is illustrated in Figure 1.

At frequencies above the peak, where the signature of $S_{n1}$ is predominantly negative, Young treats the nonlinear term implicitly by limiting growth to a saturation spectrum. This serves also to preclude the need for explicit representation of dissipation processes. However, with no negative source terms in this frequency range the source function here is solely determined by the wind input ($S_{in}$) until the saturation spectral density is reached. This results in rapid growth. To a certain extent this excess growth is offset by the retarding effect of applying an $s_{nl}$ calculated on the basis of the spectrum at the beginning of the timestep.

The balance of source terms on the rear face of the spectrum (frequencies above the spectral peak) is improved by parameterising the negative lobe of the nonlinear transfer $S_{n1}$ (this is in addition to including an explicit form for $S_{dis}$). The same triangular form as for the positive lobe is employed, and conservation of energy and action density used to determine the magnitude and position of the vertex. To avoid spurious bimodality in the spectrum sharp discontinuities in $S_{n1}$ at high frequencies must be avoided. Hence, a high frequency tail is included in $S_{n1}$ and the negative lobe is given double the directional spread of the positive lobe (hence the ow amplitudes in the negative lobe for the function at $0^\circ$ shown in Figure 1").
Figure 1. The general shape of the function $S_{nl}$ at $\Theta=\Theta_w$ showing the positive and negative lobes. The modification of the original $S_{nl}$ (fine line) for the spectrum $E_{in}(f)$ (dashed line) given the migration of peak frequency $f_m$ by an increment $df_p$ is shown by the heavy line ($S_{nl*}$).

It is recognised that the representation of the source function on the rear face of the spectrum is not sufficiently accurate to naturally limit wave growth and so a limit to growth is imposed in the form of a saturation spectrum, here specified by the Phillips' (1977) spectrum

$$E_{sat}(f,\theta) = \alpha \frac{g^2}{(2\pi f)^4} D(\theta)$$

(5)

where $\alpha$ is the Phillips' parameter determined from the total energy, and $D(\Theta)$ is the directional distribution $D(\Theta) = \frac{8}{3\pi} \cos^4(\Theta-\Theta_w)$ for $\Theta-\Theta_w<90^\circ$ and 0 elsewhere.

In order to contend with more complicated situations it is necessary to separate from the spectrum the energy density associated with locally generated waves and use this in calculating the $S_{nl}$ term. This is particularly important in situations where winds are varying or there is appreciable energy advecting into a growth area from another event (i.e. swell). Since the parameters $f_m$, $\gamma$, $\hat{\alpha}$ and $\hat{\Theta}$ used in the specification of $S_{nl}$ and $S_{dis}$, are determined from integral features of the spectrum, the inclusion of energy not related to input from the local wind can lead to distortion of wave growth through poor representation of $f_m$ and $\gamma$ and hence the positive lobe of $S_{nl}$. 
Further, calculation of the saturation spectrum from equation (5) depends on $\alpha$, which can be misrepresented by inclusion of swell energy in $E_{\text{tot}}$. This can lead to under-development.

Accordingly, $E_{\text{tot}}$ and $f_m$ for the wind sea portion of the spectrum are found. To separate the wind-sea, the frequency spectrum in the local wind direction is calculated (by interpolation if necessary) and the frequency range associated with the wind sea is isolated by finding a distinct local maximum or, if this is not possible, using a wind-speed-related lower bound (0.8 of the peak frequency associated with a fully developed spectrum). All energy within this frequency range, and within an umbrella created by a $\cos^4$ spread around this direction (with a 150% safety margin) is called local wind sea. $S_{nl}$ and $E_{\text{sat}}$ can now be calculated.

From the residual energy (swell) $f_m$, mean direction and $E_{\text{tot}}$ are also calculated. These determine an additional contribution for $S_{nl}$ which $\gamma$ is set to 1 and $\alpha$ to 0.01. Although the wave steepness, $\hat{\alpha}$, is small in decaying swell, the inclusion of $S_{\text{dis}}$, for swell necessitates a corresponding $S_{nl}$ contribution to avoid over-rapid decay.

3. TEST CASES

A number of simple tests were made to assess the performance of the model in idealised conditions. A well documented set of tests are those used by the SWAMP group (1985). The HYPA model (Gunther et al, 1979) uses parameterisations based on results of the JONSWAP field experiment (Hasselmann et al 1973) and hence is a useful target in pure growth cases. Further, a single point version of the 3rd generation WAM model (the WAMDI group, 1988) was available (PCWAM, G. von Vledder, personal communication) and this provided an additional benchmark for testing temporal aspects of the model.

Two of the tests are reported here. Specifically, Case II, which tests simple generation in the presence of a 20m/s wind, and CASE VII, in which the wind is suddenly rotated by 90° when the sea-state reaches half-development (i.e. when the peak frequency reaches twice the fully developed peak frequencies. For these tests the model was set up with 15 frequencies (specified by $0.045x1.15^{(n-1)}$, where $n=1,15$) and 18 directions (at regular 20° intervals). A timestep of 2 hours was used for the new model (20 mins was used for the WAM model).

CASE IIa: To test the temporal evolution a single-point version of the model (representing uniform development across an infinite ocean) was used. To match the SWAMP tests a 10 metre wind of 20m/s in a neutrally stable atmosphere was applied.

In Figure 2a the growth of total energy for this model and PCWAM is shown with the JONSWAP growth curve (as used in the HYPA model) and
the fully developed state specified by the Pierson-Moskowitz spectrum (EPM). The present model agrees very well with both the HYPA and PCWAM models. The inclusion of the growth curve for the case when the $S_{nl}$ term is calculated from conditions at the beginning of a timestep ($E_{no df_p}$) illustrates the retardation in growth if this consideration is neglected.

In Figure 2b the evolution of the peak frequency parameter is shown. Once again the present model agrees very well with the benchmarks and once again the slow evolution of the "peak" frequency is seen when we do not make provision for migration of the peak frequency within a timestep. Note that in Figure 2b the frequency parameter plotted is calculated from the full spectrum and so is only indirectly a function of the peak frequency used in the $S_{nl}$ calculation.

![Figure 2a](image)

Figure 2a. Growth of total energy as a function of duration for SWAMP Case II. Also shown are the fully developed Pierson-Moskowitz (EPM) level, the growth parameterisation derived from JONSWAP data (HYPA), growth given by the 1-d WAM model with a 20min timestep (PCWAM 0.3) and growth from the present model with no predictor step for $f_m$ ($E_{no df_p}$).
Figure 2b. Evolution of "peak" frequency parameter \( f_m \) as a function of duration for SWAMP Case II. Also included are the fully developed Pierson-Moskowitz (EPM) level, the evolutions for the HYPA and PCWAM models, and growth from the present model with no predictor step for \( f_m \) (no \( df_p \)).

**CASE IIb**: Using a grid with 42km spacing and the same 20m/s wind the fetch characteristics were tested by allowing the model to run until a balance was reached between the advection and source terms. Full development was not reached until about 1500km downwind of the shoreline. This is consistent with expectations from the SWAMP tests and also with more traditional fetch-growth relationships (see for example Pierson, Neumann and James, 1953 in which about 1400km is required for a fully developed sea at 20m/s).

Figure 3 shows the total energy as a function of fetch along with the full WAM results (from WAMDI group, 1988) and the JONSWAP (1973) relationship. Very good agreement is evident.
Figure 3. Growth of non-dimensional total energy $E^*$ as a function of fetch for SWAMP Case II. Also shown are the growth for the WAM model, the growth indicated by the JONSWAP data (with 5% margins) and the envelope of growth curves from the models in the SWAMP (1985) tests.

CASE VII: In this test the 20m/s wind was again used but the direction suddenly changed by 90° when “half-development” was reached. Figure 4 shows the growth of the spectral peak in the new wind direction and the decay of the peak in the old direction for the first 12 hours after the change in wind direction. These are accompanied by comparable plots from the single point PCWAM model. The noticeable differences are a less rapid growth in the new wind direction, and a slightly greater rate of decay. It is very difficult to assess the difference in decay as good data is not readily available for testing the modelling of pure decay processes.

In general the present model performs in test cases very similarly to the single point version of the WAM model with the same discretisation. This is despite the 2 hour timestep used for the former and the 20 minute timestep used for the latter. Further it agrees acceptably well with the empirical benchmarks derived from data. Similarly in the limited fetch tests using the full model.
Figure 4. Evolution of wind–sea $E_0$ (solid line) and swell $E_{90}$ (dotted line) for SWAMP case VII. The values (E) shown are the ratios of the maximum spectral density in the directions $0^\circ$ and $90^\circ$ respectively to the maximum fully developed value from the Pierson-Moskowitz spectrum).

4. MODEL CONFIGURATION

In the above tests alternative concentrations were tested. Up to 40 frequencies ($0.04 \times 1.05^{(n-1)}$ for $n=1,40$) with 24 directions were tried and the results were very similar to those shown. Some improvement in early growth was noted, which was to be expected given the improved spectral representation at frequencies above 0.35Hz.

However, such spectral resolution is unsustainable over a full grid. For both forecast and hindcast applications high frequency wave energy (above 0.35Hz) plays an insignificant role. Whilst it is an important feature in the exact nonlinear wave-wave interactions the present formulation does not rely on accurate representation in this frequency range. For these frequencies a diagnostic tail taking the form $\beta f^4$ is added to the spectrum.

Given the computing resources available and seeking a realistic balance between spatial, temporal and spectral resolution a total of 15 frequencies ranging from 0.045Hz to 0.32Hz defined by $0.045 \times 1.15^{(n-1)}$ were used. Directional resolution was set at $20^\circ$ (18 bands).

The grid was selected to cover the entire New Zealand Exclusive Economic Zone (EEZ) with sufficient space around the borders to ensure
that most events generating waves which affect the New Zealand EEZ are captured. A polar stereographic projection was used with a grid spacing of 190km at 60°S. This choice coincides with part of the grid of the Numerical Weather Prediction (NWP) model of the New Zealand Meteorological Service. The wave model grid has dimensions of 39x29 and is shown in Figure 5. The gridpoints over land are marked with grey circles.

Given that the numerical advection scheme is still accurate for movement of up to a grid space per timestep, and the group velocity of the fastest component (0.045Hz) was 17m/s (62km/hr), a propagation timestep of 2 hrs was viable. The same timestep was used in integrating the source terms.

---

Figure 5. Area covered by the model showing the grid used and the sub-grid, enclosed by the inner rectangle, in which the comparisons with GEOSAT data were made. The land points are denoted with filled circles, the grid point used in the verification by the open circle (○) and the position of the Waverider (W/R) by the cross (x).

5. VERIFICATION

The model was run for the 5 month period May to September 1989. For this run the 10 metre winds required as input were derived from the NWP model operated by the New Zealand Meteorological Service. This provides 6 hourly fields of geopotential height from which a 1000hPa
gradient wind is derived. A diagnostic boundary layer model produces the required input for the wave model. For intermediate timesteps the winds were interpolated (in speed and direction).

The 2-dimensional (frequency-directional) wave spectra produced by the model at each gridpoint yield any required wave parameter including significant wave height, mean period, peak period, secondary peak period, mean direction, peak direction, secondary peak direction and directional spread.

Over this test period two sources of wave data were available for comparison with the model results. Significant wave heights derived from the radar altimeter on board the GEOSAT satellite, although becoming a little sparse at this late stage of the mission, provided accurate measures over the full area of the grid. The only surface measurements available from exposed locations around New Zealand during this time were from a Waverider buoy which had been moored in the Western Foveaux Strait by B.T.W. Associates. Frequency spectra were derived from the buoy measurements and saved hourly. The site is marked in Figure 5\(^\text{5}\) by a cross (x).

Since the site is sheltered from the North by the southwest corner of the South Island and to the east by Stewart Island it was necessary to apply a filter to the model spectra. This was constructed by limiting the wave energy in each spectral direction to the fetch dependent maximum specified by the JONSWAP spectrum (Hasselmann et al, 1973) for the local wind-speed.

At 6 hourly intervals the significant wave height and mean frequency from the filtered spectrum at the nearest grid point to the Waverider site (marked on Figure 5\(^\text{5}\) with an open circle, \(\bigcirc\)) were compared with values derived from the measurements. Figure 6\(^\text{6}\) shows the time series of significant wave heights for these data for a 50 day period in 1989.

![Figure 6. Time series of significant wave height for Waverider (W/R) and model results (after filtering) for the site "x" marked on Figure 5\(^\text{5}\).](image)
In Table 1 some bulk statistics for the comparisons are presented, specifically, the mean difference (bias), the root-mean-square error, the correlation coefficient, and the scatter index (the ratio of root-mean-square-error to mean measured value).

To put these values in context a root-mean-square error for significant wave height of less than 1.0m or a scatter index of less than 0.3 is regarded as a satisfactory performance. Since verifications for frequency parameters are generally not quoted there is no indicative level for judging their performance here.

The GEOSAT data is limited to significant wave heights. In making comparisons with model results it is first necessary to resolve the different spatial scales over which the data are representative. The model grid spacing is of the order of 200km whilst the GEOSAT radar altimeter has a footprint with a diameter of about 7km. Therefore, after spurious data had been eliminated, the data from GEOSAT were averaged along 100km sections of sub-satellite track. Significant wave height fields from the model are then interpolated in space and time to the central point of these sections. The results of the comparisons are included in Table 1.

Whilst the wave model results during the trial look very encouraging it should always be borne in mind that the quality of results is totally dependent on the quality of the input winds. This paper has not attempted to validate these. In fact this would be a difficult task given the scarcity of surface measurements over the ocean. This is particularly so in the higher southern latitudes where so many of the waves which influence the Waverider site are generated. However, some hope lies in the potential of wind scatterometer data from satellites such as ERS-1.

6. SUMMARY

A 2nd-generation wave model has been described. This model includes all the major processes which contribute to spectral evolution. The weakly nonlinear energy transfers, which dominate this evolution, are explicitly parameterised for the range of frequencies where they have the controlling interest in the balance of source terms.

This model performs very well in controlled tests of growth under steady wind and rapidly varying winds. In fact its performance was comparable to that of the 3rd-generation WAM model. During an extended run of the model over about 5 months the results compared very favourably with measurements from a Waverider buoy and with data derived from the GEOSAT radar altimeter. The former gives a reasonable indication that the temporal development and decay of waves, at least near the Waverider site, are quite accurate. The GEOSAT data on the
other hand gives us a good expectation that the model reasonably accurately simulates the spatial characteristics of wave events.

Table 1 – Verification statistics for significant wave height ($h_s$) and mean frequency ($f_{\text{ave}}$, defined here as inverse of mean period)\text{s.} The mean and standard deviation, $\sigma$, of the observations are given followed by the bias, root-mean-square difference (RMSE), correlation coefficient ($\rho$), scatter index (SI) and the number of comparison points ($N$).

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7. ACKNOWLEDGEMENTS

I would like to thank B.T.W. Associates for kindly allowing me to use the wave data from Western Foveaux Strait.

8. REFERENCES


WAVE–CURRENT INTERACTIONS STUDY IN BRITISH COLUMBIA WATERS

Diane Masson and Peter Chandler
Institute of Ocean Sciences
Sidney, B.C.

Abstract

In the coastal waters of British Columbia, there are strong tidal currents which have the potential to significantly alter the surface wave field. A research project aimed at a better understanding of the wave–current interactions was initiated in 1990 with PERD funding. The objective of the project was to collect wave and current data in an area of strong tidal currents, and determine to what extent the observations can be adequately modelled by an operational wave model. In August 1991, a field program was carried out in the vicinity of Cape St. James over a period of three weeks. The surface currents were monitored with a newly developed CODAR type radar (Seasonde) and with surface LORAN-C drifters. Wave information was acquired with Waverider buoys and also from the radar. A preliminary data analysis indicates a strong current-induced modulation of the wave field, and reasonable agreement between the measured change in wave energy and the predictions of a simple wave–current interactions model.

1 Introduction

It is well known that, when sea waves propagate through a varying current, their form is affected as well as their velocity. For example, surface waves in a region of significant tidal currents have been shown to exhibit some modulation of their properties at the tidal period (e.g. Gonzales, 1984; Tolman, 1991). What are less known however are the actual magnitudes of such changes in the wave field. This deficiency is certainly due in part to the difficulty of adequately monitoring waves and surface currents. Also, wave-current interactions can be a complicated mathematical problem as it represents wave propagation in an inhomogeneous, dispersive, and dissipative medium which also interacts with the wave. In many aspects of marine engineering, it may be important to include the wave-current interactions to determine the wave climate (e.g. predicting extreme wave events) in a location where currents are significant (e.g. Burrows and Hedges, 1985). However, this potentially important effect is often overlooked, treating the wind waves and the currents separately. In order to complement the rather sparse data set on wave-current interactions available in the literature, a project, funded by PERD (Federal Panel on Energy Research and Development), was initiated in 1990 to collect wave and current data in an area of strong tidal currents.

In August 1991, a field program was carried out over a period of three weeks in the region of Cape St. James at the southern tip of
Moresby Island (Fig. 1). In this area, the vigorous tidal driven flow combines with an active wave climate, comprised of long swell from the Pacific Ocean as well as of local wind waves, to provide an ideal site for monitoring wave-current interactions. The surface current data were collected using a new High Frequency (HF) radar, the Seasonde, which has the ability to map surface currents on a fairly large area of the ocean, and with a series of Lagrangian drifters. The wave field was monitored by three Waverider buoys as well as by the HF radar from which can be extracted some characteristics of the wave field. In this paper, the Cape St. James field program will be described in greater detail, and the results of a preliminary analysis of the radar and buoy data presented.

2 Description of field program

The rugged coastline of the Cape St. James area leads westward to a very narrow continental shelf and a remarkably steep continental slope (10 to 15% grades), and eastward to a continental shelf of complex bathymetry, including a broad trough reaching 400 m depth. The area lies in one of the windiest regions of Canada and the coastal winds tend to blow parallel to the coast. In the summer months, prevailing winds are moderate (around 5 ms\(^{-1}\)) from the northwest, with occasional storm activity. A time series of wind vectors recorded at the Cape St. James weather station is given in Fig. 2 for the period of the field program. For most of the month, the relatively low wind speed was typically blowing from the northwest. However, two storm systems passed through the study area with high wind speeds of up to 23 ms\(^{-1}\) from the south-southeast. The wind data are one minute averages measured at 90 m above sea level. The site is open to all directions other than the north although the steepness of the island could affect the wind data by inducing vertical turbulence. The strong tidal currents around Cape St. James are mixed semi-diurnal, with maximum ebb current of the order of 1.5 ms\(^{-1}\) and large horizontal shears.

Three Waverider buoys were deployed in water depths of 115 m (Stn 295), 185 m (Stn 296), and 290 m (Stn 297) respectively (Fig. 1). Stn 295 was located about 10 km southeast of the Cape where the tidal currents are known to reach a local maximum. The second buoy, Stn 296, was deployed southeast of Lyman Point where one of the two radar units was operating. Finally, Stn 297 was located approximately 30 km to the southeast of Cape St. James just outside of the strong tidal current regime that characterizes the region around the Cape. The wave buoys recorded for 27 minutes every half hour at a sampling rate of 1.28 Hz. A Fast Fourier Transform was applied to the time series of the heave signal (8 blocks of 256 records) to give a frequency spectrum with a bandwidth of 0.005 Hz and 16 degrees of freedom. The spectral data were then reprocessed to average over several frequency bands to give an average bandwidth of about 0.015 Hz and 48 degrees of freedom.
One important aspect of the wave–current interactions phenomenon is the change in the wave field due to refraction by lateral current shears. Such an effect can produce dramatic increase in wave energy due to local focusing, and can also lead to wave trapping (e.g. Irvine, 1987). As the waves may be very sensitive to small scale features in the current field, these small features need to be measured in order to obtain a realistic estimate of the wave–current interactions. Unlike conventional current measurement techniques, the CODAR type HF radar used in the present experiment, Seasonde, has the ability to measure currents over an extended area and for an extended time. This instrument deduces current velocity from the first-order echo scattered by Bragg reflection from ocean waves of wavelength equal to one half the HF signal wavelength. The motion of the waves is seen by the radar as a translation (Doppler shift) of the frequency of the received echo signal from that of the transmitted signal due to the known phase velocity of the waves and the surface current. The radar can thus measure the component of current velocity along the line between the waves and the radar, the radial velocity. Two sites are required to obtain two radial current vector components in different directions in order to construct a total current vector at a point on the ocean surface.
Seasonde operates at a frequency of about 12 MHz which is Bragg scattered by short deep water ocean waves of 0.36 Hz frequency (12 m wavelength). If a linear vertical profile of the current is assumed, it can be shown that the radar probes the current at a depth of approximately 8% of the ocean wavelength or, here, at a 1 m depth (e.g. Teague, 1986). In the field program, two radar units were setup: the main unit where most of the data processing was done at the Cape St. James station, and a second unit on top of a rock outcrop near Lyman Point. Each radar unit operated continuously while the average radial files at each location were archived every 60 minutes. The radial current velocities were extracted for a series of range cells having a width of 2.67 km and an angular resolution of 5°. Finally, the combined current vectors were computed over a 1 x 1 km grid using a 5 km averaging radius. In Fig.3, the average radials from the two radars as well as the combined current field are given for one 60 minute period on August 21, 10:33 GMT at maximum ebb current.

Figure 2: Wind vectors measured at Cape St. James from 30 July to 24 August 1991.
Figure 3: An example of the radar data: a) the radials measured from the Cape St. James station, b) the radials measured from the Lyman Point station, and c) the combined current field.

Eight surface drifters employing LORAN-C navigation and drogued at a mid-depth of 2.5 m were deployed in the study area from 21 to 24 August. The unit relays its position every 27 minutes to a standby vessel by VHF radio telemetry. The drift tracks are shown in Fig. 1 and clearly indicate a net drift to the southwest with a maximum velocity of the order of 1.5 ms\(^{-1}\) passing south of the Cape. This is in qualitative agreement with the radar data, but a more extensive comparison between the drifter and the radar data remains to be done.
Figure 4: Significant wave height, $H_s$, measured by the three wave buoys from 30 July to 24 August 1991.

3 Preliminary data analysis

The first three moments of the frequency wave spectra derived from the buoy measurements were computed as

$$m_r = \int_0^\infty f^r E(f) df, \quad \text{for } r = 0, 1, 2,$$

with $f$ the frequency, and $E(f)$ the frequency power spectrum. From these spectral moments, the significant wave height, $H_s = 4\sqrt{m_0}$, the mean frequency, $f_m = \frac{m_1}{m_0}$, and the spectral width, $\nu = \sqrt{\frac{m_2}{m_1^2} - 1}$, were computed for every 30 minute period (Fig. 4, 5, and 6). Gaps in the time series represent a disruption of the data stream due to either unlocking of the signal in high sea states or instrument malfunction. The latter accounts for the lack of data from Stn296 after 14 August. It should be noted that the wave spectra used to compute the wave parameters have not yet been corrected to account for the Doppler shifting due to the surface current.
Figure 5: Mean frequency, $f_m$, measured by the three wave buoys from 30 July to 24 August 1991.

One striking feature of the $H_s$ time series is the signature of the two storms (2–3 and 8–9 August) during which the buoys measured high levels of wave energy at the three locations. In addition, the second storm event is characterized by a sharp drop in the mean frequency, $f_m$, for all buoys. Also evident in the spectral wave data is the relatively reduced exposure of Stn296 to the waves coming from the Pacific Ocean, with a consistently lower $H_s$ and higher $f_m$ at this location.

Of particular interest here, however, is the strong semidiurnal oscillation in the time series of the three spectral parameters. This feature becomes more pronounced in relatively low wind conditions when tidal forcing, rather than storm driven forces, predominates. Also, the semidiurnal oscillation of the wave parameters is particularly noticeable at Stn295 where the tidal currents are the strongest.

In order to further examine the effect of the tidal currents on the wave field, a time series of the current vector at one location was extracted from the hourly current maps produced by the radar. At each hour, a mean vector was computed for the area surrounding Stn295 where the surface current and its effect on the wave field appear to be the strongest. The mean current at this location was simply computed as a vector average of all current vectors extracted from the radar data.
within 2 km of the wave buoy. On Fig. 7, the resulting time series extending over the period 7–24 August shows a mixed tidal regime strongly dominated by the semidiurnal $M_2$ component. Over this period, the strong current ebbing to the southwest reaches a maximum of 1.5 ms$^{-1}$, and the much weaker flood current flows to the east at a speed of about 0.2 ms$^{-1}$. A quick inspection of both the $H_s$ and the surface current time series at Stn295 indicates that the local wave energy level generally follows the tidal oscillation of the surface currents.

![Figure 6: Spectral width, $\nu$, measured by the three wave buoys from 30 July to 24 August 1991.](image)

The way in which waves and a surface current are known to interact depends on the exact nature of the directional wave field and on the whole two dimensional structure of the surface current. However, in many cases, it may be possible to obtain a reasonable first approximation of the interactions by using a simple model in which a deep water wavetrain enters a region of current from quiescent water. Based on the principle of wave action conservation, this model predicts that, if the waves propagate into a following current, the waves lengthen and their amplitudes are reduced. On the other hand, for waves entering an opposing current, the wave components are shortened and their amplitudes increased up to a point where their growth becomes limited by the breaking process. Huang et al. (1972)
applied such an approach to the case of a random wave field, and derived an expression for the ratio of the spectrum in the current, $E'(f)$, to the spectrum outside the current, $E(f)$:

$$\frac{E'(f)}{E(f)} = \frac{4}{\left(1 + \sqrt{1 + \frac{8\pi Uf}{g}}\right)^2 \sqrt{1 + \frac{8\pi Uf}{g}}} \quad \text{if} \quad 8\pi Uf/g > -1$$

$$= 0 \quad \text{otherwise},$$

where $g$ is the gravitational acceleration, $U$ the current velocity in the direction of the waves, and $f$ the absolute frequency. Given this transfer function for the surface displacement spectrum, it is an easy matter to estimate the associated change in the significant wave height.

Figure 7: Surface current speed and direction from the radar at Stn295, from 7 August to 24 August 1991.

To apply this model to the present data set at Stn295, a 3 day period, extending from 20 August to 23 August, was selected during which $H_s$ shows no significant trend. The component of the surface current in the direction of the waves, $U$, was estimated by assuming that the waves travel in the direction of the wind. This is a
reasonable assumption as the effect of the wave-current coupling is stronger in the high frequency part of the spectrum for which the wave components have a relatively short time response to changes in the wind forcing (e.g. Masson, 1990). The resulting time series of $U$, on Fig. 8, shows a succession of episodes of opposing and following currents with maximum amplitude of 1.0 ms$^{-1}$, and 0.5 ms$^{-1}$ respectively. For the same period, a relative change of energy level of the wave field at the buoy location was computed as,

$$H'_s = \frac{H_s - \bar{H}_s}{\bar{H}_s}$$  \hspace{1cm} (3)

with $\bar{H}_s$ the average significant wave height measured at Stn295 over this period (Fig. 8). The parameter $H'_s$ is, as predicted by the model, negatively correlated to $U$, with an increase of $H_s$ with a locally opposing current, and the opposite effect in a following current.

Figure 8: Time series of $H'_s$ (solid line) and $U$ (dotted line) for the period 20–23 August.

The measured fluctuations of $H'_s$ over the studied period indicates a maximum increase (decrease) of about 40% (30%) around the mean wave
energy level. To compare these results with the current induced fluctuations predicted by the model, two values of surface current, \( U = -1.0 \), and \( +0.5 \) ms\(^{-1}\), were used in (2) to change an incident spectrum, \( E(f) \), into a wave spectrum modified by the current, \( E'(f) \). The incident wave field was chosen as having the spectrum measured on 22 August 4:00 GMT. This period was selected at that time, the measured current \( U \) was very small, and \( H_s \) was nearly equal to the mean \( \bar{H}_s \). The resulting change in significant wave height is an increase of 31% for the opposing current, and a decrease of 9% for the following one. In view of the simplicity of the model used here, these results, although underpredicting the current induced change in the wave energy level, are reasonably close to the measured change. However, because of the particularly strong horizontal shears of the surface current in the area, it is very likely that the neglected refraction effect plays here an important role in changing the wave field.

4 Conclusions

In this paper, we have described wave and radar data measured in the Cape St. James area with Waverider buoys and the Seononde radar. Significant modulations of the wave properties were observed at the tidal period. Analysis of selected data collected at Stn295 reveals a strong correlation between the tidal phase and the changes in the wave energy level. Furthermore, the measured modulation in the wave field agrees fairly well with the current induced changes predicted by a simple model (Huang et al., 1972) applied at this location. However, a more complete model, incorporating important processes such as refraction by horizontal shear in the current, would undoubtedly better reproduce the current induced changes in the wave field.

The observed modulations of the wave properties clearly indicate that the currents have here a strong effect on the local wave climate. In addition to the obvious change in the mean significant wave height, the local current also causes important changes in parameters such as the wave groupiness, resulting from a change in the measured mean spectral width. This significant change in the wave climate is due to the considerable bias in magnitude between flood and ebb in the Cape St. James area, in agreement with Burrows and Hedges, 1985.

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THE IMPACT OF COUPLING THE BOUNDARY LAYER TO OCEAN WAVES

William Perrie\textsuperscript{1} and Liangming Wang\textsuperscript{2}

\textsuperscript{1} Physical and Chemical Sciences
Scottia-Fundy Region
Department of Fisheries and Oceans
Bedford Institute of Oceanography
Dartmouth, Nova Scotia

\textsuperscript{2} National Research Center for
Marine Environmental Forecasts
No. 8, Da Hui Si
Hai Dian Division, Beijing
100081 China

ABSTRACT

We present a coupled model for the dynamics by which waves react on the atmosphere in the planetary surface layer. Sea surface roughness is related not only to the friction velocity, as suggested by Charnock (1955), but also to sea state parameters such as wave age. This is consistent with the recent HEXOS experiment of Oost, Smith and Katsaros (1988) and agrees with Kitaigorodskii (1968).

We use HEXOS parameterizations for $U^*$ and $Z_0$, as well as those of other researchers. We couple the WAM wave model to the boundary layer model of Delage (1988). Results demonstrate that a large variation in estimates for wave energy and significant wave height will follow depending on the parameterization which is assumed for $U^*$ and $Z_0$. Specifically, the Charnock (1955) relation for the roughness of a fully-developed wind sea is found to lead to low estimates for significant wave height.

1. INTRODUCTION

The recent successful launch of the ERS-1 satellite will provide a database of sea surface wind speeds and directions and concomitant directional surface wave spectra over the whole globe. Typically, operational atmospheric weather models do not presently incorporate coupling dynamics with ocean surface waves. There is no parameterization of sea surface roughness in terms of wave age, for example. Furthermore, operational wave forecast models generally do not consider any interaction with the surface boundary layer. The wind field is assumed to be the driving mechanism, and no consideration is given to the influence of the sea surface, as it evolves in time, on the surface winds that force it.

Using the third generation WAM (the Wave Modelling group supported by the European Community) wave model, Janssen et al (1988) evaluated the effect on wave hindcast skill of including SASS winds, using the SEASAT altimeter wave heights to check the wave heights predicted by the model. They concluded that SASS winds had little effect on the accuracy of their wave predictions and laid the blame at the feet of the SASS wind algorithm. Since winds and waves are strongly coupled,
it can be argued that a coupled wind-wave assimilation scheme is needed, which would allow (a) quality assessment and cross-validation of scatterometer, altimeter and even SAR data with in situ data over the oceans, (b) consistent wind-wave analysis from the atmospheric data assimilation, and (c) iterative updating of the wind field which drives the wave model.

Models for boundary layer dynamics need to be coupled to ocean surface wave models. These coupled models must then be fitted to remotely sensed wind fields and surface wave data using state-of-the-art data assimilation methods. In Section 2 we describe the wave model and the planetary boundary model. Section 3 presents coupling mechanisms. Finally, Section 4 describes the implications these coupling mechanisms have on total spectral wave energy $E_o$, wave height $H_s$, the drag coefficient and wind stress $U_*$.

2. MODELS

(i) Waves

We integrate the spectral energy balance equation for wind-generated waves in time for duration-limited growth. We use the formulations of the WAM model (Hasselmann et al: 1989) for nonlinear transfer, energy input due to the wind, and energy removed due to dissipative breaking.

The spectral energy density for surface gravity waves in deep water $E(f, \Theta)$ evolves in space and time according to the relation

$$\frac{\partial E(f, \Theta)}{\partial t} + \mathbf{C} \cdot \nabla E(f, \Theta) = \mathcal{J}_{\text{in}} + \mathcal{J}_{\text{nl}} + \mathcal{J}_{\text{ds}}$$

(2.1)

where $\mathcal{J}_{\text{in}}$ is the spectral energy input by the wind, $\mathcal{J}_{\text{ds}}$ is the dissipation due to wave breaking and white-cap formation and $\mathcal{J}_{\text{nl}}$ is the change in spectral energy due to nonlinear transfer resulting from wave-wave interactions.

Parameterizations for wind input energy $\mathcal{J}_{\text{in}}$ are heavily motivated by the observations of Snyder et al (1981). The form is

$$\mathcal{J}_{\text{in}} \equiv \beta \ E(f, \Theta)$$

(2.2)

where $\beta$, as specified by Hasselmann et al (1989), is given by

$$\beta = \max \left\{ 0, \ 0.25 \ \frac{\rho_a}{\rho_w} \left( 28 \ \frac{u_*}{\bar{e}} \ \cos \ \Theta \ - \ 1 \right) \right\} \ \omega$$

(2.3)

air density is $\rho_a$, water density $\rho_w$, friction velocity in the wave direction is $U_*$, cosine $\Theta$ with $\Theta$ the direction of the wind relative to
the wave propagation direction, phase velocity is \( C = \omega/k \) and angular frequency is related to wavenumber \( k \) through the deep water dispersion relation.

Dissipation due to wave breaking \( \varepsilon_{ds} \) is assumed to have a simple form, motivated by Hasselmann (1974), as well as numerical experiments completed in Hasselmann et al (1989), and may be written,

\[
\varepsilon_{ds} \approx g k^{-4} \mathcal{F}(k^4 F(k)) \tag{2.4}
\]

where \( k = |k| \), \( F(k) \) is the energy spectrum in vector wavenumber space \( k \) and \( \mathcal{F} \) is an appropriate functional. It is usually taken as

\[
\varepsilon_{ds} = -2.33 \times 10^{-5} \omega (\omega / \hat{\omega})^2 (\hat{\alpha} / \hat{\alpha}_{PM})^2 E(f, \theta) \tag{2.5}
\]

where

\[
\hat{\omega} = \left( E^{-1}_0 \int \int E(f, \theta) \omega^{-1} df d\theta \right)^{-1} \tag{2.6}
\]

\[
\hat{\alpha} = E_0 g^{-2} \quad E_0 = \int E(f, \theta) df d\theta \tag{2.7}
\]

and

\[
\hat{\alpha}_{PM} = \frac{2}{3} E_0 g^{-2} \left( E^{-1}_0 \int \int E(f, \theta) \omega df d\theta \right)^4 \bigg|_{\text{Pierson-Moskowitz}} \tag{2.8}
\]

\[
\approx 0.003
\]

The complete representation for nonlinear transfer due to wave-wave interactions \( \varepsilon_{nl} \) can be represented in terms of a 6-fold Boltzmann integral in wavenumber space by Hasselmann (1961),

\[
\varepsilon_{nl}(k) = \int \int \int \int \int \int \delta(k_1 + k_2 - k_3 - k_4) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) dk_1 dk_2 dk_3 dk_4
\]

\[
\cdot \delta(k_1 k_2 k_3 k_4) \tag{2.9}
\]

The WAM approximation to equation (2.9) is described in Hasselmann et al (1989) and is based on the so-called discrete interaction approximation.
We are interested in duration-limited waves, evolving in response to forcing by wind that is initiated at an initial time. For a very large ocean, observations at very large fetch (» 103 km) will not experience convective effects. We assume that

\[ C_g \cdot \nabla E(f) \ll \gamma_{in} + \gamma_{nl} + \gamma_{ds} \]  \hspace{1cm} (2.10)

which is valid for growing windsea spectra at large fetch.

(ii) The Planetary Boundary Layer

We consider the planetary boundary layer developed by Delage (1988) and colleagues at RPN Montréal. The steady state wind \( \mathcal{U}(h) \) at some height \( h \) satisfies

\[ f k \times (\mathcal{U} - \mathcal{U}_0) = \frac{\partial \tau}{\partial h} \]  \hspace{1cm} (2.11)

where \( f \) is the Coriolis parameter, \( k \) a unit vertical vector, the geostrophic wind, \( \tau \) is the horizontal shear stress given by,

\[ \tau = \kappa \frac{\partial u}{\partial h} \]  \hspace{1cm} (2.12)

and \( \kappa \) is the vertical diffusion coefficient. In neutral conditions this given by,

\[ \kappa = \ell^2 \left| \frac{\partial u}{\partial h} \right| \]  \hspace{1cm} (2.13)

The mixing length \( \ell \) of Blackdar (1962) satisfies

\[ \ell = \left[ \frac{1}{\kappa (Z + Z_0)} + \frac{1}{\lambda} \right]^{-1} \]  \hspace{1cm} (2.14)

where \( \kappa \) is the von Kármán constant, \( Z_0 \) is the roughness length and \( \lambda = c \mathcal{U}_*/f \)  \hspace{1cm} (2.15)

for an appropriate constant \( c \). For further details on solution of these equations and characteristics of these solutions, the reader is referred to Delage (1988) and associated papers.

It is important to note that given \( \mathcal{U}(h) \) and parameters characterising the sea state such as the total spectral energy \( E_0 \) and the peak frequency \( f_p \), the boundary layer model will estimate both the drag coefficient and the wind stress. This may then be incorporated in the modelling of the evolution of the surface wave field at each time
step. If the timesteps themselves are small enough, the integration will be stable and the computation of new wind stress at each timestep will be consistent with estimated wind stress used in computing the wave parameters at that time step. If timesteps are too large, the sea state will change rapidly as a function of time step. The wind stress and drag coefficient will then not be consistent with the values for wind stress and drag coefficient $C_d$ used to compute the wave parameters of the present timestep. Of course, an inconsistency in this study is that we assume that $U(\Phi)$ is unaffected by the evolution of the wave spectrum and the wind stress. In reality, as drag coefficient and roughness length $Z_o$ evolve in time, $U(\Phi)$ must also change. Spatially this is seen in the aircraft measurements of Smith and MacPherson (1987) which concluded that as the offshore winds move from land to sea and experience a large and sudden change in roughness, the wind speed also exhibits a fetch-dependent variation.

3. COUPLING

Over the last few years several new parameterizations have arisen for the dependency of roughness on sea state variables. We present a few of the more prominent of these in the following discussion. The sea state dependence of roughness is the mechanism by which we couple the planetary boundary layer and ocean surface wave models. The diversity of these parameterizations gives an indication of the difficulty associated with knowing how the coupling should properly be modelled.

(a) Charnock

The most commonly used roughness length is due to Charnock (1955) which simply puts

$$Z_o = 0.0185 \times \frac{U^2}{g}$$

(3.1)

For a constant wind speed and a drag coefficient that does not depend on sea state, this implies no dependence on sea state. The atmospheric layer is then decoupled from the waves.

(b) Smith

Smith et al (1992) have suggested, based on measurements from the HEXMAX experiment in the North Sea, that roughness be

$$Z_o = 0.48 \times \frac{U^2}{(g \times C/U^*)}$$

(3.2)

where $C$ is the wave phase velocity at the spectral peak and thus $C/U^*$ is the wave age.
Based on an analysis of laboratory and field data, Toba et al (1990) suggested that

\[ z_0 = 0.025 \times (\xi/U^*) \times \frac{U^2}{g} \]  

(3.3)

which is in some sense the inverse of Smith et al (1992)'s parameterization in equation (3.2), in putting wave age in the denominator.

Nordeng (1991) has recently suggested the more complicated expression

\[ z_0 = 0.11 \times (\xi/U^*) \times \frac{0.75}{\sqrt{1-e^{-W(1+\nu W/2+W^2)}}} \times \frac{U^2}{g} \]  

(3.4)

where \( W = 2 \times \kappa \xi / U^* \) and \( \kappa \) is the von Karman constant. He was motivated by Charnock (1955)'s original formulation, which he generalized to consider the effects of turbulent stress in a reference frame following the waves.

Hsu (1974) suggested

\[ z_0 = 0.90 \times \sqrt{c/U^*} \times \frac{U^2}{g} \]  

(3.5)

based on experimental results and dimensional considerations.

4. RESULTS

(i) Total energy \( E_0 \) and wave height \( H_s \)

We present estimates of total energy \( E_0 \) and significant wave height \( H_s \) in Figure 1 and 2. We have assumed a constant wind speed of 30 m/s at 10 m reference height and we have used the WAM model approximation to equation (1.1) assuming equation (1.10) as described in the previous section.
Figure 1. Variation in significant wave height $H_s$ (m) as a function of time (hr) forced by a wind speed of 30 m/s. The 5 different parameterizations for roughness $Z_o$ are as indicated.

Figure 2. As in Figure 1 for total spectral energy $E_o$ (m$^2$).
We find that the Charnock (1955) parameterization for roughness, as given in equation (3.1) results in a serious underestimate of the significant wave height $H_s$ and the total energy $E_o$ as compared to results generated when we use roughness parameterizations such as Smith et al. (1992) or Hsu (1974), which are dependent on sea state maturity in terms of wave age, for example. Nordeng (1991)'s parameterization gives the same result as Charnock (1955)'s and therefore is essentially an uncoupled parameterization of $Z_0$. After some 50 hr, the variation in estimated significant wave height $H_s$ is at least 50%!

(ii) Drag coefficient $C_d$ and wind stress $U^*$

Drag coefficients $C_d$ corresponding to Figures 1–2 are shown in Figure 3. The uncoupled behavior of the Charnock (1955) and Nordeng (1991) parameterizations of $Z_0$ are evident as compared to the variation of the other 3 parameterizations shown. Wind stress $U^*$ has the same variation as these curves because we have held wind speed $U(10)$ constant. A complete coupling to a full atmospheric model would cause $U(10)$ to vary in response to changing surface roughness with increasing sea state maturity, which would result in wind stress dependence different from the drag coefficient variation in time.

![Figure 3](image)

Figure 3. As in Figure 1 for drag coefficient $C_d$.

5. CONCLUSIONS

For 30 m/s wind speed, using the WAM model of Hasselmann et al (1989), we found an overestimate of as much as 50% in the predicted
values for significant wave height $H_s$, as compared to results obtained from the Charnock (1955) parameterization for sea surface roughness.

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HINDCASTING WAVES USING A COUPLED WAVE–TIDE–SURGE MODEL

Xiaoming Wu and R.A. Flather
Proudman Oceanographic Laboratory
Bidston Observatory
Birkenhead
Merseyside L43 7RA
U.K.

1. Introduction

Numerical wave prediction models have been used operationally for many years to forecast sea conditions, either globally or regionally. Very few models, however, consider the interaction between waves and tide/surge motion. It becomes clear now that the influence of changing water depth and current on wave propagation can be quite significant in shallow water continental shelf seas. Examples of the interaction between waves and tidal currents in the southern North Sea were given by Vincent (1979), who observed a tidal modulation of amplitude 25cm in wave height. Clayson and Ewing (1988) also found semi-diurnal tidal current influence on the modulation of measured waves in the North Sea. A pronounced refraction effect of the bottom topography on surface waves was found by Aranuvachapun (1977), who compared wave data obtained from a wave refraction diagram method with measurements at some North Sea stations.

Storm surges generated in close association with waves produce modified total water depth and current. Calculations (Wolf et al. 1988) showed that the refraction of waves by tide and surge currents as well as water depth changes can be significant in shallow water, with long period waves particularly affected. A number of idealised cases were studied by Hubbert and Wolf (1991) in order to investigate wave refraction due to temporally and spatially varying depth and current. Test runs with a third generation wave prediction model including a depth and current refraction scheme showed that effects of depth and current refraction are not limited to just a turning of the waves, but also involve significant changes in the shape of the wave spectra.

A combined wave and tide/surge model is being developed at the Proudman Oceanographic Laboratory (POL). This model, when fully coupled, considers various aspects of interactions between waves and tide/surge including the effect of wave refraction caused by tide/surge currents and water depth variations. It is expected that this combined model will give improved routine forecasts of waves as well as sea surface elevation and current during storms.

In this paper we first give an outline of the wave-tide-surge model. Then we examine interaction processes between waves and
tide/surge motion with particular emphasis on the influence of tide/surge on the propagation of sea surface waves. Results of simulations of recent storms will be presented.

2. The Wave-Tide-Surge Model

The wave-tide-surge model consists of two major components – a third generation wave model (WAM) including a depth and current refraction scheme and a barotropic tidesurge model, calculating directional wave energy spectra and sea surface elevation and depth-mean currents respectively. The two models run interactively, exchanging calculated data of sea parameters at regular time intervals.

2.1 Wave Model

The wave model solves for the wave action spectrum \( N(\omega_0, \Theta; \vec{x}, t) \), the conserved quantity in the presence of currents (Bretherton and Garrett, 1969), with \( \omega_0 \) intrinsic angular frequency, \( \Theta \) direction, \( \vec{x} \) two dimensional spatial coordinates and \( t \) time. Based on the equations of wave energy spectrum \( N(\omega_0, \Theta; \vec{x}, t) \) (WAMDIG, 1988), noting that \( E = \omega_0 N \), the wave action density equation in spherical polar coordinates (latitude \( \phi \), longitude \( \psi \)) is derived as follows

\[
\frac{dN}{dt} + \frac{\partial}{\partial \Theta}(N \frac{d\Theta}{dt}) + \frac{\partial}{\partial \omega_0}(N \frac{d\omega_0}{dt}) = S, \quad (1)
\]

where

\[
\frac{dN}{dt} = \frac{\partial N}{\partial t} + \nabla \cdot (u + c_\theta)N
\]

\[
= \frac{\partial N}{\partial t} + \frac{1}{R \cos \phi} \frac{\partial}{\partial \psi}[(u + c_\theta \sin \Theta)N] + \frac{1}{R \cos \phi} \frac{\partial}{\partial \phi}[(v + c_\phi \cos \Theta)N \cos \phi], \quad (2)
\]

\[
\frac{d\Theta}{dt} = \frac{\omega_0}{k^2} \frac{k \times \nabla |D| \partial \omega_0}{k^2} + \frac{k \cdot k \times \nabla |u|}{k^2}
\]

\[
= \frac{c_\theta \sin \Theta \tan \phi}{R} + \frac{\omega_0}{R \sin 2kD} \left( \sin \Theta \frac{\partial D}{\partial \phi} - \frac{\cos \Theta \partial D}{\cos \phi \partial \psi} \right)
\]

\[
+ \frac{\sin \Theta}{R} \left( \sin \Theta \frac{\partial u}{\partial \phi} + \cos \Theta \frac{\partial v}{\partial \phi} \right) - \frac{\cos \Theta}{R \cos \phi} \left( \sin \Theta \frac{\partial u}{\partial \psi} + \cos \Theta \frac{\partial v}{\partial \psi} \right), \quad (3)
\]
\[
\frac{d\omega}{dt} = -D\nabla u \cdot \frac{\partial \omega}{\partial D} - c_g \cdot (k \cdot \nabla) u
\]

\[
= -\frac{\omega_c kD}{R \sin 2kD} \left( \frac{1}{\cos \phi} \frac{\partial u}{\partial \psi} + \frac{\partial v}{\partial \phi} - \tan \phi \right) - \frac{c_g k}{R} \cos \phi \left( \sin \theta \frac{\partial u}{\partial \phi} + \cos \theta \frac{\partial v}{\partial \phi} \right)
\]

\[
+ \frac{c_g k}{R} \left[ \sin \theta \left( \sin \theta \frac{\partial u}{\partial \psi} + \cos \theta \frac{\partial v}{\partial \psi} \right) - \cos \theta \tan \phi (u \sin \theta + v \cos \theta) \right],
\]

where \( c_g \) is the wave group velocity, \( c = |c_g| \); \( k \) the wavenumber vector, \( k = |k| \); \( D \) denotes the total water depth, \( R \) the radius of the earth; \( u \) represents the depth mean current velocity \((u, v)\), \( \Theta_{sc} \) the great circle refraction term.

The source term \( S = S_{in} + S_{nl} + S_{ds} + S_{bf} \), represents the wind input, the nonlinear wave-wave interaction, the dissipation due to white-capping and bottom friction, respectively (WAMDIG, 1988). An implicit integration scheme is used for the source functions and upwind propagation scheme for the advective terms. At the open boundaries an energy forcing scheme is implemented to allow swell to propagate into the model. The importance of the open boundary condition for a regional wave model has been shown by Wu (1992).

### 2.2 Tide/Surge Model

The tide/surge model developed at Proudman Oceanographic Laboratory (Proctor and Flather, 1983) is based on the momentum and mass conservation equations in depth-averaged form,

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u + 2\Omega \times u + g \left( \frac{\zeta}{\rho g} \right) + \frac{\tau_s - \tau_B}{\rho D} + A_H \nabla^2 u = 0, \tag{5}
\]

\[
\frac{\partial \zeta}{\partial t} + \nabla (D u) = 0, \tag{6}
\]

where \( u \) is the depth averaged current velocity vector, \( \zeta \) the elevation of the sea surface above its undisturbed level; \( D \) represents the total water depth, \( \Omega \) the planetary angular velocity; \( \tau_s \) and \( \tau_B \) are the surface wind stress and bottom stress respectively; \( \rho \) is the density of water, \( p_a \), the atmospheric pressure; \( g \) denotes the gravitational acceleration and \( A_H \) is a horizontal eddy viscosity coefficient.

The equations are solved by a finite difference scheme, using tide and surge input at the open boundaries, where a gravity wave radiation condition is employed. (see Proctor and Flather, 1983).
2.3 Interaction Processes Between Waves and Tide/Surge

The coupled wave-tide-surge model considers two-way interactions between waves and tide/surge. The waves affect surges mainly through enhanced surface wind stress and bottom stress, i.e. $\tau_S$ and $\tau_B$ and in equation (5), and the nonlinear interaction through radiation stress. The radiation stress, which causes wave set-up or set-down, is important only in depths less than 10m with a grid resolution less than 10km (see Wolf et al. 1988). It can be neglected for the present wave-tide-surge model, which was set up on the European continental shelf grid (35km resolution) and the U.K. south-west coast (12km resolution), with depth at most grid points exceeding 10m. The effects of waves on surface and bottom stresses are found to be important in the continental shelf seas. Various theories exist concerning wave dependent wind stress (e.g. Kitaigorodskii, 1973; Donelan, 1991 and Janssen, 1991) and bottom friction (e.g. Christoffersen and Jonsson, 1985 and Weber, 1991). They have been assessed with applications to real storm surge simulation and results will be reported separately.

The temporally and spatially varying water depth and current caused by tide/surge propagation influence wave generation, propagation and dissipation. It may be deduced from the calculations of Wolf et al. (1988) that the effects of changing water depth and current on wave generation and dissipation are at least one order of magnitude smaller than the effects of depth and current refraction in the continental shelf seas. The present model, therefore, does not account for the influence of tide/surge motion on wave generation. Although it also uses temporally updated water depth in calculating bottom dissipation according to Hasselmann et al. (1973), the total change in waves is mainly due to the effects of depth and current refraction, which is represented by equations (2)-(4).

Coupling of the wave and tide/surge models requires the two models to run interactively, each providing updated sea status data for the other. In particular, the wave model component provides wave fields for the surge model component in calculating surface and bottom stress, at the same time it receives updated water depth and current from the surge model component to calculate the refraction.

3. Simulation of Storm Waves

The wave and tide/surge models were first set up on a latitude and longitude grid for the European continental shelf, covering an area from 45°N to 62°N and 15°W to 13°E. The computational grid for both models has a spacing of 1/3° in latitude and 1/2° in longitude, resulting in a resolution of approximately 35km. A directional resolution of 15° is adopted for the wave model in order to study the effects of depth and current refraction. The hourly meteorological
data (atmospheric pressure and surface winds) used to drive the models were extracted from the atmospheric model at the British Meterological Office. The wind speeds were converted from 19.5m to 10m assuming a logarithmic profile, and a two dimensional linear interpolation was used to map the winds to the wave model grid. Two storm events were selected, with one caused by the 'great storm' in October 1987 and the other February 1990. The wave model, initialised with the JONSWAP spectrum (Hasselmann et al.1973), was run with a time step of 10 minutes for both propagation and source integration. For each storm two runs were carried out for 4 days. The first run used the prescribed bathymetry in the region and assumed zero current, without depth and current input from the surge model. The second run was performed using the refraction scheme with total water depth and current obtained from a surge model run for the same period. The results from two model runs were compared and found almost identical. It was concluded from these two simulations that a spatial resolution of 35km is not fine enough to resolve the rather smaller scale processes of wave refraction due to tides and surges.

Finer resolution wave and tide-surge models were then set up for the U.K. south west coast. They were designed to study the interaction between waves and tide/surge, which has been considered an important factor effecting the performance of the routine forecasting storm surge models in the region. It is hoped that coupling of the wave and the surge model will result in an improved forecast of both waves and sea surface elevations during storms in the west and south coast of Britain. The computational grid for the model is shown in Figure 1. The spatial resolution is increased by a factor of 3, i.e. the grid spacing is reduced to 1/9° in latitude and 1/60 in longitude. The directional resolution of the wave model remains 15°, fine enough to resolve the refractions (Hubbert and Wolf,1991). The storm of February 1990, which caused serious flooding at Towyn on the west coast, was selected for the experiments. The wave model was run from 0z of the 24th. to 0z of the 28th. February, first using the prescribed bathymetry of the region and secondly using the time-varying water depth and current due to the presence of tides and storm surges. The difference between the two calculations was found to increase with time, which shows an accumulative influence of the refraction. The significant wave height and mean direction at 12z on the 26th. are shown in Figure 2. This is around the time when flooding occurred in Towyn. The difference between significant wave heights of the two runs at the time is plotted in Figure 3. The directional wave spectra at a location in the Bristol Channel for the two runs are plotted in Figures 4 and 5. It is evident that the depth and current refraction caused changes in both significant wave height and directional wave spectra. Changes in local wave heights are
particularly significant in shallower regions. In some areas the significant wave height is increased (the Irish coast) or reduced (the English Channel) by approximately 1 meter, with local wave heights at about 4 to 5 meters. To examine depth and current refractions individually, the model was run with depth refraction only, assuming zero current. The resulting significant wave heights throughout the region were found almost the same as those with depth and current refraction. Only very slight change was found present in wave spectra. (see Fig.5 and 6) This suggests that the refraction due to water depth changes played a dominant role in the total refraction, which may be due to the fact that the depth-averaged currents were used in the model, instead of surface currents.

4. Conclusions

In this paper, we have presented some examples of wave refraction due to changing water depth and current during a storm, using a coupled wave-tide-surge model. It has been demonstrated that the refraction of waves by tide and surge currents and water depth changes can be significant in continental shelf seas. The accumulative effects on both significant wave heights and spectra are particularly important in the prediction of wave conditions in shallow water regions. In order to provide an accurate forecast of waves in the continental shelf seas, it is necessary to consider the influence of depth and current refraction on wave propagation. The coupled wave-tide-surge model may offer a solution to the problem. However, validation of the model against measurements is necessary before the operational use of the model. This work is currently being carried out at POL. A spatial resolution of 12km and a directional resolution of 15° is probably enough to resolve such depth and current refraction. Further improvements may be achieved when surface currents are used in the wave model. This requires a 3D current model to be coupled with the wave model.

References


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swc3 model

Fig. 1 The U.K South West Coast model (SWC3) grid, with a spatial resolution 3 times finer than the continental shelf grid.
Fig. 2 Waves at 12z on the 26th. of February 1990. Contours indicate significant wave heights in meters and arrows mean wave directions.
Fig. 3  The differences in significant wave height (in meters) at 12z 26th Feb. 90, between model runs with and without depth and current refraction.
Fig. 4  Directional wave spectrum at a location (51.28°N, 3.08°W) in the Bristol Channel, at 15z 26th. Feb. 90, obtained from model run without refraction.
Fig. 5  Directional wave spectrum at a location (51.28°N, 3.08°W) in the Bristol Channel, at 15z 26th Feb 90, obtained from model run with depth and current refraction.
Fig. 6  Directional wave spectrum at a location (51.28°N, 3.08°W) in the Bristol Channel, at 15z 26th Feb. 90, obtained from model run with depth refraction only.
A COUPLED WIND–WAVE DATA ASSIMILATION SYSTEM

M.M. de las Heras\(^1\) and P.A.E.M. Janssen
Royal Netherlands Meteorological Institute, De Bilt, The Netherlands

1 Introduction and Motivation

Data assimilation has progressively gained importance as the amount and reliability of available observations has increased. The main reason for the increase of data accessibility has been the launch of satellites in the last decades. Different assimilation techniques have been developed, which try to extract all possible information, in order to improve the hindcast and forecast capability of numerical models. Nowadays, data assimilation is a common practice in numerical weather prediction. However, its application to wave prediction is still being developed. Traditionally, the most general way to improve wave predictions was the use of analysed winds – provided by the assimilation in a weather prediction model (Janssen et al. 1987). But wave observations can also be used to improve wave predictions, as was demonstrated for a swell case by Komen (1985), and for a more general case by Francis and Stratton (1990), who used the so called Optimal Interpolation technique.

Since, as a matter of fact, wave prediction models are very sensitive to errors in the wind fields, an attempt to update the wind field using wave height observations was made by Janssen et al. (1989a). In this way, satellite data – e.g. from ERS-1 – can be used to improve both winds and waves using the global WAM model (WAMDI Group, 1988) and the Optimal Interpolation (OI) assimilation algorithm.

Francis and Stratton (1990), but also Janssen et al. (1989a) neglect the dynamical aspects of the coupling between wind and waves, which is quite important. Therefore we have considered the possibility of assimilating both wind and wave data into a coupled wind-wave model system, in which both wind and wave fields depend on each other.

In most of the assimilation algorithms such as OI, the corrections are carried out at successive times, one after the other, combining the new observations available with the last forecast at each step. In this way, the dynamics of the model are taken only indirectly into account.

\(^1\)On leave from MOPT, Programa de Clima Maritimo, Madrid, Spain
An alternative way of assimilation is to try to find the model solution which best fits to the whole set of data collected. This approach has been tested already in meteorological models (Talagrand and Courtier, 1987) and in oceanic ones (Thacker and Long, 1988; Long and Thacker, 1989) with satisfactory results. The advantage of this so-called adjoint technique is that the analysed fields produced are always consistent with the dynamics of the model. When dealing with wave models, an additional advantage should be pointed out: analysed wave spectra are directly obtained, and there is no need to calculate them from the analysed wave heights. This spectral reconstruction, which is necessary in the OI technique, contains unavoidably some inaccuracies.

The development of this adjoint technique for wave models is reported here. Firstly, the results of a first attempt to assimilate observations into a coupled wind-wave model are presented in section 2. This coupled system is very simple, in order to test how the adjoint technique works in such a system. The wave model has only an input and a non-linear dissipation term. Non-linear wave-wave interactions are not considered in this first case. The wind model simulates an atmospheric one, which is driven by upper layer stresses. The winds predicted by this model depend on the wave state, through the dependency of the drag coefficient on the wave stress. As a consequence of this coupling, waves and winds predicted by this model have the property of being dynamically consistent.

Both wave and wind observations have been assimilated into this system, using the adjoint technique. The results are discussed in section 2.2. After assimilation, both wave and wind fields show a better agreement with the observations than before, and are still dynamically consistent. There is some persistency of the assimilation in the forecast period as well.

Since the results of this first attempt look promising, the possibility of extending the technique to more sophisticated models has been contemplated. As a first application, we have considered the one-dimensional WAM wave model, which is presented in section 3. WAM is a third generation wave model, which calculates the two-dimensional wave spectrum by solving explicitly the spectral energy balance equation, without any restriction on the shape of the spectrum. It takes into account not only wind input and wave dissipation by white capping, but also non-linear wave-wave interactions.

It is our intention to develop a similar wind-wave coupled assimilation scheme as in the first case, for the WAM model. But up to now, only preliminary results are available. These results are presented in section 3.2 and have been obtained for very simplified cases, such as short assimilation periods of e.g. one hour, in which
only one source function like non-linear interactions or dissipation - or no source at all! - has been taken into account. Analysed wave heights are significantly corrected in all these simple tests which have been done till now.

Since these preliminary results are encouraging, further extensions will be performed in the near future. The next step is to consider all source functions including the latest input term of Janssen (1991) which depends on the wave age, reflecting the dependence of the wind on the waves, as in the first coupled case of our study.

2 Application of the adjoint technique to a coupled wind-wave model

2.1 The wave and wind models

As a first approach, simple wave and wind models are considered. The wave model regarded is a first generation spectral model and its energy balance equation reads:

\[ \frac{\partial}{\partial t} F = \alpha F - \beta F^3 + \nu \]  

(1)

so that it has a Miles (1957) term, a Phillips (1957) term, and a non-linear dissipation one. \( F = F(f, \Theta) \) is the two-dimensional wave spectrum. The detailed description of the coefficients \( \alpha, \beta \) and \( \nu \) is given in De las Heras and Janssen (1991).

The wave model is driven by the winds predicted by the following wind model:

\[ \frac{\partial <u>}{\partial t} = \frac{1}{L}(\tau - C_D <u>^2) \]  

(2)

This model is in turn driven by the stresses \( \tau \), at the upper limit of the atmospheric layer of height \( L \); \( <u> \) represents the mean wind speed over that layer and \( C \) is the drag coefficient \( C = C(u, \tau) \), which is not only a function of the wind speed, but of the wave stress as well, as in Janssen (1989b), reflecting the dependency of the wind on the waves. The explicit expression of \( C_D \) can be found in De las Heras (1991). Through \( C_D \) both models are coupled and their results will depend both on each other.

Using these models, non-real wind and wave observations are generated, taking 'true' windfields as input for the wind model. The deviation of the model results from the observations and the first guess fields considered is expressed by the cost function \( J \).
\[ J = w_1 J_1 + w_2 J_2 + w_3 J_3 \]  \hspace{1cm} (3)

where \( J_1 \) penalizes disagreements between analysed and observed wave energies at any time, \( J_2 \) does the same for analysed and observed wind speeds and \( J_3 \) punishes deviations from first guess stress fields, which are the control variables or this problem and determine both the winds and the waves.

The assimilation of observations is done by minimizing the cost function \( J \), which means, finding the solution to both wind and wave model that fits best to both data and first guess fields considered. The minimization is achieved constructing the Lagrange function

\[ L = J + \mu \left\{ \frac{\partial}{\partial t} u_* - s(\tau, u_*, F) \right\} + \int df d\theta \lambda \left\{ \frac{\partial}{\partial t} F - S(F, u_*) \right\} \]  \hspace{1cm} (4)

and calculating the gradient of \( J \) by solving the Euler equations. (The coefficients \( \lambda \) and \( \mu \) are called Lagrange multipliers and \( S(F, u_*) \) and \( s(\tau, u_*, F) \) represent the source function of the wave and the wind model respectively.) If the gradient is not small enough, a next guess for the stresses is found by means of the conjugate gradient descent algorithm. Iteratively, the best guess for \( \tau \) is found and the analysed winds and waves are calculated by rerunning the models. The mathematical details of this procedure can be found in e.g. Talagrand and Courtier (1987) or Thacker and Long (1988).

### 2.2 Results

The wind and the wave models were run for 18 hours, assimilating observations only during the first 9 hours, so that the last 9 hours were the forecast period. Both wave and wind observations were assimilated every 10 minutes. The results of this run are shown in Figs.1, 2 and 3. On the left graph of Fig.1, a comparison between observed, first guess and analysed wave height time series is illustrated.
Figure 1: Comparison of observed, first guess and analysed wave height time series on the left and friction velocity time series on the right. Wind and wave data are assimilated every 10 minutes.

Figure 2: Comparison of observed, first guess and analysed two-dimensional spectra after 9 hours assimilation.

On the right graph, the same comparison for friction velocities is shown. In the hindcast period, both waves and winds are very much corrected by the assimilation. After stopping assimilating, the waves
and the winds relax to the respective first guess fields, since observations are no longer taken into account. However, some persistency can be noted especially in the wave forecast. It has been observed that this persistence is determined by the relaxation time of the coupled system.

A property of the application of the adjoint technique is the fact that the dynamical consistency of winds and waves is retained after assimilation.

Figs.2 and 3 show a comparison of first guess, observed and analysed two-dimensional spectra at 9 and 12 hours after the beginning of the run respectively. In Fig.2, after 9 hours of assimilation, the assimilation impact is quite large and the analysed spectrum looks very similar to the observed one.

Figure 3: As Fig. 2, but now after 3 hours of having stopped assimilating.
Three hours later (Fig.3), already in the forecast, a considerable impact can still be noticed. The reduction in the value of the cost function and its gradient during the successive iterations of the minimization procedure is plotted in Fig.4.

Another experiment has been carried out, in which only wave observations have been assimilated. As a result, a significant correction of both wind and wave fields is obtained after assimilation. The comparison of the observed, first guess and analysed wind and wave time series is illustrated in Fig.5.

Figure 4: Reduction of the value of the cost function and its gradient in every iteration of the minimization.

Figure 5: As Fig. 1, but now only wave observations are taken into account.
As a main conclusion, we can claim that, using the adjoint technique, discrete wave height and friction velocity data have been assimilated into a coupled wind-wave model, resulting in a remarkable improvement of both wave and wind fields, and in a considerable reduction of the cost. The impact of the assimilation can still be clearly noticed in the forecast period and both wave and wind fields are still dynamically consistent after the assimilation. Thanks to the coupling between the models, all this can still be achieved by assimilating wave height data only.

Since satisfactory results have been obtained for this simplified coupled system, the application of the assimilation procedure developed here to more sophisticated models seems hopeful. As a first attempt of extending this technique to a real wave model, the WAM wave model has been considered.

3 Extension of the assimilation procedure to a real wave model.

3.1 The WAM model

The WAM model is a third-generation wave model, the first which does not impose any restriction on the two-dimensional spectral shape. The application of the procedure of the last section to the WAM is not trivial. One deals with more sophisticated source terms, and with a number of other numerical complications. Among others, these are the time-centered semi-implicit integration scheme, in which the step for the spectrum is not constant but dependent on the frequency, and the dynamic high-frequency cut off from which a spectral tail is added to the spectrum.

A one-dimensional version of the WAM has been considered here, whose evolution equation reads:

\[ \frac{\partial}{\partial t} F = S(F, u) = S_{in} + S_{dis} + S_{nl} \]  

(5)

where \( S \) is the source function, which consists of three terms: \( S_{in} \) the input term, \( S_{dis} \) the dissipation due to white capping and \( S_{nl} \) the non-linear wave-wave interactions one. The exact formulation of each of these terms is as follows: first, the input term \( S_{in} \) is not the usual Snyder function (Snyder et al., 1981), which depends linearly on \( F \), but has the form:

\[ S_{in} = \max\{0, \varepsilon c_1 \chi^2 \mu \min\{0, \log \mu\}^4\} \omega F \]  

(6)

where \( \varepsilon = 1.225 \times 10^{-3}, \ c_1 = 7.1386, \ \chi = \max\{0, \cos(\Theta - \phi)\}\left(\frac{u_*}{c} + z_{alp}\right), \)

with \( \Theta \) and \( \phi \) the wave and wind directions, \( u_* \) the friction velocity, \( c \) the phase speed, \( Z_{alp} = 0.011 \), the angular frequency and
\[ \mu = \frac{g z_0}{c^2} \min(20, \frac{k}{z_1}) \]  

(7)

where \( g \) is the gravitational acceleration, \( k = 0.41 \) is the von Karman constant, \( z_1 = \max(0.01, \cos(\Theta - \phi)) \left( \frac{u_0}{c} + z_{alp} \right) \) and \( z_0 \) is the roughness length, which depends in this case in the wave state - this is the reason of calling our model 'coupled' - and has the following expression:

\[ z_0 = \frac{\alpha \tau}{g \sqrt{1 - \frac{\tau w}{\tau}}} \]  

(8)

where \( \alpha = 0.009 \) is the Charnock constant, \( \tau = u_*^2 \) is the surface stress, and \( \tau w = (\tau_x, \tau_y) \) is the wave stress, in which \( \tau_x = \int \omega S_{in}(F, u) \sin \Theta \, df \, d\Theta \) and \( \tau_y = \int \omega S_{in}(F, u) \cos \Theta \, df \, d\Theta \). As can be seen, \( S_{in} \) is an implicit functional of the wave spectrum \( F \).

The dissipation source function of our model has a quasi-linear dependence on \( F \), and reads:

\[ S_{dis} = C \bar{k}^4 \bar{E}^2 \left((1 - x) \frac{k}{k} + x \left( \frac{k}{k} \right)^2 \right) F \]  

(9)

where \( C \) is a constant, \( \bar{k} \) is the mean frequency, \( \bar{E} \) the mean wave number, \( E \) the energy density, and \( \chi = 0.5 \).

The non-linear wave-wave interactions source function has the original expression of the WAM model which is given in WAMDI Group (1988)

\[ S_{ni}(k) = \int \omega \sigma(k_1 + k_2 - k_3 - k) \delta(\omega_1 + \omega_2 - \omega_3 - \omega) [n_1 n_2 (n_3 + n) - n_3 n_1 (n_1 + n_2)] \, dk_1 \, dk_2 \, dk_3 \]  

(10)

where \( n_i = \frac{F(k_i)}{\omega_i} \) denotes the action spectrum and the coefficient \( \sigma(k_1, k_2, k_3, k) \) stands for the four-wave transition probability.

Some observations have been assimilated into the model, in a couple of very simple cases, which will be explained in detail in the next subsection. The same procedure as before is followed to assimilate them, that means, first a cost function is minimized, which in this case has the form

\[ J(t) = \omega_1 \sum_{t_i=\text{obs.time}} (E(t_i) - E_{obs})^2 \cdot \delta(t - t_i) + \omega_2 \sum_{k,m} (F_{km}^{t=0} - F_{1stkm}^{t=0})^2 \]  

(11)
being $E_{\text{obs}}$ and $E$ the observed and modelled energy density, $F_{1st}$ and $F$ the first guess and current initial spectra, and $k$ and $m$ the direction and frequency indices respectively.

To achieve this, the adjoint of the WAM model is computed by means of differentiating the corresponding Lagrange function, which reads:

$$\mathcal{L} = \int dt \{ J + \int df \, d\theta \lambda \left( \frac{\partial k'}{\partial t} - S(F, u) \right) \}$$  \hspace{1cm} (12)

The analytical adjoint equations of the WAM in this particular case are as follows, for time $t \neq 0$:

$$\frac{\delta \mathcal{L}}{\delta F} = 2\omega_1 \sum_{t_i=\text{obs.time}} (E(t_i) - E_{\text{obs}}) \frac{\delta E}{\delta F} \delta (t - t_i) - \frac{\partial}{\partial t} \lambda - \lambda \frac{\partial}{\partial F} S = 0$$  \hspace{1cm} (13)

so the discretised equations of both WAM and ‘adjoint–WAM’ read:

$$F_{km}^t = F_{km}^{t-1} + \frac{S_{km}^{t-1} \Delta t}{1 - \frac{1}{2} D_{km}^{t-1} \Delta t}$$  \hspace{1cm} (14)

for the WAM, and

$$2\omega_1 (E^T - E_{\text{obs}}) \frac{\delta E^T}{\delta F_{KM}} + \sum_{km} \lambda_{km}^T \mathcal{F}_{kmKM}^T + \lambda_{km}^{T+1} (-\mathcal{F}_{kmKM}^T - \frac{S_{kmKM}^T \Delta t (1 - \frac{1}{2} D_{km}^T \Delta t) + \frac{1}{2} S_{kmKM}^T \Delta t^2 \Lambda_{kmKM}^T}{(1 - \frac{1}{2} D_{km}^T \Delta t)^2}) = 0$$  \hspace{1cm} (15)

for its adjoint, due to the implicit integration scheme used. Here $\mathcal{F}_{kmKM}^T$ denotes the diagonal of the matrix $S_{kmKM} = \frac{\delta S_{km}}{\delta F_{KM}}$ at time $t$, $\mathcal{F}_{kmKM}$ is the matrix $\frac{\partial D_{km}}{\partial F_{KM}}$ and $\Lambda_{kmKM}$ is the matrix of the second derivative of the source function, i.e. the derivative of the diagonal $D_{km}$, $\frac{\partial D_{km}}{\partial F_{KM}}$.

Equation (15) is solved for $\lambda$ backwards in time, for the times $T \neq 0$. In the case of $T = 0$ the left hand side of the equation will not be equal to zero but results in the value of the gradient of $J$. Once the gradient is known, we use the conjugate gradient descent algorithm to
get a next guess for the two-dimensional spectrum at initial time – which is our control variable in this case – and so to get new wave height fields.

Calculating the adjoint of the WAM model is not a trivial task. The main difference with the case presented in the previous section is the presence of the non-linear interactions term. WAM does not apply the exact wave-wave interaction source term as is operated in the wave model EXACT-NL (Hasselmann, 1981), but simplifies the approach using only one type of interacting quadruplet and its mirror symmetrical one, as illustrated in Fig.6a. This approach is called discrete interaction approximation and has been tested with satisfactory results (Hasselmann, 1985; or Young et al. 1987). This means that every point in the frequency direction plane interacts with the 15 points which constitute the quadruplets shown in Fig.6a, including the central one.

The adjoint model is not solved forwards but backwards in time. Therefore, speaking in adjoint terms, one is not interested in which points will interact with a chosen one, but in the interacting points the chosen one was affected by. Since second derivatives are present in the adjoint expressions, the number of points which take part in the adjoint interaction increases to 93. All these points are plotted in the plane \((f, \Theta)\) in Fig.6b, in which three different types of points are indicated. The gridpoints marked by the light hatched areas constitute the main quadruplet and its symmetrical, for which the point \((M,K)\) is the central one. The dark hatched areas indicate the centers of all other quadruplets in which \((M,K)\) takes part, and finally the dotted areas show the rest of the points which constitute the quadruplets centered in the previous ones. All these 93 points have to be taken into account in the adjoint of the non-linear interaction term of the WAM. This makes things more complicated than in the case of the previous section.
3.2 Preliminary results

Initialising the model with the Jonswap spectrum (Hasselmann et al. 1973), an initial observed spectrum has been generated running the model with constant 18.45 m/s wind speed \( u_{10} \) for one day. In the same way, with constant 12 m/s wind speed, an initial first-guess spectrum has been created after a one day run. Next, the wind has been stopped in both runs and observations and first guess wave height fields have been generated for a one day decay, every 20 minutes.

Subsequently, those observations have been assimilated into the model, in several test cases. The results are preliminary, but show that the assimilation of wave data via the adjoint technique into the WAM model is feasible.

For the first test case only the dissipation source term has been considered - i.e. \( S = S_{\text{dis}} \) - in order to check first the behaviour of the most simple term of the WAM. Observations have been assimilated every 20 minutes during one day. The analysed wave height time series is shown in Fig. 7a. For comparison, also first guesses and observations are plotted. After only two iterations of the
minimization procedure, the value of the cost function has been reduced by a factor of 9. This is shown in Fig. 7b.

For the next test, the non-linear wave-wave interaction term has been also taken into account, so that now is \( S = S_{\text{dis}} + S_{\text{nl}} \). A similar comparison of wave height time series before and after assimilation is plotted in Fig. 8a, and now the reduction factor of the cost after four iterations is 11. This can be observed in Fig. 8b.

Finally, the last test case has been carried out considering the WAM input source term as well, i.e. taking \( S = S_{\text{in}} + S_{\text{dis}} + S_{\text{nl}} \) and again observations were assimilated every 20 minutes in a three hour run this time. The results are shown in similar Figs. 9a and 9b. Now, only 7 iterations are sufficient to reduce the cost by a factor of 49.

As can be noted in the previous graphs, the correction of wave height fields after assimilation is very remarkable in all the cases. Furthermore, the procedure seems to be quite efficient, since generally only a few iterations are needed to obtain a significant reduction in the cost.

![Figure 7: (a) Comparison of observed, first guess and analysed wave height time series taking \( S = S_{\text{dis}} \) in the WAM. (b) Reduction of the value of the cost function and its gradient in every iteration of the minimization, for the case \( S = S_{\text{dis}} \).](image)
4 Concluding remarks

Even though preliminary, the obtained results look promising as can be seen in the figures.

The first part of the work introduced here demonstrated the possibility of the application of the adjoint technique for data assimilation into a coupled wind-wave model system. Both wave and wind fields were satisfactory corrected after assimilation, showing still
some impact of the assimilation in the forecast period. The dynamical consistency between wind and waves was retained due to the adjoint technique of assimilation. Moreover thanks to the coupling between both models, similar results could be obtained disregarding wind data by assimilating wave height data only.

In the second part of the work, the same assimilation technique has been applied to the WAM, for several test cases. All source terms have been tried out in one or another case, always using the implicit integration scheme of the model. The correction of the wave fields after assimilation and the reduction of the value of the cost function are significant. In addition, the procedure seems to work efficiently requiring only a few iterations.

The results, though preliminary, are satisfactory. Our main conclusion is that the application of the adjoint technique to assimilate wave observations into the WAM wave model is possible and realistic.

5 Acknowledgements

Useful discussions with Gerrit Burgers are gratefully acknowledged.

References


1. INTRODUCTION

The use of remote sensing techniques to measure wind and wave data has led to recent progress in the study of oceanographic data assimilation in numerical wind and wave prediction models. Satellite measurements provide extensive oceanographic data which can be used for wave model verification as well as for global scale assimilation into numerical models (Hasselmann (1985); Komen (1985)). Micro-wave radars allow for the collection of data with high spatial coverage but their calibration still presents a difficult task.

The main objective of the present research is to implement and test the inversion and assimilation of the Synthetic Aperture Radar (SAR) image spectra for use with the European Space Agency (ESA) ERS-1 satellite data. The inversion is based on the method developed by Hasselmann and Hasselmann (1990 and 1991). At this stage the wind input to the operational wave forecast model is assumed to be correct and observational wave data are used to optimally update the model wave field only. However, to preserve the corrections in the wind part of the wave it is necessary to simultaneously correct the input wind fields. This can be done in the wave data assimilation scheme in which the corrected wave fields are obtained by the optimization of the wind forcing (Hasselmann (1988), Janssen (1987 and 1989). The work presented herein constitutes the first step in the development of a SAR-capable Canadian operational wind/wave model which will be ready for real-time forecasting operation before the launch of Canada’s RADSAT.

The data from the ERS-1 Geophysical Calibration/Validation (ERS-1 CAL/VAL) field experiment which took place from November 10-26, 1991, on the Grand Banks of Newfoundland will be used to test and tune the SAR inversion-assimilation software.

2. WAVE DATA ASSIMILATION

2.1 General classification of data assimilation methods

In general all data assimilation methods require the solution of a variational problem and may be subject to additional constraints, often defined by model equations. Additional constraints are needed in
order to remove the indeterminacy of the optimization problem which is a result of the attempt to reconstruct a continuous field using discrete experimental data. The "cost" function to be optimized represents a measure of the discrepancy between the observational data and their model counterparts.

There is a number of different assimilation techniques available. They can be classified as statistical or deterministic, sequential or non sequential, and linear or non linear.

In most statistical methods the present state is constructed as an average of the current observation and the current forecast which are treated as random variables. The weights used in averaging are constructed from error covariances of the forecast and the observations in such a way that the variance of the combination is minimized. Statistical methods include Kalman Filtering, optimal interpolation (kriging) and the successive correction method. These correspond to optimization with weak constraints (Sasaki, 1970) because they assume that a statistical error term is added to the model equations. Statistical methods sequentially assimilate data gathered over an interval of time and can not be used to optimally recover the time evolution of the model state. An excellent review and comparison of various statistical data assimilation methods is presented by Lorenc (1986).

Deterministic methods minimize an error-free measure of the discrepancy between model fields and data in order to determine the optimal model state. They often use a requirement of smoothness of the solutions as an additional constraint. If the system dynamics is included then deterministic methods use model equations as strong constraints (Sasaki, 1970) Various methods of solving large constrained optimization problems include the penalty algorithm, the duality algorithm, the augmented Lagrange method and the adjoint method, (Gill et al, 1981). Extensive information on vocational dynamical model fitting techniques can be found in Thacker (1988 and 1992) and Thacker and Long (1988).

Assimilation methods can also be divided into linear and non linear categories. This division is based on the validity of the assumption that model equations and equations which relate model fields and measured data are linear. When error distributions are approximately Gaussian and both the model equations and die mapping relations are linear, then there exists an explicit, noniterative solution to the unconstrained optimization problem. When the above assumptions are not fulfilled, the solution is necessarily iterative and a descent algorithm has to be used. In the linear case the adjoint method is equivalent to the Kalman filter method and unconstrained deterministic methods produce results equivalent to those given by statistical methods.
2.2 Assimilation of SAR image spectra into numerical wave prediction models

An optimal reconstruction of the complete global wind and wave fields requires the use of all available data together with the application of the surface wind and wave models. In general, measured data can be of different types. They can be scalars, like the significant wave height, or they can be two-dimensional arrays, like SAR or marine radar image spectra.

To be able to assimilate the data into the numerical model it is necessary to know the mapping relation between the model field and the model counterparts of the data. The most theoretically sound way of producing analyzed fields is to proceed with the constrained optimization of the cost function with all the available data in their original form, non-inverted into model variables. Such an approach is, in most cases, too complex for practical applications, especially when mapping relations are not well known. This is certainly the case for SAR image spectra, where the basic mechanisms of image formation are not yet well understood. Therefore, the implementation of SAR image spectra assimilation is divided into two separate tasks. Firstly, the "in situ" inversion of SAR image spectra into the corresponding wave spectra is performed, and secondly, the analyzed wave field is constructed.

In most of the existing literature on wave data assimilation into oceanographic models (Thomas (1988), Jenssen (1997 and 1989), Francis and Stratton (1989 and 1990), Esteva (1989), and Lalbeharry et al (1989 and 1990)) wave data consist of significant wave heights. To utilize these data for wave models, which require a two-dimensional wave considerable effort has to be made to properly reconstruct the energy distribution from the integrated spectral parameters. When measured two-dimensional spectra are available, eg. from the inversion of SAR image spectra, the corrections to the model field at the point of observation are known for each frequency and direction and the main task of the assimilation is to optimally interpolate these corrections into the model grid.

In the present research it is assumed that the wind input is correct and a sequential, deterministic variational method is used to construct analyzed wave fields. The deterministic formulation of the variational problem is used in this study, because it can be formally extended to a nonsequential constrained case. In addition, the statistics of the data and the forecast model are usually not well known. When error covariances are known they can be easily incorporated into the deterministic definition of the cost function.

Analyzed wave fields are restricted to (at least) continuous functions in order to remove the indeterminacy of the problem. The rigorous
mathematical foundations of this formulation (i.e. norm spline solution to the variational problem) was presented by McIntosh (1990), who also showed that this method is directly analogous to the optimal interpolation and that a simple analytical solution can be constructed at the price of introducing an empirical length scale pan, meter. This situation closely resembles the trade-offs of the successive corrections method.

2.3 SAR non linear inversion algorithm

Due to the complexity of imaging mechanisms, calibration of SAR image spectra is a very difficult task. Recently there has been a growing understanding of the physical processes leading to SAR image formation (Brüning (1990)). A closed integral relation for the nonlinear transformation of ocean wave s into a SAR image spectrum and its inversion have been derived by Hasselmann and Hasselmann (1990).

Our SAR image spectrum inversion software implements this SAR non linear inversion algorithm and is, to a large extent, based on inversion programs supplied by S. Hasselmann. Ibis software will be tested and tuned using ERS-1 SAR data which have recently become available. The Ocean Data Gathering Program (ODGP) model and the Canadian Spectral Ocean Wave Model (CSOWM) spectra will be used as the first guess spectra which are required by the variational inversion technique. As the model spectra corresponding to the ERS-1 CAL/VAL experiment are not presently available, the results of the SAR image spectrum inversion will be illustrated using the test SEASAT SAR image for 19 August 1978, and the corresponding first guess WAM model wave spectrum.

The straightforward application of the inversion method may yield wave spectra which show non-physical dislocations. This is due to the presence of the azimuthal cut-off bands in the SAR image spectrum. As a result, part of the initial first guess wave spectrum remains unaffected by the inverse mapping of the SAR image spectrum. As a general wind and wave assimilation scheme, in which all modifications of the wave field result from the modifications in the wind field, (and as a result affect all the spectral components in the wind-sea range of the spectrum) has not yet been developed a simple method is applied. The initial guess model wave spectrum is uniformly modified prior to the actual inversion to yield the best fit of spectral peaks of the observed and first guess (computed) SAR image spectra.
Figure 1. Nonlinear n'th order inversion of the SAR image spectrum into wave spectrum.
Figure 2. Nonlinear n’th order inversion of the SAR image spectrum into wave spectrum after rotation ($\Theta$) and scaling ($A,B$) of the first guess spectrum.
This transformation of the initial guess wave consists of rotation (by the angle $\Theta$), zooming (with a factor $B$) and energy scaling (with the factor $A$). The effect of the first guess wave spectrum transformation on the inversion results is illustrated in Figures 1 and 2. In both figures $k_x$ grows in the azimuthal direction while $k_y$ is directed opposite to the range direction. Figure 1 shows results of the inversion based on the unmodified first guess wave while Figure 2 shows the inversion results based on the transformed first guess wave spectrum. The rotation of the initial guess spectrum removes the plainly visible distortion of the best fit wave spectrum in Figure 1. The energy scaling parameter $A$ corrects the value of the azimuthal cut-off of the best fit SAR image spectrum but it also seems to overestimate the significant wave height of the best fit wave. Further tests and comparisons with the in situ data are required.

3. PRELIMINARY COMPARISON OF ERS-1 SAR IMAGE SPECTRA WITH ERS-1 CAL/VAL FIELD DATA

A preliminary comparison of the selected ERS-1 SAR and the MacLaren Plansearch Limited marine radar (MACRADAR) image spectra, and WAVEC and Waverider buoy spectra was performed at times as close as possible to each other. The results of this comparison are shown in Figures 3 and 4 for 21 November 1991, and in Figures 5 and 6 for 23 November 1991. The WAVEC directional energy spectra were generated from the variable bandwidth co-ind quad-spectra, using the Maximum Entropy (MEM) method and were then interpolated into the grid in wavenumber space. MACRADAR image spectra are not calibrated. The work on the empirical transfer functions for MACRADAR is under way.

Almost all WAVEC spectra collected during the period of ERS-1 satellite passes show multimodal structure. Not all of these modes are equally well resolved by the ERS-1 and MACRADAR radars and by the WAVEVEC buoy. Usually both radars properly measured the long wave swell component. On 21 November there is an excellent agreement between the MACRADAR and WAVEC spectra, while the ERS-1 SAR image spectrum shows the presence of only one of the two shorter waves recorded by both other sensors (see Figure 3). On 23 November a swell of approximately 200 m from about 10 is evident in both radar spectra and slightly visible in the WAVEC spectrum. A 100 m wavelength component from 260 and 270 can be detected in both radar spectra and the WAVEC spectrum. A systematic analysis of the ERS-1 CAL/VAL data is in progress.

4. SUMMARY

In the present work we have demonstrated the utility of inversion and data assimilation methods for the analysis of ocean wave spectra. The
present SAR wave data assimilation software can be further refined, using observational data.

Both the SAR image spectra inversion and the following assimilation of the SAR wave information require extensive tuning of all empirical parameters, like the cost function weights and the length scaling (range of influence) parameter which is, in general frequency dependent. The validity of the SAR inversion algorithm should be verified using well calibrated in situ data from buoys and other sources such as marine wave radar. Marine radar spectra are particularly useful for removing ambiguities resulting from the limited resolution of the wave estimates obtained from directional buoys.

In a wind–sea dominated situation, the corrections to the model wave field will be lost unless the forcing wind field becomes the independent variable to be optimized. Such a coupled wind and wave scheme requires solving a constrained optimization problem, where the constraints are the model equations themselves. However, the solution of such a problem requires the use of sophisticated optimization methods (e.g. adjoint method) and extensive computer resources. Approximate methods might be required in practical applications.

5. ACKNOWLEDGMENTS

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6. REFERENCES


Figure 3. ERS-1 CAL/VAL, 21 November 1991. (a) SAR image spectrum for 01:33 GMT, (b) MACRadar image spectrum (coming from) for 02:00 GMT, and (c) WAVE buoy spectrum (coming from) for 02:50 GMT. The inner circle represents 200 m wavelength, and the outer circle 100 m wavelength.
ERS-1 CAL/VAL, 21 November 1991. Marine radar image spectra for 02:00 GMT, (solid line), (a) and (b). WAVEC buoy spectra for 02:50 GMT, (solid line), (c) and (d). Waverider buoy spectra for 01:58 GMT, (broken line). In figure (d) $H_s=3$ m (from WAVEC and Waverider).
Figure 5. ERS-1 CAL/VAL, 23 November 1991. (a) SAR image spectrum for 14:14 GMT, (b) MACRADAR image spectrum (coming from) for 11:30 GMT, and (c) WAVEC buoy spectrum (coming from) for 11:50 GMT. The inner circle represents 200 m wavelength, and the outer circle 100 m wavelength.
Figure 6. ERS-1 CAL/VAL, 23 November 1991. Marine radar image spectra for 11:30 GMT, (solid line), (a) and (b). WAVEC buoy spectra for 11:50 GMT, (solid line), (c) and (d). Waverider buoy spectra for 12:41 GMT, (broken line). In figure (d) $H_s=3.8$ m (WAVEC) and 3.6 m (Waverider).


ENHANCED WAVE PREDICTIONS FROM ASSIMILATION OF WIND SPEEDS AND WAVE HEIGHTS FROM GEOSAT DURING LEWEX

by

W. Perrie and B. Toulany
Physical and Chemical Sciences
Scotia-Fundy Region
Department of Fisheries and Oceans
Bedford Institute of Oceanography
Dartmouth, Nova Scotia

ABSTRACT

LEWEX (the Labrador Extreme Wave Experiment 1987) attempted to intercompare wave models, in situ measurements and remotely sensed wave spectra as described by Beal (1991). A problem was that on March 18 of the experiment, a storm system in the Labrador Sea was unaccounted for in the so-called common wind fields. Therefore, although the wave measurements reflect the wind forcing at that time, the wave models do not. However, the March 18 winds and waves are evident in GEOSAT observations. We therefore do an analysis of the GEOSAT wind speeds, using objective analysis to assimilate them into the LEWEX common winds. At the same time, we assimilate the GEOSAT wind speeds into wave model estimates using empirical relations.

1. INTRODUCTION

LEWEX (Labrador Extreme Wave Experiment) concerns two locations in the Labrador Sea where observations were collected in early March 1987. Ship, aircraft and satellite estimates of wind and waves were enhanced by a number of numerical wave model estimates. The Canadian ship CFAV Quest, and Dutch ship HNLMS Tydeman used buoys and radars on the sea surface. A CCRS Canadian CV-580 aircraft and a NASA P-3 aircraft used radar remote sensors. The U.S. oceanographic satellite GEOSAT monitored wind speed and wave height with its radar altimeter. The CV-580 used a C-band synthetic aperture radar (SAR) at two altitudes (two range-to-velocity ratios), the NASA P-3 used both a surface contour radar (SCR) and a radar ocean wave spectrometer (ROWS) and the ships used moored (such as the pitch-roll WAVESCAN) and drifting buoys. Each of six agencies used numerical wave models and wind fields to estimate the wave spectra at the ship locations. These and an additional three agencies later used common wind fields to estimate a second set of hindcasts to provide a further comparison of models. From 1200 UT on 12 March until 1200 UT on 19 March about 2000 spectra estimates were made with up to 25 nearly simultaneous estimates at each of the ship locations.

This set of spectra was given a common format and not surprisingly, no set of spectral estimates from a single source is identical to
those from a single other source. The Reader is referred to the volume compiled by Beal (1991) for further discussion on the implications and details concerning these comparisons. With improved wave models, directional wind estimates and wave spectra from satellites, skill in wave forecasting should improve. This assumes there is a proper assimilation of the satellite data, of course. In general there will be biases and uncertainties and LEWEX and Beal (1991) presents a discussion of these problems.

We consider the common wind fields used in LEWEX in conjunction with the wind speed and wave height estimates from GEOSAT. We first assimilate the wind speed observations using the algorithm of Thomas (1988) to scale the wave height estimates and we show that relative to GEOSAT wave height estimates, this represents an improvement. Secondly, we develop error correlation functions for the common wind fields. This allows us to give new estimates for what the common fields should be, using GEOSAT observations. In principle, this should lead to improved wave height estimates over our original estimates.

2. Data Assimilation of GEOSAT Winds

Following the approach of Thomas (1988), we note the duration-limited growth relation implicit in the JONSWAP results of Hasselmann et al (1973),

\[ E_{WS} \approx 4.3 \times 10^{-10} \ g^{-4/7} \ U^{18/7} \ t^{10/7} \] (2.1)

where \( E_{WS} \) is the total energy of the wind-generated waves, the wind-sea, \( g \) is gravitational acceleration, \( U \) is the wind speed and \( t \) is time. Denoting modelled common winds as \( U_M \), GEOSAT measured winds as \( U_G \), modelled wind-sea as \( E_M \) and corrected wind-sea as \( E_C \) then from equation (2.1) we write

\[
E_C = E_M \left( \frac{U_C}{U_M} \right)^{18/7} \] (2.2)

This is our ratio for scaling the modelled wave energy.

GEOSAT also provides estimates for significant wave height \( H_s \) (of the wind-sea). In Figure 1 we present the time series of the wave model \( H_s \), without assimilation of wind data as in equation (2.2), as compared to GEOSAT \( H_s \). Figure 2 shows the wave model \( H_s \), when GEOSAT winds have been assimilated according to equation (2.2), as compared to GEOSAT \( H_s \). There is a systematic improvement which we quantify in Tables 1 and 2.
Figure 1. Time series of the wave model $H_s$, without assimilation of wind data as in equation (2.2), as compared to GEOSAT $H_s$.

Figure 2. As in Figure 1 when GEOSAT winds have been assimilated.
Table 1.

Comparison between wave model significant height $H_s$ and GEOSAT $H_s$ when there is no assimilation of GEOSAT winds.

<table>
<thead>
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<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Std Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>model</td>
<td>0.31</td>
<td>10.51</td>
<td>4.20</td>
<td>2.37</td>
</tr>
<tr>
<td>GEOSAT</td>
<td>0.53</td>
<td>5.63</td>
<td>2.53</td>
<td>0.87</td>
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<tr>
<td>RMSH-error</td>
<td>-4.81</td>
<td>7.96</td>
<td>1.67</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 2.

As in Table 1 when there is assimilation of GEOSAT winds.

<table>
<thead>
<tr>
<th></th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Std Dev.</th>
</tr>
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<tbody>
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<td>model</td>
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<td>10.49</td>
<td>3.75</td>
<td>2.02</td>
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<tr>
<td>GEOSAT</td>
<td>0.53</td>
<td>5.63</td>
<td>2.53</td>
<td>0.87</td>
</tr>
<tr>
<td>RMSH-error</td>
<td>-4.86</td>
<td>7.93</td>
<td>1.23</td>
<td>1.15</td>
</tr>
</tbody>
</table>

We see that assimilation of the GEOSAT winds gives a reduction in

1. the mean bias, or the difference between the wave model and the GEOSAT mean significant wave height $H_s$, by about 0.44 m or 26%, and

2. the root mean square height (RMSH) error, or the difference between the wave model and GEOSAT RMSH, by about 0.35 m or 23%

The RMSH (standard deviation) is defined as

$$\text{RMSH} = \left\{ \frac{1}{N} \sum_{n=1}^{N} \left[ H_s(n) - \langle H_s \rangle \right]^2 \right\}^{1/2} \quad (2.3)$$

The Reader is referred to Gerling (1991) for a discussion of the storm systems that occurred during LEWEX and the ability of the common wind fields to properly model them. Figures 3–4 show the two-dimensional wave spectra for the 18 March at 0000 UT, with and without the assimilation of GEOSAT wind speeds. During this period, Labrador Sea swell was unaccounted for in the common wind fields and in the wave models, as shown in Figure 3. In principle, the GEOSAT
altimeter observed this swell. When the GEOSAT altimeter winds were assimilated, the wave model results shown in Figure 4 should show a better comparison with the WAVESCAN measurements (which are presented in Gerling: 1991). Because of the position of the GEOSAT data and our method of assimilation, this was not achieved.

Figure 3. The wind speed data provided by the GEOSAT altimeter for the LEWEX period. Satellite tracks are indicated by the lines. Each point represents the position of a 31s average.

Figure 4. Number of averaged GEOSAT observations as shown in Figure 5 as function of time.
Modifications to wave energy using equation (2.2) result in modified estimates for the wind-sea spectral energy. Thereafter, the spectral energy density for surface gravity waves in deep water \( E(f, \theta) \) evolves in space and time according to the relation

\[
\frac{\partial E(f, \theta)}{\partial t} + \mathbf{g} \cdot \nabla E(f, \theta) = \mathcal{J}_{\text{in}} + \mathcal{J}_{\text{nl}} + \mathcal{J}_{\text{ds}}
\]  

(2.3)

where \( \mathcal{J}_{\text{in}} \) is the spectral energy input by the wind, \( \mathcal{J}_{\text{ds}} \) is the dissipation due to wave breaking and white-cap formation and \( \mathcal{J}_{\text{nl}} \) is the change in spectral energy due to nonlinear transfer resulting from wave-wave interactions. The nonlinear interactions drive this evolution and are the principle elements in transferring energy within regions of the spectrum. In particular, the inhomogeneities introduced through implementation of equation (2.2) are diminished by this nonlinear transfer throughout the spectrum.

As GEOSAT also provides wind speeds, it is possible to make a comparison between the common wind fields and the wind observations of GEOSAT. We present this comparison in Table 3.

Table 3.

Comparison between common wind fields and GEOSAT wind measurements

<table>
<thead>
<tr>
<th></th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Std Dev.</th>
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<td>4.17</td>
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<td>-0.05</td>
<td>1.24</td>
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</tbody>
</table>

It would be possible to construct a ratio of GEOSAT wind speeds to common wind speeds and apply them to the common winds as a whole in an attempt to create enhanced winds to drive the wave model. However, GEOSAT data does not provide complete coverage of the North Atlantic. As shown in Figures 5-6, the data for a typical wind map is sparse. Therefore without proper correlation functions to smooth the impact of the insertion of GEOSAT data, the resultant wind field would be ‘jagged’. This would result in unrealistic chaotic wave fields. Therefore, this study also considered the assimilation of wind speeds using wind speed correlation functions and GEOSAT wind measurements to improve the common fields. Resultant wave height estimates will be compared to GEOSAT and WAVESCAN observations when they have been computed.
Figure 5. The wind speed data provided by the GEOSAT altimeter for the LEWEX period. Satellite tracks are indicated by the lines. Each point represents the position of a 31a average.

Figure 6. Number of averaged GEOSAT observations as shown in Figure 5 as function of time.
3. Conclusions

We have presented an assimilation of GEOSAT wind speeds into the significant wave height $H_s$ estimates as provided by a wave model. Statistically we have shown that our results are an improvement over the estimates that the wave model gives when the wind speed are not assimilated. Of course, if we stopped the assimilation at some particular time, the wave height forecast estimates would quickly (within a few hours) relax back to the results obtained when wind speeds were not assimilated. This is because the model is still driven by the common wind fields.

We show that our data assimilation method and GEOSAT wind speeds did not improve the estimates of wave spectra at 0000 UT on 18 March. Labrador Sea swell was not accounted for in the common wind fields, or consequently in the estimates of wave spectra from the wave model.

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WAVE DATA ASSIMILATION FOR OPERATIONAL WAVE FORECASTING AT THE NORTH SEA

Gerrit Burgers, Vladimir Makin, Gao Quanduo and Miriam de las Heras
Royal Netherlands Meteorological Institute (KNMI), De Bilt, The Netherlands

1 On leave from MOPT, Programa de Clima Maritimo, Madrid, Spain

1 Introduction

In this paper, we study the possibility of wave-data assimilation into the regional model NEDWAM which is used for operational wave forecasting in the North Sea by the Royal Netherlands Meteorological Institute, KNMI.

For the North Sea as compared to the open ocean, both the content of the information from wave observations and the content of information in the driving surface wind field is larger, because the density of conventional wave observations is high in the North Sea – over 250 per day – and the quality of analysed wind fields is quite good in the North Sea.

The desired benefit of wave data assimilation is to improve the forecasts of the wave field for standard forecast periods of +12 hours and +24 hours. But one should realize that wind sea waves have a characteristic growth time scale of some hours and that the characteristic time scale of wind variability is about 10 hours. That means that for wind sea the expected impact of data assimilation for a +12 hour forecast is limited already. A larger impact of data assimilation would be expected for events with characteristic time scales comparative to the forecast period, that is swell and decaying storms. But the North Sea is so small, that only waves from northern directions can propagate for more than a day before hitting the shore.

The method used for wave-data assimilation was originally suggested by Janssen et al. (1989) and Lionello and Janssen (1990). for application to the open ocean. The method consists of two steps. The first step is the optimal interpolation of wave heights and periods, which gives analysed wave height and period fields. The second step, called the update of the spectrum, is needed to obtain a complete analysed spectrum. An advantage of the method is that it is fast and requires little data storage. Therefore, this method could be used operationally.

Since November 1991 the model has been running semi-operationally accepting conventional observations and ERS-1 altimeter heights measurements. Here a thorough analysis of data assimilation NEDWAM (DA NEDWAM) model results is presented for the period of December 16-31 1991, which contains a storm at December.
2 The North-Sea wave model NEDWAM

Wind-generated ocean waves are characterized by the wave spectrum $F(f, \Theta)$, which is a function of tile frequency $f$ and the direction $\Theta$ and also depends on position and time.

The wave variance $E$ is the integral of the wave spectrum: $E = \int df \ d\Theta F(f, \Theta)$. The significant wave height $H$, is directly related to $E$: $H = 4E^{1/2}$. The mean period is defined as $T = E^{-1} \int df \ d\Theta F(f, \Theta)f^{-1}$. The terms "wave height‘ and "wave period“ will always stand for the significant wave height and the mean period throughout this paper.

The evolution of the wave spectrum is described by the energy-balance equation, which can be written as

$$\frac{\partial F}{\partial t} + c_g \cdot \nabla F = S,$$

if we neglect refraction terms. The l.h.s of eq. (1) describes the advection of the spectral components. $c_g = c_g(f, D)$, where $D = D(\mathbf{x})$ is the local depth of the sea, denotes the group speed of spectral component $F(f, \Theta)$. The r.h.s is the source term which gets contributions from wind input (generation of wave components by the wind), dissipation at tile surface through white-capping and turbulence, dissipation at the bottom through bottom friction, and non-linear wave-wave interactions.

NEDWAM (Burgers 1990) is a North Sea version of the global ocean wave model WAM (WAMDI 1988). The stereographic grid covers the North Sea and a large part of the Norwegian sea. It contains 612 gridpoints. The grid spacing is approximately 75 km. The purpose of NEDWAM is to produce good wave forecasts for the North Sea, especially for the Southern part. The Northern part of the grid is included only in order to handle properly cases where swell enters the North Sea from the North West.

The wave spectrum in NEDWAM is defined on a grid of 25 frequencies by 12 directions.

The propagation time step of the model is 30 minutes. The wind-fields are provided by the Limited Area Model of the KNMI. The wind-time step is 3 hours.

The errors in the NEDWAM wave height field are of the order of 0.5m, and in the period field of the order of 1s, for the analyses of the version without wave-data assimilation. The performance of NEDWAM is mainly limited by the quality of the driving surface wind field and the resolution of the coast line. For longer waves, with periods of
10s or more, refraction effects and bottom dissipation effects have also an impact on the performance of the model. Refraction effects cannot be resolved with the present resolution and are neglected in NEDWAM. Bottom dissipation effects, which can be quite substantial, are parameterized in a rather crude way in NEDWAM, because their precise dependence on wave spectrum is not known.

3 Observational data and quality control

The low reliability of conventional wave data, in particular of the visually observed ones, is a well known problem. It is evident that a good quality control is crucial to the assimilation of conventional wave data.

A real-time quality control scheme of wave heights and periods observations from ships, buoys and platforms in the North Sea and Norwegian Sea was developed at KNMI by Etala (1991). The method follows ideas from Comprehensive Quality Control (Gandin 1988).

Every three hours wave heights and periods are assimilated in the model.

Observations are obtained from three sources: a) SHIPS bulletins of the Global Telecommunication System (GTS). The number of observations is of the order of thirty. Very few observations come from the Norwegian Sea, since there are not many ships over there. b) Dutch North Sea Network (MNZ) provides data from platforms and buoys, which have a reputation for good quality. The number of observations is of order of ten. The measured wave spectra which are available from some platforms are not assimilated, but are sometimes used for verification purposes. c) ERS-1 altimeter height measurements. During the period studied, the ERS-1 tracks crossed the North Sea every third day, providing heights with spacing of 7 km along the track. The accuracy of altimeter measurements is reported to be less than 10\% in heights.

4 The optimal interpolation of wave heights and periods

The model yields a first-guess estimate of the wave height and period at every model gridpoint.

Observed and first-guess values are blended into analysed values by means of standard Optimal Interpolation (see e.g. Ghil (1989)):

$$H^{(am)} = H^{(fg)} + M C^T (C M C^T + O)^{-1} (H^{(ob)} - C H^{(fg)})$$

(2)

Here $H^{(am)}$, $H^{(fg)}$ and $H^{(ob)}$ denote the vectors of the analysed, the model first-guess and the observed values, respectively. $C$ is a $N^{(mod)} \times N^{(ob)}$ matrix which relates the values at the $N^{(mod)} = 612$ model grid points to model estimates at the $N^{(ob)}$ observation points. The $N^{(mod)} \times N^{(mod)}$ matrix $M$ is the error covariance matrix of the model first
guesses, and the $N^{(ob)} \times N^{(ob)}$ matrix $O$ is the error covariance matrix of the observations.

For $C$ we have chosen the simplest possibility, projection of each observation point to the nearest model sea grid point.

The standard problem of Optimal Interpolation is what to take for the standard deviations of the observations and the model and what to take for the error covariance matrix of the model. Here we follow a relatively simply approach where e.g. we do not distinguish between different positions in the grid and we do not take into account the wind field or the mean wave direction. The actual values of the parameters in the equations below were suggested by verification studies of the NEDWAM model.

The ERS–1 observations come with an estimate for the standard deviation. For the standard deviations of the model, the MNZ observations and the GTS observations we took

$$\sigma_{H,\text{mod}} = 0.3 + 0.1H$$
$$\sigma_{H,\text{obs}} = w(0.3+0.1H)$$

$$\sigma_{T,\text{mod}} = \max(0.6, 0.2T - 0.6)$$
$$\sigma_{T,\text{obs}} = w \max(0.6, 0.15T-0.3)$$

In the above expressions $H$ is in m and $T$ is in s. The factor $w$ which is equal to 1 for GTS observations and 0.6 for MNZ observations reflects that we trust the latter observations more than the former.

We neglect correlations in the errors between different observations. Thus the matrix $O$ is diagonal. We have used the following expression for $O$:

$$O_{kl} = \delta_{kl} (\sigma^{\text{obs}})^2$$

To get some insight into the model error correlations, we have determined the correlations in the differences between model wave heights obtained by +24 h forecasted winds and model wave heights obtained by analysed winds. One can expect these correlations to be similar to the correlations in the model error. The correlation could be represented significantly better by a function of the form $(1 + a r) \exp(-a r)$ then by a function of the form $\exp(-b r)$. The correlation length for periods was found to some 10% smaller than for heights.

For the model error covariance matrix $M$ we have taken

$$M_{ij} = \begin{cases} 
\sigma_i \sigma_j (1 + d_{ij}/d_{\text{corr}}) \exp(-d_{ij}/d_{\text{corr}}) & \text{if } d_{ij} \leq d_{\text{max}} \\
0 & \text{if } d_{ij} > d_{\text{max}} 
\end{cases}$$
where $\sigma_i$ is the model standard deviation in model grid point $i$, $d_{ij}$ is the distance over sea between gridpoints $i$ and $j$. $d_{\text{corr}} = 1.8$ both for heights and periods. $d_{\text{max}} = 7$ grid units is a cut-off length introduced to simplify the calculation.

5 The update of the wave spectrum

So far, we have only described how to get analyzed wave heights from the first-guess fields and observations. In this section we will describe how to update the wave-spectrum field and the wind field.

In contrast to the case of the interpolation of wave heights and periods, where the field was considered as a whole, the spectral update procedure is performed for each gridpoint separately. The wave-spectrum field has 300 components per gridpoint, and the wind field has two components per gridpoint. Clearly, the analyzed wave spectrum should give the analyzed wave height. This amounts to

$$\int df d\theta F^{(an)}(f, \theta) = \left( \frac{H^{(an)}}{H^{(fg)}} \right)^2 \int df d\theta F^{(fg)}(f, \theta),$$

but this constraint does not fix uniquely the way to obtain all 300+2 analyzed quantities.

After Lionello and Janssen (1990), we scale both the magnitude and the frequency dependence of the spectrum,

$$F^{(an)}(f, \theta) = B \left( \frac{H^{(an)}}{H^{(fg)}} \right)^2 F^{(fg)}(Bf, \theta).$$

We have less period observations than height observations. If there are period observations close to the grid points considered, we simply take

$$B = \frac{T^{(an)}}{T^{(fg)}}.$$ \hspace{1cm} (13)

If not, we take

$$B = \left( \frac{H^{(an)}}{H^{(fg)}} \right)^{1/2}.$$ \hspace{1cm} (14)

The latter choice for $B$ conserves in deep water the mean wave steepness squared $s = (k^2)E$, where $(k^2)$ is the wave number squared.
averaged over the wave spectrum, and $E$ is the wave variance. Conservation of the wave steepness implies a change in the peak frequency: increasing the wave height goes hand in hand with decreasing the peak frequency. As large wave heights are correlated with low frequencies, this is a nice property.

The above scaling formula is applied to all of the spectrum, treating wind-sea and swell on an equal footing.

If there is wind sea, then the wind-field is updated as follows. The wind-direction is not changed and the wind-speed is multiplied by $H(a_m/H(g))$. For the wind-fields of the forecast period the multiplication factor is relaxed to unity with a decay time of 8 hours.

6 Results and discussion

A thorough analysis of DA NEDWAM model results was done for the period December 16–31 1991. In this period, there was a storm at December 19–21. The analysis consists of statistics for analysed period (fp=+0) and forecast period of 6 hours (fp=+6), summarized at Table 1. Only model results and observations in the North Sea are considered. Events with pronounced impact of data assimilation (the difference between wave heights of DA NEDWAM and NEDWAM being more than 1 meter) are analysed using time series (Figures 1 and 2) of significant wave height, mean period and height of low-frequency waves $H_{s10}$. The latter is defined as

$$H_{s10} = 4 \left( \int_0^{0.1} df \int d\theta F(f, \theta) \right)^{1/2}.$$

As expected, after data assimilation the analysed wave state (fp=+0) is closer to the observed wave state, than before data assimilation, i.e. for standard NEDWAM run (Table 1).
As was already mentioned, for wind sea we did not expect much impact of data assimilation for a forecast period of \( f_p = +12 \) hours, as the characteristic time scales for wave growth and evolution of wind field are less than forecast period. More effect is to be expected for swell coming from northern directions or decaying storms, events which are quite rare in the North Sea. At the other extreme, for off-shore winds and locations near the coast one does not expect any impact on the forecasted waves at all from data assimilation. In fact, for a forecast period of 6 hours, there is hardly any effect of data assimilation anymore in the overall statistics, as can be seen from Table 1.

From the period studied, we selected two events with significant impact of data assimilation on analysed and forecasted wave heights, that is decaying storm at the location of the North Cormorant platform (coordinates 61.14 N and 1.09 E) for December 21, 12 hours UTC, and swell coming from the north at the location of AUK platform (coordinates 56.28 N and 2.17 E) for December 22, 18 hours UTC.

In figs. 1 and 2) the timeseries are shown for the analysis from 12 hours before until 18 hours after the mentioned dates. For the analysis, the wave model was driven by wind-field analyses and wave data were assimilated in the DA NEDWAM version. Also shown are 18 hour forecasts which start from the wave-state analysis of the mentioned dates. In the forecasts runs the wave model was driven by wind-field forecasts and no wave data were assimilated. The observed data corresponds to measurements from platforms.

The North Cormorant case is a good example of decaying storm with constantly decreasing winds. The impact of data assimilation can be
traced up to the +12 hours forecast both for wave and low-frequency wave heights, though significant improvement of 0.75 m compared to NEDWAM is traced only up to the +6 hours forecast. Assimilated periods have a constant shift of about 2 s from observed at North Cormorant. This is due to the fact that the assimilated values were also influenced by several GTS observations quite close with reported periods of about 8 sec. So, the question whether the assimilated periods are good or not is also a question of what kind of observations are to be trusted more and what observations are to be taken as a reference for comparison.
Figure 1: Timeseries of significant wave height $H_s$ in $m$, low-frequency wave heights $H_{s10}$ in $m$ and mean periods $T$ in $s$ at station North Cormorant for the period Dec. 21 1991, 00UTC until Dec. 22 1991, 06UTC. The "0" at the time axis corresponds to Dec. 21 1991, 12UTC, when the forecasts start. Solid lines – observations, dashed lines – DA NEDWAM forecast, dash-dotted – NEDWAM analysis, dash-dotted with stars – NEDWAM forecast.
Figure 2: Same as fig. 1, but for station AUK for the period Dec. 22 1991, 06UTC until Dec. 23 1991, 12UTC. The "0" at the time axis corresponds to Dec. 22 1991, 18UTC, when the forecasts start.
At AUK the wind sea began to grow on swell coming from the north. For the analysis of December 22, 18 hours UTC the value of wave height is considerably improved, up to 1.5m, the difference with NEDWAM is still about 1m for forecast of +6 hours, but observed value falls between. When the wind sea began to grow the values of periods Hlo of NEDWAM show a better agreement with AUK observations than data assimilated periods. This is because of several nearby stations which reported much smaller periods than AUK.

A good example how rapid variability of wind field ruins the forecast is seen for December 23, 03 hours UTC, when the sudden unpredicted drop of wind speed (forecast – 18m/s, in reality – 8m/s) made the forecasted heights completely wrong.

7 Summary

There is a positive impact of data assimilation on statistically averaged wave heights, the bias goes essentially to zero from 0.2m for the no data-assimilation case, and the r.m.s. difference drops from 0.7m to 0.3m. Periods are a more difficult quantity to handle, and the reduction in the bias and r.m.s difference is less spectacular than for heights. The impact of data assimilation on the quality of forecast is rather small, and usually has disappeared after 6 hours. In some special cases an impact up to 12 hours can be seen.

There is still a lot of uncertainty in how to treat and to trust different kinds of observations, in particular for periods. Apparently, some interaction with the people who make the wave observations is called for.

References

The WAM model - A third generation ocean wave prediction model. 
THE IMPACT OF ALTIMETER DATA ASSIMILATION FOR WAVE FORECASTING IN THE MEDITERRANEAN SEA

J.M. Lefèvre
METEO–FRANCE
Toulouse, France.

1. SUMMARY

Recent works on data assimilation techniques for wave modelling (Janssen and al. 1989, Lionello and al. 1991) led to the implementation of an assimilation scheme in the third generation wave model WAM. A sequential method with optimal interpolation is used to assimilate altimeter satellite data. The effect of the assimilation of altimeter derived wave height has been investigated by Lionello and al. (1991) on the global version of WAM. In their approach, only large scale phenomena are accounted for. The purpose of our study is to investigate the impact of the assimilation for limited area and meso-scale phenomena. Taking advantage of the recent launch of the European satellite ERS1 in July 1991 and the new resolution (about 60 km) of the atmospheric model at the European Center for Medium-range Weather Forecast (ECMWF) since September 1991, we carried out experiments to qualify the ability of the WAM model to catch and keep the high variability of Mediterranean phenomena using satellite data.

2. INTRODUCTION

The recent launch of the satellite ERS-1 and the new resolution of the ECMWF atmospheric model (about 60 km) allows one to test the potentiality of data assimilation in the Mediterranean sea. Data assimilation techniques have been processed for a long time in atmospheric model for which initial conditions mainly determine the forecast. Since then, new techniques have been investigated (4-D variational analysis) to improve the analysis of the atmosphere initial state. On the contrary, the wave field tends to lose the memory of the initial conditions because of the strong forcing by the wind and the breaking of waves on the coasts. The impact of data assimilation is therefore expected to be weak for medium range forecasting. However, it is of interest to investigate how fast the wave model loses the information it gained through a data assimilation scheme. Such investigations have been carried out by many authors (Janssen et al. 1989, Lionello et al. 1991) using altimeter data and the global WAM model (resolution 3x3 degrees). The purpose of this paper is to investigate the potentiality of using altimeter data to improve wave forecasting in the Mediterranean sea where the complexity of the surrounding orography generates a high wind variability. Mediterranean phenomena are characterized by a strong intensity, a sudden strengthening, and a limited extension. Their evolution over the sea is greatly influenced by the orography of the main islands.
Studies (Guillaume et al 1990, Lefevre 1991) have shown the ability of limited fine mesh model PERIDOT to reproduce accurately the wind fields in the Mediterranean sea. A good description of the orography in a model is crucial to get its influence on the wind fields. Since end of 1991, the ECMWF atmospheric model has reduced the resolution from about 120 km to 60 km. A large improvement of the wind description has been noticed by the french marine forecast service in the Mediterranean sea. It is therefore of interest to use the ECMWF wind fields for driving the WAM model implemented over the Mediterranean sea. We used the data assimilation techniques processed and implemented in the WAM model by Lionello et al. (1991). Their approach is described in the next section. The present work is mainly focused on the temporal effect of the assimilation on the forecast, the intensity of the possible improvement, the advantages and the limitations of the method, using only altimeter data.

3. THE ASSIMILATION SCHEME

The data assimilation scheme implemented in the WAM model belongs to the class of sequential methods. At different times, all the observations available in a given time window are used to correct the model’s forecast used as first guess using optimal interpolation. Although more sophisticated methods of assimilation are able to produce analyses that are consistent with the dynamic of the model (4-D variational analysis), the approach chosen in the WAM model enables to reveal the potentiality of data altimeter assimilation. Moreover, the minor computer resources required to implement such a scheme allow one to carry out experiments easily. The schematic description of the method is shown in figure (1). The Significant wave heights (SWH) and derived wind speeds are collected along the satellite track in the time window centered at the time of the guess field (model forecast). The altimeter data are processed in order to control their quality (in order to eliminate bad data as data contaminated by neighbouring island). Data are then averaged in a box of size of the mesh and centered on each grid point. The standard deviation is regarded as an indicator of the spatial homogeneity of the measurements. The averaged value is rejected if the standard deviation is too large. The rejection criterion must depend on the size of the box. we chose 0.5 m or 25% of the mean value. Only points where more than 30% of the data are reliable are kept in the analysis. For our experiment, those parameters have been tuned to take into account the particularities of the Mediterranean sea, namely the presence of islands and the high variability of the meteorological phenomena. Then, optimal interpolation using provide an analysis of the total energy field $E_a$ through the relations:
where $c_p^i$ is the root mean square error in the model prediction, $P_{kj}$ is the $(k,j)$ element of the prediction error matrix scaled by $c_p$, and $O_{kj}$ is the $(k,j)$ element of the observation error matrix scaled by $c_p$.

It was assumed that the prediction error matrix of prediction has the following classical form: $P_{kj} = \exp(\frac{\text{abs}(x_k-x_j)}{L_{\text{max}}})$ where $L_{\text{max}}$ is the correlation length for prediction error of SWH. The observation errors are assumed to be random and uncorrelated. The analyzed spectrum is computed proportionally to the ratio $E_a/E_p$ using the relation: $F_a(f,\Theta) = A.F_p(Bf,\Theta)$. The A and B coefficients are processed in different ways depending on whether wind sea is found in the guess spectrum or not. If wind sea is found, dimensionless relations (Kidagorotii 1962) are used to estimate the duration of the wind sea:

$$E^* = \left(\frac{g^2}{u^2}\right) E \ t^* = \left(\frac{g}{u^2}\right) t$$

In the WAM model the dimensionless growth curve is:

$$E^* = 955 \tanh (6.02.10^{-5}.t^0.695) \text{ and } E^*(f^*) = 1.68x10^{-4} \text{ fm}^{-3.27}$$

The duration of the wind sea, the wind stress consistent with this duration and the analyzed wind sea energy $E_a$ are deduced from the previous dimensionless relations.

If wind sea is more energetic than swell then $A = \frac{E_a}{E_p}$ and $B = \frac{f_{mp}}{f_{ma}}$, where $f_{m}$ is the mean frequency. If dominant swell is found, A and B are processed in such a way that the average steepness $s = \frac{E_k}{4\pi^2}$ of the spectrum is not modified except for large variations of SWH. Therefore:

$$A = \frac{(E_a/E_p)}{B} \ , \ B = C\left(\frac{E_a}{E_p}\right)^{1/4} \quad C = 1 - \left(\frac{2}{\sqrt{E_a}}\frac{2}{\sqrt{E_p}}\right)$$

The method described above relies on three main assumptions:
1) the ratio among parts of the first guess spectrum having a different origin is correct
2) the duration of the wind sea is correct
3) the wind direction is correct

The Lionello et al (1991) approach is different from the one proposed by Thomas (1988). In Lionello approach, the derived altimeter wind speeds are used only as a consistent criteria instead of being introduced in the data assimilation scheme to estimate the wind sea. The Thomas approach enables therefore to modify the ratio of the swell
to the wind sea. However, it was preferred not to rely on altimeter wind speed in order not to introduce shocks in the model. Consequently, if some swell is missing, the method would not be able to compensate this error. The impact of such assumptions are discussed in a further section.

4. THE EXPERIMENT

In our experiment, we used only analyzed wind fields to drive the WAM model. However, the outputs of the model are referred to as forecasts (in the classical way for wave modelling they are incorrectly referred as analysis), as long as data are introduced in the model through the assimilation scheme. The purpose of our study is not to qualify the wind forecasting. So, analyzed wind fields are adequate for our purpose. The model has been running from the 15th to 30th of December. During the three first days the model was spun up and the study period stated on December 18 at 1200 UT. Two experiments were carried out. A reference run was done without assimilation, then the altimeter data were introduced in the analysis. The period we chose for our experiment exhibited extreme conditions with an evolution of the weather patterns that led to a succession of exceptional storms in the Mediterranean sea. This windy period started on December 19 at 00 UTC as a cold front crossing the eastern Europe from North–West to South West reached the Mediterranean sea. At the rear of the front, a strong Mistral was canalized between Baleare and Corsica. A cyclogenesis in the gulf of Genova strengthened the intensity of the wind. As this depression travelled across the Mediterranean sea, another front associated with another cyclogenesis was responsible for an other intensification of the Mistral on December 20 early in the morning. Two other cyclogenesis occurred respectively near Libya on December 25 and near Sicilia on December 24 and generated strong winds mainly in the Eastern part of the Mediterranean sea. The maximum intensity of the wind analyzed by the ECMWF model was more than 25 m/s on 21 at 12 UT. In some area, the wind was often stronger that 20 m/s during the whole Period. The trajectories of the four storms are shown in fig (5).

The wind fields used in this experiment were produced by the analysis of the ECMWF operational model whose horizontal resolution is about 70 km. The ten meters winds were extracted at the main synoptic hours (00, 06, 12, 18 UT) from the operational archives on a regular latitude-longitude grid with 0.5x0.5 degree resolution. These winds are used as the input of the WAM model implemented on the Mediterranean sea with the same resolution as the winds. The Cycle 4 of the WAM model, including the physic introduced by Janssen (1990) to correct the total stress as depending on the sea state, has been used. The discretized spectrum consists of 25 frequencies in geometric progression (f1 = 0.05 Hz, fn+1 = 1.1.fn) and 12 directional bands
with 300 resolution. The shallow water effects are parameterized depending only on the bottom friction.

The ERS-1 altimeter derived SWH and wind speed are computed every second along the track. Due to the speed of the satellite, the distance between two data is 0.05 degrees corresponding to a 5 km resolution. The wind speed has been derived from the radar cross section using the Chelton-MacCabe algorithm. Altimeter sea state measurements are computed on board of the satellite from the slope of the mean return signal. A statistical comparison of the ECMWF and WAM hindcasts with the ERS-1 data set during the period 16-28 of December has been carried out. Results are summarized on figure (4). These results are fully consistent with those obtained with other models (particularly the operational french models). Recent results of comparisons between ERS1 altimeter data and buoys moored in the North sea for the ERS-1 calibration experiment RENE91 (September-December 1991) exhibit similar statistics. The underestimation of high waves by the Geosat altimeter has been also noticed by Guillaume et al. (1990) and Guillaume et al. (1991). In the last paper, it was underlined that the ability of both fine mesh model and altimeter data to describe the high variability in the Mediterranean sea was mostly due to the complexity of the surrounding topography. However a higher variability of the satellite data is observed after the data have been averaged to match the model grid size. This gives an indication of the potential interest of assimilate altimeter data. The examinations of the structures of the SWH field leads to an estimation of the correlation length for the prediction error. The normalised autocorrelation function \( c(h) \) has been processed:

\[
c(h) = \frac{E(H^*_{\text{mod}}(x+h)H_{\text{mod}}(x))}{E(H^*_{\text{mod}}(x))^2}
\]

where \( H^*_{\text{mod}}(x) = H_{\text{mod}}(x) - E(H_{\text{mod}}(x)) \)

The function (fig. 3) is consistent with the prediction error matrix with \( L_{\text{max}} = 4 \) if we assume that the prediction error correlation are related to the spatial correlation of the model SWH corrected of the "climatology". As no dramatic impact was found (Lionello et al. 1991) on the value of the ratio of the model errors to the observation errors, we chose \( c_{pi} = c_{oi} = 0.5 \) m.

5. RESULTS

It would be too long to describe here in details, the result of our experiment at each output time step (6 hours). Therefore, typical cases have been selected to describe the major features of the assimilation experiment results. For each cases the surface wind and the wave fields from the reference run and the differences with satellite data along the tracks are shown. To investigate how the
model loses the extra information it gained through the assimilation scheme, the differences between SWH after and before assimilation are computed at different times (figures 6–10). The retrieved wind field (the total wind stress is modified in case of wind sea) is also shown. In the first case (December 23 at 1800 UT) a straight effect between Greece and Crete is strongly underestimated for both the wind and wave model fields. Two ship reports are in good agreement with the satellite data. The track is crossing the young D3 depression that took place near the Lybian coast 12 hours before. The extra information for SWH reached 0.85 m. Under the enhancement of the wind due to a cyclogenesis, the benefit is reduced to 25 cm 6 hours later and to 10 cm 12 hours later. Only points where wind sea is dominant have been concerned by the correction of the wind. In the Paper of Janssen et al. (1989), it was found that the typical time scale for losing the extra information was 12 hours. In our experiment, the strong intensity of the phenomena is responsible for a lower time scale (between 6 and 12 hours). No significant improvement is found using the retrieved wind speed to drive the model during a half wind input time step (3 hours). This result agrees with the typical time scale for generating new wind sea as shown in fig. (2). On December 24 at 12 UT, the track is crossing the axis of the Mistral between Corsica and Sardaigna. The maximum SWH at the middle of the track is underestimated by the altimeter data whereas the winds are in fair agreement. In the southern part of the track, swell is dominant so no correction has been done on the wind. The strengthening of the wind due to the cyclogenesis in relation with the depression D4 near Sardigna is responsible for the benefit of assimilation to vanish within 12 hours. The 21 at 0600 UT, swell (according to ship reports) is underestimated by the model. In the extreme eastern part of the basin, the wind remains weak during a 12 hours period. Swell introduced by the assimilation scheme is travelling eastward until it reaches the coast. 12 hours later, the loss of extra information is only reduced by 15%. The wind is not modified and this is an indication that only swell was produced by the model and that is consistent with some ship reports. In the two cases(19 at 25 at 1200 UT) shown in fig (6,8), though the ECMWF winds are in good agreement with the ERS-1 altimeter winds, the wave model predictions are too low in some parts of the tracks were the fetch is very limited (in the Adriatic sea, and between Italia and Sicilia). Consequently, a dramatic overestimation of the retrieved winds is noticed. This also happens when the swell is missing in the reference run (27 at 1200 UT) in such a way that the ratio of wind sea to the swell is completely wrong. The assimilation scheme produces wind sea instead of swell to compensate the SWH error. The retrieved winds are not realistic according to the ship reports. These three last cases underline the limitations of the method. To summarize the results, correlations have
been computed for the reference run and for the result of assimilation:

<table>
<thead>
<tr>
<th></th>
<th>reference</th>
<th>assimilation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>r(Usat,Umod)</td>
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<td>0.79</td>
</tr>
<tr>
<td>r(Hsat,Hmod)</td>
<td>0.87</td>
<td>0.97</td>
</tr>
<tr>
<td>r(Umod,Hmod)</td>
<td>0.76</td>
<td>0.65</td>
</tr>
<tr>
<td>r(Usat,Hsat)</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>r(Usat–Umod,Hsat–Hmod)</td>
<td>0.47</td>
<td>0.19</td>
</tr>
</tbody>
</table>

The benefit of the assimilation for SWH is clearly shown by the increasing of the correlation between $H_{\text{sat}}$ and $H_{\text{mod}}$ (0.97 instead of 0.87). However no improvement was found in the correlation between retrieved wind speed and altimeter derived wind speed. This is consistent with the poor correlation found between the SWH error and the wind error (0.47). Because swell is responsible for the partial no correlation between wind and waves (respectively, 0.76 and 0.77 for the model and the satellite in the reference run) the improvement we can expect for the retrieved wind speed cannot produce a correlation coefficient very close to one, as it is the case for the SWH after assimilation. An other important reason for the absence of improvement of the statistics for the retrieved wind speed might be due to wrong assumptions in the assimilation scheme. We have verified this fact in two kinds of situations: the first one, when some swell is missing in the first guess in such a way that the ratio of the wind sea to the swell is wrong, and the second one, when the model does not reproduce the exact wind sea due for example to a bad estimation of the duration. Some failures in the model might induce such bias in the retrieved wind speed too. A possible explanation for the wrong evaluation of the swell contribution to SWH could be the excessive diffusion noticed in the WAM model (Lionello et al 1991) due to a first order scheme used for propagation and a possible incorrect dissipation function tuned to reproduce the Pierson–Moskowitz spectrum at the final stage of the wave growth (Lionello et al 1991). So it was found that the WAM model overestimates the wind sea and underestimates the swell.

6. CONCLUSION

We found that the method is an interesting tool to validate a wave model (propagation, growth, dissipation ...) through the cross validation of altimeter derived wind speed with the retrieved wind speed.

In this first study it appears that altimeter data assimilation has a benefic impact, especially during the wave growth period. we found that the benefit of the assimilation is kept during a maximum time when the sea state mainly consist of swell. For the swell, the time scale for losing the extra information that the model gained
tough the assimilation process is only limited by the size of the Mediterranean basin. Unfortunately, the method is not able to compensate large inaccuracies in the prediction of swell. However, this problem could be partly solved by rectifying the spectrum at the time and place where wind sea was generated and where the method is very efficient because of the strong coupling between the waves and the wind field. This implies a large coverage in time and in space of altimeter data, depending on the scale of the meteorological phenomena. So the impact of assimilation must be more evident in the Atlantic and Pacific oceans where storms have a larger spatial extent than they have in the Mediterranean sea.

In the future it would be of interest to investigate the impact of data assimilation for extreme conditions over oceans (with hurricanes).

Because swell has a long memory, its estimation by measurements is important to improve wave forecasting, and radar ocean wave spectrum measurements will be of importance in future works about data assimilation in wave models.

7. ACKNOWLEDGMENTS

I would like to thank Dr Heinz Gunther for making the software, the documentation for WAM and the draft of the manuscript of the assimilation study (Lionello et al 1991) available and also for his kind help during my visit at the ECMWF.

8. REFERENCES


Figure 1. Schematic description of the data assimilation scheme.
Figure 2. Dimensionless wave energy as a function of the dimensionless time (A) and wave energy as a function of time for a 10 m/s wind speed (B).

Figure 3. Autocorrelation for the model SWH along the satellite tracks as a function of the distance.
Figure 4. Comparison of model (without assimilation) and altimeter data: for SWH, bias (A) and standard deviation (B); and for wind speed bias (D), and standard deviation (D).

Figure 5. Tracks of the satellite during on repeat period cycle (3 days) over the Mediterranean sea (A), trajectories of the storms during the period 15-30th of December 1991.
Figure 6. ECMWF wind (A), WAM wave height (B), retrieved wind (C), extra information gained after assimilation for SWH (D), compared with altimeter measurements (black squares) along the satellite track.
Figure 7. As figure 6, but for December 21 at 06 UT.
Figure 8. As figure 6, but for December 24 at 12 UT.
Figure 9. As figure 6, but for December 25 at 12 UT.
Figure 10. As figure 6, but for December 27 at 06 UT.
1. INTRODUCTION

Measured wave data for New Zealand waters are scarce. The coastline is long, the wave climate is varied and in many instances quite severe. However, there is only one site (the Maui–A oil and gas production rig off the west coast of Taranaki) for which anything approaching a respectably long wave record exists. Further, it is an expensive proposition to establish an adequate wave measurement programme from scratch and such a programme would take at least a decade to bear fruit.

The successes of using numerical wave modelling to synthesise wave climatologies have been evident elsewhere and are well documented. See for example Ewing et al, 1979, Panchang et al (1990), Swail et al (1989). It is apparent that attempts to establish a wave climatology for New Zealand waters will rely heavily on this method.

To successfully embark on such a project there are three major requirements. Firstly, accurate wind-fields over the entire hindcast period are essential. Whilst many wave models claim to do an effective job, they are all totally dependent on the input winds. The wind is the only external forcing field and wave modelling is a prime example of the adage that “rubbish in gives rubbish out”. Unfortunately, modelling the waters around New Zealand is handicapped by a paucity of surface observations, making it often awkward to assess the winds accurately.

Secondly, a good wave model must be developed. The model must accurately represent wave growth, decay and propagation, and must be able to respond to extreme wind forcing. Wave modelling is sufficiently well advanced for there to be a number of good models available, all of which perform reasonably well in varied conditions. The more sophisticated models such as the 3rd-generation WAM model (the WAMDI group, 1988) gain much of their performance from an ability to accurately calculate the redistribution of energy throughout the spectrum i.e. to model the effects of weakly nonlinear wave-wave interactions. Unfortunately, such models require considerable computing power. For modelers with more modest resources a necessary compromise is to seek simple parameterisations of the nonlinear interactions which can be effectively implemented in wave models. Such a model has been developed and is described in a companion paper (Laing, 1992).
Finally, some confidence in the model is necessary and this may only be obtained by verifying it against measured data.

This paper describes the hindcast procedures for a 10 year hindcast of waves in New Zealand waters. It outlines the methods used to analyse wind-fields over the area covered by the model, and details a pilot study made covering a 5 month period in 1989.

2. WIND FIELD ANALYSIS

It is imperative to supply wave models with the best possible input winds. Errors in winds are magnified in the waves they produce. For example, an error of 10% in wind speed can give errors of 20% in wave height.

For the full hindcast period, surface pressure fields are available over the wave model area. These originated either from analyses made to initialise the numerical weather prediction (NWP) model of the New Zealand Meteorological Service (NZMS) or from analyses made by the European Centre for Medium range Weather Forecasting (ECMWF). For the pilot study period of May to September 1989 the NZMS analyses were available on the same grid as the wave model.

These fields were used as first guesses in a re-analysis procedure. This procedure involved several steps. Firstly, the pressure field was displayed on a VAX 3100/38 workstation to which a digitiser was attached. A copy of the manual analysis of the pressure field was placed on the digitising tablet and a further display of all synoptic data was presented on a second screen. This included buoy and ship data which may not have been available at the original analysis time and so was not plotted on the manual analysis. Satellite images for the relevant time were also on hand. A check of the manual analysis was made and a “best” manual analysis produced. This was then compared to the “first-guess” from the NWP model and sections of the “first-guess” requiring modification were traced from the manual hardcopy using the digitiser stylus. Spot values or sections of isobaric contour could be entered. These repairs were automatically displayed on the monitor. The contributing information is shown in the schematic Figure 1.
Figure 1. Schematic of the data sources employed in re-analysing surface fields.

The digitised data were then treated as new data and assimilated into the first-guess field. Assimilation was by optimal interpolation. Some control was exerted over the scheme via a number of parameters including the domain over which the data could influence a grid point, the limiting of data points allowed to influence a grid-point and the data rejection criteria. Since the new data was to be given high weighting, so that it over-rode existing values in the "first-guess" field, its error was assumed low. This enabled surface pressure fields to be forced into the shape determined by the new data. The domain of influence of the data was kept quite small so that local changes could be made without distorting the field at a distance.

Surface wind speeds derived from the radar altimeter on board the GEOSAT satellite were available during the pilot study period. These were derived from the altimeter-backscattered radar cross-section using the "smoothed Brown" algorithm (Goldhirsh and Dobson, 1985). For a window of 3 hours either side of the analysis time these winds were also plotted on the display (as numbers at regular intervals along the sub-satellite track). Care had to be taken when using this source of data as there are occasional blemishes which must be recognised. Once these are eliminated the data compare quite favourably to surface measurements from buoys. Dobson et al (1987) noted root-mean-square differences of 1.7 m/s for measurements separated by less than 50km.
Once a satisfactory surface pressure analysis was obtained the 10 metre winds were deduced. This entailed calculating the gradient wind-field and applying a diagnostic boundary layer model (Cardone, 1969) to derive the near surface winds. Discrepancies in the surface wind field could then be amended and the winds filed for direct input into the wave model.

Since the initial NWP fields were only available 6 hourly, interpolation was necessary to provide winds for the 2 hour timestep used in the model. Mean winds defined as constant over a long timestep (6 hours) imply steady growth for this timestep. However, during this period the wind will fluctuate considerably and wave growth during the period will be dominated by the occasions when the wind was higher than the mean. Further, wind-stress, which determines the input of momentum into the wave-field, is proportional to the square of the wind-speed. Accordingly, it is appropriate to use a root-mean-square interpolation for the wind-speed.

3. WAVE MODEL OPERATION

The wave model used in the hindcast is a second-generation spectral wave model with explicit parameterisation of the weakly nonlinear wave-wave interactions. The energy density spectra for the wave-fields are defined in the frequency-direction space. The model is described by Laing (1992). For this hindcast application, high frequency wave energy (above 0.35Hz) plays an insignificant role. Further, given the computing resources available and seeking a realistic balance between spatial, temporal and spectral resolution a total of 15 frequencies ranging from 0.045Hz to 0.32Hz defined by 0.045x1.15(n-1) are used. Directional resolution is set at 20° (18 bands). The integration timestep is 2hrs.

The grid was selected to cover the entire New Zealand Exclusive Economic Zone (EEZ) with sufficient space around the borders to ensure that most events generating waves which affect the New Zealand EEZ are captured. The grid was taken from a polar stereographic projection with a grid spacing of 190km at 60°S. This was chosen to coincide with part of the grid of the NWP model of the NZMS. The wave model grid has dimensions of 39x29.

Since there are no inflow data at the boundaries of the model, results near the open ocean boundaries will be suspect. Hence, best results are only expected at the centre of the grid. As this is also the main area of interest a subgrid is defined and only within this region are full model spectra saved. This subgrid is shown in Figure 2. The gridpoints over land are marked with grey circles. The wave climatology will eventually cover this area.
4. PILOT STUDY

The re-analysis procedure was followed for every 6 hourly analysis during the pilot period (mid-May to September 1989) and the wave model was then driven by the winds thus derived. The results were archived and various spectral parameters diagnosed for each gridpoint. For the 580 analysis times considered some modification to the machine “first-guess” analysis was made on 169 occasions. This does not imply that the analysts were totally satisfied with the analyses for all the other times but rather that they were satisfied with those features which could generate waves affecting New Zealand waters.

During this period two sources of data were available for verification. A Waverider buoy, which had been deployed in Western Foveaux Strait (see Figure 2) provided a time series of one-dimensional (frequency) spectral data. As noted in Laing (1992) the site was not fully open to the ocean in all directions it was necessary to apply a filter to the model spectra before reasonable comparisons could be made.

Also, as noted previously, data from the GEOSAT radar altimeter were available. Significant wave heights are the only wave parameter commonly utilised from this source but they are considered reliable with root-mean-square (rms) deviations from buoy measurements of less than 0.5m (Dobson et al, 1987). Significant wave heights derived from the model were compared to the GEOSAT data as described in Laing (1992).

Since surface-based wave data were only available from a single site, and even then partially sheltered, the comparisons against GEOSAT data provided valuable additional information about the performance of the hindcast procedures, particularly in open ocean. Further, whilst the buoy data gave a temporal comparison the satellite comparisons tested spatial characteristics of the model: each satellite pass is almost instantaneous (a few minutes) compared to the timescales involved in the model, and the comparisons covered the whole region shown in Figure 2.
Figure 2. Central sub-grid of the wave model grid. In this area full frequency-direction spectra are archived. The Waverider site is marked with the "x" and the open circle ○ indicates the gridpoint nearest the site.

Some of the statistics from comparisons of model results with data from both sources are included in Table 1 (upper value in each entry).

It is instructive to quantify the impact of the wind analysis procedures. For this purpose a separate run of the wave model was made using winds derived directly from the NWP "first-guess" fields. The results of this run were then compared with those obtained from the run where the winds were derived from the procedure incorporating full manual intervention. The results are included in Table 1 in the second row for each entry.
Table 1 - Verification statistics for significant wave height ($h_s$) and mean frequency ($f_{ave}$, defined here as inverse of mean period). The mean and standard deviation, $\sigma$, of the observations are given followed by the bias, root-mean-square difference (RMSE) correlation coefficient ($\rho$), scatter index (SI) and the number of comparison points ($N$). The upper entry of each pair is for the model operating with winds derived using the re-analysis procedure and the lower with winds directly, from the "first-guess" field.

<table>
<thead>
<tr>
<th>MEASURED</th>
<th>MODEL PERFORMANCE</th>
</tr>
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<tbody>
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<td>MEASURED</td>
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<tr>
<td></td>
<td>Mean $\sigma$</td>
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<tr>
<td>h_s (m)</td>
<td>2.52 0.97</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>f_{ave} (Hz)</td>
<td>0.096 0.014</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>h_s (m)</td>
<td>3.10 1.16</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At the Waverider buoy site the root-mean-square (rms) difference for significant wave height was reduced from 0.68m to 0.60m and the correlation coefficient increased from 0.78 to 0.80. The former is an obvious improvement and the slight increase in correlation is statistically significant at the 76% level (according to the distribution of the variate derived from the correlation coefficient by applying the Fisher variance stabilising transformation). For the mean frequency the increase in correlation coefficient from 0.35 to 0.43 was significant at the 96% level.

Although these statistics show improvement, it is also evident that the mean frequency increases slightly, increasing the bias and the rms difference. This points to a possible weakness in the model and for this site care should be taken when interpreting modelled frequency parameters. The bias accounts for 36% of the rms difference leaving a residual of 0.014Hz. It is possible that the filter applied at this site could be partly responsible, but for the model in general there is no simple explanation. This will be further discussed in the next section.
The frequency (probability) distributions for various sea-state conditions are often used as a means for displaying aspects of a wave climatology. In Figure 3 the distributions for significant wave height are shown for the Waverider buoy data (solid line) and the model data as filtered for this site (dashed line). The comparisons reflect a problem with the model failing to pick out certain events where the significant wave height was in the range 4.5 to 5.5 metres. The model appears to have compensated in the range 3.5 to 4.0 metres. This is borne out by a close examination of the full time series for the pilot period. Possible factors influencing this will be discussed in the next section.

The significance of the difference between the two distributions can be quantified by considering the random variable

\[ \sum_{j=1}^{N_{\text{box}}} \frac{(N_{i1} - N_{i2})^2}{(N_{i1} + N_{i2})} \]

where \( N_{ij} \) are the frequencies in the \( i \)th box and \( j=1,2 \) refer to the model and buoy data respectively. This random variable is chi-squared with \( (N_{\text{box}}-3) \) degrees of freedom. \( N_{\text{box}} \) is the total number of boxes where either distribution is non-zero for that box. For the data shown in Figure 3, where each box spans 0.5m, the value of the random variable is 25 with 15 degrees of freedom. This difference would be consistent with an assumption of the same population for both distributions only if a 95% significance level were adopted.

5. DISCUSSION

Notwithstanding any limitations on the wind-fields, the construction of a wave climatology requires that the wave model does not give biased results. The comparisons in Table 1 indicate that the present mix of input winds and wave model do not bias the height of waves for
the one fixed verification (buoy) site considered here although the frequencies are biased high by 0.017Hz. It is possible that this is a result of the crude filter applied to the model spectra. A more sophisticated filter requires accurate local wind information and this is not available. Thus, only further open-ocean verification sites would resolve the problem.

The comparisons with GEOSAT derived wave heights indicated that the bias averaged over the whole region is only a quarter of a metre. The magnitude and sign of this bias is consistent with that found in comparisons between the GEOSAT data and buoy measurements by Dobson et al (1987) which indicated on average a higher value for buoys by 0.36m.

The model must also replicate extreme events. As noted in the previous section there was some apparent difficulty in underestimating events in the 4.5m–5.5m significant wave height range. During the pilot period all situations for which there was a discrepancy of more than 1.5m at the Waverider buoy site were further investigated. The situations fell into two general classes; those where extreme winds were very local to the region immediately around the site and those where the analyses showed no obvious fetch from where the waves could originate. In the former case the wave model is unable to resolve such localised features and the hindcast is not expected to give the accuracy in inshore waters that we hope to achieve offshore. The latter case, provides more problems since the wind events were not identified in the analyses. In all cases a data search indicated that there was a total lack of measured information in the regions from which the waves must have originated. The analyses were therefore too smooth and winds lighter than necessary to produce the observed waves. Although it is theoretically possible to use wave measurements to infer winds, even at a distance, it is impractical given the computing required. Furthermore, there is little wave data to go on. The prospect of wind-scatterometers aboard such satellites as ERS-1 provide some hope that this situation will be improved in the near future.

The mediocre performance of the model in reproducing frequency parameters, given the good results in simple model tests as indicated in Laing (1992) leads to the possibility that the balance of energy in the spectrum may be significantly influenced for this site by a large swell component. Whilst the model may be failing to detect events in the Southern Ocean which produce low frequency wave energy affecting this site, it is possible that wave energy generated outside the grid area has a significant presence even at this distance from the boundary.

Having highlighted the value of optimising the input wind fields it is also evident that there are limits to the accuracy which can be
achieved. This is particularly pertinent in a region where there is little accurate meteorological information over the sea. The consequent limitations on wave hindcasts derived on the basis of surface analyses in this region have to be accepted.

The importance of specifying input winds does not relate only to the accuracy of the input wind but also to the appropriateness of the wind statistic used to represent the wind over a timestep. A wave model may perform very well for winds specified over very short timesteps (such as 15 mins). However, if the wind is only specified over a much longer period (6 hours) the mean wind over this period is not necessarily the most appropriate wind statistic to use. Wave generation is not a linear response to the wind. The wind fluctuates and the growth during a timestep will be dominated by the instances when the wind is higher than the mean. Thus, whilst the winds derived from synoptic analyses may be accurate, assuming such winds as a steady mean over the whole period or linearly interpolating to smaller timesteps may well result in underestimation of wave conditions. Wave models are, in one way or other, often tuned to the timescales of wind specification as well as propagation and source term integration. The winds used to drive the model in a particular application must be specified consistently with the model tuning.

6. HINDCAST STORM SELECTION

Whilst the dearth of data in the Southern Ocean limits the analyses and hence the determination of wind fields, the largest return for effort in the re-analysis procedure is still with the more severe events. Modifications during moderate wind conditions have little or no effect on the resulting wave climatology. Thus, in view of the impracticability of re-analysing every single wind-field for a 10 year hindcast, the strategy adopted was to identify and re-analyse events which will promote extreme wave conditions in New Zealand waters. For the rest, the input winds can be adequately derived directly from the "first guess" fields taken from NWP models.

The object of the selection process was to identify those storms over the 10 year period 1980-1989 which had the most potential for generating large waves in the waters immediately surrounding New Zealand. Thus, it was necessary to identify storms with winds exceeding some specified threshold where the winds were either active in these waters or were directed towards them.

A three stage process was used. Firstly, some simple objective selection criteria were specified and the 12 hourly surface pressure analyses from ECMWF were searched. Two thresholds were set to identify dates of possible interest:

(a) The gradient wind was "aimed" at New Zealand waters and had a speed exceeding 60 knots (32m/s)
(b) Pressure centres in the region 27.5°S – 55°S and 140°E – 160°W were lower than some set latitude-dependent values. A simple formula was derived which had the values 955hPa at 55°S, 965hPa at 50°S, 980hPa at 40°S and 990hPa at 30°S.

Secondly, an independent manual search was made of daily charts for events which the analyst considered may be of interest. The two sets of dates were then given a closer examination to ensure that events would indeed be significant in the waters around New Zealand, and to ensure a reasonable spread of events affecting both the east and west coastlines. From these a total of 44 events were finally identified for the years 1980-1988 these are listed in Table 2. 1989 events were largely covered by the pilot study.

A number of broad categories were identified into which the events could be classified. These are listed below with some of their common attributes.

W A westerly belt at about 45°-55°S; often giving gales from a westerly quarter on the west coast of the South Island.

TT Tasman trough; northwest to southwest winds on the west coast of New Zealand

LMT Mid-Tasman low; often moving southeast and giving northwesterly winds on the west coast of the North Island and sometimes northeasterlies on the northeast coast.

LE Low to east of New Zealand; often characterised by southerly gales on east coast.

LST Low of subtropical origin; usually moving to the east and giving northeast to easterly winds on the east coast of the North Island

ETC Ex-tropical cyclone; often with quite a southerly component to its movement.

C Coastal feature; orographically enhanced and close to coast giving very localised event.

These categories are matched against the events in Table 2. Also noted are the windspeed/direction or the pressure/latitude depending on which criterion caused the event to be selected. In cases where manual selection by the analyst identified the event an ‘M’ is noted in the wind column along with the wind direction (as a compass point). For some events both criteria were met and so wind and pressure are both given. Since the selection criteria were often met for several times during an event, the most extreme condition during the period is quoted.
Table 2. The events identified for special analysis. The categories referred to are explained in the text. The gradient wind speed (in m/s) and direction are as identified by the objective selection from ECMWF analyses. An 'M' in this column (sometimes followed by a compass point) indicates that the selection was made manually by the analyst. The final column gives the pressure and latitude as identified in the objective selection criterion.

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</tr>
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<tr>
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<td></td>
<td>973/45</td>
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<tr>
<td>7 November 1980</td>
<td>C</td>
<td></td>
<td></td>
</tr>
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<td>M WSW</td>
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</tr>
<tr>
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<tr>
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<td>M</td>
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<tr>
<td>15-18 September 1985</td>
<td>MTL</td>
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7. SUMMARY

The procedures used in optimising the input winds for hindcasting wave conditions during a pilot study period have been detailed and the results achieved during that study have been discussed. Since there were only two rather different sources of information for verifying the model it is difficult to generalise. The Waverider buoy site was exposed primarily to waves originating in the Southern Ocean and so any deficiencies in the analyses for that region were highlighted. The interpretation was complicated by the limited window to the open ocean from that site necessitating the use of a filter on the model spectra.

Further, the purpose of the project is to derive the "offshore" wave climatology. Hence it is not expected that accurate estimates will be derived inshore (within 1 gridlength of the coast). Specific site studies will be needed for these waters with the present climatology providing the offshore boundary conditions.
Whilst the satellite data from GEOSAT gave much better spatial coverage it gave no information on frequency parameters.

The full hindcast will cover a 10 year period from 1980–1989. 44 events which may generate severe wave conditions in New Zealand waters have been identified during this period. These have been re-analysed and a data base of surface wind fields established for driving the wave hindcast.

8. ACKNOWLEDGMENTS

I would like to thank Cliff Revell for his analyses during both the pilot study and the selected storms and Garth England for his contribution to the former. I also appreciate the permission of B.T.W. Associates to use the wave data from Western Foveaux Strait.

9. REFERENCES


VALIDATION OF THE HINDCAST APPROACH

TO THE

SPECIFICATION OF WAVE CONDITIONS AT THE MAUI LOCATION

OFF THE WEST COAST OF NEW ZEALAND

1V. J. Cardone and 2K. C. Ewans

1Oceanweather Inc
Cos Cob, CT

2Shell Todd Oil Services
New Plymouth, NZ

1. INTRODUCTION

The hindcast approach has been widely embraced by the coastal and offshore industry as the only reliable way to obtain extreme data for coastal and offshore structure design. For example, major hindcast studies have been completed recently which considered the east coast of Canada for Hibernia (Cardone et. al, 1989), the west coast of Canada (Swail et al, 1991), the Gulf of Mexico (Joint Industry Projects (JIP) GUMSHOE and WINX) and the North Sea and adjacent waters (the JIP known as NESS).

Most hindcast studies usually include a validation phase in which high-quality wind and wave measurements are used to validate, and if necessary, tune the wind and wave hindcast method to be used before they are applied to a population of historical storms. Two types of verification statistics are usually reported: so called peak-peak measures of difference (most commonly bias, and scatter index, SI, which is the standard deviation of the differences divided by the mean of the observations) between hindcast and measured storm peak surface winds, significant wave height (HS) and peak spectral period (TP) or average period (TM), and the same measures based upon comparison of full time history comparisons of the same variables. Comparisons of frequency and directional spectra in storm hindcasts are less commonly reported. It has become quite common for validation studies involving hindcasts of tropical and extratropical storms in NH basins to yield mean errors of less than 0.5 m (1 sec) and SI of 10–15% (10–15%) in HS (TP) (e.g. Reece and Cardone, 1982; Swail et al. 1991).

There are special difficulties associated with the application of the hindcast approach in the SH, including: (1) the sparsity of historical marine meteorological data relative to the data base of mid-latitude NH basins; (2) the inhomogeneity in time in the quantity and quality of historical marine data over the past two decades, associated with the introduction of polar and geostationary satellites, the massive
deployment of drifting buoys in 1979 during the First GARP Global Experiment (FGGE) and the ensuing diminution of the FGGE network; (3) the potential large impact of the temporal variability and accuracy of historical archived surface weather maps on the process of identification and ranking of historical storms to be subjected to hindcasting; (4) the lack of guidance from previous studies involving validated models and measured data as to the sufficient attributes of a wave hindcast model and the maximum accuracy achievable in historical storms.

A vital component of the Maui Hindcast Study was therefore the validation phase. Shell Todd oil Services Ltd. (STOS), who commissioned the study, had been acquiring accurate wind and wave data since 1976, associated with the operation of the Maui A offshore platform; these data were used to validate the hindcast approach, by comparing numerical model predictions of the wind and wave conditions in the Maui field during storms in which wind and wave data had been recorded.

2. THE MAUI LOCATION

The Maui gas and condensate field is located approximately 30 km off the West Coast of the North Island of New Zealand in around 110m of water. Past analyses of the Maui environment (Kibblewhite et al., 1982) established that the wave field can vary significantly throughout the area due to the orographic influences of Cook Strait to the southeast and the South Island to the south and to a high level of long period southwesterly swell which has its origins in the Southern Ocean and exists as a more or less permanent component of the wave field (Ewans et al., 1988). Locally generated waves from the storms within the region are superimposed upon this background swell.

To adequately account for both swell and local seas, the wave hindcast model was adapted to a nested gridding scheme employing a coarse grid covering 20S–60S; 115E – 175E (Figure 1) and a fine grid covering 37S – 42S; 169E – 176E (Figure 2).

3. AVAILABLE DATA

The repositories of historical meteorological data of the following centers were utilized for data of the indicated types:

A. New Zealand Meteorological Service (NZMS), Wellington, NZ:

1. 6-hourly regional mean sea-level pressure (MSLP) analyses for selected periods;

2. synoptic surface weather reports for selected land stations and periods;
3. synoptic surface weather reports from transient ships for selected areas and periods;

4. hourly-averaged surface winds derived from anemogram records of selected land stations during selected periods;

5. hourly-averaged surface winds derived from anemogram records at Maui-A for the period of record.

B. Australian Bureau of Meteorology (ABM), Melbourne, Australia:

1. 12-hourly hemisphere MSLP analyses for period of record;

2. 6-hourly regional MSLP analyses for selected periods;

C. National Oceanic and Atmospheric Administration, (NOAA) National Climatic Data Center (NCDC), Asheville, North Carolina, USA:

1. 12-hourly hemisphere MSLP analyses (1965–1969);

2. 12-hourly (6-hourly 1979 on) low cloud motion tropical strip analyses;

3. synoptic surface weather reports from transient ships.

The NZMS and NOAA charts were found to complement each other. Plotted surface data from land and ships are legible on both series, while the NOAA charts add low-level winds derived from cloud motions. These winds may be reduced to nearly the equivalent of winds observed from transient ships. The ABM hemispheric MSLP charts provide less detail than the NZMS or NOAA NMC maps. The isobar interval is 10 mb, versus 5 mb for the NZMS charts (the NOAA maps show streamflow lines), and are evidently based upon less underlying plotted surface data.

Ship reports are probably the single most important data type. Of most significance is the considerable non-overlap between NZMS and NOAA sources. The sum of both data sets provides a data base of ship reports which is considerably larger than that available in real time to either the NZMS or NOAA analysts. There is no particular trend with time discernable in the total number of 6-hourly ship observations available over the decade in which the validation storms occurred (1977–1987). While the basic density of ship reports in the area of interest is sparse compared with the density of ship reports in mid-latitude NH basins, the available reports combined with cloud-motion-derived winds seem to provide, at least north of 50S, a reasonable data base for reanalysis of a time and space continuous pressure field in historical storms.

The stations for which land station data were gathered were selected on the basis of perceived representativeness of the adjacent marine areas and completeness of the archived record on magnetic tape. The
land stations selected were Cape Egmont, New Plymouth, Ohakea, Wellington, Westport, Farewell Spit, and Hokitika. Hourly winds averaged from anemograms were obtained for these stations and also for the Maui platform.

In addition to the meteorological data provided by the Meteorological agencies, SBPT provided wind and wave data acquired on and in the vicinity of the Maui platform. Included in these data were:

1. Significant wave height and mean wave period parameters derived from measurements made at Maui-A and the proposed site of the Maui-B platform over the period 1976 to 1985.
2. Maui-A and Maui-B wave spectra
3. Maui-A directional wave spectra.

STOS data were used to validate the wind and wave hindcast methods in the selected storms. The continuous 1976-1985 data were also used to help select the validation cases, to refine certain aspects of the wind analysis procedures, and to help define the historical storm selection process.

4. Wind Field Specification

4.1 Background

Three methods for specification of surface wind fields in historical extratropical cyclones are described and intercompared in detail by Cardone et al. (1980). Two of the methods involved the application of a marine planetary boundary layer model (MPBL) to the calculation of surface winds from MSLP fields. In one of these methods the MSLP fields are derived by objective analysis of real-time data (as carried out at most NWP centers); in the second method, MSLP fields are subjectively reanalyzed using enhanced sets of historical surface weather observations. The third method available is direct kinematic analysis of the wind field, a subjective analysis process which relies heavily on wind observations, sometimes taking guidance from MPBL winds in data sparse regions. The relative performance of these methods has also been described more recently by Cardone (1991).

The standard analysis methodology adopted by Oceanweather in all of it’s NH studies of extratropical weather regimes is a mixture of methods 2 and 3. First, pressure charts are reanalyzed (typically at 6-hourly intervals) by an experienced analyst, after referring to all alternative analyses and the combined raw data sources. The reanalysis benefits greatly from the ability in a hindcast to impose continuity of evolution of major systems from backward and forward extrapolation (a real-time analyst can only look back in time) of observed data. The reanalyzed charts are placed on an x-y digitizing tablet whereon the isobars, each pressure center location and central pressure are
digitized. Gridded pressures at grid points are recovered by employing an objective fitting algorithm. The pressure gradient at each point, calculated by centered differences, is then entered to the MPBL along with the other required external variables to yield the surface winds.

The calculation of winds from pressure fields makes use of the MPBL developed by Cardone (1969) as later updated (Cardone, 1978). The model links the following external factors governing the near-surface flow in a steady-state, horizontally homogeneous MPBL: latitude (Coriolis parameter, \( f \)), surface roughness parameter \( Z_0 \), air-sea temperature difference \( (T_a - T_s) \), geostrophic wind vector \( (V_g) \) and horizontal temperature gradient \( (T_a) \). The roughness parameter is calculated implicitly as a function of friction velocity, \( U_x \). The air-sea temperature difference is supplied at each grid point through a parametric dependence on the local geostrophic wind direction and the horizontal temperature gradient is supplied from climatology.

As shown in previous studies, kinematic analysis provides the most accurate and least biased winds, primarily because the method allows a thorough reanalysis of the evolution of the wind field. Kinematic analysis also allows the wind fields to represent effects not well modelled by pressure-wind transformation techniques such as inertial accelerations associated with large spatial and temporal variations in surface pressure gradients and deformation in surface winds near and downstream of coasts.

Kinematic analysis is a tedious, time consuming, and costly manual process that therefore should be used sparingly. The approach used in problems involving extratropical cyclones in mid-latitude NH basin has been to derive MPBL winds for storm spin-up and spin down and perform kinematic analysis for the time period that the Generating area of storm waves is active at the region of interest. However, because the density of synoptic ship observations is typically too small to justify the application of kinematic analysis to the great expanse of the portion of the Southern Ocean to be treated in the Maui study, a somewhat modified approach was adopted.

4.2 The Maui Approach

Two phases are employed. In the first phase, a wind field is produced from the MSLP field through the application of the MBPL. These pressure-to-wind, or PRESTO winds, are developed at 6-hourly intervals directly on the COARSE and FINE wave model grids. In this phase, the MSLP fields are manually reanalyzed prior to digitization. In the second phase, winds on the FINE grid are developed for a selected interval of the total hindcast period through a manual kinematic analysis.

PRESTO Winds
The steps involved in the derivation of the PRESTO winds are:

1. definition of hindcast period (including the kinematic analysis sub-period);
2. the synthesis of synoptic data assembled; (in which data from all sources were hand plotted on the NZMS 6-hourly maps);
3. the production of a continuity chart (this depicts the movement and intensity of major centers of action);
4. the reanalysis of the isobar field (correcting for any gross departures from continuity or irregularities in isobar spacing);
5. the digitization of the isobars;
6. the computation of the MPBL winds.

KINEMA Winds

The steps involved in the derivation of kinematic or KINEMA winds are:

1. the preparation of the kinematic analysis base maps (consisting of the PRESTO winds plotted on the FINE grid points as standard wind barbs, and the hand plotted wind data from coastal stations, ships and Maui-A);
2. the kinematic analyses (in which the streamlines and isotachs in 5-knot intervals are constructed, and the wind speed and direction are gridded);
3. the entering and assimilation of the kinematic winds (involving the replacement of COARSE grid PRESTO winds with KINEMA winds at common points);
4. the production of KINEMA winds to the FINE grid wave model and to 3-hourly intervals.

Coastal Wind Transformations

Of the coastal stations for which data were obtained in the validation storms, only winds measured at Cape Egmont, New Plymouth, and Farewell Spit appeared to be correlated significantly with the measured winds at Maui-A. For these stations, and using only data available for the validation storms, a comparison analysis on pairs of measured winds formed from Maui-A and each coastal station was carried out. The results of the correlations provided a basis for the use of the coastal winds in the kinematic analysis for storms in which measurements at Maui-A were not available.

Standardization of Winds to Effective-Neutral 20-m Marine Winds

Wind variables supplied at wave model grid points represent winds in which turbulent fluctuations of time scales less than about one hour and spatial scales less than 100 km (except near the coast) have been filtered. The winds also are referred to height of 20 m. The simple concept of the “effective neutral” wind speed introduced by Cardone
(1969) is used to describe the effects of thermal stratification in the marine boundary layer on wave generation. The effective neutral wind speed is simply the wind which would produce the same surface stress at the sea surface in a neutrally stratified boundary layer as the wind speed in a boundary layer of a given stratification. The MPBL is set up to provide the effective neutral 20-m wind speed. Reports of wind speed from ships and rigs equipped with anemometers are transformed using the reported air-sea temperature difference and anemometer height. For ships which use the Beaufort system to estimate wind speed, a revised table of wind speed equivalents is used to retrieve the 20-m wind speed (Cardone et al., 1990).

**Interpolation of 3-hourly wind fields from 6-hourly wind fields**

The default algorithm for interpolation of winds from 6 hours to 3 hours is linear interpolation of zonal and meridional components and of the fourth power of wind speed. The adopted algorithm, which is applied first so that the default algorithm operates only in areas remote from low centers, regards the location and radius of the principal low centers identified for a storm (entered manually by the analyst). The program linearly interpolates the latitude, longitude, and radius to the intermediate 3-hour field under construction. For each grid point within the interpolated circle, winds are interpolated to appropriate non-grid points at times T–3 and T+3; the interpolation matches the bearing of the target point from the low center and the ratio of the distance from the low to the target to the radius of the cyclonic system; finally, a-midway linear interpolation is made between the interpolated circles at T–3 and T+3.

5. Wave Hindcast Models

5.1 Background

Fully-discrete spectral first (1G) and second (2G) generation models were applied. The third generation model was not tested since oceanweather’s adaptation of same was not available in time for use, and in addition it would have required an order of magnitude greater computer time to run than the 1G or 2G models. The 1G ODGP model (Cardone, Pierson, and Ward, 1976), is part of the family of fully-discrete spectral models originally proposed by Pierson, Tick, and Baer (1966). The formulation of the ODGP model has been described in detail in past studies, most recently in MacLaren (1985). The skill of the model has also been documented in numerous studies, including Reece and Cardone (1982), and more recently by Cardone and Greenwood (1987), wherein the characteristics of the model are compared to those of recent 2G and 3G models.

A 2G model developed by Oceanweather for an international wave model comparison program (SWAMP, 1985), and known as the SAIL model
has been calibrated against the same data base used for the ODGP model, and was also tested against the validation storm wave measurements in this study, along with the ODGP source term algorithm.

5.2 Spectral Model Adaptation

The 1G ODGP and 2G SAIL spectral models were adapted to the basin of interest on a nested-grid system consisting of a COARSE grid of 150-km spacing and a FINE grid of 30-km spacing. The grid systems are rectangular arrays of points laid out on a transverse Mercator projection. The models were adapted such that either 1G or 2G deep-water source term algorithms could be selected on the coarse grid, with both options sharing a common spectral discretization, propagation, and archiving schemes. Shallow-water propagation and source term physics are modelled on the FINE grid only. Also, on the FINE grid only, an alternate propagation system known as CAIPS (Capes and Islands Propagation System) was adapted to provide more accurate resolution of the irregular shoreline geometry, as it affects wave generation in fetch-limited situations and wave propagation nearshore. (CAIPS is an algorithm which provides an array of transmissivities for each frequency-direction band at fine-mesh points, and which account for propagation on-an implied hyper-fine grid, usually taken as one-third the grid spacing of the fine grid).

COARSE Grid

The COARSE grid wave model attributes are given in Table 1. The COARSE grid wave model provided a suitable framework for testing of alternate wind fields and growth algorithms. The COARSE grid wave model provides two-dimensional spectra at the boundary of the FINE grid, as required to carry out FINE grid hindcasts, as well as solutions within the interior of the FINE grid domain, including a COARSE grid point at the Maui-B site.

FINE Grid

The FINE grid attributes are also given in Table 1. Shallow water propagation and growth/dissipation effects are modelled on the FINE grid, which is nested within the COARSE. As in the COARSE wave model, propagation is modelled through an interpolatory scheme, (Greenwood et al., 1985), the coefficients required for which at each grid point are precomputed. Alternative tables of propagation coefficients were developed: one which included the CAIPS sheltering algorithm at grid points within two grid distances of the coast, and one which did not.

Spectral Growth Algorithms

Both the 1G ODGP algorithm (incorporated in the program as subroutine CMPE27) and the 2G algorithm (called subroutine CMPE41) were
implemented on the COARSE grid while only the ODGP shallow-water spectral growth algorithm was included in the FINE wave model.

5.3 Input

The COARSE wave models are driven basically by two files: (1) the input wind field; (2) the initial wave spectrum. The wind field is supplied in the form of files of effective-neutral 20-m level wind speed and direction at each COARSE grid point at 3-hourly intervals. The spectrum is assumed to be zero at all points at the start of a COARSE run.

The FINE model is also driven by two files: (1) the input wind field at 3-hourly intervals on the FINE grid; (2) a file consisting of two-dimensional spectra at all COARSE grid points along the open boundary of the FINE grid at all time steps of the COARSE model. The spectra in this file are interpolated to every FINE grid boundary point within the FINE wave model program.

5.4 Output

Each run of the wave models on a validation storm provides digital archive files of COARSE and FINE grid results of integrated properties of the wave spectrum at three-hourly intervals (significant wave height, peak spectral period, average period, significant wave period, dominant wave direction, directional spreading parameters). Directional spectra are archived only at FINE grid points.

6. VALIDATION

6.1 Validation Storm Selection

The validation storms were selected mainly on the basis of study of the wind and wave measurements made at Maui-A and wave measurements at Maui-B. A coarse screen of all storm occurrences observed in which the peak significant wave height equalled or exceeded 5 m yielded a total of 107 possible cases between September 1976 and December 1984. A preliminary assignment of these candidates to one of three possible directional sectors was made on the basis of the observed (visual) wave direction, except that where wave direction was lacking, wind direction was used. For the purposes of this study, storms are assigned to directional sector based upon the supposed approach direction of maximum wave energy according to the following definitions:

- westerly: from 210 to 290 true
- northerly: from 290 to 045 true
- southeasterly: from 100 to 210 true

For the top-ranked storms in each directional sector, weather map sequences from the NOAA tropical strip and the ABM microfilm file of
SH analyses were studied both to aid in the selection of high-ranked events and to refine the directional sector assignment. The candidate list was eventually distilled to six storms consisting of three westerly storms, two southeasterly storms, and one northerly storm.

To the population of six storms selected between 1976 - 1984, was added a winter 1985 northerly storm in which spectra were available at Maui-B. Finally, after the validation work was begun, a final validation storm was selected from the few candidates available during a period in which directional wave measurements were acquired from a WAVEC buoy. This westerly storm of January 1987 brought the total population of storms available for wave hindcast validation to eight, six of which were also used to validate the wind analysis procedure.

### 6.2 Wind Analysis Scheme Validation

The validation of the wind analysis scheme is based exclusively upon the comparisons of modelled winds at grid point 1755 (the Maui-B point) and winds measured at Maui-A. Of course, the accuracy of the wind fields over the much larger domain of the wave hindcast grids is indirectly validated through the assessment of wave hindcast skill at Maui.

Since the wind analysis schemes are designed to provide effective-neutral 20-m winds, the Maui wind estimates were also reduced to 20-m neutral wind speeds before comparisons were made with analysis winds. Further, since our wind analysis scheme is a two-phased process, each phase of which provides a wind estimate at Maui, separate comparisons were made for each estimate.

#### Assessment of Wind Field Accuracy

The Maui-A wind measurements provide a reasonably objective basis for assessment of accuracy of PRESTO winds. Of course, the kinematic analysis refers to the Maui-A measured winds, making an assessment of accuracy of the KINEMA wind fields more difficult.

A summary of comparison statistics at Maui-A for wind validation storms for both PRESTO and KINEMA winds is given in Table 2.

The error characteristics of PRESTO wind fields at Maui-A are reasonably similar to errors found by Cardone et al. (1980) in Northern Hemisphere mid-latitudes. Except in southeasterlies, PRESTO wind speeds tend to be biased low by about 1 m/sec, and RMS differences are about 2.5 m/sec. Mean differences in wind direction are generally less than 10 degrees, while RMS errors average about 20 degrees. Large errors in PRESTO winds at Maui-A were evident only in the southeasterler of 7907. Errors in PRESTO winds in the open basin west of Maui-A are probably larger than exhibited at Maui-A because the pressure
reanalysis there is based upon somewhat sparser data. We suspect, however, that in the reach between New Zealand and Australia, errors in PRESTO winds are comparable to those of wind fields derived by comparable techniques in open NH ocean basins.

Errors in KINEMA winds at Maui-A are exceptionally low (mean difference of less than 0.5 m/sec and 5 degrees, RMS differences of about 1.5 m/sec and 10 degrees) because the Maui-A observations have been assimilated effectively. However, by imposition of space-time continuity in the kinematic analysis process, the low errors implied near Maui-A probably apply over a greater domain of the FINE grid, especially areas along streamlines upstream of Maui-A.

An example time-history plot of predicted PRESTO and KINEMA wind speeds and direction versus measurements (3-hourly plotting interval) reduced to 20-m neutral wind speeds for the 7807 storm is given in Figure 3□.

6.3 Wave Hindcast Method Validation

The rather flexible structure of the wave model programs adapted in this study allowed the investigation of the sensitivity of wave hindcast skill on factors such as: source of wind fields (PRESTO or KINEMA); grid resolution (COARSE or FINE); spectral growth/dissipation physics (1G or 2G); and propagation scheme (CAIPS or standard). Therefore, a total of six hindcasts were made of each of eight validation storms and comparisons were made of the results of each hindcast at the Maui grid point, and available Maui-A and Maui-B wave measurements. The validation emphasized the specification of peak significant wave height (HS) and associated wave period at Maui, since these are the quantities most relevant to the intended use of the results of the production hindcasts in the extremal analysis. However, time history comparisons of measured and hindcast HS and wave period were made in all cases, and where available, measured and hindcast frequency spectra were compared at and near the time of occurrence of storm peaks. Comparisons of time histories of hindcast and observed dominant wave direction were also made whenever possible, though only in the validation storm of January 1987 were instrumental wave direction properties available.

Summary of Peak-Peak Comparisons

COARSE Runs

Table 3□ provides a concise summary of comparisons of peak hindcast and peak measured significant wave height (HS) at Maui in the validation storms. Comparisons are made at the time of respective (hindcast, measured) peak occurrence and at available measurements and time steps immediately before and after the time of peak occurrence.
COARSE grid hindcasts were executed from both PRESTO and KINEMA winds and for both ODGP and SAIL spectral growth algorithms. Comparisons of average period (TM) associated with the indicated HS are also shown.

The COARSE ODGP hindcasts of peak HS and TM from PRESTO (C27P) and KINEMA (C27K) winds and the COARSE SAIL hindcasts from KINEMA winds (C41K) at Maui-A are compared graphically (scatter plots) and statistically with the measured peak HS and associated TM in Figure 4 and Table 4. All COARSE hindcasts are quite skillful, with scatter index in HS of at most .12 in the SAIL model series. The C27K series are, by a slight margin, the most skillful series.

FINE Runs

Table 5 provides a summary of comparisons of peak hindcast and peak measured HS and associated TM as in Table 3, except for the FINE grid ODGP shallow-water runs. Comparisons are given for both the nominal FINE model with the CAIPS sheltering algorithm, F27KS, and the alternate version without the CAIPS algorithm, F27KN. The scatter plots, on HS and TM for these runs are shown in Figures 6 and 7, and the difference statistics are given in Table 6.

Differences between the F27KS and F27KN runs are slight, except in the southeaster of 8105, in which the CAIPS algorithm evidently contributes to skill. The overspecification of F27KN in this southeaster actually leads to a smaller mean error in HS for the F27KN runs compared with the F27KS series. The F27KN runs, however, exhibit slightly greater scatter than the F27KS runs. The hindcast skill exhibited by both FINE models, however, is comparable to the maximum skill achieved in prior hindcast studies of this kind in Northern Hemisphere basins.

Comparison of Analysis and Measured Time Histories

Figure 8 shows time histories of F27KS model hindcast and measured HS and TM, at Maui-A for a fairly skillful hindcast (8505). Similarly skillful time histories were also found in the storms of 7907 and 8701. In the other storms, while peak sea states are generally well represented, the comparisons reveal temporally coherent hindcast-measurement differences similar to those that characterize even the most careful wave hindcast studies carried out in mid-latitude Northern Hemisphere basins. We attribute most of these differences to our inability to reduce wind field errors to negligible levels.

Comparison of Frequency Spectra

Frequency spectra are available in the validation storms of 8409 and 8508. The principal objective of the spectral comparisons is to
evaluate the ability of the hindcast model to correctly specify spectral shape. Thus, spectral comparisons are meaningful only if the total spectral energy is specified to within about 20% (HS within about 10%) of that observed. For example, in the storm of 8409, the hindcast and measured HS are in best agreement at 1500 GMT September 13 which is 3 hours before the observed peak HS. Hindcast and estimated frequency spectra at Maui–A and Maui–B compared at this time (not shown), exhibited excellent agreement in spectral peak frequency location and width, and in the shape of the forward and rear faces of the spectrum.

In the storm of 8508, two kinematic analysis intervals were adopted. The first was centered on a northerly sector wave peak, the second on a separate westerly sector wave peak. The F27KS hindcast time histories (see Figure 8) of HS and TM track the measured closely in both events. Comparison of frequency spectra hindcast peak HS in each peak and corresponding measured spectra (for which hindcast and estimated peak HS agree to within 5%) showed close agreement (e.g. Figure 9).

**Comparison of Directional Spectra**

Directional wave measurements were acquired from a WAVEC buoy in the directional validation storm of 8701. The signals on vertical elevation and two slope components of the sea surface provided by the WAVEC are processed to vertical variance density spectra, mean direction, and directional width (spreading) following the method of Kuik and van Vledder (1984). The WAVEC acquired two wave records of 20-minute length each hour. The frequency spectra and directional estimates were smoothed to 3-hourly averages and rebinned to the frequency resolution of the hindcast model before the comparisons were made. Mean direction and spectral width were estimated from the hindcast two-dimensional spectrum in each frequency band, in a manner consistent with the definition of the corresponding measured derived quantities.

The time history comparisons (Figure 10) for this storm show excellent agreement between hindcast and measured HS during the kinematic analysis interval. A typical comparisons of hindcast and measured frequency spectrum and directional parameters is shown in Figure 11. Agreement between hindcast and measured mean direction and spreading is generally excellent. The small occasional differences in wave direction in the rear face appear to be related to differences between modelled and measured local wind direction. The systematic variation of wave direction with wave frequency in the main part of the spectrum is very well modelled.

**Overall Assessment of Wave Hindcast Accuracy**
Most of the published data on model performance in situations of severe atmospheric forcing pertain to first-generation models. Reece and Cardone (1982) evaluated the skill of ODGP model hindcasts of HS and its associated TP at a site in a storm, specified naturally as part of basin-wide simulations of complete storm histories. In over 60 individual comparisons in 19 different tropical and extratropical cyclones, the model hindcasts exhibited negligible bias and root-mean-square errors of less than 1 meter in height and 1 second in peak spectral period. Comparisons of measured and hindcast directional wave spectra in three of the hurricanes showed excellent agreement. The scatter index in HS was 11.9% in the comparisons cited above. More recent applications of the ODGP model in validation studies in the North Atlantic, North Pacific, Bering Sea and Gulf of Mexico have shown similar skill.

Several modelers have achieved comparable success with first- and second-generation models in hindcasts of historical storms after surface wind fields have been carefully reconstructed from source data. For example, the Shallow Water Intercomparison Model (SWIM) project models produced scatter indices of 19, 14, and 24% in deep-water hindcasts of two severe North Sea storms (SWIM, 1985).

So far, the third-generation model has undergone more limited evaluation in terms of storms. However, six extratropical storms that occurred in 1983 and 1984 on the western European continental shelf were hindcast and evaluated at several measurement sites. Mean errors in HS were generally less than 0.5 meter with scatter indices between 10 and 20%. The third-generation WAM model has also been used to hindcast three Gulf of Mexico hurricanes, including the intense Hurricane Camille in 1969, with excellent results.

Viewed within the context of these and other proprietary studies which are not citable at the present time, the errors in the F27KS hindcasts of HS and TM at Maui-A in the validation storms are among the lowest achieved to date with any model in any basin. Indeed, except for the WAVEC measurements, the scatter index in HS of 9% is the minimum which could be expected, even for perfect hindcasts, considering the typical sampling variability of estimates of HS derived from 17-minute wave records (typically +/- 12%, see Donelan and Pierson, 1983).

The validation results supported the extension of the study to a production phase, with the wind field analysis procedures adapted in this study to the total hindcast period, and with production wave hindcasts carried out in all instances with the C27K/F27KS model combination.

7. CONCLUSIONS

The validation phase of the Maui Hindcast Study has shown that the wind and peak sea state during storm events in the Maui region can be
predicted using numerical models of the type described with an accuracy comparable to that achieved in NH basins. This validates the hindcast approach as an appropriate technique for establishing accurate wind and wave design criteria for offshore engineering facilities in the Maui region and generally for most of the region off the west coast of New Zealand.

It may not be inferred from this study that all basins in the SH are as susceptible to the hindcast approach as the area considered here. The critical consideration is the availability of ship, island and coastal synoptic weather data at least as plentiful as along the west coast of New Zealand and in the Tasman Sea. This condition appears to be satisfied in the waters surrounding eastern and southern Australia, and much of the South Atlantic basin. It is not surprising therefore, that major hindcast studies have been undertaken recently for areas such as Bass Strait of Australia, offshore Brazil and the west coast of Africa.

ACKNOWLEDGEMENTS

This study was carried out for shell Todd Oil Services Ltd, acting on behalf of Maui Development Ltd. Their permission to publish the paper is gratefully acknowledged. Special thanks is also due to R. K. Falconer of GeoResearch Associates Ltd, New Zealand, who assisted in the assembly and analysis of data.

REFERENCES


Table 1. COARSE and FINE wave model attributes.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>COARSE grid</th>
<th>FINE grid</th>
</tr>
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<tr>
<td>Domain</td>
<td>20°S - 40°N; 115°W - 175°E</td>
<td>37°44'S - 42°S; 169°E - 179°W</td>
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<tr>
<td>Spacing</td>
<td>160 km, square</td>
<td>30 km, square</td>
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<tr>
<td>Projection</td>
<td>transverse Mercator</td>
<td>transverse Mercator</td>
</tr>
<tr>
<td>Time step</td>
<td>3 hours (1.5 hours grow, 3 hour propagation, 1.5 hours grow)</td>
<td>45 minutes (22.5 min. grow, 45 min. propagation, 22.5 min grow)</td>
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<td>24 directions, 15 degree bandwidth</td>
<td>24 directions, 15 degree bandwidth</td>
</tr>
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<td>15 frequencies</td>
<td>15 frequencies</td>
</tr>
<tr>
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<td>GOOS (shallow water); SAIL (shallow water)</td>
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<td>Interpolatory, shallow water with CAIP, optimal delete CAIP</td>
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Table 2. Wind validation results at Maui-A.

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<th>C27P</th>
<th>C41P</th>
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<td>kts</td>
<td>(m/s)</td>
<td>kts</td>
<td>(m/s)</td>
</tr>
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<td>10.4</td>
<td>10.8</td>
<td>30/06</td>
<td>NA</td>
</tr>
<tr>
<td>7807</td>
<td>9.2</td>
<td>8.9</td>
<td>24/16</td>
<td>NA</td>
</tr>
<tr>
<td>7907</td>
<td>6.3</td>
<td>7.4</td>
<td>15/04</td>
<td>5.2</td>
</tr>
<tr>
<td>8105</td>
<td>7.8</td>
<td>8.6</td>
<td>20/21</td>
<td>5.7</td>
</tr>
<tr>
<td>8307</td>
<td>5.3</td>
<td>7.0</td>
<td>09/12</td>
<td>4.9</td>
</tr>
<tr>
<td>8409</td>
<td>6.9</td>
<td>9.1</td>
<td>13/18</td>
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<tr>
<td>8508a</td>
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<td>19/00</td>
<td>&quot;noisy&quot;</td>
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</tr>
<tr>
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Table 3. Measured and COARSE deepwater model hindcast storm peaks.

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<th>Observed</th>
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</tr>
<tr>
<td>7807</td>
<td>9.2</td>
<td>8.9</td>
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<tr>
<td>7907</td>
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<td>7.4</td>
</tr>
<tr>
<td>8105</td>
<td>7.8</td>
<td>8.6</td>
</tr>
<tr>
<td>8307</td>
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<td>7.0</td>
</tr>
<tr>
<td>8409</td>
<td>6.9</td>
<td>9.1</td>
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<tr>
<td>8508a</td>
<td>6.2</td>
<td>7.5</td>
</tr>
<tr>
<td>8508b</td>
<td>7.0</td>
<td>9.2</td>
</tr>
<tr>
<td>8701</td>
<td>4.8</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Code: C - COARSE; D - DEEP; S - SAIL; P - PREDICT; K - Kinematic Winds
NA - not available; M - missing
*Peak spectral period used throughout for 8701 storm
Table 4. Storm peak comparisons for COARSE runs.

<table>
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<th>G37R</th>
<th>G41E</th>
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</thead>
<tbody>
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<td>Total number of points</td>
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<td>8.00</td>
<td>9.00</td>
</tr>
<tr>
<td>Average hindcast</td>
<td>8.42</td>
<td>8.67</td>
<td>7.11</td>
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<tr>
<td>Average measurement</td>
<td>6.38</td>
<td>6.45</td>
<td>6.40</td>
</tr>
<tr>
<td>Mean difference</td>
<td>-0.16</td>
<td>0.02</td>
<td>-0.34</td>
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<tr>
<td>Root mean square</td>
<td>0.92</td>
<td>0.56</td>
<td>0.79</td>
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<tr>
<td>Standard deviation</td>
<td>0.73</td>
<td>0.57</td>
<td>0.78</td>
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<tr>
<td>Scatter index</td>
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<td>0.07</td>
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<tr>
<td>Ratio</td>
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<td>0.38</td>
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Table 5. Measured and FINE shallow-water model hindcast storm peaks.

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<tr>
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<th>Tm (s)</th>
<th>D/H (M)</th>
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<td>7705</td>
<td>10.4</td>
<td>10.8</td>
<td>30/06</td>
</tr>
<tr>
<td>-3</td>
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<td>9.0</td>
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<td>7907</td>
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<td>15/08</td>
</tr>
<tr>
<td>8105</td>
<td>6.7</td>
<td>8.6</td>
<td>20/21</td>
</tr>
<tr>
<td>+3</td>
<td>6.2</td>
<td>8.9</td>
<td>21/00</td>
</tr>
<tr>
<td>8307</td>
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<td>09/12</td>
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<tr>
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<td>7.8</td>
<td>09/15</td>
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<td>13/18</td>
</tr>
<tr>
<td>-5</td>
<td>8.0</td>
<td>8.4</td>
<td>13/15</td>
</tr>
<tr>
<td>850a</td>
<td>6.2</td>
<td>7.5</td>
<td>09/07</td>
</tr>
<tr>
<td>+3</td>
<td>5.6</td>
<td>7.4</td>
<td>09/03</td>
</tr>
<tr>
<td>850b</td>
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<td>21/06</td>
</tr>
<tr>
<td>-5</td>
<td>6.2</td>
<td>8.6</td>
<td>21/09</td>
</tr>
<tr>
<td>8701</td>
<td>4.8</td>
<td>10.0</td>
<td>22/09</td>
</tr>
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</table>

Table 6. Storm peak comparisons for FINE runs.

<table>
<thead>
<tr>
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<th>F37aE</th>
<th>F37aR</th>
<th>F37aE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of points</td>
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<td>8.00</td>
<td>9.00</td>
</tr>
<tr>
<td>Average hindcast</td>
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<td>8.86</td>
<td>7.00</td>
</tr>
<tr>
<td>Average measurement</td>
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<td>6.65</td>
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<tr>
<td>Mean difference</td>
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<td>0.21</td>
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<td>Root mean square</td>
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<td>Scatter index</td>
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<td>0.09</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.33</td>
<td>0.38</td>
<td>0.33</td>
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Table 5. Measured and FINE shallow-water model hindcast storm peaks.

<table>
<thead>
<tr>
<th>Store</th>
<th>Hs (m)</th>
<th>Tm (s)</th>
<th>D/H (M)</th>
</tr>
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<tbody>
<tr>
<td>Maui-A</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Maui-B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F27XK</td>
<td></td>
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<td></td>
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<tr>
<td>F27XK</td>
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</table>

Code: F - FINE; 27 - GUP; K - Kinematic winds; S - Shallow with CAIPS; N - Shallow without CAIPS
NA - not available; M - missing
Peak spectral period used throughout for 8701 storm.
Fig. 1  COARSE wave model grid.

Fig. 2  FINE wave model grid.

Fig. 3  Comparison of PRESTO (small dashed), KINEMA (long dashed) and measured (solid) winds at MAUI-A in storm of 7807.
Fig. 4 Scatter plot of COARSE hindcast and measured peak significant wave height.

Fig. 5 Scatter plot of COARSE hindcast and measured storm peak mean period.

Fig. 6 Scatter plot of FINE hindcast and measured peak significant wave height.

Fig. 7 Scatter plot of FINE hindcast and measured storm peak mean period.
Fig. 8 Comparison of FINE/KINEMA hindcast and measured sig. wave height and mean period at MAUI in storm of 8508.

Fig. 10 Comparison of FINE/KINEMA hindcast and measured sig. wave height at MAUL in storm of 8701.

Fig. 9 Comparison of FINE/KINEMA peak hindcast and measured spectrum at MAUI in storm of 8508.

Fig. 11 Comparison of FINE/KINEMA hindcast and measured frequency spectrum (upper), wave direction (middle), and directional width (lower) at 2100 UT, 870122.
HINDCAST WAVE INFORMATION FOR THE U.S. ATLANTIC COAST

J. M. Hubertz
U.S. Army Corps of Engineers
Coastal Engineering Research Center
Vicksburg, MS

1. INTRODUCTION

Studies by the Corps of Engineers (CE) Wave Information Study (WIS) have supplied wave climate information at locations along the U.S. Atlantic coast based on the 20 year period 1956–1975, (Corson, et. al., 1981), (Corson, et. al., 1982), (Jensen, 1983). This information met a critical need for wave information in coastal engineering studies in the 1980’s. Wave measurements made by the National Oceanic and Atmospheric Administration (NOAA) and CE during the 1980’s made verification of these WIS results possible by comparing statistics and the distributions of wave heights, periods, and directions from the two different time periods. The statistics and distributions differed enough to require revision of the previous hindcast information using the CE present hindcast model. This paper discusses the revised regional hindcast and available results.

2. WINDS

Hindcast wind speeds and directions for the period 1956–1975 were calculated from surface atmospheric pressures and observations of wind speeds from ships. The distribution of hindcast (1956–1975) wind speeds and directions in various speed and direction categories were compared to measurements from five NOAA buoy made in the 1980’s, in order to judge the accuracy of the wind climatology developed in the original hindcast. The locations of the five buoys and model grid points used for the wind comparisons are shown in Figure 1 (a–c). The buoys are; 44011, 44004, 41001, 41002, 41006. Buoy data were obtained from Gilhousen et. al., (1990). Buoy data were generally available from the late 1970’s to the late 1980’s. WIS winds are available from 1956–1975. Thus the time periods are different, but the assumption is made that the wind climatology for the two periods is similar, thus the distribution of speeds and directions should be similar. Speeds and directions were divided into categories and the percent of occurrences falling in each category was calculated from the buoy and hindcast information at each site. The difference between these percents (buoy – hindcast) in each category is shown in Table 1 for each location.

The distributions of speeds and directions are quite similar at all locations. It is concluded from these comparisons that the original hindcast winds are an accurate representation of the wind climate over
the Atlantic during 1956–1975. Thus, the original winds were judged acceptable and used for the revised hindcast.

3. WAVE MODEL

The latest version of the CE wave model, WISWAVE 2.0, developed by Dr. Donald Resio, (Hubertz, in publication) was used with the CRAY computer system at the Coastal Engineering Research Center (CERC). The nested grid option of WISWAVE was employed using two levels of resolution. Level 1 covered the North Atlantic Ocean with a grid spacing of 1 degree in latitude and longitude.

Figure 1a. Location of Hindcast Stations, Buoys, and Model Land/Water Boundary

Figure 1b. Location of Hindcast Stations, Buoys, and Model Land/Water Boundary
Figure 1c. Location of Hindcast Stations, Buoys and Model Land/Water Boundary

Figure 1d. Location of Hindcast Stations, Buoys and Model Land/Water Boundary
This grid is shown in Figure 2a. The circled points along the US East coast in Figure 2a represent level 1 grid points which supply information to the level 2 grid. These points are also shown in Figure 2b which shows the level 2 grid. Level 2 covered the continental shelf with a grid spacing of 1/4 degree. Level 2 grid points between the coast and boundary input points from level 1 are water points with associated depths. Grid points to the east of the boundary input points are considered land points to reduce the number of computational water points.

Locations at which wave information is available are shown in Figure 1 (a-d) as numbered dots along the coast. Deep water is assumed in level 1, and bathymetry at mean low water is used in level 2. The islands and shoals off the south east coast of Florida are included in the depth grid for the revised hindcast. The Bahamas Banks and shoals were not included in the original hindcast. Thus, the revised hindcast should more accurately represent these features as well as representation of the coastline in the numerical grid because of the increased resolution from 30 nm in the old hindcast to 15 nm in the new.

The values of the various coefficients in WISWAVE are the same as used in recently completed hindcasts for each of the Great Lakes and are discussed in individual reports for each lake, for example (Hubertz, Driver, and Reinhard, 1991). The deep water version of WISWAVE (WISWAVE 1.0. referred to as DWAVE in the Great Lakes

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(1) Buoy 44011 (41.1N, 66.6W) / WIS (41N, 67W)
(2) Buoy 44004 (38.5N, 70.7W) / WIS (38N, 71W)
(3) Buoy 41001 (34.9N, 72.9W) / WIS (35N, 73W)
(4) Buoy 41002 (32.3N, 75.3W) / WIS (32N, 75W)
(5) Buoy 41006 (29.3N, 77.3W) / WIS (29N, 77W)
hindcasts) is identical to the present version WISWAVE 2.0, but with the addition of propagation routines which can use arbitrary depths.

4. VERIFICATION – A Hindcast for 1990

A one year hindcast for 1990 was completed prior to recalculating the 20 years of wave information in order to verify the model and procedures used in the hindcast. Winds for the hindcast were obtained from the Navy’s Fleet Numerical Oceanographic Center. Model results were compared to measurements at 14 locations along the Atlantic coast, Figure 1(a–c). These comparisons are summarized in Table 2 below. Values by month from which the annual values below are calculated are presented in (Hubertz, et al., in preparation).

Figure 2a. Hindcast Model Grid for Level 1
Figure 2b. Hindcast Model Grid for Level 2

Table 2

Comparison of 1990 Hindcast and Measured Wind and Wave Parameters

<table>
<thead>
<tr>
<th>Buoy</th>
<th>HmO(m)</th>
<th>Tp(sec)</th>
<th>WndSpd(m/sec)</th>
<th>Hmo(m)</th>
<th>Tp(sec)</th>
<th>WndSpd(m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41006</td>
<td>-0.2</td>
<td>-0.9</td>
<td>0.9</td>
<td>0.5</td>
<td>2.5</td>
<td>1.9</td>
</tr>
<tr>
<td>41008</td>
<td>-0.2</td>
<td>-1.5</td>
<td>1.0</td>
<td>0.4</td>
<td>3.0</td>
<td>2.2</td>
</tr>
<tr>
<td>41002</td>
<td>-0.4</td>
<td>-1.0</td>
<td>0.8</td>
<td>0.5</td>
<td>2.4</td>
<td>1.8</td>
</tr>
<tr>
<td>41001</td>
<td>-0.1</td>
<td>-0.8</td>
<td>1.7</td>
<td>0.5</td>
<td>2.4</td>
<td>2.6</td>
</tr>
<tr>
<td>44014</td>
<td>-0.1</td>
<td>-0.5</td>
<td>-0.3</td>
<td>0.5</td>
<td>2.4</td>
<td>-</td>
</tr>
<tr>
<td>44015</td>
<td>-0.5</td>
<td>-1.8</td>
<td>1.2</td>
<td>1.0</td>
<td>3.3</td>
<td>3.0</td>
</tr>
<tr>
<td>44009</td>
<td>-0.3</td>
<td>-0.4</td>
<td>1.2</td>
<td>0.6</td>
<td>2.9</td>
<td>2.8</td>
</tr>
<tr>
<td>44001</td>
<td>0.0</td>
<td>-0.4</td>
<td>0.4</td>
<td>0.5</td>
<td>2.4</td>
<td>-</td>
</tr>
<tr>
<td>44004</td>
<td>-0.1</td>
<td>-0.7</td>
<td>1.1</td>
<td>0.6</td>
<td>2.5</td>
<td>2.7</td>
</tr>
<tr>
<td>44012</td>
<td>-0.2</td>
<td>-1.5</td>
<td>-</td>
<td>0.7</td>
<td>3.5</td>
<td>-</td>
</tr>
<tr>
<td>44008</td>
<td>-0.2</td>
<td>0.0</td>
<td>1.7</td>
<td>0.6</td>
<td>2.2</td>
<td>2.8</td>
</tr>
<tr>
<td>44011</td>
<td>-0.2</td>
<td>-0.4</td>
<td>0.2</td>
<td>0.6</td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td>44005</td>
<td>-0.2</td>
<td>-0.8</td>
<td>1.1</td>
<td>0.6</td>
<td>2.4</td>
<td>2.2</td>
</tr>
<tr>
<td>44007</td>
<td>-0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>0.6</td>
<td>2.8</td>
<td>2.5</td>
</tr>
</tbody>
</table>

(-)No data available

The difference between measured and hindcast (buoy-model) monthly mean spectral wave height varies from 0.3 to -0.8 meters. The difference between measured and hindcast (buoy-model) monthly mean peak wave period varies from 1.8 to -3.6 seconds. The range of root...
mean square differences for height and period are 0.3–1.1 and 0.4–4.8 respectively. Thus, the results of this one year hindcast indicate one could expect hindcast wave heights to be slightly high (0.2 m) with respect to measurements along the coast, and at any time to differ from measured values by about 0.6 m. The accuracy of the buoy wave height measurements is +/- 0.2 m or 5% for waves above 4.0 m, (Gilhousen, 1990). Hindcast wave peak (dominant) periods are slightly high, but generally less than a second with respect to measured values along the coast, and at any time to differ from measured values by about 2–3 sec. The accuracy of the buoy wave peak period measurements is +/- 1.0 sec, (Gilhousen, 1990).

Spectral wave heights are an integrated quantity. That is, they are based on the sum of energy under the discrete spectrum. Wave peak mean directions are similar, but less an integral quantity, in that they are the energy weighted mean of all wave directions in the peak frequency band. Wave peak periods, however are not an integrated or mean value. They are based only on the location along the frequency axis of the peak spectral energy. Thus, if the spectrum is double peaked, representing sea and swell for example, the peak frequency can shift from say 4 to 10 seconds depending on the relative magnitude of the two peaks. This tends to introduce large differences in period comparisons, when for example, a sea peak is largest in a gage record and the swell peak is largest in a hindcast for the same period or vice versa. Mean periods, which are energy weighted means over all frequencies, can be compared, but for a double peaked spectrum, the mean period may be between the sea and swell periods, say 7 seconds in the example above, and thus not be representative of either sea or swell wave period. These points need to be considered when making comparisons of peak period and peak mean direction. Next, hindcast results from the period 1956–1975 are compared to measured results from buoys at five locations along the coast where old and new WIS hindcast stations and buoys are close to the same location and at nearly the same depth.

Hindcast for 1956–1975

There are five buoys along the coast which are at locations close to points where WIS results are saved from the 20 year hindcast for both the old and new hindcasts. The buoy and WIS station locations are listed in Table 3 below for both new and old hindcasts along with depths. Figure 1(a–c) shows the location of the buoys with respect to the WIS stations, the actual coastline, and the land/water boundary as represented in the model.
Measured and hindcast wave conditions can be affected by local features especially near the coast. Examples of these features are depth differences between measurement and model sites, differences in actual location of the coastline and its representation in the model, presence of currents in nature which are absent in the model and possibly others. Such features which may affect comparisons at the five buoy sites are discussed below.

Buoy 41008 is approximately 8 nautical miles farther offshore than WIS station 28 and is in 18 m of water versus 11 m at the WIS station, Figure 1c. Both locations are open and unsheltered by either the actual coastline or model representation of the land/water boundary. Both should be unaffected by currents since they are away from the Gulf Stream which is the major current in this region. The effects of the Gulf Stream or local tidal currents near bays and inlets are not included in the hindcast. Buoy CHLV2 is closest to WIS station 59, Figure 1b. The buoy is in a depth of 12 m and the depth at the WIS station is 14 m. Both locations are open and unsheltered by either the actual coastline or model representation of the land/water boundary. The buoy is near the entrance to Chesapeake Bay and thus may be exposed to the ebb and flood currents from the bay and ocean. These currents have a maximum magnitude of about 1 m/sec in the entrance. No currents were used in the hindcast. Thus, any effects of the ebb and flood currents on the waves which would be measured by the buoy, such as steepening and breaking, is not represented in the hindcast results. No quantitative measure of the effect of currents such as these on wave conditions is available, so their possible effects are unknown. Buoy 44012 at a depth of 24 m is located in the vicinity of
the entrance to Delaware Bay closest to WIS station 66 at a depth of 18 m, Figure 1b. The currents in the bay entrance are again on the order of 1 m/sec, Buoy 44013 is near Massachusetts Bay, Figure 1a, at a depth of 30 m. WIS station 94 was chosen for comparison since it is at a depth of 27 m and the depth at station 95, also close by, is 55 m. The location of 44013 is sheltered by the configuration of the coastline. The grid spacing in the model does not allow resolution of features such as Cape Ann which is the small cape to the north of 44013. Station 94 is not sheltered from the north as the land/water boundary of the model (solid line in Figure 1) is configured. Finally, buoy 44007 is at a depth of 47 m and is closest to WIS station 99 at a depth of 18 m, Figure 1a. Sheltering from the actual coastline and model representation is nearly equivalent and should not introduce any bias in comparisons.

Tables 4 and 5 present a summary of average and maximum values of wave height and period from measurements and new and old hindcast results at the five locations discussed above. Note that the time periods, from which the averages and maximum values are derived from measurements, and the hindcast are different. The hindcast values are from the 20 year period 1956–1975, while the measurements are from periods of 4 to 9 years between 1982–1991.

<table>
<thead>
<tr>
<th>Buoy ID</th>
<th>Period WIS</th>
<th>Period Years</th>
<th>Hm0 (m)</th>
<th>Tp (sec)</th>
<th>Hmax (m)</th>
<th>Tmax (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41008</td>
<td>88-91</td>
<td>56-75</td>
<td>1.0</td>
<td>0.9</td>
<td>8</td>
<td>4.6</td>
</tr>
<tr>
<td>CHLV2</td>
<td>85-91</td>
<td>59-75</td>
<td>0.7</td>
<td>1.0</td>
<td>6</td>
<td>6.2</td>
</tr>
<tr>
<td>44012</td>
<td>84-91</td>
<td>66-75</td>
<td>0.6</td>
<td>0.9</td>
<td>4</td>
<td>6.0</td>
</tr>
<tr>
<td>44013</td>
<td>84-91</td>
<td>94-75</td>
<td>0.5</td>
<td>1.0</td>
<td>6</td>
<td>5.0</td>
</tr>
<tr>
<td>44007</td>
<td>82-91</td>
<td>99-75</td>
<td>0.8</td>
<td>1.0</td>
<td>8</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Buoy ID</th>
<th>Period WIS</th>
<th>Period Years</th>
<th>Hm0 (m)</th>
<th>Tp (sec)</th>
<th>Tm (sec)</th>
<th>Hmax (m)</th>
<th>Tmax (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41008</td>
<td>88-91</td>
<td>132-75</td>
<td>1.0</td>
<td>0.6</td>
<td>8</td>
<td>4.6</td>
<td>3.8</td>
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<tr>
<td>CHLV2</td>
<td>85-91</td>
<td>76-75</td>
<td>0.7</td>
<td>0.6</td>
<td>6</td>
<td>6.2</td>
<td>5.2</td>
</tr>
<tr>
<td>44012</td>
<td>84-91</td>
<td>64-75</td>
<td>0.6</td>
<td>0.6</td>
<td>4</td>
<td>6.0</td>
<td>4.2</td>
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<tr>
<td>44013</td>
<td>84-91</td>
<td>25-75</td>
<td>0.5</td>
<td>0.3</td>
<td>6</td>
<td>5.0</td>
<td>4.4</td>
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<tr>
<td>44007</td>
<td>82-91</td>
<td>15-75</td>
<td>0.8</td>
<td>0.5</td>
<td>8</td>
<td>7.3</td>
<td>4.4</td>
</tr>
</tbody>
</table>

The hindcast results are continuous in time every three hours. The buoy data may have gaps in the record of various lengths of time throughout the years. The buoys measure all waves passing them, while
the hindcast only generates waves within the model grid and the wind fields used exclude tropical storms and hurricanes. Thus swell propagating from the South Atlantic and waves generated by tropical storms and hurricanes are not present in the hindcast results.

First, consider the results from the new hindcast in Table 4. Mean wave heights at the five sites generally agree, considering the bias of 0.2 m determined from the hindcast for 1990 and the accuracy of the buoy measurements of 0.2 m or 5 percent. The largest difference of 0.5 m is at buoy 44013 in Massachusetts Bay where the measured values are on the average lower than hindcast values. The differential sheltering discussed above may contribute to this trend. The effect of currents near the entrances to Chesapeake and Delaware Bays may affect the wave climate at these locations. Mean generally agree within 0–2 sec or close to the accuracy of the buoy.

Maximum waves heights generally agree within 1.0 m. There is maximum hindcast values to be higher or lower than measured maximum difference is at the Massachusetts Bay site where the buoy value is hindcast by 2.6 m. The differential sheltering discussed above may this difference. Maximum measured and hindcast peak periods agree within 0–3 sec with the exception of the Cape Elizabeth site where the difference is 5 sec.

The results from the old hindcast in Table 5 indicate that the old hindcast mean wave height results are generally in agreement with buoy measurements while maximum values are Consistently low. Both mean and maximum periods are low, but the old hindcast periods are mean periods while the buoy values are peak. Peak periods were not available from the old hindcast.

Mean and maximum values of spectral wave heights and peak periods from the new 20 year hindcast agree well with measured values at five sites along the coast from Georgia to Maine. These comparisons and the results from the one year hindcast discussed above verify that the new hindcast values of wave height and peak period accurately represent these wave parameters along the coast. Next, hindcast wave directions are compared to directional wave measurements.

There are few directional wave measurements of long duration in deeper water offshore away from nearshore refraction effects. One set of such measurements for the period 1988–1991 is available from NOAA buoy 41008 about 20 miles off the coast of Georgia, Figure 1c. The distribution of peak mean direction in 10 deg increments from this set of measurements is shown in Figure 3. Also plotted are values of the same parameter from the revised hindcast for the period 1956–1975 at WIS station 28, (RAL2, Revised Atlantic Level 2). The distribution of mean wave directions from Phase III (30.81 N, 81.45 W, depth 10 m) of
the original hindcast are also plotted (AP3, Atlantic Phase 3). Note that the Phase III results are mean directions versus peak mean directions and only wave energy with a component toward shore is present.

The directional distribution from the revised hindcast results agree well with the shape of the curve from the buoy data. Note that the buoy data is from the period 1988-1991, while the WIS results are from 1956-1975. To conclude that the new WIS results are accurate from this information, one must assume the directional wave climate is the same for both periods of time. This directional comparison, though limited because of lack of other data, shows that the revised WIS results accurately represent the directional wave climate at this location.

5. RESULTS

Hindcast results are available at the locations shown in Figure 1 a-d. Results are available as time series for the 20 year period or as tabular summaries similar to previous WIS reports. Time series results are available every three hours. Each record contains, in order; station number, date-time, spectral wave height, peak and mean period, mean direction, wind speed and direction, the frequency spectrum, and the mean direction in each frequency band. The format and documentation of the output records are described in detail in WIS Report 27 or in a README.DOC file accompanying the data. The record format was designed to provide basic wave information such as height, period and direction, as well as a more complete spectral description of wave conditions. The entire 20 year time series at a station can be provided in a compressed format on two high density 3 1/2 inch disks for use on personal computers. Also available are two personal computer programs VIEWTSAT (View Time Series Atlantic) and VIEWSPAT (View Spectra Atlantic) which can be used to view time series of wave parameters or wave spectra respectively. Time series and the programs to view the results are available upon request to the WIS project office (601-634-2028).

6. SUMMARY TABLES

Information for each station is contained on two pages (front and back) in an appendix to the summary report, (Hubertz, et. al., in preparation). The first page contains the distribution by number of occurrences of spectral wave height, peak period and peak mean direction in half meter height, one second period, and 22.5 degree direction categories by month for the 20 year period. These tables are useful in showing the distribution of height, period and direction through the range of their values and in time through the year.
Figure 3. Measured and Hindcast Directional Distribution of Wave Energy

The next set of tables shows the number of occurrences by one meter height and two second period categories for eight different direction bands each 45 degrees in width and a final table for all directions. These tables are useful to find the dominant direction from which wave energy is approaching a location and how it is distributed in height and period.

The distribution of wind in 2.5 m/sec and 45 degree, speed and direction categories on a monthly basis is presented. These tables are useful for understanding the climatology of winds at a site. Local sea conditions and wind driven currents can be inferred from the wind climatology.

The last two tables summarize mean and maximum wave heights by month for each of the 20 years hindcast. The maximum wave height table also includes the peak period and peak mean wave direction associated with the maximum wave height occurrence. Means and maximums by month for all years and by years for all months are summarized on the bottom row and right hand column respectively. Associated peak periods and peak mean directions are included in the summary row and column for the maximum table. The last two lines of the summary tables for each
station contain the maximum height at the station for the 20 year period and the associated peak period, peak mean direction, and date of occurrence, and the maximum wind speed, associated direction and date of occurrence. A separate appendix contains return period wave heights for intervals of 2, 5, 10, 20, and 50 years calculated using Fisher-Tippett type I and II distribution functions.

These summary tables and the model output records from which they were derived are a verified source of information on the wind and wave climate along the U.S. East coast. This information can be used to gain a basic understanding of wind and wave conditions at a site and also as input to more specific modeling associated with coastal engineering projects. Present coastal engineering practice requires site specific investigations with possible alternative model applications. Thus, higher resolution wave information close to the shore is more efficiently produced, as required, making use of offshore wave information, such as provided by this hindcast, rather than applying a higher resolution level of modeling along the entire coastline. Thus, this one set of hindcast results replaces the three phases of the previous study.

7. ACKNOWLEDGEMENTS

Three members of the WIS group contributed to this study. Ms. Rebecca Brooks processed the wind input, hindcast the 20 years of wave information, and prepared the summary tables. Ms. Willie Brandon performed the verification hindcast for 1990, and Ms. Barbara Tracy assisted in verification comparisons to measurements. This work was carried out under the Coastal Field Data Collection Program, Coastal Engineering functional area of Civil Works Research and Development. The author acknowledges the Office of the Chief of Engineers, U. S. Army Corps of Engineers, for authorizing publication of this paper.

8. REFERENCES


A HINDCAST STUDY OF EXTREME WAVE CONDITIONS IN THE COAST OF THE IBERIAN PENINSULA

Juan C. Carretero
Programa de Clima Maritimo (PCM), Madrid

1. INTRODUCTION

One of the most important requirements for design of coastal and off-shore structures is to have probability estimates of the extreme wave heights and associated periods for a given return period, PCM initiated a hindcast study in 1989, the final aim of this study was to evaluate these parameters along the Coast of the Iberian Peninsula, Canary Islands and Azores Islands.

The temporal and spatial coverage of observed wave data is sparse and recent, so long-term wave statistics have to be estimated from sets of data obtained simulating events of the past. Wave fields are to be reproduced by means of mathematical models driven by wind fields derived from the available surface pressure fields. There are two possible approaches to the problem:

(1) Continuous hindcast for a long period of time.
(2) Selecting and hindcasting the most severe storms occurred during a given period.

Practical and economical constraints led to the second one. The successive steps needed to obtain the extreme wave height data set are briefly described in the following items. Figure 1 depicts the 7 different partially overlapping parts into which the studied area was divided and Figure 2 the coverage of the different grids used for pressure, wind and wave fields in the Atlantic Ocean. For these last fields a nested grid scheme was needed and the coarse and fine grid are shown in the figure.

Figure 1: Analyzed regions
Figure 2: Coverage of the grids
This hindcast and subsequent extreme analysis is now been carried on, Regions 1 and 2 have been finished and preliminary results are shown in this paper. The obtained values for the most southern part of Region 2 are partial results due to the lack of the expected contribution from storms selected for Region 3 in the above mentioned area.

2. STORM SELECTION

The aim of the storm selection was to identify the 40–50 most severe storms in the last 25–35 years for each of the different regions covering the Atlantic and Mediterranean Coast of the Iberian Peninsula, including Canary and Azores Islands. This work was done for PCM by Oceanweather Inc. The main steps of the selection procedure were as follows:
1. Surface weather observations and off-shore wave observations archived or assembled by PCM as well as other data sources were studied and a preliminary set of storms were selected.
2. A scoring system based on the correlation of specific storm properties with observed peak significant wave height (from now on H_s) was developed from the previous data.
3. A rank was assigned to each storm.
3. The storms ranked above a certain threshold were selected as the final storm population for each region.

The final set of storms for Region 1 and 2 were frontal depressions moving eastwards typically between Latitudes 40° and 50° North being most active in the coast when moving approximately between Longitudes 20° and 10° West (storms specifically selected for Region 2 were the most meridional from the set), and being the estimated approach direction to the coast of maximum wave energy from the North and Northwest. The duration of these storms was typically of 4 days. Due to the similarity of both populations, with many common storms, and as the physical process was clearly the same, it was decided to perform the extreme analysis for both regions together considering both sets as the whole population.

Storms dated from 1954 up to 1986, and a total of 40 were selected for Region 1 and 54 selected for Region 2, being 18 storms common to both Regions 1 and 2. The population was distributed between September and March with 50% of the storms happening in December or January, being more frequent since 1972 probably due to increased detectability of storms.

3. PRESSURE FIELDS

Wind fields for the selected storms are derived from surface weather maps. The U.S. NOAA’s Northern Hemisphere Surface Charts have been found to be the most complete and extensive source of pressure fields.
(available on microfilm since May 1954). This charts are digitized and isobars interpolated on to a uniform 50 Km spacing grid. The digitized period for a typical storm is 8 days: the whole period of the storm plus 3 previous days to “spin up” the model and 1 posterior day to collect residual energy. As 4 charts per day of storm are digitized plus one every 12 hours for the previous and posterior days, typically 24 charts per storm are needed. Special software was developed for this purpose, the isobars are traced along with some reference points, this image is captured by a TV camera and the traced lines automatically followed and digitized by a program. These charts are drawn on a polar stereographic projection and interpolation is done onto a Lambert projection. From now on all the grids are built on this last projection.

4. WIND FIELDS

A simplified procedure is used to derive wind fields from pressure fields as: 1) extreme storms selected for Regions 1 and 2 were not affected by the orography in the studied area, 2) an objective analysis package was subsequently used to assimilate all the available experimental data into this “first guess” wind fields.

Modified geostrophic wind fields are computed from the pressure gradients and from its temporal and spatial partial derivatives, H. C. Bijvoet (1957). The following motion equation is solved:

\[
\frac{d\vec{v}}{dt} = \vec{G} + f \vec{v} \times \vec{k} + \vec{R} = -\frac{1}{\xi} \nabla p + \vec{R} = \mu f \vec{v}
\]

\(\xi=\) Air density \(p=\) Pressure \(f=\) Coriolis coefficient

Winds are computed over a 50 Km spacing Cartesian grid built on a Lambert projection with a correction in grid spacing due to the Latitude to compute the pressure gradients. To derive \(U_{10}\) from the geostrophic wind fields, a constant \(T_a-T_s\) is considered: -2 degrees in winter and 0 degrees in summer.

These wind fields are improved by means of NCDC Is set of marine observations known as ‘Comprehensive Ocean Atmosphere Data Set’. An analysis package (produced by the British Meteorological Office) inserts these observations made at irregular points ensuring a smooth final field. The analysis consists of a number of ‘scans’. During each scan the field is first modified with observations and then a high powered polynomial surface is fitted to the modified field. There are quality control tests on all the observations; those that differ significantly from the latest background field are not used further in the scan.
5. WAVE MODEL

The wave model selected for the hindcast is HYPAR, H. Günther (1981). This is a second generation wave model, parametric for wind-sea and coupled with a characteristic ray method for swell waves, developed by GKSS-Forschungszentrum Geesthacht GMBH. A Cartesian nested grid scheme is used, being 25 km the resolution of the fine grid covering the analyzed area, and 100 km for the coarse grid extended to the North Atlantic Ocean. Wind fields, available every 6 hours are interpolated for every 3600 s. The following specifications are used:

Coarse grid (100 Km spacing) - Integration time step 3600 s
- Directional bins 24
- Frequency bins 20, 0.0425 s - 0.4075 s

Fine grid (25 Km spacing) - Integration time step 900 s
- Directional bins 24
- Frequency bins 20, 0.0425 s - 0.4075 s

6. EXTREME EVENTS

After all storms had been hindcasted, the extreme event series for all points in the nested fine grid within Regions 1 and 2 were defined as follows:
1) All \( H_s \) peaks per grid point were selected.
2) As the typical length of a storm was of 8 days, a minimum of 192 hours was required as separation between extreme events to assure the independence of the events.
3) After several trials with different thresholds, testing depending variables as for example: sensibility of the return values, number of obtained peaks and some other results, a threshold per grid point was set as 1/2 of the mean value of the three biggest peaks and consistently events above this threshold were selected as the final set. Results were nearly
the same using a threshold equal to half the biggest peak except for grid point in some well defined small areas with “suspicious” peaks much bigger than the rest of the set.
4) Associated series of wind speed \( (U_{10}) \) and mean zero-downcrossing period \( (T_z) \) were stored for all of the selected \( H_s \) peaks. \( T_z \) and \( H_s \) are derived from the spectral output of the model in the usual way:

\[
H_s = 4\left(\frac{m_0}{m_2}\right)\frac{1}{2} T_{02} = \left(\frac{m_0}{m_2}\right)\frac{1}{2}\text{ being } m_n = \int_0^\infty f^n E(f) df
\]

with \( E(f) \) the energy density spectrum.

Figure 3 depicts the applied threshold, the mean value of the three biggest peaks in the area can be inferred from this plot. Figure 4
shows the final number of peaks. In both regions, and for the nearest grid points to the coast line, the number of peaks and their distribution along the hindcasted period is near to be 1 per year, tending to 2 per year in the upper part of the grid far away from the coast. As has already been said, there is a tendency towards increased storminess in the past added to the fact that some of the older storms could not be digitized due to the low quality of the charts.

The mean direction of the peaks in the Cantabric Sea is well defined to the East (Figure 5) being South East near the Coast (Figures 6 and 7). In regard to the Atlantic Coast (Figures 8, 9 and 10), the propagation directions are scattered between 80° and 110° (being 90° the East with the origin in the North). The location of points P1-P6 can be seen in Figure 3.

Figure 3: Threshold  
Figure 4: Number of peaks  
Figure 5: P1, angular distribution  
Figure 6: P2, angular distribution
Figure 11 shows the % of peaks coming from storms selected for Region 1 that is specifically selected for Region 1 plus common storms to both regions. It can be seen that in Region 1 around 45% of the peaks come from storms specifically selected for Region 2, Figure 12 shows the % of peaks coming from storms selected for Region 2 (specifically selected for Region 2 plus common storms to both regions). In Region 2 around 25% of the peaks come from storms specifically selected for Region 1. We did not find a significant change in this % with bigger thresholds.
7. VERIFICATION

A preliminary test on the "quality" of the selected extreme Hs peaks was done through their steepness. Values above 1/10 are rarely found in measurements from open waters while values around 1/18 are usually observed for extreme waves in open ocean sites, WMO–No 702 (1988). For around 30 selected grid points in both regions, steepness for all peaks:

\[ \xi = \frac{2 \pi H_s}{g T_z} \]

(with tanh \(kh=1\) in the dispersion relation)

were computed to see how realistic were these values. This preliminary test was considered encouraging. Results in % are resumed in Table 1 for Region 1 normalized with the total number of data, None of the peaks are above 1/10 and most of them are in the 1/18 steepness "bins" (this table is a crude estimate, since \(H_s\) and \(T_z\) are approximated to the nearest integer). Table 2 depicts \(H_s\) against \(U_{10}\) for Region 1 (in % normalized against total data), values inside the "bins" correspond to a fully developed sea. It can be seen that most of the peaks which are between the threshold and around 11 m are swell while the few bigger peaks seem to be windsea. Results for Region 2 are very similar.

<table>
<thead>
<tr>
<th>TABLE 1: Wave steepness ((H_s - T_z)) - Region 1</th>
</tr>
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<tbody>
<tr>
<td>Normalized distribution</td>
</tr>
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</table>

<table>
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<tr>
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<th>1/18</th>
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<table>
<thead>
<tr>
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<td>Normalized distribution</td>
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<table>
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<tr>
<th>(H_s (m))</th>
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After this previous quality control test, results for a 22 days continuous period (85/12/18–86/01/04) containing 3 of the selected storms were tested against results produced by the third generation WAM model, WAMDI Group (1988), driven by wind fields provided by the European Centre for Medium Range Weather Forecast, Reading, U. K. This is a well tested scheme. Zambresky (1989), and so a reliable validation for both our wind and wave fields.

The grid used by the WAM model was a regular 19 spacing grid built on a Mercator projection covering the North Atlantic Ocean. Hs time series (HYPA and WAM) for a total of 4 different locations were compared. Figure 13 shows results at coordinates 4.0°W and 46.0°N in the Cantabric Sea about 250 Km off-shore Santander (Region 1) and Figure 14 is for coordinates 12.0°W and 40.0°N in the Atlantic Ocean about 250 Km off-shore Oporto (Region 2). The biggest Hs difference found in peaks for the 4 time series was slightly bigger than 1m on a 11m peak which was considered acceptable (the thin line is for HYPA model).

Unfortunately, no data from directional buoys were available for the hindcast period, and most of available data from scalar buoys was either measured very near to the coast or in very shallow water. After a selection process, 6 buoys were considered good enough for verification purposes, these buoys are moored at depths from 23 m up
to 105 m with data available for 13 storms although not from all the buoys. On the whole, 18 time series were finally available distributed among both regions. As the deep water version of HYPA model was used to hindcast the storms, a "gross" correction had to be applied to the model results for this comparison. Kitaigorodskii et al. (1975) recommended the use of a function of depth and frequency to modify a Jonswap spectra for shallow water. This function was applied in a simplified manner to the model results.

The scatter plot shown in Figure 15 depicts the 18 model Hs peaks versus these same 18 buoy peaks before the correction and Figure 16 after it. The positive BIAS of 0.76 m of the model results was reduced this way to an acceptable -0.19 and the correlation coefficient was left as 0.75. Although the verification process is not considered closed, the obtained results are considered acceptable.

8. EXTREME ANALYSIS

The aim of this hindcast is to define the return value for a 50 years return period of the following parameters: Hs, Tz and U10. Return values of Tz and U10 should be understood as the extreme value to be expected along with an Hs peak, that is, the return values for these two last parameters are not derived from the extreme event series of Tz and U10 but from the associated values to the Hs extreme event series.

The extreme analysis was performed under the following assumptions:

A) The two sets of storms selected for Region 1 and 2 constitute the same population as they belong to the same physical process, being many of them common to both sets. This first assumption allows us to fit a distribution to the whole set mixing the extreme events for each grid point without any consideration about their origin.

B) No different directional extreme analysis are needed as the biggest peaks have a well defined direction in both regions.
C) The extreme events series per grid point this way selected does not correspond to the annual maxima along the hindcast period, as they are data censored by the storm selection and by the applied threshold, but this "upper tail" of the data contains nearly all of the annual maxima as extreme storms for these two regions where found always in winter during the hindcast period and one or sometimes two storms were selected from nearly all of them, on the other hand the applied threshold basically only deletes small peaks caused in Region 1 by storms selected specifically for Region 2 and vice versa. For the evaluation of return values, this type of data should be properly fitted by a compound distribution:

\[
F(x) = \sum P(k)G(x) \quad R(x) = \frac{1}{1-F(x)}
\]

with \( F(x) \) the probability of non-exceedance of \( x \) in a random year, \( P(k) \) the probability of \( k \) storms occurring in any particular year, \( G(x) \) the distribution of wave heights above the threshold and \( R(x) \) the return period. Nevertheless it can be shown that for long return periods, return values are very well approximated by:

\[
F(x) = G(x) \quad C(x) = 1 - \frac{1}{\lambda R(x)}
\]

with \( G(x) \) a probability distribution fitted to the observed data and \( \lambda \) the number of samples divided by the number of hindcasted years. This last method has been applied to all the samples in the analyzed region.

Three probability distributions have been fitted to the samples using the least squares method: Weibull distribution and two extreme value distributions, FT-I (Gumbel) and FT-III. The obtained results are shown in Figures 17, 18 and 19, and Figure 17 depicts the maximum difference between return values. Most of the differences are below 2 m, for return values of around 15 m means deviations below 13%. In two areas there are big differences: A) the lower part of the grid, where as it has already been said, results are preliminary as the contribution of storms selected for Region 3 is missing (no further analysis is been done here) and B) a small area in the Cantabric Coast with differences above 3 m. In this area, some few peaks very much higher than the rest shift upwards the fitting of the Weibull distribution which has proved to be very sensible to the biggest peaks of each sample with the applied plotting positions.

Figures 21 and 22 show the fitting of the Weibull distribution and the Gumbel distribution for a grid point in this area. Due to the
uncertainty of results inherent to the hindcast method, results obtained from the Weibull distribution with the applied fitting method are not considered as reliable as the fitting is extremely sensible to the two or three biggest peaks.

**Figure 17:** $H_s$ Gumbel  
**Figure 18:** $H_s$ Weibull  
**Figure 19:** $H_s$ FT-III  
**Figure 20:** Maximum difference  

**Figure 21:** $H_s$, Weibull fitting  
**Figure 22:** $H_s$, Gumbel fitting
Table 3 shows the distribution giving the biggest return value, and Table 4 the distribution giving the smallest value. It was found that in areas where the fittings were bad, the Weibull return value was usually the biggest giving Gumbel the smallest. In areas where the fitting was good the results where the opposite, this was determined by the consistency of the biggest peaks with the rest of the samples.

Confidence intervals have been computed for each grid point and for each distribution, in most of the two regions the obtained values are below 1.5 m except for the two mentioned areas where fittings are bad and the confidence intervals unacceptable. The applied method is based on the simulation of n samples fitting the applied distributions with same parameters for each grid point, the corrected correlation of the fitting of each sample is computed and a probability distribution of correlations is developed for each distribution and used to chose the best fitting one in each grid point. The confidence interval for a given distribution and for a given probability is the range of return values of the n simulated samples for the distribution within 90% of the area of the probability distribution of return values, C. Petruskas & P.M. Aagaard (1970) and Y. Goda (1988).

Return values for $T_z$ and $U_{10}$ are very similar for the three distributions and nearly constant in both regions, so no figures are included. For $T_z$, 12.5 s was obtained as return value for Region 2 and 13 s for Region 1. For $U_{10}$, values of 22 m/s in the southern part of Region 2, increasing uniformly up to 26 m/s in Region 1 were obtained.
9. REFERENCES

Bijvoet, H. C., 1957: A new overlay for the determination of the surface wind over sea from surface weather charts, KNMI, Mededelingen en Verhandelingen, n 71.


EFFECTS OF UNCERTAINTIES IN HINDCAST DATA ON THE ESTIMATED FATIGUE DAMAGE AND FAILURE PROBABILITY OF MARINE STRUCTURES

Sverre Haver
Statoil, R&D-dep.
Trondheim, Norway

ABSTRACT

An important application of hindcast data is to apply them as basis for structural design – both with respect to fatigue assessments and ultimate load calculations. If this is to be successful, it is crucial that the hindcast data are of a sufficient accuracy, i.e. the results obtained should be so close to the expected "true values" that possible differences are accounted for by standard load coefficients. One possible way of assessing this topic is to compare the response distributions obtained using hindcast data with those obtained using wave measurements. Data, wave measurements and hindcast values, from two North Sea locations, Ekofisk (1980–90) and Statfjord (1976–89), are considered. As the long term distribution is determined, an estimate for the fatigue damage can be calculated and an estimate of the n-year value is also easily obtained. These quantities are typically governed by different regions of the wave scatter diagram, i.e. the present study will provide an overall quality assessment of the hindcast data. The study will involve two structural concepts. One concept represents a quasistatic system, while the other is heavily influenced by dynamics. The latter is thus expected to be more sensitive to lower sea states with a short spectral peak period since such seas may yield a very strong dynamic amplification. The annual failure probability can be estimated from the long term distributions. However, it can alternatively be approximated by only consider the annual largest storm event. Adopting this approach for a drag dominated structure, the effects of the uncertainties in hindcast data with respect to the failure probability will be compared with the effects of other sources of uncertainties.

1. INTRODUCTION

The overall aim of the design process of a marine structure is to ensure that the structure can withstand the largest environmental forces during its lifetime with an adequate degree of safety. For most structures, the force level is governed by the wave induced forces. Accordingly, a reliable prediction of these forces becomes crucial regarding a reliable design. Due to this emphasis is herein given to the wave induced forces.

If a sufficient amount of wave measurements are available for a particular offshore site, the design conditions will usually be predicted based solely on these. If only a limited amount of
measurements are available, hindcast data represents a possible source of additional information. The adequacy of the hindcast values can then be established from the overlapping data and this information can be used for modifying the extremes predicted from the hindcast data. The advantage of the hindcast data is that they typically cover a rather long time period (25–35 years), while the disadvantage of course is the fact that the data are generated by means of idealized numerical models. Over the years several authors have considered the adequacy of modern hindcast models. The main impression is that modern hindcast data is a useful source of information in connection with structural design, see e.g. Haver (1986) and especially a series of Hibernian papers, e.g. Szabo et al. (1989), Cardone et al. (1989).

The purpose of this paper is not to compare the wave conditions obtained from hindcast models with those being established from measurements. Herein we will rather focus on possible differences in the predicted structural response caused by differences between hindcast data and measurements. The long term response distributions will be established for both a quasistatic platform and a platform heavily influenced by dynamics. From these distributions, the differences caused by the choice of data source will be discussed both with respect to extremes and fatigue damage. Concerning fatigue, the discussion will be of a qualitative nature. For extreme value calculations, one may alternatively adopt a peak-over-threshold technique and describe the wave conditions by merely include storm events exceeding a given threshold. The consequences of the differences between measurements and hindcast data in connection with such an approach will be addressed by considering the shear force at mudline of a drag dominated offshore structure.

Two North Sea locations are considered, namely Ekofisk and Statfjord. Hindcast data from the Norwegian hindcast database which is established using WINCH, Cardone (1984), Eide et al. (1985), are included. Herein we will merely present the main results. For more details and background data reference is made to Haver (1992), where also data from the North European Storm Study (NESS), Francis (1986), are considered.

2. ENVIRONMENTAL CONDITIONS

Long term Wave Climate

One way of describing the long term wave climate is to establish a joint probability density function for the main sea state characteristics; significant wave height, spectral peak direction, and, possibly, the main wave direction. This is usually done by fitting selected probabilistic models to available scatter diagrams, see e.g. Haver and Nyhus (1986) and Bitner-Gregersen and Haver (1989). Herein we are mainly interested in a relative comparison and we will
therefore simply adopt the observed scatter diagrams as the joint distributions. It should be stressed that this will typically result in extreme values which are somewhat underestimated. Scatter diagrams for Ekofisk and Statfjord corresponding to the cold season of the year (Oct.–March) are given in Haver (1992) both with respect to measurements and hindcast data.

Storm Wave Climate

A storm wave climate for Ekofisk and Statfjord based on measurements is suggested by Haver (1991).

The distribution of the maximum significant wave height for storms exceeding a certain threshold, $h_o$, is modelled by a truncated Weibull model, i.e.:

$$F_{H_{h_o}}(h) = 1 - \exp \left\{ -\left( \frac{h}{\theta} \right)^\gamma + \left( \frac{h_o}{\theta} \right)^\gamma \right\} \quad h \geq h_o$$

(1)

where $\theta$ and $\gamma$ have to be estimated from available data. The corresponding spectral peak period is characterized by the following expressions for the expected value and the standard deviation:

$$\bar{t}_p = a_0 + a_1 h^a_2 \quad \text{and} \quad s_{t_p} = 0.09 \bar{t}_p$$

(2)

Finally, the annual number of storms, $N$, exceeding the threshold, $h_o$, is modelled by a Poisson distribution where $\phi$ denotes the expected no. of storms per year, i.e.:

$$P_N(n) = \frac{\phi^n}{n!} e^{-\phi}$$

(3)

The key parameters for the storm wave climate for Ekofisk and Statfjord are given in Table 1, where also various uncertainties are quantified.

For the WINCH-data all storms exceeding 6.5 m at Ekofisk and 7.5 m at Statfjord are identified. Only the cold season of the year is included. For Ekofisk we have included the time period from the summer 1980 until the summer 1989, while the years from 1976 until 1989 are covered for Statfjord. The time periods are not identical to those corresponding to wave measurements, however, they are so close that the results should in principle be comparable. The parameters of the truncated Weibull model are estimated using the maximum likelihood principle and the fitted models are compared to the empirical
distribution functions in Fig. 1. The parameters for the mean spectral peak period, Eq. (2) are determined by a least square fit. The fitted relation for the Ekofisk case is shown in Fig. 2. The storm climate parameters obtained from the WINCH-data are given in Table 1.

Based on the adopted data sources we obtain the following 100-year sea states:

<table>
<thead>
<tr>
<th></th>
<th>Ekofisk</th>
<th>Statfjord</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurements:</td>
<td>13.5 m, 15.2 s</td>
<td>14.0 m, 16.2 s</td>
</tr>
<tr>
<td>WINCH:</td>
<td>15.8 m, 17.5 s</td>
<td>14.7 m, 16.4 s</td>
</tr>
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</table>

It is seen that the results are not too different for Statfjord, while at Ekofisk the 100-year height is rather different. WINCH seems to represent a severe overestimation for this area. This should possibly be expected since the WINCH-model is a deep water model. Concerning the results obtained from measurements it should be mentioned that the Ekofisk 100-year value is reduced from 14.0 m to 13.5 m in order to account for the scatter in the 20-min. mean values as compared to the 3-hour values. A similar modification is not introduced for Statfjord since a comparison between the fitted model and the empirical model suggests that this to some extent is accounted for by the particular result of the fitting process.

3. LONG TERM DISTRIBUTION OF RESPONSE MAXIMA

The long term distribution of crest heights, $Y$, of a response process is given by;

$$F_Y(y)=\frac{1}{<v_0^+>}\int_{h,t} f_{Y|H_\infty T_p}(y|h,t) f_{H_\infty T_p}(h,t) dh dt$$

where $F_Y|H_\infty T_p(y|h,t)$ is the short term distribution, $f_{H_\infty T_p}(h,t)$ is the joint probability density function of $H_\infty$ and $T_p$, $v_0^+(h,t)$ is the expected zero-up-crossing frequency for the response process within a stationary sea state, and $<v_0^+>$ is the long term mean value of $v_0^+(h,t)$.

Provided that the response process is a Gaussian process, the short term distribution is reasonably well modelled by a Rayleigh distribution. This distribution involves the standard deviation of the response process as the parameter and this quantity is easily estimated from the response spectrum. The latter is determined as a product between the wave spectrum and the modulus part of the transfer function squared. Herein the long term distribution are obtained using a Pierson Moskowitz wave spectrum for all sea states.
From Eq. (5), the n-year response value, $y_n$, is estimated by the value corresponding to an exceedance probability of $1/q_n$, where $q_n$ is the expected no. of zero-up-crossings during n years. Concerning fatigue it is rather the cycle width, $s$, that is of interest. For fatigue purposes one can often assume that the width is equal to twice the crest height. Assuming that the no. of cycles to fatigue failure is given by $cs^{-3}$ where $c$ is a proper coefficient, it can be shown that the expected fatigue damage is proportional to the third order moment of the long term distributions, Langen and Sigbjørnsson (1979). The effect of using hindcast data or measurements for fatigue calculations is therefore considered by comparing the third order moment of $Y$ for the various sources of wave data. Two response quantities are considered herein. The shear force at mudline for a Condeep type platform and the axial stress in the leg of a deep water jacket. The transfer function for the shear force at mudline reflects an essentially quasi-static behaviour. This means that the most extreme sea states are of main interest concerning ultimate loads. On the other hand, the jacket structure has a natural period as large as 5.8 s, and sea states leading to resonance may be of importance.

The 100-year values and third order moments for the various distributions are given in Table 2. Concerning base shear, it is seen that WINCH overestimate the 100-year value considerably for the Ekofisk area, while a similar underestimation is obtained for the Statfjord field. The Ekofisk result is according to our expectations, while we at first were surprised by the Statfjord result. However, the reason is most probably due to a very long spectral peak period of the highest measured sea state. For the axial stress, which due to dynamics is likely to be affected by the presence of low and moderate sea states with a peak period in the vicinity of the largest structural period, the results are slightly different. With respect to Ekofisk, WINCH-data yield a value which is about 8% less than the value obtained from the measured scatter diagram. At Statfjord, on the other hand, WINCH-data result in a 100-value about 17% less than the target value. The reason for this is most probably that the WINCH-model yields a much lower number of steep sea, which may be important due to the strong dynamic amplification for such seas.

The adequacy of the hindcast data concerning fatigue calculation can be qualitatively indicated by considering the third order moment of the long term distributions. For a quasistatic response quantity, WINCH data yield conservative results. The degree of conservatism is very large for the Ekofisk location. However, for the response quantity governed by dynamics, the WINCH scatter diagram yields a very accurate result for Ekofisk and a somewhat too low value for Statfjord. Again the adequacy of the results is the resulting effect
of the adequacy of the hindcast significant wave height level and the accuracy of the corresponding range for the spectral peak period.

4. ANNUAL EXCEEDANCE PROBABILITY FOR THE BASE SHEAR OF A DRAG DOMINATED STRUCTURE

Introduction

Based on Eqs. (1 and 3) it can be shown that the distribution function of the annual largest storm, \( H_{\text{mo,1}} \), reads:

\[
F_{H_{\text{mo,1}}}(h) = \exp \left\{-\phi \exp \left[-\left(\frac{h}{\theta}\right)^\gamma + \left(\frac{h_o}{\theta}\right)^\gamma\right]\right\}
\]

(6)

The conditional distribution of the spectral peak period given the significant wave height is modelled by a log-normal distribution, i.e. the probability density function reads:

\[
f_{T_p|H_{\text{mo,1}}}(t|h) = \frac{1}{\sqrt{2\pi}\sigma_{\ln T_p}} t \exp \left\{-\frac{1}{2} \left(\frac{\ln t - \mu_{\ln T_p}}{\sigma_{\ln T_p}}\right)^2\right\}
\]

(7)

\[
\mu_{\ln T_p} = \ln \bar{t}_p - \frac{1}{2} \ln \left(1 + \left(\frac{S_T}{\bar{c}_p}\right)^2\right) = \ln \bar{t}_p - 0.004
\]

(8)

\[
\sigma_{\ln T_p} = \left\{\ln \left[1 + \left(\frac{S_T}{\bar{c}_p}\right)^2\right]\right\}^{1/2} = 0.09
\]

(9)

where \( t_p \) and \( S_T \) are given by Eq. (2).

The load calculation procedure adopted herein involves the expected zero up-crossing wave period, \( T_z \), and the wave period corresponding to the highest wave crest, \( T \). We will assume that both these variables can be described by a log-normal distribution and, furthermore, that they are given by \( T_z = 0.74 T_p \) and \( T = 0.90 T_p \).

The conditional distribution function for the largest crest height, \( C_{\text{max}} \), during a storm event can for a Gaussian surface be estimated by, Madsen et al. (1986):
where $\Delta t$ is the duration of the event. $\Delta t = 3 \text{ hrs} = 10800 \text{ s}$ is herein adopted as a base case value.

Uncertainties related to the wave climate description are discussed by Haver (1991). Emphasis is given to the uncertainties associated with the probabilistic modelling of the significant wave height. Model parameters and corresponding uncertainties are summarized in Table 1. Uncertainties in the significant wave height distribution are accounted for by introducing the shape parameter, $\gamma$, as a random variable, $\Gamma$. The variability is due to model uncertainties as well as statistical uncertainties. Accordingly, we write; $\Gamma = \gamma + \Gamma_s + \Gamma_m$, where $\Gamma_s$ and $\Gamma_m$ account for statistical uncertainties and model uncertainties, respectively. $\Gamma$ is assumed to follow a lognormal distribution. $\Gamma_s$ and $\Gamma_m$ are assumed to be independent and their standard deviations are given in Table 1. The scale parameter is calculated by a closed form expression which also is given in the table.

Shear force at mudline

The annual largest shear force of a drag dominated structure is assumed to occur as the largest wave of the annual largest storm passes the structure. The maximum shear force at mudline is assumed to be about 5 times the load on a single vertical pile, and it is given by, Haver (1992):

$$Q_{\text{max}} = 5 \rho r C_d w_f^2 \left\{ \frac{\pi}{2} \left[ \frac{4 \pi d}{L} + \sinh \left( \frac{4 \pi d}{L} \right) \right] C_{\text{max}}^2 \right\} + \frac{\pi}{2} \left( \frac{4 \pi d + L \sinh \left( \frac{4 \pi d}{L} \right)}{L} \right) \frac{C_{\text{max}}^3}{2 d^2 \sinh^2 \left( \frac{2 \pi d}{L} \right)}$$

where $d$ is the water depth, $T (-0.9 T_p)$ is the period of the maximum wave, $L$ is the corresponding wave length, $C_{\text{max}}$ is the crest height, $\rho$ is density of sea water, $r$ is leg radius, $C_d$ is the drag coefficient, $\omega$ is the wave frequency, $k$ is the wave number, and $w_f$ is a factor which is introduced in order to account for uncertainties in the wave speed. The wave speed is calculated using Wheeler stretching, Wheeler (1970).

Assume that a critical shear force level is $q_o$. The annual probability of exceeding this level is conveniently estimated using a First Order Reliability Method (FORM), Madsen et al. (1986). The limit state function for such a purpose is given by:
\[ g(H_{mo}, T, C_{max}) = q_o - Q_{max} \] (12)

It is seen that the limit state function becomes negative as the critical level is exceeded. The exceedance probability is estimated by using the FORM-mode of the computer program, PROBAN, Tvedt (1989).

The present problem formulation involves 3 environmental characteristics which essentially are of a random nature. However, a particular estimation of the exceedance probability requires that all distribution parameters and physical coefficients are known, and that mathematical models are formulated for the underlying physical mechanisms. Neither the mathematical models nor the distribution parameters/physical coefficients are perfectly true. A varying degree of uncertainty will be related to the various assumptions/choices depending on the amount of available data and/or the degree of understanding of the underlying mechanisms. Herein we will indicate the effect of such uncertainties by introducing the drag coefficient, \( c_d \), and the speed factor, \( w_f \), as random variables. They are both assumed to be Gaussian variables with a mean value equal to 1 and a coefficient of variation of 15%. Additionally, uncertainties are also introduced for the shape parameter as mentioned previously. These are assumed to be the same both for measurements and hindcast data.

The main purpose of this part of the paper is to indicate the differences in the distribution of \( Q_{max} \) whether hindcast data or measurements are used to characterize the environmental conditions. However, we will also assess the relative importance of the uncertainties related to the hindcast data as compared to other sources of uncertainties. For this purpose we can assume that the true value of the significant wave height \( h_{mo}(t) \), is related to the hindcast value as follows;

\[ h_{mo}^{(t)} = h_{mo}^{(hc)} - x \] (13)

where \( x \) is a realization of a random variable, \( X \), describing the error related to the hindcast model. Assuming that the measurements, \( h_{mo}^{(m)} \), can be taken as good estimates for the true values, realizations of \( X \) are obtained from the particular storm samples. For the two offshore locations, the results are shown by Fig. 3. The mean deviation is estimated by a linear regression approach. The standard deviation about this line is found to vary between 1 and 1.4. We will for the present study adopt a standard deviation of 1.25 m for both sites and see the effect of this uncertainty on the estimated exceedance probability.

4.3 **Annual exceedance probability**

The annual exceedance probability is calculated both for the Ekofisk and Statfjord locations. The results are shown in Fig. 4. It is seen
that the hindcast model slightly overestimate the exceedance probability for Statfjord, while it significantly overestimates the exceedance probability regarding Ekofisk.

Two sources of errors in the hindcast data can be considered separately. The adequacy of the hindcast height level (see Fig. 3) is most probably the most important error. However, the fitted relation for the conditional mean period, Eq. (2), may also affect the result. For the Ekofisk area, the WINCH-case has been analysed using the conditional mean period as obtained from the measurements. It is seen from Fig. 4(a) that the effect of changing the period relation is rather small. The mean error in the hindcast significant wave height is shown versus the hindcast wave height in Fig. 3. Annual exceedance probabilities are estimated when the hindcast height is corrected according to this mean error. The results are shown with broken-dotted lines in Fig. 4. It is interesting to note that the annual exceedance probabilities are too much “corrected” by introducing the mean error expressions. The reason for this can be twofold. At first we neglect the scatter around the regression line and, secondly, the extrapolation of the linear regression lines should always be questioned when the correlation is as low as for these cases.

A more proper correction for the hindcast data is obtained by requiring the hindcast distribution function to equal that established from measurements. Solving this equation with respect to the argument of the measured distribution and assuming that the measurements are close to the true values, the following correction formula is obtained for Ekofisk:

\[
h = 2.03 \left( \frac{h_{\text{WINCH}}}{0.273} \right)^{0.654} - 3.58 \right)^{0.8}
\]

Eq. (14) is established from the fitted annual extreme value distributions given by Eq. (6) and Table 1. The exceedance probabilities obtained when the hindcast data are corrected according to this relation are compared to those obtained from measurements in Fig. 4(a). As expected a rather good fit is observed. This suggests that when correcting hindcast data one should rather use expressions obtained from comparing distribution functions than simple regression relations obtained directly from simultaneous observations.

If the hindcast data are used without any calibration, they seem to yield slightly conservative characteristic loads for the Statfjord area. The degree of conservatism at a 10^{-2} level is about 10% for WINCH. Regarding the Ekofisk area, the hindcast data should not be
used for design purposes without being calibrated against measurements. WINCH overestimates the characteristic load level (annual probability of exceedance of $10^{-2}$) with nearly 50%.

The characteristic load is usually determined using fixed values for the various parameters. This goes both for physical parameters of the various probabilistic models. Most parameters, however, will be associated with uncertainties both of a systematic nature and a random nature. Correcting for systematic errors, may in principle either reduce or increase the estimated failure probabilities. However, if a sound engineering judgement has been used when design parameters are selected, the correction for systematic errors should reduce the nominal failure probability. Random errors on the other hand will always increase the exceedance probabilities of large load levels.

This is clearly demonstrated by Fig. 5, where the effects of introducing uncertainties related to the measurements are demonstrated. The uncertainties actually introduced in the above consideration are;

- drag coefficient, $c_d$. The drag coefficient is assumed to be normal distributed with a mean value of 1.0 and a standard deviation of 0.15.

- speed coefficient, $w_f$, see Eq. (11). This coefficient is also assumed to be normal distributed with a mean value of 1.0 and a standard deviation of 0.15.

- shape parameter, $\gamma$, of the distribution of storm peaks. $\Gamma$ (a priori notation for $\gamma$) is assumed to be lognormal distributed with a mean value $\gamma$ according to Table 1 and a standard deviation given by:

\[
\sigma_T = \left( \sigma_{\Gamma_m}^2 + \sigma_{\Gamma_s}^2 \right)^{1/2}
\]

where $\sigma_{\Gamma_m}$ and $\sigma_{\Gamma_s}$, are given in Table 1. The corresponding location parameter is calculated by the relation which is given in this table.

An interesting result provided by the FORM calculation is the importance factors for the various random variables and uncertain parameters, see e.g. Madsen et al. (1986). The importance factors can be interpreted as a measure of the relative contribution to the failure probability from the various random variables/uncertain parameters. If the importance factor is close to zero for a variable, then the estimated failure probability is not changed by replacing the random variable by its mean value. It should be stressed that the mean value may still be an important quantity concerning the load level.

The importance factors will typically change with the load level and this is illustrated by Table 3. If hindcast data are used for
assessing the exceedance probabilities, the relative importance of the uncertainties related to these data can be indicated by calculating the importance factor for the error variable, X, see Eq. (13). Uncertainties in the shape parameter of the storm peak distribution is assumed to be the same as for the measurements. However, the corresponding scale parameter is calculated by the respective expressions given in Table 1.

It is seen that the results obtained using hindcast remind very much of those obtained from measurements. The main contribution to the exceedance probability of large load levels comes from the inherent variability of the annual largest significant wave height. As compared with the measurements the Ekofisk value is increased, while the Statfjord number has decreased somewhat. The reason for the increase in the importance factor for $H_{mo}$ for Ekofisk in spite of the introduction of the error variable is most probably as follows. The annual extreme value distribution for Ekofisk corresponds to a very fat tail. This means that rather large values may occur for this site. Since the mean error is assumed to be a linear function of the significant wave height, the variation in the mean (which is associated with the variability of $H_{mo}$) may in case of a wide variability in $H_{mo}$ be more important than the scatter around the mean curve.

The most interesting result for the present study, however, is the importance factor for the error variable, X. It is seen that if hindcast is used for calculating the design loads, the adequacy of the hindcast data is one of the most important sources of uncertainties. Accordingly, further efforts on improving the fit between hindcast and measurements are recommended, especially if the design is to be based solely on hindcast data. The first step should be to eliminate the height dependent bias and, thereafter, a long term aim should be to reduce the scatter between the hindcast values and the measurements.

CONCLUSIONS

The results of response calculations using both wave measurements and hindcast data as input data are compared. Two North Sea locations (Ekofisk and Statfjord) are considered and hindcast data produced by WINCH have been included.

At first a comparison involving all overlapping data points is carried out for both a quasistatic response quantity and a response quantity heavily influenced by dynamics. Since the investigation is carried out using the scatter diagrams directly (no smoothing is introduced), the predicted extremes are rather sensitive to the characteristics of some few extreme sea states. Due to this we should be careful in generalizing the present results. However, we made the following
observations concerning the predicted 100-year response values: The hindcast data yield a 10% overestimation for the quasistatic response at Ekofisk while a similar underestimation is the case for Statfjord. For the dynamic response quantity, an underestimation of about 5–10% seems to be the case for Ekofisk while a rather strong underestimation (15–20%) takes place for Statfjord. It should be stressed that the deviations from what is obtained from measurements is the combined effect of the accuracy of the hindcast wave height and the accuracy of the hindcast spectral peak period. The scatter in the spectral peak period at a given height level is typically somewhat smaller for the hindcast data than what is measured, i.e. hindcast data yield fewer and lower sea states with a spectral peak period close to the natural period. Regarding fatigue calculations, WINCH data tend to overestimate the fatigue damage of a quasistatic structure considerably, while the results are not too bad for a structure exposed to dynamics. In order to obtain a comparison less sensitive to some few extreme events, a similar comparison should also be carried out using smoothed scatter diagrams.

The main effort is herein given to a consideration of annual exceedance probability of the shear force at mudline of a drag dominated structure. From the present considerations the following conclusions apply:

* for the Ekofisk area, the annual exceedance probability is very much overestimated when WINCH data are used. The 100-year shear force is overestimated by about 50% by WINCH. For this area, the hindcast data should not be applied for design purposes without being calibrated against measurements.

* regarding Statfjord, the results obtained using hindcast data are not too bad. The hindcast data slightly overestimate the failure probability. The 100-year value is overestimated with about 10%.

* if hindcast data are to be calibrated against measurements one should be careful in using a regression approach directly on simultaneous data. A correction formula should rather be established by comparing distribution functions. This will ensure that the scatter around a regression curve is properly accounted for.

* uncertainties related to the adequacy of hindcast data yield important contributions to an estimated nominal failure probability. The relative importance of these uncertainties is comparable to or even larger than the uncertainties related to the calculation of wave kinematics, the choice of drag coefficient, and the modelling the annual largest storm peak.

As a final recommendation we will stress the need for further efforts on improving hindcast data if design is to be based solely on such
data. First priority should be given to an elimination of the bias, thereafter one should aim at reducing the random error. However, if a sufficient amount of overlapping data are available, then hindcast data together with measurements will represent a very good background for design.

Finally, it should be mentioned that the measurements represent 20-min. mean values, while the hindcast data correspond to a length of averaging of about a couple of hours. This difference will of course introduce some noise with respect to a comparative study. However, it is not expected to change the main findings of the present study.

ACKNOWLEDGEMENT

Statoil is acknowledged for the permission to publish this paper. The opinions expressed herein are those of the author and they should not be construed as reflecting the views of Statoil.

REFERENCES


Table 1: Key parameters for the storm wave climate descriptions for Statfjord and Ekofisk.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>EKOFSK</th>
<th>STATFJORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storm threshold</td>
<td>6.5 m</td>
<td>7.5 m</td>
</tr>
<tr>
<td>Measurements / Hindcast</td>
<td>(m)</td>
<td>(hc)</td>
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<td>Expected annual no. of storms, φ</td>
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<td>Statistical uncertainty in φ</td>
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<td>C.O.V. (φ)</td>
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<td>Significant wave height</td>
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<td></td>
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<td>θ̄</td>
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<td></td>
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<td>Scale parameter accounting for uncertainties</td>
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<tr>
<td></td>
<td>(7.69γ-6.5)γ + (7.79γ-6.5)(γ)</td>
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<tr>
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<td>(8.79γ-7.5)γ + (8.95γ-7.5)γ</td>
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Table 2  
100-year values and third order moment obtained from the long term distributions of response.

<table>
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<th>Response quantity</th>
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<th>Environmental data</th>
<th>100-year value</th>
<th>Third order moment</th>
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<td>Measurements WINCH</td>
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<td>Measurements WINCH</td>
<td>35.8 29.8</td>
<td>138.8 114.6</td>
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Table 3  
Importance factors (%) for the various variables. 
m: Measurements, hc: hindcast (WINCH)

<table>
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<td>Statfjord</td>
<td>Ekofisk</td>
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<td></td>
<td>m</td>
<td>hc</td>
<td>m</td>
</tr>
<tr>
<td>Inherent randomness</td>
<td>H_m0</td>
<td>C_max</td>
<td>T</td>
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<td></td>
<td>76</td>
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<tr>
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<td>w_r</td>
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<td></td>
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<tr>
<td></td>
<td>4</td>
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<td>2</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>16</td>
<td>-</td>
</tr>
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Fig. 1  Distribution functions of storm peaks for Ekofisk and Statfjord. Hindcast data.

Fig. 2  Expected spectral peak period versus height of storm peak, Ekofisk hindcast data.

Fig. 3  Error in hindcast data versus hindcast level for Ekofisk (a) and Statfjord (b).
EXTREMAL ANALYSIS OF HIBERNIA WAVE HINDCAST DATA

G.Z. Gu
E.P. Berek
F.J. Dello Stritto
D. Szabo
H.V. Leder

Mobil Research and Development Corporation, Dallas E&P Engineering, 13777 Midway Road, Dallas, Texas 75244

ABSTRACT

Extreme wave criteria for the Hibernia site were based on extremal analysis of peak significant wave heights from storm hindcasts. In addition to the storms hindcast for the criteria development, other Hibernia storms occurring during approximately the same time period were hindcasted in a newly completed hindcast study, using the same numerical model. From the complete set of available hindcast peak seastates, detailed investigations of the extreme population were performed, which could not have been done meaningfully in the past. These included comparisons of different extremal distributions, fitting methods and measures of goodness-of-fit. Two separate issues of extreme waves, of practical importance in offshore engineering, are also addressed. First, load factors used in the recent CSA offshore standards are based in part on the assumed variability in the distribution of annual extremes. The data set of hindcast extremes is used to test that assumption. Second, measured wave height data are analyzed with a Weibull distribution to assess extremal estimates based on very limited data.

1.0 INTRODUCTION

Offshore development requires site-specific extreme environmental criteria. Perhaps the most subjective and uncertain element in developing criteria is extremal analysis, i.e., the extrapolation of finite data sets to estimate long return period values. Several extreme criteria studies have been performed for the Hibernia site. The combined results of these independent efforts allow more detailed assessments of extremal analysis techniques and results than previously possible.

The Hibernia site is located in approximately 80m water depth, on the Grand Banks, off Newfoundland. Its nominal location is 46°46′N, 48°46′W. Estimates of extreme wave criteria have been made from three separate, though not entirely independent, investigations.

Extreme wave criteria were developed for the Hibernia Development Project in a series of studies spanning 1980 to 1986, known collectively as Hibernia Wave Hindcast Project (HWHP) [3]. HWHP yields
15.9m and 29.3m as the 100-year Hs (significant wave height) and H\text{max} (maximum wave height). These criteria are ultimately based on hindcasts and extremal analysis of 29 severe storms occurring over a 34 year period (1951–1984). The peak significant wave heights of storms in the hindcast were fitted to the Borgman extremal distribution, with 15.9m as the 90% confidence limit of the mean 100-year value (14.4m).

In 1991, results of a wind/wave hindcast study for the entire east coast of Canada performed by Oceanweather Inc. (OWI) for the Canadian Climate Center became available [2]. In this hindcast, referred to in this study as Canadian Waves Project (CWP), 68 storms over a period of 32 years (1957–1988) were hindcasted. OWI also performed hindcasts for HWHP, using various versions of the ODGP numerical model. The grid system used in CWP covered a much larger area (including the Grand Banks, Scotian Shelf, and Georges Bank) than that in HWHP. At the Hibernia site, CWP yields the 100-year mean Hs and H\text{max} of 14.3m and 26.7m with the Borgman and 14.6m and 27.4m with Gumbel distributions.

For extreme analysis, numerous techniques exist and have been described in the literature. Different techniques are used by different industry disciplines for different purposes. A number of extremal analysis methods and techniques appropriate to the offshore industry have been reviewed and summarized in Ref.[7]. In this study, many of these techniques and methods are applied to the combined HWHP and CWP hindcast data sets to assess the accuracy of the existing Hibernia wave criteria and to test the applicability of the techniques. In the course of the analysis, a group of estimator equations of the maximum likelihood method have been developed for compound extreme distributions, which are not readily available in the literature.

From waverider buoy measurements made during drilling at or near the Hibernia site, an almost continuous five-year span of recorded seastate data have been assembled from waverider buoy measurements made during drilling. The measured data were fitted to a Weibull distribution to determine extreme wave heights for short (monthly to 1-year), as well as long return periods [6]. The best fit 100-year value (14.3m) agrees quite well with the results based on the hindcast data, though [6] describes the limitations in the value of that comparison.

In the concluding sections of this paper, two tangential, but related, topics are addressed. The combined annual extremes of the hindcast data are used to assess the implicit assumptions about annual maxima in the new Canadian Standards Association Offshore Standard. The 5-year measured data are used to study the sensitivity of the extreme estimates to the duration of the data set when a continuous data set
is used. This analysis addresses a common, practical problem in offshore applications: basing long term extremes on very limited spans of measured data.

2.0 WAVE HEIGHT DATA

Table 1 lists peak Hs values from the HWHP and CWP hindcast studies. HWHP (1951-1984) and CWP (1957-1988) overlap, and 16 of the HWHP storms were also hindcasted in CWP. Due to changes of time step and grid in the, numerical wave model, and principally due to differences in windfield specification, the two peak Hs values for the same storm often differ, on average by 1.2m. These differences are indicative of uncertainty in wind specification, model accuracy, and sensitivity to grid size and time steps - valid topics beyond the scope of this study.

Where HWHP and CWP differ, the higher Hs was used in the following analyses. The combined hindcast data set has 81 storm-peak Hs values, ranging from 3.1m to 13.4m and spanning 37 solstice years (6/1951-6/1988, see Table 1). Based on this combined Hs data set, two data files were created: Annual Extreme Values (AEV) and Peaks Over Threshold (POT). Of the 37 years, peak seastates are available for 31; for 6 of the years neither HWHP or CWP identified severe storm meriting hindcasts. Peak seastates for these years are assumed to be less than 6.0m. No attempt has been made to infill the missing years, and the AEV data set is therefore assumed to span a 31-year period. Hs values in the AEV data file range from 5.6m to 13.4m, as listed in Table 2. The POT data set covers a period of 38 calendar years with a threshold value of 6.0m.

The measured Hs data cover a five-year period, 1980 through 1984. The significant wave height is approximately a 20-minute average, measured at 3-hour intervals throughout the 5 years. The maximum measured significant wave height is 11.9m.

3.0 EXTREME VALUE DISTRIBUTIONS

Since the extreme value distribution which best describes the peak Hs in storms is not known a priori, the data were fitted to a number of different distributions. In this study, six extreme value distributions were tested for the hindcast Hs data sets, and only the Weibull distribution was used for the continuous data.

The six extremal distributions for the hindcast data used in this paper are:

1) Gumbel (FT-I, i.e., Fisher-Tippet Type I) distribution
2) Borgman distribution
3) FT-II distribution
4) FT-III distribution
5) Weibull distribution
6) Exponential distribution.

Each distribution has two different expressions according to the data type: one for AEV data and one for POT data. The expressions for AEV data are termed simple extreme value distributions in this paper and those for POT data are termed compound extreme value distributions. A compound distribution $F(x)$ is generally expressed by the convolution of a discrete distribution, which describes the number of events in a unit time (usually a year), and a simple extreme value distribution. A compound extreme value distribution is given by

$$F(x) = \sum_{k=0}^{\infty} P(k) F_s(x) \tag{1}$$

where $F_s(x)$ is the so-called simple extreme value distribution, $F(x)$ is the corresponding compound extreme value distribution, and $P(k)$ is the discrete distribution for the number of events in a year.

When the Poisson distribution is used to describe the number of storms in a year, the relationship between the two types of distributions can be expressed as

$$F(x) = \exp \left\{ -\lambda [1 - F_s(x)] \right\} \tag{2}$$

where $\lambda$ is the number of events per year.

The expressions of various compound extreme value distributions are listed in the Appendix.

For continuous data, fitting to the Weibull distribution is widely used to estimate extremes, though it is not a true extremal analysis (i.e., an analysis of extremes). Accordingly, the resulting return period extremes (e.g., 100-year value) are not the same statistic as that produced by fitting independent maxima (i.e., individual storm peaks), though it bears the same name [6]. However, in offshore development, a common necessity is the estimation of long term extremes from very limited data. Data fits to Weibull distributions produce as accurate an estimate as can be obtained within such restrictions. The Weibull distribution (two parameter Weibull) has the following form:

$$F(x) = 1 - \exp \left[ -\left( \frac{x}{B} \right)^c \right] \tag{3}$$
where B and C are constants to be determined by fitting to the data.

4.0 FITTING METHOD

All the distributions listed above contain parameters whose values must be determined by fitting the function to the data. In this study, two fitting methods are employed: the Linear Least Squares (LLS) method and the Maximum Likelihood Estimate (MLE) method.

4.1 Linear Least Squares Method (LLS)

LLS method is still the most popular fitting method, regardless of its recognized bias [9]. In LLS, both the distribution function and the data are first transformed into the so-called reduced variates so that the distribution function can be expressed as a linear equation. The unknown parameters in the distribution are then determined by minimizing the total squared error between the reduced variates of the data and the distribution function.

One of the weaknesses of the LLS method is the necessity of using a so-called plotting position formula for calculating the probabilities associated with each data point. The formula is not rigorously and uniquely defined. Two of the most commonly used plotting positions in the analysis of wave data were tested in this study:

1) Gumbel:

\[ P_i = \frac{i}{N+1} \]  

(4)

2) Gringorten's approximation:

\[ P_i = \frac{i - 0.44}{N + 0.12} \]  

(5)

where i is the rank of the data point in an ascendingly ranked data set and N is the total number of the data points.

From this point on, the Gumbel plotting position is used unless otherwise specified.

According to the data type (AEV or POT) and the distribution (simple or compound), the linear equation of reduced variates (see Appendix) is different. For example, for AEV data and the simple Gumbel distribution, the equation is

\[ X = A - B \ln \{-\ln [F_s(x)]\} \]  

(6)

but for POT data and the Poisson-Gumbel compound distribution, the equation of reduced variates becomes

\[ x = A - B \ln \left\{-\ln \left[1 + \frac{1}{\lambda} \ln (F(x))\right]\right\} \]  

(7)
where $\lambda$ is the average number of events in a year.

In the equation for the compound distribution, the factor $1+1/\lambda \ln(F)$ could become negative for small values of $F$; and such data points must be discarded. This is related to the data censorship and will be discussed later.

The linear equations of reduced variates for all six distributions are listed in the Appendix.

### 4.2 Maximum Likelihood Estimate Method (MLE)

MLE is based on the maximization of the so-called likelihood function. A likelihood function is defined as the joint probability distribution of all the data points which is the product of the density functions for all the data:

$$L = \prod_{i=1}^{N} f(x_i, \theta)$$

where $\theta$ is a vector representing a group of parameters in the distribution, e.g., $\theta = \{A, B\}$ for the Gumbel distribution.

The parameters are determined by maximizing $L$, or, for convenience, $\ln(L)$. Setting the derivatives with respect to the parameters equal to zero yields the estimator equations, which have different forms for simple (for AEV data) and compound (for POT data) distributions. In this study, all the distributions are treated as 2-parameter distributions. For the 3-parameter distributions, such as FT-III and Weibull distributions, the position parameter is treated as a known value and determined by trial and error to obtain the optimum fit. The estimator equations for all six distributions are listed in the Appendix.

### 5.0 DATA CENSORSHIP

Censoring data is often necessary in using compound distributions or in optimizing fit. Most commonly, data are Type-II censored, i.e., only the $r$ largest values are retained from the complete sample of $n$ values ($n>r$). In this study, the hindcast data are considered type-II censored only when the compound distributions are fitted by LLS method. Those lower ranked data points for which the factor $1+1/\lambda \ln(F)$ in eq.(7) becomes negative are discarded. To minimize the effect of data censorship and to acknowledge the fact that only the $r$ largest values are retained, the value “$N$” in the plotting position formulas is set to be $n$ instead of $r$, e.g., for the Gumbel plotting position, eq.(4) becomes
\[ P_i = \frac{1}{n+1} \]

instead of

\[ P_i = \frac{i}{r+1} \]

This treatment prevents the "shift" caused by data censorship and acknowledges the fact that the r largest values are used.

6.0 GOODNESS OF FIT TESTS

A number of parameters have been proposed for testing the goodness of fit. It was found that the following five parameters, in most cases, are well behaved and provide reasonable indications of the goodness of fit:

a. Linear correlation coefficient (r) defined as:

\[ r = \sqrt{\frac{\sum XY - \sum X \sum Y}{\sum X^2 - (\sum X)^2/N} \frac{\sum Y^2 - (\sum Y)^2/N}{N}} \]

where \( X \) and \( Y \) are the reduced variates of the random variables -- \( H_s \) and \( P \), respectively.

b. Mean Square Error (M.S.E.) defined as

\[ M.S.E. = \frac{1}{N} \sum_{i=1}^{N} (A + BX_i - Y_i)^2 \]

where \( A \) and \( B \) are the constants determined by linear least square (LLS) fit to the data, \( X_i \) and \( Y_i \) are the reduced variates of \( H_s \) and the plotting position \( P_i \), respectively.

c. Reduced Maximum Likelihood Function (M.L.F.) defined as

\[ M.L.F. = \frac{1}{N} \ln(L) = \frac{1}{N} \sum_{i=1}^{N} \ln[f(x_i)] \]

where \( x_i \) is the \( i \)-th \( H_s \) and \( f(x) \) is the probability density function.

d. Cramer–Von Mises statistic defined as:

\[ W_n = \sum \left[ F_0(x_i) - \frac{i-1/2}{n} \right]^2 + \frac{1}{12n} \]

where \( F_0(x_i) \) is the "true" distribution fitted to the data and \( i \) is the ascending order of the data.
e. Kolmogorov–Smimov statistic defined as:

$$D_n = \max \left\{ \max_{1 \leq i \leq n} \left\{ \frac{i}{n} - F_0(x_i) \right\}, \max_{1 \leq i \leq n} \left\{ F_0(x_i) - \frac{i-1}{n} \right\} \right\}$$  \hspace{1cm} (15)$$

Among them, $r$, M.S.E. are valid only for LLS method and M.L.F. is only for MLE method. The other two can be used for both methods. In general, higher $r$ and M.L.F values, and lower M.S.E., $W_n$ and $D_n$ values indicate better fits. In most cases examined in this study, the parameters are consistent, i.e., when $r$ or M.L.F. is high, the others are usually low.

7.0 EXTREME WAVE HEIGHTS FROM HINDCAST DATA

The AEV and POT data sets are analyzed with techniques described above. The probability associated with the desired return period $T_r$ (in years) used in this paper is

$$F_x = 1 - \frac{1}{T_r}$$  \hspace{1cm} (16)$$

for both simple (for AEV data) and compound (for POT data) distributions.

The results are in Tables 3 through 8. The goodness of fit parameters in the tables show that the so-called “best fit” distribution is different for different fitting methods and different data types.

Table 3 summarizes the results of LLS analyses of the 31 annual extreme values. The range of the estimated 100-year return period $H_s$ value is from 13.6m to 18.1 m. Many of the analyses conclude that the data is not well represented by most of the distributions. Only the FT-III and Weibull (with Gringorten’s plotting position) distributions provide “best” fits compared to the others. Plots of these distributions as well as the data points are given in Figures 1 and 2.

Table 3 also lists the estimated 31-year return period $H_s$ value obtained using the various extreme value distributions. The largest annual value in the 31 year’s data used in the AEV analysis is 13.4m (during February 1982) which should be qualitatively close to the estimated 31-year return period value. The fact that this is only true for the FT-III and Weibull (with p.p.2) distributions further confirms better data representation by these two distributions.

Table 4 summarizes the results of the LLS analyses of the 72 hindcast peaks exceeding 6.0m over the 38-year hindcast period. In
this table, the Borgman (with plotting position 2), Gumbel, FT-III, and Weibull (with plotting position 2) distributions are classified as from fair to good fits. Plots of these distributions are given in Figures 3 through 6. The estimated 38-year return period $H_s$ values are within $\pm 1.0$ meter of the peak hindcast value (13.4m) for all the distributions, hence, this qualitative test provides no further insight in choosing the best distribution. According to the goodness of fit parameters $r$ and MSE, only FT-II and Exponential can be excluded as poor fits. The 100-year return period $H_s$ predicted by the 4 remaining (Gumbel, Borgman, FT-III, and Weibull), distributions range from 13.9m to 14.7m, i.e., within 3% of the mean value. The 38-year return period values are from 13.1 m to 13.5m, very close to 13.4m: the highest $H_s$ in the 38 year data.

Table 5 summarizes the results of MLE analyses of the 31 annual extreme values (AEV data). Only the FT-III and Weibull distributions provide good and fair fits (small MLF) and reasonable estimates of the 31-year return period $H_s$. The two distributions are plotted in Figure 1 and 2. The dashed line should be compared with the distribution fitted by LLS method instead of the data points because the data are plotted with plotting positions (MLE does not use the plotting position). For this reason, the best fit lines do not look as good.

Table 6 summarizes the results of MLE analyses of the peak $H_s$ values over a threshold (POT data). Only FT-III can be classified as a fair fit. The 100-year return period $H_s$ predicted by FT-III is 14.0m.

In Tables 3 through 6, it is clear that the absolute values of $\omega_n$ and $D_n$ are different for different fitting methods. Therefore, it is not appropriate to choose fitting method according to the goodness of fit parameters, as stated by Muir et al (1986).

The results listed in Table 3 and 5 are based on the AEV data of 31 solstice years (Table 2). The same analyses were also performed on AEV data of 31 calendar years. The results showed very slight variation from those based on solstice year. For POT data, 8.0m threshold was also tested and the resulting 100-year $H_s$ values were very close to those in Table 4 and 6.

Table 7 summarizes the extreme $H_s$ values estimated with four selected extreme value distributions, Gumbel, Borgman, FT-III, and Weibull, and the combination of analysis techniques. Five return period $H_s$ values are listed. The Gumbel distribution with POT data and LLS method produces the largest 100-year extreme $H_s$ (14.7m) and FT-III with AEV data gives the lowest (13.4m). It appears that the AEV data produce a rather "flat" curve for the FT-III distribution. That is, the AEV data with the FT-III distribution produce higher estimates for
short return period Hs values and lower estimates for long return period ones, as compared to the others. Among the four acceptable distributions for this set of data, the Gumbel and FT-III have, respectively, the steepest and the mildest slope.

8.0 ANNUAL EXTREMES AND CSA LOAD FACTORS

The new Canadian Standards Association (CSA) offshore standards [10] are based in part on ultimate limit states design (ULS). ULS means the factored strength or resistance of a structure must exceed the factored loads. For a safety class 1 structure (i.e., loss of the structure poses a threat to personnel safety and the environment), the CSA factor on the 100-year wave load is 1.35. This value is derived from rather detailed investigations and analyses [11], but is implicitly based on two quantities:

1) Target annual probability of failure (for structural failure, ignoring system ductility, $10^{-4}$ in CSA)

2) Variability in the annual maxima of the loads (for environmental loads, the coefficient of variation, COV—the standard deviation normalized by the mean—is assumed as 0.2 to 0.4).

Simply stated (perhaps too simple), the load factor value is chosen such that the factored 100-year wave load is approximately equal to the 10,000 year load. As COV increases, so must the load factor, though not according to any simple relationship.

Loads are beyond the scope of this paper. But COV of the annual wave height maxima is easily computed from the AEV data (Table 2). The mean and standard deviation of the 31 annual maxima are 10.1m and 2.2m, respectively. The resulting COV is 0.21. If the years with no hindcast data are infilled with Hs = 6.0m (assuming the missing maxima to be rather low), the mean, standard deviation and COV become 9.5m, 2.5m and 0.26. The two COV’S, 0.21 and 0.26, probably bracket the true value. Thus, the COV of the annual maximum wave is consistent with the CSA assumption.

9.0 EXTREMES FROM MEASURED DATA

The Weibull distribution is commonly used to estimate extremes when long-term data are not available. For most practical applications, the precision of extreme value extrapolations is restricted by the short time span of the data instead of the distribution itself. Because the data could capture predominately mild or severe climates in a short time window, the extremes predicted based on such data often contain bias.

Estimates of 100-year wave heights based on the five year continuous wave record on the Grand Banks are summarized in Ref.[6]. In the
present study, the same data set is used to examine the variability in the extrapolations by the Weibull distribution due to shorter time spans of data, since a five-year continuous record is a luxury in most locations. The five-year continuous data were broken into five, one-year data sets (based on calendar year) and a Weibull distribution (two parameter Weibull) was fitted to each of the annual records by the LLS method. The extreme values are listed in Table 8 along with the five-year means (they are approximately the same as those by the extremal analysis of the five-year data) and the percentage variabilities.

Weibull estimates based on the data from each calendar year indicate significant variability from the five-year means. Extremes predicted based solely on 1982 data overestimate the mean values by as much as 19%. The bias is fairly uniform for different return periods in each column. The extreme Hs values based on the five-year data are very close to the mean return period values predicted using hindcast data. Three of the five sets of extreme estimates based on one-year data listed in Table 8 also give reasonable results. The standard deviation and coefficient of variation of the five 100-year estimates are 1.69m and 0.12, respectively.

10.0 CONCLUSIONS
The following conclusions can be drawn from the above analysis:

1) For a specific data set, the extremal distribution giving the best fit varies depending on the data type and fitting method. Therefore, different distributions, fitting methods and data types must be assessed to produce defensible extreme value estimates.

2) The mean 100 return period Hs estimated in this study based on the combined hindcast data ranges from 13.4m to 14.7m. (Note: the mean 100 year value from HWHP is 14.4m, from CWP it is 14.3m and 14.6m.)

3) FT-III is one of the best fit distributions in both fitting methods with both data types. Although it provides good fits in most cases, it has a rather flat tail (Table 7) and can result in rather low 100-year extremes.

4) Conversely, Gumbel tends to be the “steepest” distribution. It generates the highest 100 year return period Hs and lowest 5 year return period values (Table 7). For short return period extremes, Gumbel is probably not conservative.

5) The coefficient of variation, COV, of the annual wave maxima, based on the hindcast data, is between 0.21 to 0.26. This range is consistent with the implicit assumptions in the recent CSA offshore standards, though the COV of annual wave load maxima is not addressed here.
6) Based on 5-years of measured data at Hibernia, 100-year estimates derived from one-year of data have a coefficient of variation of 11.6%. Long term extremes based on such short data sets must reflect this intrinsic uncertainty.

REFERENCES


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### Table 3
L.L.S. Method with AEV data
100 year and 31 year Return Period Hs(m) and
Goodness of Fit Parameters

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<th>M.S.E.</th>
<th>Wn</th>
<th>Dn</th>
<th>A</th>
<th>Fitness</th>
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<td>0.1584</td>
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<tr>
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### Table 4
L.L.S. Method with POT data (threshold = 6.0m)
100 year and 38 year Return Period Hs(m) and
Goodness of fit parameters

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<th>Distribution</th>
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<th>Hs(38y)</th>
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<th>Wn</th>
<th>Dn</th>
<th>A</th>
<th>Fitness</th>
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### Table 5
**M.L.E. Method with AEV data**
100 year and 31 year Return Period Hs (m) with Goodness of Fit Parameter

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<th>Dn</th>
<th>A</th>
<th>Fitness</th>
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### Table 6
**M.L.E. Method with POT data (threshold = 6.0m)**
100 year and 38 year Return period Hs (m) with Goodness of Fit Parameter

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<tr>
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### Table 7

**Extreme Value Estimates by Different Methods**  
Threshold = 6.0m with 72 Storm Peaks for POT Data  
31 Annual Extreme Values for AEV Data

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### Table 8

**Comparison of Extreme Significant Wave Heights from Five One-Year Records and The Five Year Means**  
\[
D\% = \frac{\text{Hs(1yr) - Hs(5yr)}}{\text{Hs(5yr)}} \times 100
\]

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<th>1982</th>
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<th>1983</th>
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Fig. 1 FT-III fits to AEV data

Fig. 2 Weibull fits to AEV data

Fig. 3 Gumbel fits to POT data

Fig. 4 Borgman fits to POT data

Fig. 5 FT-III fits to POT data

Fig. 6 Weibull fits to POT data
APPENDIX

In this appendix, the simple and compound extreme value distributions, their inverse equations, linear equations of reduced variates and maximum likelihood estimators are listed.

1. Gumbel

a) Simple cumulative distribution function

\[ P(H_s \leq x) = F_s(x) = \exp\left(-\exp\left(-\frac{x-A}{B}\right)\right) \]  \quad (17)

Probability density function

\[ f_s(x) = B^{-1}\exp\left(-\frac{x-A}{B}\right)\exp\left(-\frac{x-A}{B}\right) \]  \quad (18)

Inverse equation

\[ x = A - B\ln(-\ln(F_s)) \]  \quad (19)

Linear equation of reduced variates

\[ X = A - B\ln(-\ln(F_s)) \]  \quad (20)

Maximum likelihood estimators

\[ B = \bar{x} - \frac{1}{N} \sum_{i=1}^{N} x_i \exp\left(-\frac{x_i}{B}\right) \]  \quad (21)

\[ A = -B\ln\left(\frac{1}{N} \sum_{i=1}^{N} \exp\left(-\frac{x_i}{B}\right)\right) \]  \quad (22)

b) Compound (Poisson) Gumbel cumulative distribution

\[ P_{\lambda}(H_s \leq x) = F(x) = \exp\left(-\lambda\left[1 - \exp\left(-\frac{x-A}{B}\right)\right]\right) \]  \quad (23)

Inverse equation

\[ x = A - B\ln\left(-\ln\left[1 + \frac{1}{\lambda} \ln(F)\right]\right) \]  \quad (24)

Linear equation of reduced variates

\[ x = A - B\ln\left(-\ln\left[1 + \frac{1}{\lambda} \ln(F)\right]\right) \]  \quad (25)
Maximum likelihood estimators

\[
B = \overline{X} - \sum_{i=1}^{N}(1 + \lambda F_{s1})x_i \exp\left(-\frac{x_i}{B}\right)
\]

\[
A = -B \ln\left[\frac{1}{N} \sum_{i=1}^{N}(1 + \lambda F_{s1}) \exp\left(-\frac{x_i}{B}\right)\right]
\]  

where

\[F_{s1} = F_s(x_i)\]  

2. Borgnian

a) Simple cumulative distribution function

\[P(H_s \leq x) = F_s(x) = \exp\left(-\exp\left(-\frac{x^2-A}{B}\right)\right)\]  

Probability density function

\[f_s(x) = \frac{2x}{B} \exp\left(-\frac{x^2-A}{B}\right) \exp\left(-\frac{x^2-A}{B}\right)\]  

Invers equation

\[x = \sqrt{A - B \ln(-\ln(F_s))}\]  

Linear equation of reduced variates

\[x^2 = A - B \ln(-\ln(F_s))\]  

Maximum likelihood estimators

\[A = -B \ln\left[\frac{1}{N} \sum_{i=1}^{N} \exp\left(-\frac{x_i^2}{B}\right)\right]\]  

\[B = \overline{x^2} - \left[\frac{1}{N} \sum_{i=1}^{N} \exp\left(-\frac{x_i^2}{B}\right)\right]^{-1} \sum_{i=1}^{N} x_i^2 \exp\left(-\frac{x_i^2}{B}\right)\]  

b) Compound (Poisson) Borgman
\[ P(H_s \leq x) = F(x) = \exp\left( -\lambda \left[ 1 - \exp\left( -\frac{x^2 - A}{B} \right) \right] \right) \] (35)

Invers equation

\[ x = \sqrt{A - B \ln\left( -\ln\left[ 1 + \frac{1}{\lambda} \ln(F) \right] \right)} \] (36)

Linear equation of reduced variates

\[ x^2 = A - B \ln\left( -\ln\left[ 1 + \frac{1}{\lambda} \ln(F) \right] \right) \] (37)

Maximum likelihood estimators

\[ B = x^2 - \sum_{i=1}^{N} (1 + \lambda F_{s_i}) x_i \exp\left( -\frac{x_i^2}{B} \right) \left[ \frac{N}{\sum_{i=1}^{N} (1 + \lambda F_{s_i})} \exp\left( -\frac{x_i^2}{B} \right) \right]^{-1} \] (38)

\[ A = -B \ln\left( \frac{1}{N} \sum_{i=1}^{N} (1 + \lambda F_{s_i}) \exp\left( -\frac{x_i^2}{B} \right) \right) \] (39)

where

\[ F_{s_i} = F_s(X_i) \] (40)

3. FT-II

a) Simple cumulative distribution function

\[ P(H_s \leq x) = F_s(x) = \exp\left[ -\left( \frac{x-A}{B} \right)^C \right] \] (41)

Probability density function

\[ f_s(x) = \frac{C}{B} \left( \frac{B}{x-A} \right)^{1+C} \exp\left[ -\left( \frac{x-A}{B} \right)^C \right] \] (42)

Invers equation

\[ x = A + B \left( -\ln(F_s) \right)^{-\frac{1}{C}} \] (43)
Linear equation of reduced variates

\[ \ln(x-A) = -\frac{1}{C} \ln[-\ln(F_s)] + \ln(B) \] (44)

Maximum likelihood estimators

\[ C^{-1} = -\sum_{i=1}^{N} (x_i-A)^{-C} \ln(x_i-A) \left[ \sum_{i=1}^{N} (x_i-A)^{-C} \right]^{-1} + \frac{1}{N} \sum_{i=1}^{N} \ln(x_i-A) \] (45)

\[ B = \left[ \frac{1}{N} \sum_{i=1}^{N} (x_i-A)^{-C} \right]^{-\frac{1}{C}} \] (46)

b) Compound (Poisson) FT-II

\[ P_x(h_s \leq x) = F(x) = \exp \left( -\lambda \left( 1 - \exp \left[ -\left( \frac{x-A}{B} \right)^C \right] \right) \right) \] (47)

Invers equation

\[ x = A + B \left[ -\ln \left( 1 + \frac{1}{\lambda} \ln(F) \right) \right]^{-\frac{1}{C}} \] (48)

Linear equation of reduced variates

\[ \ln(x-A) = -\frac{1}{C} \ln[-\ln(1 + \frac{1}{\lambda} \ln(F))] + \ln(B) \] (49)

Maximum likelihood estimators

\[ C^{-1} = -\sum_{i=1}^{N} (1 + \lambda F_{si})(x_i-A)^{-C} \ln(x_i-A) \left[ \sum_{i=1}^{N} (1 + \lambda F_{si})(x_i-A)^{-C} \right]^{-1} + \frac{1}{N} \sum_{i=1}^{N} \ln(x_i-A) \] (50)

\[ B = \left[ \frac{1}{N} \sum_{i=1}^{N} (1 + \lambda F_{si})(x_i-A)^{-C} \right]^{-\frac{1}{C}} \] (51)

where

\[ F_{si} = F_{s}(X_i) \] (52)
4. FT–III

a) Simple cumulative distribution function

\[ P(H_s \leq x) = F_s(x) = \exp\left(-\left(\frac{A-x}{B}\right)^C\right) \]  

(53)

Probability density function

\[ f_s(x) = \frac{C}{B} \left(\frac{A-x}{B}\right)^{C-1} \exp\left(-\left(\frac{A-x}{B}\right)^C\right) \]  

(54)

Invers equation

\[ x = A - B \left( -\ln(F_s) \right)^\frac{1}{C} \]  

(55)

Linear equation of reduced variates

\[ \ln(A-x) = \frac{1}{C} \ln(-\ln(F_s)) + \ln(B) \]  

(56)

Maximum likelihood estimators

\[ C^{-1} = \frac{1}{N} \sum_{i=1}^{N} (A-x_i) \ln(A-x_i) \left[ \sum_{i=1}^{N} (A-x_i) \right]^{-1} - \frac{1}{N} \sum_{i=1}^{N} \ln(A-x_i) \]  

(57)

\[ B = \left[ \frac{1}{N} \sum_{i=1}^{N} (A-x_i) \right]^{\frac{1}{C}} \]  

(58)

b) Compound (Poisson) FT–III

\[ P_x(H_s \leq x) = F(x) = \exp\left(-\lambda \left[1 - \exp\left(-\left(\frac{A-x}{B}\right)^C\right)\right]\right) \]  

(59)

Invers equation

\[ x = A - B \left[-\ln\left(1 + \frac{1}{\lambda} \ln(F)\right)\right]^\frac{1}{C} \]  

(60)

Linear equation of reduced variates

\[ \ln(A-x) = \frac{1}{C} \ln\left[-\ln\left(1 + \frac{1}{\lambda} \ln(F)\right)\right] + \ln(B) \]  

(61)
Maximum likelihood estimators

\[ C^{-1} = \sum_{i=1}^{N} (1 + \lambda F_{si})(A - X_i)^c \ln(A - X_i) \left[ \sum_{i=1}^{N} (1 + \lambda F_{si})(A - X_i)^c \right]^{-1} - \frac{1}{N} \sum_{i=1}^{N} \ln(A - X_i) \]  

(62)

\[ B = \frac{1}{N} \sum_{i=1}^{N} (1 + \lambda F_{si})(A - X_i)^c \right]^{\frac{1}{c}} \]  

(63)

where

\[ F_{si} = F_s(x_i) \]  

(64)

5. Weibull

a) Simple cumulative distribution function

\[ P(H_s \leq X) = F_s(X) = 1 - \exp \left[ -\left( \frac{X - A}{B} \right)^c \right] \]  

(65)

Probability density function

\[ f_s(x) = C \left( \frac{X - A}{B} \right)^{c-1} \exp \left[ -\left( \frac{X - A}{B} \right)^c \right] \]  

(66)

Invers equation

\[ X = A + B \left[ -\ln(1 - F_s) \right]^{\frac{1}{c}} \]  

(67)

Linear equation of reduced variates

\[ \ln(X - A) = \frac{1}{C} \ln(-\ln(1 - F_s)) + \ln(B) \]  

(68)

Maximum likelihood estimators

\[ C^{-1} = \sum_{i=1}^{N} (X_i - A)^c \ln(X_i - A) \left[ \sum_{i=1}^{N} (X_i - A)^c \right]^{-1} - \frac{1}{N} \sum_{i=1}^{N} \ln(X_i - A) \]  

(69)

\[ B = \left[ \frac{1}{N} \sum_{i=1}^{N} (X_i - A)^c \right]^{\frac{1}{c}} \]  

(70)
b) Compound (Poisson) Weibull

\[ P_x(H_s \leq x) = F(x) = \exp\left\{ -\lambda \exp\left[ -\left( \frac{x-A}{B} \right)^c \right] \right\} \quad (71) \]

Invers equation

\[ x = A + B \left( -\frac{1}{\lambda} \ln(\frac{1}{F}) \right)^{\frac{1}{c}} \quad (72) \]

Linear equation of reduced variates

\[ \ln(x-A) = \frac{1}{c} \ln\left( -\frac{1}{\lambda} \ln(\frac{1}{F}) \right) + \ln(B) \quad (73) \]

Maximum likelihood estimators

\[ C^{-1} = \sum_{i=1}^{N} \left[ 1 - \lambda(1-F_{sl})(x_i-A) \right] \ln(x_i-A) \left\{ \sum_{i=1}^{N} \left[ 1 - \lambda(1-F_{sl})(x_i-A) \right] \right\}^{-1} - \frac{1}{N} \sum_{i=1}^{N} \ln(x_i-A) \quad (74) \]

\[ B = \left[ \frac{1}{N} \sum_{i=1}^{N} \left[ 1 - \lambda(1-F_{sl})(x_i-A) \right] \right]^{\frac{1}{c}} \quad (75) \]

where

\[ F_{si} = F_s(x_i) \quad (76) \]

6. Exponential

a) Simple cumulative distribution function

\[ P(H_s \leq x) = F_s(x) = 1 - \exp\left( -\frac{x-A}{B} \right) \quad (77) \]

Probability density function

\[ f_s(x) = B^{-1} \exp\left( -\frac{x-A}{B} \right) \quad (78) \]

Invers equation

\[ x = A - B \left[ \ln(1 - F_s) \right] \quad (79) \]

Linear equation of reduced variates
\[ x = A - B[-\ln(F_s)] \] (80)

Maximum likelihood estimators
\[ B = \frac{1}{N} \sum_{i=1}^{N} (X_i - A) = \frac{\bar{X} - A}{N} \] (81)

b) Compound (Poisson) Exponential
\[ P_r(H_s \leq x) = F(x) = \exp\left[-\lambda \exp\left(-\frac{x - A}{B}\right)\right] \] (82)

Invers equation
\[ x = A - B \ln\left[-\frac{1}{\lambda} \ln(F)\right] \] (83)

Linear equation of reduced variates
\[ x = A - B \ln\left[-\frac{1}{\lambda} \ln(F)\right] \] (84)

Maximum likelihood estimators
\[ B = \bar{X} - \sum_{i=1}^{N} x_i \exp\left(-\frac{X_i}{B}\right) \left[\sum_{i=1}^{N} \exp\left(-\frac{X_i}{B}\right)\right]^{-1} \] (85)
\[ A = -B \left\{ \ln \lambda + \ln \left[\frac{1}{N} \sum_{i=1}^{N} \exp\left(-\frac{X_i}{B}\right)\right]\right\} \] (86)

where
\[ F_{si} = F_s(x_i) \] (87)
OPERATIONAL WAVE FORECASTING FOR THE MEDITERRANEAN SEA

Paul D. Farrar
and
Andrew Johnson, Jr.

Naval Oceanographic Office, Code OPT
Stennis Space Center, MS 39522-5001, USA
Phone: (601) 688-5732, Fax: (601) 688-5605

ABSTRACT

The Naval Oceanographic Office (NAVOCEANO), which supports the U.S. Navy with oceanographic products and services, has the task of providing operational wave forecasts for all of the world’s marginal seas and enclosed ocean basins. The first phase of implementing this worldwide capability at NAVOCEANO began with the Mediterranean Sea. This paper describes the application of the numerical model, and the validations performed before the model implementation is released for operational use. The wave forecasts use wind forecasts supplied twice daily by the U.S. Navy’s Fleet Numerical Oceanography Center. Once operational, wave forecasts will be provided twice daily for 48-hour periods.

1. INTRODUCTION

The Naval Oceanographic Office (NAVOCEANO), located at Stennis Space Center, Mississippi, provides operational U.S. Navy units with oceanographic products and services. At NAVOCEANO many of these products will soon be generated directly from numerical ocean models. The geographic areas of responsibility include all the world’s marginal seas and semi-enclosed basins. The initial wave model product will be an operational wave forecast system for the Mediterranean Sea. This system is presently being prepared for full operational use later in this year.

2. WAVE MODEL

The wave model selected for this project was the third cycle release of the WAM model (WAMDI Group, 1988). WAM is a third generation spectral wave model incorporating the effects of wave refraction, diffraction, dispersion, wind energy input, wave dissipation, and the transfer of energy within the spectrum by weak wave-wave interactions. In the model, the directional wave spectrum is divided into discrete frequency and direction bands. For our application, there are twelve direction bands at 30 deg intervals and 26 frequency bands, starting at 0.0418 Hz (24 s period) and increasing in frequency by a factor of 1.1 to 0.4526 Hz (2.2 s period).

The model grid is 166 by 63 points with a resolution of 1/4 deg, covering the entire Mediterranean Sea area. This grid resolution is
required primarily to accurately represent the complicated geometric form of the Mediterranean shoreline and to accommodate many small islands.

a. Wind Input

The source of the surface wind input data for the model implementation is the Navy Operational Regional Atmospheric Prediction System (NORAPS) of the Fleet Numerical Oceanography Center (FNOC), Monterey, CA, NORAPS is a regional scale model running on a 109 (east–west) by 82 (north–south) grid centered on the Mediterranean. The grid resolution is 80 km on a Lambert Conformal conic projection. The wind predictions, which are run twice daily, at 0000 UTC and 1200 UTC, are available for a 48-hour forecast period. At 0600 UTC and 1800 UTC a wind field analysis (without a forecast) is also available. The NORAPS output results are distributed by FNOC in a 63 by 63 polar stereographic grid with a spacing of 92.5 km at 60N. Shortly after completion, the NORAPS results are transmitted electronically to NAVOCEANO, where the surface winds are interpolated to the WAM model grid and converted to WAM input format.

b. Model Runs and Products

Every 12 hours (0000 UTC and 1200 UTC; tau = Ohr) WAM forecast is produced for a 54-hour period, starting at tau = -6hr, and continuing until tau = 48 hr, which is the limit imposed by the wind forecasts. The reason for the start at tau = -6hr is to take advantage of the 0600 UTC and 1800 UTC wind analyses. Forecasts are produced for tau = 01 12f 24, 36, and 48hr. Fig. 1-3 show model output for one forecast. The significant wave height contours are in feet (1 foot = 0.3048m). The vectors show the mean direction, and have a length proportional to the mean period. An additional product consisting of separate sea and swell data (not shown) is also produced.

4. MODEL VALIDATION

a. Prediction consistency checks.

The consistency of the model was checked by comparing the tau = 0 values with the 12 hr forecasts done 12 hours earlier, and similarly for 24, 36, and 48 hours. Fig. 4 is a typical example for one site. In general, there was no significant bias (over- or under-prediction) in the forecasts, and scatter was reasonable. This indicates that the wind and wave model combination we are using does not exhibit systematic errors.

b. Checks against FNOC forecasts and observations.

Fig. 5 shows a comparison of this model (solid line) against an existing 1/2-degree WAM implementation by FNOC (dashed line) and a
ship observation (triangle). These comparisons are now in progress for more locations and longer periods of time. Ship observations will be used, although they are often sparse (as in Fig. 5) and unreliable. NAVOCEANO is attempting to obtain wave gauge and ERS-1 satellite data for further model evaluations.

5. OPERATIONAL IMPLEMENTATION

When the model has completed its operational tests later this year, it will be operated by NAVOCEANO’s Operational oceanography Center. Operational forecasts will be distributed twice daily through Navy communications channels to the Naval Oceanography Command Center in Rota, Spain and where the data will be incorporated into standard Navy wave forecast products there.

6. REFERENCES

SIGNIFICANT WAVE HEIGHT (FT) 0 HR ANAL VT 12Z 9 MAR 1992

Figure 1

SIGNIFICANT WAVE HEIGHT (FT) 24 HR FCST VT 12Z 10 MAR 1992

Figure 2

SIGNIFICANT WAVE HEIGHT (FT) 48 HR FCST VT 12Z 11 MAR 1992

Figure 3
Wave Height (m)

\[ y = ax + b \]
\[ a = 0.996 \]
\[ b = 0.084 \]
\[ R = 0.886 \]

Figure 4

40N, 5E, Tau = 0 hr

Figure 5
MU–WAVE: AN OPERATIONAL WAVE FORECASTING SYSTEM FOR THE BELGIAN COAST

Dries Van den Eynde

Management Unit of the Mathematical Models of the North Sea and the Scheidt estuary Ministry of Public Health and Environment Guiledelle 100, B-1200 Brussels, Belgium

1. INTRODUCTION

In November 1991, an operational wave forecasting system for the Belgian coast was implemented at the Service of the Coastal Harbours of the Flemish Ministry of Public Works and Traffic. The model, developed at the Management Unit of the Mathematical Models of the North Sea and the Scheidt estuary (M.U.M.M.), is used as a management tool for ship routing towards the Belgian sea harbours through the fairly shallow waters of the southern Bight of the North Sea. The goal is to use the model to minimise the risk of grounding of large sea vessels and to optimise the time frame, during which the ships can safely enter and leave the harbours. Also dredging works can be optimised, using the operational wave prediction model, by allowing an efficient use of the available water depth. The model is incorporated in the “Hydro–Meteo–System”, an operational data gathering system of the Flemish Ministry of Public Works and Traffic, which describes the hydrodynamics] climate in the Belgian coastal zone. Since the model is especially used for the guidance of sea vessels towards the Belgian sea harbours, the emphasis for the model lies in a good prediction of the waves—especially the low frequency waves, which are important for the movements of vessels—along the fairways.

In this presentation, the structure of the model and the operational implementation will be discussed first. Then, some results of the validation of the model and of some operational tests will be presented.

2. STRUCTURE OF THE WAVE FORECASTING SYSTEM MU–WAVE

In the framework of the project, different models are coupled. The core of the model is formed by the wave model ‘HYPAS’ developed by Dr. W. Rosenthal and Dr. H. Günther, GKSS Forschungszentrum GmbH, Geesthacht, F.R.G. –Günther et al. (1979); Günther and Rosenthal (1985); Hermans (1989)–. The HYPAS model is a second generation wave model, which combines the traditional approach of independent calculation of swell energy for each frequency and direction through a ray technique, with a paranietical wind sea model, using the parameters of the JONSWAP spectrum and the mean wind sea direction as prognostic variables. Some shallow water effects, such as shoaling, are included in the model.
For the current application, the model is implemented on two nested grids. The first grid has a resolution of 50 x 50 km$^2$ (stereographic projection) and covers the entire North Sea. It is extended enough to intercept the swell before the Belgian coast, which may be generated far away. The open sea boundaries of this model are treated as closed boundaries, where fetch laws are used. In the southern North Sea a higher resolution is needed to account for the complex bathymetry in the area. A fine 10 x 10 km$^2$ resolution grid is used. The two grids are coupled through open boundaries. The extent and the position of the grids are shown in Figure 1. In both wave model applications, the swell is represented by 20 frequency and 24 direction bins.

The refraction effect—the bending of the wave rays under the influence of depth gradients, especially of the low frequency waves—is not included in the wave model HYPAS. Since in the shallow Belgian coastal zone, refraction has a significant effect on the wave characteristics, the fine grid wave model is further coupled to a spectral refraction model SPECIN—Van den Eynde and Monbaliu (1989); Van den Eynde (1991). This refraction model allows correction at
predefined points of the results, obtained with the fine grid wave model, for the refraction effect.

For this purpose, at each selected point on the hyper fine refraction grid resolution 500 x 500 m$^2$ - a so called "belt" of points is constructed, i.e. an imaginary edge of a refraction grid for that point. These belt points, which are formed by high resolution HYPAS model grid points, are supposed to be lying in deeper water, where the refraction phenomenon is less important and where the fine grid HYPAS model results are accurate enough. The time for the wave energy to travel from the belt points to the inshore point is almost constant and equal to one hour.

For each combination of a set of frequencies and directions, the wave rays are then constructed in reverse from the point of interest, until the belt points are reached. There, the energy spectra are accurately predicted by the fine grid HYPAS model. The wave energy is then transferred along the constructed wave rays from the belt to the inshore point, where the two-dimensional spectrum is reconstructed. The coupling is illustrated in Figure 2.

Further, the refraction model is coupled with the 2D storm-surge model MU-STORM -Adam (1979)- to take into account the effect of the varying water depth, due to the tides and storm surges, during the calculation.
of the refraction of the wave rays. The range of the tides, which near the Belgian coast is 4 to 5 m, is of the same order of magnitude as the water depth (10 to 20 m) and the tides can therefore have an important effect on the refraction.

The structure of the model is given in Figure 3.

3. OPERATIONAL USE OF THE MODEL

The swell prediction model is operated twice a day to predict the wave climate in the southern North Sea and more specifically near the Belgian coast. The wind and atmospheric pressure forecasts, which are used for the operational calculations, are predicted and distributed through the Global Telecommunication System (G.T.S.) by the United Kingdom Meteorological Office, Bracknell, U.K. (U.K.M.O.). This type of atmospheric input data was selected after some comparative tests. The data include the northerly and easterly wind components on a high resolution grid (1.25° by 1.25°) for the first 36 hours, whereafter the data are provided on the coarser 2.5° by 5° grid.

*Figure 3: General structure of the swell prediction system MU-WAVE.*
The data are transferred from U.K.M.O. to M.U.M.M. via the national meteorological institute K.M.I. each day around 7 GMT and 21 GMT. The forecasts are then made for four days in advance.

When the meteo data arrive, the data are first decoded and interpolated to the different model grids. A backup of the produced results is performed automatically. The forecasting calculations are then performed and a preliminary postprocessing is done, to allow for the use of a postprocessor to visualise the results, is done. To make it possible to investigate the results of the model as fast as possible, the four day forecast is split into blocks of twelve hours each. In this manner, the predictions for the first twelve hours can already be investigated after one eighth of the total calculation time for the four day simulation.

When the model forecasts are prepared, the results can be visualised with the postprocessor. The results can be presented both numerically and graphically. Results of the four models can be looked at. The graphical output includes ‘animations’ for some parameters – different colour plots, which describe the parameter over a certain grid at succeeding moments, are rapidly changed to give a good picture of the progress of the parameter over the area–, time series of different parameters and at different wave stations and spectral data at selected wave stations.

The models and the postprocessor are installed at the Service of the Coastal Harbours on two linked workstations, a HP9000-834 and a Cyber910B-430. While the wave predictions are executed on the powerful Hewlett-Packard workstation, the hydrodynamical forecasts and the postprocessor are run on the Cyber. Also the graphical output is displayed on the Cyber workstation. On the present computer system, the total four day forecast consumes about 3h30’ calculation time.

The operational setup of the swell prediction model MU-WAVE then is shown in Figure 4.
4. VALIDATION OF THE MODEL

4.1. Hindcast calculations

To evaluate the quality of the wave model, a simulation was executed for the six month period October 1987 to March 1988. This period was selected on the basis of the availability of atmospheric and wave data and high wave activity in the southern Bight of the North Sea.

The wind data, which were used to drive the model, were the data which would have been used operationally, i.e. the U.K.M.O. wind data, as they were predicted at that time. Wave data were obtained from the Services of the Flemish Executive, Public Works and Traffic, Service of the Coastal Harbours, Belgium and from Rijkswaterstaat. Tidal Waters Division, the Netherlands. They include wave data over thirteen wave stations near the Belgian and Dutch coast. The location of the different wave stations is illustrated in Figure 5. The data were
averaged over a three hour interval to compare them with the results of the wave model.

The simulation was then performed and the model results were compared to the wave data. The error statistics, which were prepared for the significant wave height, the mean period and the low frequency wave height, include the bias, the root-mean-square-error (RMSE) and the ratio of the RMSE to the mean of the observations, i.e. the scatter index (S.I.). The statistics are presented for the results of the fine grid HYPAS model – for all wave stations in the fine grid and for the wave stations very close to the Belgian coast (the refraction grid stations) and for the results of the refraction model. Note that, due to practical reasons, the influence of the varying water level on the refraction effect was not taken into account during the validation exercise.

The results, obtained for the significant wave height, for the mean period and for the low frequency wave height are presented in Table 1, Table 2 and Table 3 respectively.

From Table 1, one can see that the model tends to underpredict the significant wave height. The main reason for this probably is the underprediction of the wind speed, which is used as model input. This underprediction of the input wind speed is clearly illustrated in Figure 6, where a plot is given of the correlation between the
observed wind speed and the model input wind speed for the five stations, for which wind measurements were available. On average, the wind speed is biased 2.2 m/s too low. The mean scatter index however still is very satisfactory. The mean scatter index of 31.8%, found for the results of the fine grid HYPAS model, can be considered, for an operational second generation wave model, as very good. An example of a time series for the significant wave height, obtained with the model, is presented in Figure 7.

<table>
<thead>
<tr>
<th>Period</th>
<th>fine grid wave model</th>
<th>refraction model</th>
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<tbody>
<tr>
<td></td>
<td>fine grid stations</td>
<td>refraction grid stations</td>
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<tr>
<td></td>
<td>bias (cm)</td>
<td>RMSE (cm)</td>
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<td>-26.0</td>
<td>32.6</td>
</tr>
<tr>
<td>Full period</td>
<td>-13.6</td>
<td>35.4</td>
</tr>
</tbody>
</table>

Table 1: Statistical analysis of significant wave height results.

Figure 6: Correlation between observed and model input wind speed.
The measurements are given only as integers.
Further remark that the results, found with the refraction model SPECIN, are of the same quality as the results, found with the fine grid HYPAS model. The significant wave height is underestimated as well, with a mean scatter index of 34.1 %. The refraction model does not improve the results of the HYPAS fine grid model. The main reason for the negative bias is the negative bias of the input for the refraction model, i.e. the results of the fine grid HYPAS model. The fact that the results were not improved may be due to the rather sparse physics included in the refraction model –indeed, in the model, the atmospheric energy input and the bottom friction are not taken into account–, although Dhellemmes (1984) obtained, with a similar refraction model, the best results, when only refraction was accounted for. Current refraction may influence the wave characteristics near the Belgian coast as well. Further improvement of the refraction model results may be obtained in the future.

The results obtained for the mean period are also satisfactory. The bias is kept small, while a mean scatter index around 26% is found. Notice that, for the mean period, the results are improved by the refraction model. For the refraction model results, the bias is still very low, while the mean scatter index is decreased to 21.7% only. This implies that the refraction effect does have an influence on the wave characteristics in the very shallow area in front of the Belgian coast. The waves are bent correctly by the refraction model, but the energy level is not adjusted properly, to obtain corrected significant wave heights –see below–.

![significant wave height graph](image-url)  
*Figure 7: Time series for the significant wave height at stations A2-buoy during the month February 1988.*
The low frequency wave height, at last, is underpredicted by the model –see Table 3–. This is as one could expect from the previously reported underprediction of the significant wave height. As a result, the ability of the model to predict the exceedance of a predefined critical value, e.g. 50 cm, is not optimal. To overcome this problem, more accurate predictions of the atmospheric forecasts are necessary. The obtained root-mean-square-errors and scatter indexes however are satisfactory. An example of a time series for the modelled low frequency wave height is presented in Figure 8.
4.2. Operational tests

In addition to the validation, some operational tests were executed as well. Two periods were arbitrary selected, which are the periods April, 20th to 26th, 1991 and August, 27th to September, 2nd, 1991. For these periods, the forecasting runs were executed twice a day for four days ahead. For each moment in time, eight times, a forecast of the waves at that time is performed: a first forecasts is made four days on beforehand, while a last (eighth) forecast is calculated just 12 hours before the time actually arrives. This is illustrated in Figure 9.

From the results of these simulations, new time series were produced by gathering the forecasts over a certain fixed period from the different simulations –see Figure 9–. By comparing these new time series with the wave data, one can estimate the quality of the forecasts over this period and can examine the decay of the quality of the wave predictions further ahead.
From these tests, one could conclude that up to 36 hours ahead, the quality remains almost constant and equal to the results found during the validation. Forecasts over a longer period are much less reliable. The reason for this is obviously the less accurate atmospheric data forecasts, which are only provided on the coarse 2.5° by 5° grid –see section 3—. The decay of the quality of the forecasts is illustrated in Figure 10, where the scatter indexes for the significant wave height are presented for different forecasting periods.
5. FUTURE WORK

In the future, a further continuous validation and quality control of the operational wave model MU-WAVE will be performed, to allow for the more accurate determination of the strong and weak points of the wave model. This should help the operators of the model to interpret results and to use them for the routing of sea vessels through the fairways.

Further, some additional tests on the quality of the wind fields and on the influence of these wind fields on the results can be undertaken.

Finally, a separate validation of the refraction model and an investigation of the influence of the varying water level on refraction can be performed, to improve the results of the refraction routine over the HYPAS wave model results. If necessary, the current refraction could be taken into account to further improve the results of the refraction routine.

6. ACKNOWLEDGEMENTS

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All people who contributed to the project are gratefully acknowledged. Especially, I want to thank Josd Ozer and Kevin Ruddick, for their
constructive remarks, the computer scientists of the CAMME Computer Centre, for their great work in the construction of the operational system, and Wolfgang Rosenthal and Gerhard Gayer of the GKSS Forschungszentrum GmbH, for their support.

7. REFERENCES


A BRIEF OVERVIEW OF ENVIRONMENT CANADA’S OPERATIONAL OCEAN WAVE MODEL
M.L. Khandekar and R. Lalbeharry
Atmospheric Environment Service, Downsview, Ontario

1. INTRODUCTION

A state-of-the-art ocean wave model called CSOWM (Canadian Spectral Ocean Wave Model) has been developed by the Meteorological Services Research Branch of the Atmospheric Environment Service (AES). The model has been designed to produce sea-state analysis and prediction over two separate oceanic regions, namely the northwest Atlantic and the northeast Pacific. Over each of the two regions, the model operates on a coarse grid and a nested fine grid which covers the nearshore regions of the Canadian Atlantic and the Canadian Pacific. For the fine grid region of the Canadian Atlantic, shallow-water processes namely wave refraction, wave shoaling, bottom friction and wave number scaling are modelled. No shallow-water processes are modelled for the northeast Pacific since the near shore regions of the Canadian Pacific are deep enough that shallow-water processes are not considered important. Further, the wave model also includes the third generation (3G) source terms as represented by the nonlinear wave-wave interaction processes. The 3G source terms are included as an optional package of the main computer source code of the CSOWM. Brief details of the model are presented in the next section following which model verification statistics based on a three-month period and based on a case study are presented.

2. THE MODEL

The grids over which the model operates are displayed in Figure 1. The grids are laid out on a transverse mercator projection with assumed equator at 51° W for the Atlantic grid and at 167° E for the Pacific grid. The coarse grid spacing is 1.084 degrees of longitude for the Atlantic grid and 1.228 degrees of longitude for the Pacific grid. For each of the two coarse grids, a nested fine grid is designed with a grid spacing about one-third of the coarse grid. For the northeast Atlantic, there are 2081 coarse grid points of which 1650 are water points while on the fine grid there are 2479 grid points of which 1622 are water points. For the northeast Pacific, there are 1705 coarse grid points of which 1301 are water points while on the fine grid there are 1233 grid points of which 962 are water points. Over the fine grid region of the Atlantic, bathymetry is specified at all 1622 water points where shallow-water physics is applied as one of the optional packages of the model code.

In the basic form, the model physics is specified by the first generation (1G) source terms which include the linear and the exponential wave growth mechanisms of Phillips (1957) and Miles.
The linear growth term $A$ represents Phillips' resonant interaction between waves and turbulent pressure fluctuations in the overlying atmosphere, while the exponential wave growth term $B$ is expressed as a function of $U^*$ the friction velocity which is obtained following the study of Large and Pond (1981). The nonlinear wave-wave interaction terms are not explicitly included in the 1G version of the CSOWM, however its effects are modelled implicitly by modifying the atmospheric forcing term using the concept of a fully developed wind-sea as defined by the P-M (Pierson-Moskowitz) spectrum which is expressed as:

$$E_{PM}(f) = \frac{0.0081}{(2\pi)^4 f_5^5} g^2 \exp\left[-1.25\left(\frac{f_p}{f}\right)^4\right]$$

Here $g$ is the gravitational acceleration, $f_p = 0.14g/U_{19.5}$ is the frequency corresponding to the peak in the spectral energy of the P-M spectrum and $U_{19.5}$ is the local wind speed (ms$^{-1}$) at 19.5m above the sea level. The concept of the P-M spectrum is also used to partition the total energy spectrum into its wind-sea and swell-sea components.

The two dimensional energy spectrum of the CSOWM has 23 frequencies and 24 directions. The 23 frequencies range from 0.039Hz to 0.32Hz increasing in geometric progression with a constant ratio of 1.0064, applicable to both the coarse and the fine grids. The 24 directions have a bandwidth of 15° everywhere. For numerical integration, the model uses a time step of 1.5 hour on the coarse grid and 1 hour (0.75 hour) on the fine grid to satisfy the computational stability criterion for propagation in deep (shallow) water. The time-stepping cycle is so designed that the growth and the propagation mechanisms are calculated alternately in a three-hour time step after which a new set of wind fields is applied to the model and the whole cycle is repeated for the next time step.

In the third generation (3G) mode, the nonlinear interactions are included explicitly by calculating the same through the use of Discrete Interaction Approximation applied to the full two-dimensional spectrum of 23 frequencies and 24 directions; further, over the Canadian Atlantic, the calculation of the 3G source term is extended to include shallow-water effects through the addition of a simple enhancement of the nonlinear source term as a function of water depth, together with a bottom friction dissipation term incorporating an empirical friction factor. In the 3G version of the CSOWM, the wind input source terms are exactly the same as those used in the WAM model (see The WAMDI Group, 1988).
Figure 1: The Atlantic and the Pacific grid of the Canadian Spectral Ocean Wave Model. The nested fine grids covering the nearshore regions of the Canadian Atlantic and the Canadian Pacific are also shown.
3. MODEL VERIFICATION

A basic version of the model (coarse grid with 1G deep-water physics) has been implemented in an operational mode at the Canadian Meteorological Centre (CMC) in Montreal since December, 1990. The Atlantic version of the model is driven by surface winds generated by the regional finite element weather prediction model of the CMC; these surface winds are generated so as to be applicable at 10 m above the mean sea level. For the Pacific grid, the model is driven by 1000 mb winds obtainable from the hemispheric spectral weather prediction model of the CMC. For both the oceanic regions, the operational model is run twice a day (00 GMT and 12 GMT) and a four panel wave height chart containing significant wave height contours as well as surface wind, wind-wave and swell-wave height and period is prepared and disseminated by the CMC. The four panels depict analyzed wave chart (valid at T+0 hour) plus three 12-hourly forecast charts valid at T+120 T+24 and T+36 hour respectively.

A preliminary verification of the model based on a three-month period (May–July, 1990) is presented in Table I. The verification is made in respect of three model parameters, namely wind speed, significant wave height and peak period; these three parameters are verified against observed values at 10 buoy locations in the northwest Atlantic and about 14 buoy locations in the northeast Pacific. For each of the two oceanic regions, four error statistics, namely Mean Error (ME), Root Mean Square Error (RMSE), Scatter Index (SI) and r (the linear correlation coefficient between model and observed value) are calculated in respect of wind speed, significant wave height and peak period. The buoy anemometers were located at different heights varying from 5 m to 13.8 m. Consequently, two sets of error statistics were calculated in respect of wind speed. In one set, the buoy winds as reported were used to compute the various error statistics, while in the other set, buoy winds were adjusted to the 19.5 m level using a logarithmic wind profile. Both the sets of error statistics in respect of wind speed are shown in Table I together with error statistics for wave height and peak period.

Several important findings of the verification can be summarized as follows:

1. The positive Mean Error (or bias) in the wind speed for the Atlantic grid suggests that the finite element weather prediction model generates boundary layer wind which may be representative of a level higher than the mean height of all buoy anemometers in the Atlantic. When buoy winds are adjusted to 19.5 m level, the reduction in ME for the Atlantic grid suggests that the boundary layer winds generated by the CMC finite element weather prediction model are closer to the 19.5 m level. Over the Pacific grid, the negative wind
speed bias has increased further when buoy winds are adjusted to 19.5 m, suggesting that the CMC spectral weather prediction model produces boundary layer winds which are at a level considerably below the 19.5 m level.

### TABLE I: Evaluation of the ABS spectral wave model CSOWM

#### Summary of Error Statistics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WIND SPEED (m/s)</td>
<td>WAVE HEIGHT (m)</td>
</tr>
<tr>
<td></td>
<td>Forecast Time 00 hr 12 hr 24 hr 36 hr</td>
<td>Forecast Time 00 hr 12 hr 24 hr 36 hr</td>
</tr>
<tr>
<td>ME</td>
<td>1.7 (1.0) 0.7 (0.4) 1.0 (1.1) 1.1 (0.0)</td>
<td>1.19 (1.0) 0.13 (0.14) 0.14 (0.19) 0.19 (0.10)</td>
</tr>
<tr>
<td>RMSE</td>
<td>3.2 (2.7) 2.7 (2.8) 2.9 (2.9) 3.1 (3.1)</td>
<td>0.56 (0.52) 0.59 (0.59) 0.65 (0.65) 2.4 (2.4)</td>
</tr>
<tr>
<td>SI</td>
<td>59 (42) 50 (42) 50 (45) 53 (46)</td>
<td>42 (40) 44 (44) 49 (49) 32 (31)</td>
</tr>
<tr>
<td>r</td>
<td>0.68 (0.72) 0.62 (0.66) 0.62 (0.62) 0.55 (0.58)</td>
<td>0.80 (0.81) 0.76 (0.72) 0.74 (0.74) 0.30 (0.31)</td>
</tr>
<tr>
<td>N</td>
<td>871 (890) 909 (927) 927 (974) 974 (995)</td>
<td>974 (1016) 1003 (1033) 1033 (918) 918 (935)</td>
</tr>
</tbody>
</table>

(Numbers in parentheses represent error parameters calculated with buoy winds adjusted to 19.5 m level)

2. The RMS error in wave height varies from 0.55 m (at analysis time) to about 0.65 m (at 36-hour forecast time); this compares quite favourably with error statistics of some of the operational wave models of Europe and U.S.A (see Khandekar, 1989).

3. The scatter index for wave heights in the Atlantic is considerably higher than the corresponding value in the Pacific; this appears to be a reflection of the higher scatter index for the wind speed in the Atlantic than in the Pacific.

4. The error statistics in respect of peak period are poor everywhere, in particular over the Pacific where the RMS errors are large and correlations are small. Two factors may be attributed to these poor error statistics: (a) Peak period values reported by some of the buoys were found to be highly variable and thus unreliable. (b) During late spring and early summer, the north Pacific is sometimes influenced by
long period swells travelling from the south Pacific; these long period swells may have influenced some of the buoy data resulting in larger negative bias and lower values of correlation.

4. VERIFICATION BASED ON A CASE STUDY

An intense winter storm moved over the southern Grand Banks area of Newfoundland on January 11, 1991. The storm began as a closed low pressure area northeast of Cape Hatteras, North Carolina (USA) with a central pressure of 1010 mb on January 10, 1991, 00 GMT. During the next 24 hours, the low pressure area moved rapidly over the Scotian Shelf region developing into an intense storm with a central pressure of 976 mb. The storm continued to move northeastwards reaching its peak intensity on January 12, 1991, 00 GMT, with central pressure of 956 mb and hurricane force winds being reported by a couple of ships in the northwest Atlantic. In the wake of this intense storm, extreme sea-states with wave heights ranging from 10 to 15 m were generated in the Grand Banks area. Of the available buoy data, two buoys located in the southern Grand Banks area reported wave heights of over 13 m during the 12-hours period from January 11, 1991, 12 GMT to January 12, 1991, 00 GMT, when the storm reached its peak intensity.

The wave heights generated by the operational (1G coarse) version of the CSOWM are shown in the wave height charts of Figure 2. These charts are part of the four panel charts generated by the operational wave model and disseminated by the CMC. The top of Figure 2 is the wave height analysis chart for January 12, 1991, 00 GMT, and shows maximum wave heights of 12 m generated in the southern Grand Banks area in response to strong northwesterly winds prevailing over most of Newfoundland. The bottom of Figure 2 shows the 12-hourly forecast chart valid for January 12, 1991, 12 GMT. The forecast wave chart is obtained by driving the wave model with three-hourly forecast winds obtainable from the finite element weather prediction model. The forecast chart shows a wave height field with a maximum value of 11 m located about 800 km southeast of Newfoundland; this wave height field is in response to predicted northwesterly winds of 40 knots or higher over Newfoundland and the southern Grand Banks area.

The various versions of the CSOWM (ex. coarse and fine grid, deep and shallow water physics, 1G or 3G source term algorithm) were driven using the same set of CMC wind fields and model products were generated covering the storm period from January 10, 1991, 00GMT, to January 12, 1991, 18 GMT. These model products were evaluated against buoy data which were available only at three buoy locations and that too for only part of the storm period. Consequently additional wave height “observations” were generated by digitizing hand-analyzed wave height charts prepared by the METOC (Meteorology ad Oceanography) Centre in Halifax, Nova Scotia. These METOC wave charts make use of
observed ship wave data and are prepared using the principle of continuity of wave fields; accordingly, these METOC wave charts can be interpreted as providing a fairly realistic synoptic view of the wave fields associated with a storm. A total of 12 six-hourly METOC charts were digitized, each at 16 selected locations covering the extreme seastate region of the storm. These 16 locations together with three buoy locations are shown in Figure 3. In all, there were 19 locations which provided a total of 250 data points (58 buoy data points and 192 METOC data points) for verification of the model performance.

Figure 2: Wave height chart at analysis time, January 12, 1991, 0000 GMT (Top) and forecast wave chart valid at January 12, 1991, 1200 GMT (Bottom) generated by the present operational version of the CSOWM. The charts also depict wind speed and direction, wind-wave height and period, swell-wave height and period and swell direction at selected locations (see inset for the wave plot model).
At each of the 19 locations of Figure 3, model generated wave heights were evaluated against "observed" (buoy or METOC) wave heights and as before four parameters namely ME, RMSE, SI and r were worked out using all available data covering the storm period. These parameters were worked out using the buoy data points as well as the METOC data points and the results of the verification for all six versions of the wave model CSOWM are presented in Table II. The error parameters of Table II reveal several interesting aspects of the wave model verification. 1. The mean error is in general negative everywhere indicating that the various versions of the CSOWM underestimate the wave heights generated by the storm; this is most certainly due to the fact that the CMC winds which were used to drive the various versions of the wave model are underestimated in the storm region. 2. the RMS errors for various versions of the CSOWM appear to be much larger than the RMS errors in Table I, however the scatter index values are in general quite comparable to those in Table I which are obtained for a three-month period. 3. The coarse grid version of the model with either 1G or 3G source terms appear to provide the best error statistics. 4. The inclusion of shallow-water physics has no positive impact on the error statistics; this is due to the fact that the grid points where the model wave heights were
evaluated are all located over deep waters with average water depth of over 2000 metres.

**TABLE II:** Evaluation of the model CSOWM; Error statistics for significant wave heights generated during the Grand Banks Storm, January 1991.

<table>
<thead>
<tr>
<th>Model version</th>
<th>Buoys Data, N=58</th>
<th>METOC Data, N=192</th>
<th>Combined Data, N=250</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ME (m)</td>
<td>RMSE (m)</td>
<td>SI</td>
</tr>
<tr>
<td>1G Coarse</td>
<td>-0.19</td>
<td>2.24</td>
<td>25</td>
</tr>
<tr>
<td>1G Fine</td>
<td>+0.18</td>
<td>2.39</td>
<td>26</td>
</tr>
<tr>
<td>1G Shallow</td>
<td>-0.76</td>
<td>2.43</td>
<td>27</td>
</tr>
<tr>
<td>3G Coarse</td>
<td>+0.50</td>
<td>1.98</td>
<td>22</td>
</tr>
<tr>
<td>3G Fine</td>
<td>-1.50</td>
<td>2.67</td>
<td>29</td>
</tr>
<tr>
<td>3G Shallow</td>
<td>-1.27</td>
<td>2.66</td>
<td>29</td>
</tr>
</tbody>
</table>

5. **CONCLUDING REMARKS**

The present operational version of the CSOWM appears to provide adequate numerical guidance for sea-state analysis and forecasting over the northwest Atlantic and the northeast Pacific. The wave charts generated by the present operational wave model appear to have operational utility in terms of identifying regions of extreme sea-states. At present, the upgraded versions of the CSOWM (ex. shallow-water physics, 3G source term algorithm) are being tested; these upgraded versions of the CSOWM are expected to become operational in the near future.

6. **ACKNOWLEDGEMENT**

We would like to express our appreciation to Chris Ullrich (a co-op student from the University of Waterloo) for his assistance in preparing some of the diagrams and computations.

7. **REFERENCES**


AN INTERACTIVE GRAPHICS APPROACH
TO WAVE ANALYSIS AND FORECASTING

V.R. Swail, B. deLorenzis, C. Doe, R. Bigio and C. Calnan

Atmospheric Environment Service,
Downsview, Ont.

MacLaren Plansearch Ltd.,
Halifax, N.S.

1. INTRODUCTION

It has long been recognized that meteorological analyses which incorporate subjective input from trained professionals are a marked improvement over strictly objective products generated through computer models alone. This is most often noticed in the operational surface pressure analyses; however, a much more dramatic effect can be seen in the wind fields derived from those pressure analyses, and subsequent ocean response (waves, currents, oil spills) produced from those winds. The major drawback to subjective analysis is that it has been a very labour-intensive, time-consuming activity; this conflicts with the ever-increasing demand for timely operational products, with fewer resources. Thus, the trend has been to sacrifice the accuracy and detail associated with the subjective, expert analysis in favour of the speed and automation of the computer model.

In the following pages a system is described which uses interactive graphical analysis techniques on a computer workstation to modify the objective products of the operational models on a regional basis, and thus produce improved analyses and forecasts of winds and waves in particular, although the wind information may be easily applied to oil spill trajectory models or other application. This system is now being used operationally to provide wind and wave analyses and forecasts twice per day for the north Pacific Ocean and the northwest Atlantic Ocean; in addition, the wind and wave analysis portions of the operational runs are being archived to produce a continuous data base for climatological purposes. This will eventually result in a "normal" climatology to complement the extremes climatologies for the east and west coasts of Canada (Swail et al., 1989; Swail et al., 1992). A similar 3-year "normal" climatology was developed previously for the east coast (Eid et al., 1989).

A schematic representation of the process is shown in Figure 1.

2. METEOROLOGICAL INPUT

The starting point for the analysis and forecast system in terms of meteorological data is that data set which often represents the final product in conventional operational systems, i.e. the modelled surface
pressure analyses and prognoses produced by the Canadian Meteorological Centre (CMC). The system also accesses surface air temperature analyses and prognoses, and sea surface temperature analyses. If other fields are eventually required for the analysis, for example 850 mb temperatures, these too can be accessed; at present, however, only the surface pressure and temperature fields are used.

Figure 1 Wave Forecasting System Flow Chart
The pressure and temperature fields are acquired every 12 hours on about an 800 km grid spacing over the model domain, for the present analysis time, and forecast times to T+48 hours. For the north Pacific this domain extends from 30°N to 60°N, 120°W to 140°E; for the north Atlantic the domain covers 25°N to 67.5°N, and from 20°W to the North American coast. The input pressure and temperature fields are then fitted by a cubic B-spline surface; this changes the geophysical field values to mathematical coefficients, and allows complex manipulation of the surfaces even out to the edges of the grid. Our experience with these splines has shown that they can represent the fields very well, although there may be a slight tendency to smooth out the central values of low and high pressure centres.

3. INTERACTIVE GRAPHICS EDITOR

The Interactive Graphics Editor (INGRED), originally described by deLorenzis (1988), is the crux of the entire analysis and forecast system. This is where the "man" in the "man-machine mix" is applied. It operates on the B-spline coefficients derived from the original geophysical data. Details of the operations performed by INGRED are given in the following sections.

3.1 BASIC OPERATIONS

When the analysis for a particular forecast valid time is loaded, the pressure and temperature fields (as represented by the splines) are contoured on the screen. If the air-sea temperature difference field is desired, it can also be displayed. The basic operations available include the labelling of contours, the sampling of the field value at any specified point, the display of either geostrophic wind barbs or marine boundary layer wind barbs (19.5 m, effective neutral wind, as described by Cardone, 1969) at selected points or on a regular grid.

3.2 EDITING OPERATIONS

Objective surface pressure fields produced by national meteorological centres such as CMC rarely correspond exactly to those analysed by professional meteorologists in regional weather centres. In particular, numerical models have a tendency to truncate the peaks and valleys of high and low pressure areas compared to subjective analysis; the impact of this on wind field analysis can be marked. The other common complaint is that the low and high pressure centres are not exactly in the right location. The most commonly used editing feature of INGRED is to depress the central pressure of lows and increase the central pressure of highs. In the Pacific this is typically 5–6 mb for lows, but may be as much as 20 mb in some cases. The differences are much smaller for highs. The second most common edit is to move the locations of the low and high centres. Both of these edits are easily done in INGRED. The spline is analogous to a
membrane, so that it can be pushed and pulled at will; changes made at one location will influence the coefficients over some pre-defined radius, so that smooth gradients are maintained.

While the most common edit is to alter the central pressure of lows and highs, the pressure (or temperature) can be adjusted at any location on the map. This may be used to change the shape of an analysis. Similarly, a block move may be used to shift the position of a feature such as a ridge or trough line. Fortunately, the system also includes an UNDO feature for most operations! At any point in the editing process, the results can be labelled or sampled, including winds, as described above. When the analysis matches the analyst’s perception of the true surface pressure and temperature fields, he/she proceeds to the next step.

3.3 KINEMATIC ANALYSIS

While the winds created in the manner described above, from accurate surface analyses, using a reliable boundary layer model will produce a much improved wind field, there will still be cases where this model surface wind will not agree with actual surface winds, as observed by buoys, ships and satellite. Reasons for this discrepancy may include topographic effects (especially off the west coast of Canada), mesoscale effects, frontal zones, excessively stable or unstable conditions which introduce scatter into the boundary layer model results, etc. The system has the ability to modify the boundary layer model winds, much in the way of a kinematic analysis. Kinematic analysis has been shown (Cardone et al., 1980) to significantly improve surface wind estimates over objective methods alone. Areas may be drawn directly on the screen where the wind speed and/or wind direction is increased or decreased by an amount determined by the analyst. Contours may be nested for different levels of adjustment for different areas. For example, in the case of southwesterly winds blowing up against the mountains along the coast of British Columbia, the wind directions could be backed by 45°, and speeds increased by 40%, to reflect the coastal jet which often occurs in these situations. Similarly, in strongly baroclinic regions associated with extratropical cyclones, the wind speeds might be increased by 20% over a particular area, if it was felt that the boundary layer model was not handling the situation adequately.

A zoom feature has been built into the system to allow for accurate drawing of kinematic areas. This feature is also very useful in precise identification of specific locations in verification studies.

4. INTERPOLATION OF FIELDS

Once the surface analyses and prognoses have been edited as desired, the fields must be interpolated to the timestep of the wave (or other
Before interpolation the analyst is requested to link features (lows, highs, ridges, cols) on successive maps. This is done by simply clicking the pen on the feature location on successive maps; the maps are displayed automatically in sequence during this process. This maintains the gradients and characteristics of the systems, rather than blending and smoothing, especially of intense features. Kinematic adjustments, as described in the previous section, may also be linked and interpolated. This may be desirable for features associated with pressure systems, e.g. the baroclinic case described above; however, it would not likely be appropriate for the topographic case, so that feature would not be linked.

Animation of the sequence of 12-hourly charts can be examined prior to identifying features to be linked; after interpolation, animation of the resultant 2-hourly sequences can be viewed.

5. GENERATION OF WIND FIELDS

The interpolation step generates the surface pressure and temperature fields, as well as any kinematic modification areas which may have been included, on a 2-hour time step. A routine called SAMPLER then computes the final wind speed on the grid required by the wave model (or other ocean response model). The winds are computed in this step using the same algorithms as in the wind sampling routines in INGRID; therefore, you can see beforehand exactly what wind fields are going into the wave model. There is no interactive component to this activity, just intensive computation. At present, the computation performed by this step takes about 20 minutes for a forecast from T-12 to T+48 hours. This should be reduced by an order of magnitude with the generation of workstation now available.

The facility exists to examine listings of time series of wind fields at series of pre-specified points, or at any given point. This is particularly useful in comparing model winds with high-quality surface observations, such as from meteorological buoys.

6. WAVE ANALYSIS AND FORECASTS

Once the wind fields have been created on the appropriate grid at a 2-hour time step for input to the wave model in the step described above, the wave model run is initiated. The wave model used in both the north Atlantic and north Pacific implementations of the system is the ODGP 1-G deep-water model described by Cardone et al.(1976). This model has been widely used in both hindcast and forecast studies, and was the model used in the previous studies described in Section 1. As with the wind field production, this is not an interactive step,
but merely computational. On the east coast of Canada, an additional step may be required to produce a digital file of sea ice extent in the winter months. This file is produced as required simply by editing a list of the wave model grid points which are covered by more than 5/10 ice cover. This step occurs outside the execution of the wind and wave runs described above. Ice cover is not a concern off the west coast of Canada. On the present workstation the wave model for T–12 to T+48 takes less than 14 minutes; with available hardware this could be reduced by an order of magnitude.

Once the wave fields have been created, the output can be directed back into INGRED to produce contour plots on the screen of significant wave height and spectral peak period, and arrows representing the vector mean wave direction. The facility also exists to examine listings of time series of wave fields at series of pre-specified points, or at any given point. Wave spectra can also be examined, in tabular form.

7. WIND AND WAVE CLIMATOLOGY

In the previous steps wind and wave analyses have been created for the north Atlantic or north Pacific basins every 2 hours from T–12 to T+0 the forecasts are irrelevant for the climatology. Subsequent operational runs will produce the next 12 hours of analyses, and so on until eventually a continuous data base of winds and waves is created at 2-hour intervals. The fields archived include: wind speed and direction, significant wave height, spectral peak period and vector mean wave direction at all points on both the fine and coarse mesh grids. The coarse mesh grid is 1.25° latitude by 2.5° longitude; the fine mesh grid, which is restricted to the coastal areas, is half that spacing. In addition, full 2-D spectra (24 directions by 15 frequencies) are archived for a selection of points.

The wave fields, while created at operational centres in Vancouver and Halifax, are archived at the Marine Environmental Data Service in Ottawa. Since the wave files, especially the spectral files, are very large, it is too cumbersome to transmit them to MEDS. Therefore, the wave model is installed separately at MEDS, and the much smaller wind (and ice where required) files are transmitted to MEDS daily. The wave model is then run at MEDS and the analysis fields are input directly into the on-line archive.

For severe storms, the final digital surface analyses from INGRED are sent to the Canadian Climate Centre for inclusion in the extremes climatology data base, after further detailed re-analysis.

8. SUMMARY

In summary, a technique has been developed which uses interactive graphics procedures on a Hewlett-Packard 9000 workstation in the
real-time analysis and prediction of wind and wave fields. A continuous data base of wind and wave analyses is created as a by-product for climatological analysis. The system has been applied to both the northwest Atlantic and northeast Pacific oceans; a degree of generality has been built into the system to allow extension to any ocean (or lake) basin.

One enhancement which would further improve the system is the ability to display synoptic reports from meteorological buoys, ships, and eventually satellite on the screen, as part of the kinematic analysis procedure. Another improvement which is possible is the linking through an artificial intelligence model of a mesoscale wind model, particularly for topographically sensitive areas such as the west coast of Canada. A final improvement to the system as a whole would be the incorporation of data assimilation into the wind and wave analysis procedures, especially feedback of the wave height information into the surface wind stress calculations.

9. REFERENCES


AN EXTREMES WIND AND WAVE HINDCAST
OFF THE WEST COAST OF CANADA

V.R. Swail, V.J. Cardone and B. Eid

1Atmospheric Environment Service,
   Downsview, Ont.

2Oceanweather, Inc.,
   Cos Cob, CT

3MacLaren Plansearch Ltd.,
   Halifax, N.S.

1. INTRODUCTION

The objective of this study was to specify the extreme wave climate off the west coast of Canada, by hindcasting wave fields in the 50 top-ranked wave-producing storms.

The target areas for the study, which cover most Canadian west coast offshore exploration sites, are shown in Figure 1.

Previous experience with the historical meteorological data base of the N.E. Pacific Ocean basin supports selection of storms from about the past 30 years. The data base for earlier periods is much less extensive and wind fields may not be specified as accurately. In addition, the Canadian Meteorological Centre charts on microfilm go back only to 1957. Therefore, the historical period considered in this study extends from 1957 to 1989.

The following paragraphs describe briefly the data bases used, the storm selection methodology, the wind and wave hindcast procedures and verification, and the extremal analysis used to produce the final output product for the study, namely the design wave estimates at specified probability levels for the areas in question.

2. DATA BASE ASSEMBLY

A comprehensive set of historical meteorological data was assembled for the specification of surface wind fields for the production of historical storms. The data fall into two basic categories:

2.1 Historical surface weather maps

The following map series of weather conditions were utilized for storm selection and hindcasting (1957-89 except as noted):

- Canadian Meteorological Centre (CMC) surface analysis charts;
- Pacific Weather Centre (PWC) surface analysis charts (from 1970);
- National Meteorological Center (NMC) 3-hourly North American surface analyses;
Figure 1. Target area of west coast hindcast.
National Meteorological Center (NMC) 6-hourly Northern Hemisphere surface analyses,

### 2.2 Digital Data Bases

The following digital data sets, comprising observations from ships and buoys, and from previous hindcasts for the west coast of Canada, were utilized in this study for storm identification, hindcasting and verification:

- Comprehensive Ocean Atmosphere Data Set (COADS) ship observations 1854-1979;
- Canadian co-operating ships 1980-89;
- NOAA buoy data 1972-89;
- AES land observations 1953-1989;
- Ocean Weather Station (OWS) Papa (1951-81);
- Marine Environmental Data Service buoy data;
- U.S. National Climatic Data Center (NCDC):
  - "Marine Deck" 1854-1969;
  - "Decade of the 1970’s" 1970-1979;
  - "Tape Deck 1129" 1980-present;
- Geostrophic Wind Climatology (Swail, 1985);
- U.S. Navy SOWM North Pacific hindcast.

There is some overlap in the data sets listed above; however, to avoid missing important data, all sources were investigated.

### 3. STORM SELECTION

The storm selection was accomplished in three main steps: (1) selection of potentially severe wave-producing storms in the past 33 years; (2) storm verification and cross-checking between different data sources; (3) storm ranking and final selection.

#### 3.1 Identification of severe storms

Potentially severe wave-producing storms in the period 1957-1989 were identified by scanning all of the digital data bases listed in Section 2. All wind and wave data in the target area greater than or equal to specified thresholds (45-50 knots for wind speed and 7-8 metres for significant wave height) were listed in chronological order. For each storm identified, the starting and ending dates and selected wind and wave values, including maximum wind speed, duration of wind speed above the threshold, and maximum significant wave height measured, observed or predicted from previous hindcast studies, were abstracted from the data records.

Previous storm selection and hindcast studies for the west coast areas also produced lists of severe storms for specific sites or areas.
The preliminary lists derived from the above screenings were synthesized into a single master candidate coarse storm list (MCL) containing 500 separate events.
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3.2 Storm ranking and final list

Reduction of the MCL to the target level of 50 storms for hindcasting proceeded in three stages:

1. Assignment of a quasi-objective storm ranking parameter to each member of the MCL (based on maximum pressure gradient in the study area; duration of at least 75% of the maximum pressure gradient; pressure difference between Alaska low and California high; swell-generating parameters);

2. Subjective reduction of the MCL through further reference to the individual storm screening data (i.e. peak wind speed, duration, severity index, peak wave heights, and previous storm studies);

3. Intensive subjective study of the 6-hourly map sequences, to develop an estimate of the maximum wind speed and duration in each fetch zone susceptible to extreme wave generation within the regions of interest, to serve as a critical discriminant between otherwise comparable storms.

The final list of the 50 most severe wave-producing storms for the west coast of Canada is shown in Table 1. A severe storm occurred on October 26, 1990, during the analysis phase of the project; it was subsequently added to the storm list, as the 51st storm.

4. WIND FIELD ANALYSIS

The model domain and grid specification used for these wind and wave hindcasts are shown below. The model is comprised of two nested grids, a coarse grid and fine grid.

<table>
<thead>
<tr>
<th></th>
<th>Coarse</th>
<th>Fine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain:</td>
<td>30°N - 60°N</td>
<td>45°N - 60°N</td>
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<tr>
<td></td>
<td>120°W - 220°W</td>
<td>142.5°W - coast</td>
</tr>
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<td>Spacing:</td>
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<td>0.625° lat.</td>
</tr>
<tr>
<td></td>
<td>2.5° long.</td>
<td>1.25° long.</td>
</tr>
<tr>
<td>Grid Points:</td>
<td>755</td>
<td>173</td>
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<tr>
<td>Time Step:</td>
<td>2 hours</td>
<td>2 hours</td>
</tr>
</tbody>
</table>

Table 2. Parameters of wind and wave grid

The method used for hindcasting wind fields for the selected storms is based on man-machine mix intensive wind field analysis using a blend of surface pressure analysis and kinematic analysis wind fields.

The hindcast period of each storm consists of the following sub-periods: (a) period of spinup of background seas in the model
domain in which principal wave generation occurs prior to the peak of the selected storm; (b) period during which the selected storm generates seas in the study areas and including always the period within ±12 hours of expected occurrence of peak states; (c) 24-hour period following (b) in which peak seas continue to decay.

For the spinup and the decay periods, the approach was to specify winds from the sea level pressure analyses alone. Gridded pressures were converted to “effective neutral” 20-m winds through the marine planetary boundary layer (MPBL) model developed by Cardone (1969, 1978). The “effective neutral” wind speed is simply the wind which would produce the same surface stress at the sea surface in a neutrally stratified boundary layer as the wind speed in a boundary layer of a given stratification. This is consistent with the similarity approach and produces analogous functions. The baroclinic forcing term is supplied at each grid point from climatological horizontal air temperature gradients appropriate to the North Pacific Ocean in the cold season. The atmospheric stability term is specified as a function of local geostrophic wind direction.

Kinematic winds are extracted from the streamline/isotach analyses at the fine mesh grid point locations for the period (b), the peak of the storm, and represent the effective 1-hour average 20-m level neutral wind. This kinematic analysis is used in conjunction with the winds derived from the sea level pressure analyses. Reports of wind speed from buoys and ships equipped with anemometers are transformed into the effective neutral 20 m values (Cardone et al., 1990). For ships which use estimated wind speeds, values are adjusted according to the Scientific Beaufort scale. The kinematic winds replace the winds derived from the pressure field in the interior of the kinematic domain, and are blended with the pressure-derived winds along the boundaries of the domain.

Kinematic winds are by far the most accurate and least biased winds, primarily because the method allows a thorough re-analysis of the evolution of the wind fields. Kinematic analysis also allows the wind fields to represent effects not well modelled by pressure–wind transformation techniques, such as inertial accelerations associated with large spatial and temporal variations in surface pressure gradients and deformation in surface winds near and downstream of coasts.

The final step in the wind field analysis is interpolation from 6 hours to 2 hours (as required to drive the wave model). Linear interpolation in time of zonal and meridional wind components is used for wind direction, while the fourth power of wind speed is used for interpolation of wind speed. Further interpolation is done near centers of rapidly propagating cyclones to avoid errors due to
excessive smoothing of winds. Gridded wind fields are produced on the ODGP wave model grid.

5. WAVE HINDCASTS

The ODGP wave model is a deep-water fully spectral model (24 directions by 15 frequencies), which evolved from the U.S. Navy Spectral Ocean Wave Model (SOWM). The ODGP model has been used in hindcast studies of extreme wave regimes of many different types (e.g. winter cyclones, typhoons, hurricanes, and monsoon surges). Reece and Cardone (1982) summarized this extensive model experience and reported a record of hindcast skill unequaled by alternate models. The model, when driven by wind fields of accuracy about ±2 m/s in speed, ±20° in direction, provided unbiased specifications of significant wave height and peak frequency with a scatter of about 12%, which, incidentally, is comparable to the scatter in estimates of these quantities from measured 20-minute wave records. Recent applications on the east coast of Canada have shown deep-water wave height predictions of negligible bias with scatter indices less than 13% (Cardone et al., 1989; Swail et al., 1989). A detailed description of the model physical and numerical algorithms is given in Cardone et al. (1976). A special version of the model which accounts for sheltering by capes and islands (CAIPS) was used in this application.

6. VERIFICATION

In order to assess the quality of wave model predictions, it is necessary to isolate the errors in the input winds which are used to drive the wave model, and in the output from the wave model.

Eleven storms were selected for model validation. All available wave measurements were obtained from automatic wave recording systems (e.g. waverider buoys, non-directional and directional, NOAA buoys), including 1-D and 2-D spectral data. Wind speed and direction (and air and surface water temperature) records were obtained from all the MEDS and NOAA buoys which were in the study area during each of the ten validation storms. All measured winds were converted to "effective neutral" winds at 20 m above the mean sea level, similar to those used in running the wave model.

The time series of hindcast winds and waves were plotted against the corresponding observations at all evaluation sites for the validation storms. A quantitative statistical analysis of the peak-to-peak comparison was also carried out to provide an overall evaluation of the hindcast. Spectral plots (1-D, 2-D) of the respective observed and modelled spectra were produced at peak wave height. For continuous wave measurements a 7 point moving average was used on the recorded data (i.e. smoothed).
For an extremal analysis, the most important aspect of the model is its ability to predict the storm peak accurately; therefore, the peak-to-peak comparisons are the most important evaluation criteria.

The verification sites were grouped into three different categories: offshore-deep (O-D), inshore-deep (I-D), inshore-deep-sheltered (I-S). The overall statistics of the verification analysis for significant wave height ($H_s$) and peak period ($T_p$), including mean and rms error, scatter index (SI) and correlation coefficient (CC), are shown in Tables 3 - 6, for both smoothed and unsmoothed cases. Time series plots of hindcast versus measurement are shown in Figures 2 and 3, for buoy locations 46036 and 46205; the locations are shown in Figure 1.

<table>
<thead>
<tr>
<th>Sites</th>
<th>No.</th>
<th>Mean</th>
<th>RMS</th>
<th>SI (m)</th>
<th>CC</th>
</tr>
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<tbody>
<tr>
<td>O-D</td>
<td>22</td>
<td>-0.09</td>
<td>1.19</td>
<td>12.1</td>
<td>0.82</td>
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<tr>
<td>I-D</td>
<td>14</td>
<td>-0.40</td>
<td>1.64</td>
<td>19.5</td>
<td>0.86</td>
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<tr>
<td>I-S</td>
<td>9</td>
<td>0.18</td>
<td>1.46</td>
<td>19.1</td>
<td>0.69</td>
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Table 3. $H_s$ comparisons (unsmoothed)

<table>
<thead>
<tr>
<th>Sites</th>
<th>No.</th>
<th>Mean</th>
<th>RMS</th>
<th>SI (sec)</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-D</td>
<td>11</td>
<td>0.63</td>
<td>1.40</td>
<td>15.0</td>
<td>0.80</td>
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<tr>
<td>I-D</td>
<td>7</td>
<td>-0.86</td>
<td>1.56</td>
<td>17.6</td>
<td>0.88</td>
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<tr>
<td>I-S</td>
<td>4</td>
<td>-0.07</td>
<td>0.36</td>
<td>4.4</td>
<td>0.96</td>
</tr>
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Table 4. $H_s$ comparisons (smoothed)

<table>
<thead>
<tr>
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<th>Mean</th>
<th>RMS</th>
<th>SI (sec)</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-D</td>
<td>22</td>
<td>0.55</td>
<td>1.63</td>
<td>11.3</td>
<td>0.54</td>
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<tr>
<td>I-D</td>
<td>14</td>
<td>-0.14</td>
<td>1.26</td>
<td>8.5</td>
<td>0.68</td>
</tr>
<tr>
<td>I-S</td>
<td>9</td>
<td>0.63</td>
<td>2.89</td>
<td>21.0</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 5. $T_p$ comparisons (unsmoothed)

<table>
<thead>
<tr>
<th>Sites</th>
<th>No.</th>
<th>Mean</th>
<th>RMS</th>
<th>SI (sec)</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-D</td>
<td>11</td>
<td>0.94</td>
<td>1.64</td>
<td>11.6</td>
<td>0.48</td>
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<tr>
<td>I-D</td>
<td>7</td>
<td>-0.47</td>
<td>1.38</td>
<td>9.1</td>
<td>0.59</td>
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<tr>
<td>I-S</td>
<td>4</td>
<td>-0.83</td>
<td>1.07</td>
<td>7.0</td>
<td>0.62</td>
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</table>

Table 6. $T_p$ comparisons (smoothed)

These statistics compare favourably with other comprehensive hindcast studies carried out with calibrated spectral wave models. Scatter
indices in the range of 10–15% for $H_s$ and $T_p$ appear to represent the maximum skill achievable given ultimate limitations in the meteorological data base which limit wind field accuracy, and the sampling variability of conventional wave measurements, which implies an uncertainty of about 10% in actual measurements of $H_s$ and about 7% in $T_p$. The smoothed offshore deep water statistics are comparable, though a little worse than those for the unsmoothed data, and that effect alone could be responsible for the slight differences in the verification statistics. The verification statistics suggest lower accuracy for the inshore deep category than achieved in deep offshore sites, though hindcasts are still very skillful. The most likely explanation for the increased scatter nearshore is that wind errors are larger inshore due to the well-known effect of the coastal mountain ranges on the wind field inshore. Due to sparse data inshore, the kinematic analysis can only partially account for this effect in the final wind fields. However, we cannot rule out that the differences are mainly statistical variability stemming from the fact that the sample size for this category is much smaller than for the deep offshore category. The same may be said of the inshore sheltered category, which includes only 9 comparisons.

Although the model tends to over-predict, on average, there were sites where the average measured $H_s$ was greater than the average predicted values. This is mainly due to the effect of sheltering of the islands.

On the time series plots, the measured wave heights may be higher than the hindcast in the early stages of the storm, while the wave model is spinning up from a calm state.

Verification of wave direction at the one directional wave measurement site showed negligible bias ($-7^\circ$) and an RMS difference of $51^\circ$.

7. PRODUCTION OF HINDCASTS

The production of wind and wave hindcasts was executed following the wind and wave model validation phases.

Wind speed (effective neutral 20-m winds), wind direction, significant wave height, peak period, and vector mean wave direction were archived at all ODGP grid points, both coarse and fine grids, every 2 hours, for each storm.

Directional (2-D) spectral variance (15 frequencies x 24 directions) were archived every 6 hours at 118 selected points.

8. EXTREMAL ANALYSIS

From the wind/wave hindcast model, we have available at all points and at each time step, the following quantities:
Figure 2. Comparison of measured and hindcast winds and waves - offshore deep.

WEST COAST STORM VERIFICATION
GRID POINT 682 - WR 46036
November 17, 1988 to December 6, 1988

- - - - Model N
- - - - Model C
- - - - Observed
Figure 3. Comparison of measured and hindcast winds and waves - inshore deep.

WEST COAST STORM VERIFICATION

GRID POINT 1365 - WR 46205

November 17, 1988 to December 6, 1988

- - - - Model N
-. - Model C
--- - Observed
Figure 4. 100-year return period maximum wave height (m).

Figure 5. 100-year return period wind speed corresponding to extreme waves (m/s).
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**H**ₙ significant wave height  
**T**ₚ spectral peak period  
**Θ**ₜ vector mean wave direction  
**W**ₙ 1-hour average wind speed  
**W**ₜ wind direction

Extremes of **H**ₙ and **W**ₙ were specified at all grid points within the contiguous domain of the fine-grid, using the Gumbel extreme value distribution fitted by method-of-moments. Confidence limits (90%) were also calculated.

At a selected subset of 25 grid points on the fine grid, a more detailed analysis of extremes was carried out. The maximum individual wave height was estimated in each storm from the hindcast zeroth and first spectral moments following Borgman’s (1973) integral expression, which accounts for storm buildup and decay. The integral was evaluated for two assumed maximum individual wave height distributions: (1) Rayleigh; (2) Forristall (1978).

The same approach was used to estimate the maximum crest height at a site in a storm using the empirical crest-height distribution of Haring and Heideman (1978). The median of the resulting distributions of **H**ₘ and **H**ₐ was taken as the characteristic maximum single value in a storm. The mean ratios of **H**ₘ/**H**ₙ and **H**ₐ/**H**ₙ were calculated and used to develop a mean ratio to provide extremes of **H**ₘ and **H**ₐ from fields of extreme **H**ₙ.

Figure 4 shows the 100-year return period analysis of the maximum wave height for the west coast of Canada, as derived from the information produced by this hindcast. Figure 5 shows the 100-year return period analysis of the wind speed occurring at the time of the maximum wave (not the independent 100-year wind speed). Comparison of the 100-year return period significant wave heights produced from the hindcast and from measured data at the NOAA buoy 46004 location, shows similar results (15.3 m from the buoy record, 15.0 m from the hindcast).

9. REFERENCES


on the United States continental shelf for application to an oil slick trajectory forecast program. Contract T-35430, NOAA, U.S. Dept. of Commerce, Silver Spring, Maryland.


BEAUFORT SEA EXTREME WIND/WAVE HINDCAST STUDY

Bassem M. Eid¹, Vincent J. Cardone², and Val R. Swail³

¹ MacLaren Plansearch (1991) Limited
   A Division of SNC-Lavalin Inc.
   Halifax, Nova Scotia

² Oceanweather Inc.
   Cos Cob, Connecticut, U.S.A.

³ Atmospheric Environment Services
   Environment Canada
   Downsview, Ontario

1. INTRODUCTION

An accurate description of marine climate is required for proper design and operation of offshore and coastal marine structures. Extreme wind and wave climate information is therefore needed for these purposes.

The objective of this study was to develop new and definitive estimates of the extreme wave climate in the Canadian Beaufort Sea, with emphasis on offshore exploration areas in deep and shallow water. A hindcast approach was adopted, which includes the following traditional steps: (1) assembly of a comprehensive data base of archived historical meteorological data, wave measurements and ice cover; (2) identification and ranking of historical storm occurrences during the potential open-water season, over as long an historical period as allowed by the data, and selection of a population of storms for hindcasting; (3) adaptation and validation of the most accurate numerical hindcasting procedures using a well tested and validated model to specify time histories of surface wind fields, surface wave fields and directional spectra in each hindcast storm; (4) hindcast a sufficiently large number of the top-ranked severe historical storms; (5) statistical analysis of hindcast extremes at selected model grid points in order to estimate the significant wave height, maximum individual wave height and crest height, and associated wind speed and wave period, associated with rare return intervals.

The Beaufort Sea presents a number of special problems, not normally encountered in extreme wave climate studies of Northern Hemisphere mid-latitude basins. The main problems are: (1) the relative scarcity of historical meteorological data, including almost a total absence of transient ship reports, which are the main data source in mid-latitude areas; (2) the highly variable and complex nature of sea-ice cover, which can be expected to exert a significant control over the wave field. The lack of data complicates both the storm selection process,
and the ability to accurately specify wind fields in selected historical storms. The presence of sea ice also complicates the storm selection process, and the hindcast process, since accurate hindcasts depend to some extent on the ability to specify ice-cover in selected events accurately.

The review of all known previous wind/wave climate studies of the Canadian Beaufort confirmed the need for a new study. For example, estimates of the 100-year maximum significant wave height in deep water varied among the studies published to date from about 4m to nearly 16m with no indication that a consensus was emerging from the many studies carried out over the past decade (e.g., Murray and Maes (1986)). Previous studies, however, did contribute information useful to the data assembly and storm selection tasks.

This study included an extensive literature review, assembly of historical meteorological data and offshore data including wave measurements and sea-ice data, storm identification and selection process, selection of an appropriate spectral ocean wave model which includes shallow water effects, and adaption and validation of the hindcast methodology and finally, the hindcast production and the extremal analysis. The study area covers the Canadian Beaufort region extending from 120° W to 162° W longitude and from 76° latitude to the shoreline to the south (Figure 1).

2. DATA BASE ASSEMBLY AND STORM SELECTION

The data base assembly was intended to be comprehensive. In addition to data contributed in previous studies, the data assembly tapped into raw data sources in so far as possible, including the archives of the Atmospheric Environment Service (AES), the U.S. National Oceanic and Atmospheric Administration (NOAA), National Climatic Data Centre (NCDC), the Marine Environmental Data Service (MEDS), and the offshore industry. The data base assembled includes microfilm series of weather maps prepared in real-time at the AES Beaufort Weather Office and NOAA’s National Meteorological Centre (NMC), digital files of surface observations from land stations, transient ships, and offshore drilling rigs, and wave observations from MEDS buoys moored near exploratory rigs.

Severe storm selection for hindcasting extreme wave criteria is an important though inherently subjective, step in determining extreme wave climate. The storm selection work was designed to identify historical storms based upon their ability to generate high sea-states within the study area. Thus, while a number of storms which may be high ranked for their ability to generate strong ocean currents and cause significant erosion of artificial islands are included in the storm selection, the hindcast population does not necessarily include
the top-ranked members of the population of "erosion" storms. This is the subject of a separate study which is currently underway.

The first step in the process was to identify all storms which occurred in the potential ice-free part of the year (June 15 – Nov 15), between 1957 – 1988. The first pass through all of the data sources noted above provided a Master Candidate List (MCL) of 1,087 events. The MCL was distilled in stages to a final list of 50 storm candidates from which the final population of 30 top-ranked storms was selected (Table 1). The distillation process used both objective storm intensity and ranking procedures, and subjective assessments made by experienced synoptic meteorologists.

Table 1. Final Top 30 Severe Storms

<table>
<thead>
<tr>
<th>Number</th>
<th>Start Date</th>
<th>End Date</th>
</tr>
</thead>
<tbody>
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<td>70 09 13 00</td>
<td>70 09 16 12</td>
</tr>
<tr>
<td>2</td>
<td>75 08 09 00</td>
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The presence of ice complicated the storm selection, since it is not known whether during the warm season, the storm climatology and ice cover climatology of the basin are coupled. The location of the ice edge relative to long term normals was evaluated in the 50 storms selected, and the ice edge was found to lie offshore of the mean ice edge. This could be attributed to the fact that measured wave heights influenced the storm selection. To better account for the variability and uncertainty of extremes associated with ice edge effects, it was decided to hindcast each storm with four different ice-edge specifications, taking in each instance the 5/10 concentration as the limiting boundary for wave generation and propagation purposes. The four ice edge specifications were: (1) the actual ice edge during the
storm, taken as fixed during the whole event; (2) climatological ice edges for three probability levels: 98%, 50% and 30% occurrences. Actual ice edges were produced from careful analysis of the AES daily and weekly ice charts, whereas climatological ice edges were taken from the semi-monthly charts also produced at AES. Separate extremal analyses were carried out for each population of hindcasts, and for the combined probabilistic ice edge hindcasts.

3. WIND AND WAVE HINDCAST

The wind and wave hindcast methodology adapted to the basin has already undergone substantial refinement and validation in previous studies of this type, including several studies in Arctic basins, including the Chukchi Sea and U.S. Beaufort Sea. The wind field analysis procedure has also been applied recently in several Canadian Beaufort studies (e.g., MacLaren Plansearch Limited, 1987 and 1989). The specification of wind fields includes a complete reanalysis of the evolution of the surface pressure field, starting with the best archived maps assembled, and adding additional ship and offshore rig data which may not have been available in real time. Wind fields were calculated from the pressure fields using a proven marine planetary boundary layer model (MPB4 Cardone (1969,1978). The domain of the analysis is 68°-76N, 120°-162°W on a grid of points spaced 1 degree latitude by 3 degrees longitude. In areas where direct wind observations reveal deficiencies in the MPBL winds, kinematic analysis is carried out the resulting streamline and isotach analyses are hand-gridded and the kinematic winds then supersede the MPBL winds.

The wave hindcast model adapted for this study is a special version of the ODGP which includes shallow water formulation. This model is a so-called fully-discrete spectral wave model. That is, the wave spectrum is resolved in discrete frequency-direction bins, a grid of points is laid out to represent the basin of interest and a solution is obtained based upon integration of the spectral energy balance equation, a process which successively simulates, at each model grid point and for each time step, the physical processes of wave growth and dissipation (through the source term of the energy balance) and wave propagation.

The ODGP wave model was adapted in this problem on a very high resolution grid system (37.3 km resolution) covering the domain shown in Figure 1. The spectrum is resolved into 24 direction bands (15 degrees band width) and 15 frequencies. The model algorithm is described in MacLaren Plansearch Limited (1992).

4. EVALUATION/VALIDATION OF MODEL PREDICTIONS

The accuracy of model hindcasts in the Beaufort Sea was verified through comparisons of modelled and measured data at all measuring
locations in the study area. Assessed parameters included wind speed and direction, significant wave height ($H_s$) and peak wave period ($T_p$), throughout storm histories and at the storm peaks.

While the model has been validated in several previous studies carried out in Canada, including studies associated with the East Coast and West Coast extreme wave climate studies (e.g., Eid et al. (1989), Canadian Climate Centre, (1991), MacLaren Plansearch Limited and Oceanweather Inc.,(1992)), this study included a substantial validation of the wave hindcasts in the Canadian Beaufort. The validation involved hindcasting a number of storms of the types which characterize the selected storm population, and comparing hindcast and measured sea states at several sites located in different water depths, in each event.

The validation showed that when wind fields verify well against measured winds at offshore sites, and the ice edge location is well known and sharply defined, the wave hindcasts verify well. Comparisons of measured and hindcast time histories indicate hindcast errors of 24% in significant wave height ($H_s$) and 25% in spectral peak period ($T_p$) or root mean square error (RMSE) of 0.44m and 2s, respectively. The statistical and time series comparisons show a high degree of agreement between the measured and hindcast wave parameters.

For external analysis, however, the most important aspect of the model is its ability to predict the storm peak accurately. Therefore, the peak to peak comparisons are considered to be of significant importance for evaluating model predictions. Comparisons of hindcast and measured storm peaks at evaluation sites, yield an average bias (mean difference) of $-0.06m$ in $H_s$ and $+0.24s$ in $T_p$, and RMS differences of 0.38m in $H_s$ and 1.2s in $T_p$ with scatter indices of 14.7% and 15.5% in $H_s$ and $T_p$ respectively. These results, taken together with skillful time history comparisons, compare favourably with those exhibited in other recent comprehensive hindcast studies carried out in mid-latitude regions.

5. HINDCAST PRODUCTION AND EXTREMAL ANALYSIS

The production phase of the study included the hindcast of the top-ranked 30 storms, which for four perturbations of ice edge, required 120 separate runs. Time histories of wind fields and selected integrated properties of the wave spectrum were archived at all model grid points for each run. At a subset of 51 grid points, distributed mainly over the parts of the Canadian Beaufort of interest to offshore hydrocarbon exploration operations (Figure 2), more detailed model results were archived, including all of the integrated properties as well as the full directional wave spectrum.

The extremal analysis was carried out at each of the 51 points on a site-specific basis of hindcast peaks-over-threshold (POT). That is,
no site-averaging or smoothing of extremes was deemed necessary given the fairly smooth spatial distribution of hindcast storm peaks, which itself is believed to be due to the scale of forcing wind field and the regularity of the bottom topography. At each point, five separate populations of storm peaks were subjected to the analysis, one for each of the four ice edge treatments, and one which combined the populations of the hindcasts for the three climatological ice edge specifications, the latter serving to approximate the true extremal wave distribution under the assumption that the storm climatology and the ice cover climatology are independent.

While resolution of the extremes into directional sectors was investigated, it was deemed that only omnidirectional extremes could be reliably estimated. Prior to the site-specific analysis, peaks of maximum individual wave height ($H_M$) and crest height ($H_C$) were calculated for each storm at each point using well known statistical distributions, which operate on the entire time history of sea state at a site in a storm. These results were used to estimate the effective ratios of $H_M/H_S$ and $H_C/H_S$ at each point, to be applied later to extrapolated $H_S$.

The extrapolation of hindcast peak $H_S$ and maximum wind speed ($W_M$) for each subpopulation of peaks at each point was based upon the GUMBEL distribution, using the method of moments (MOM) to fit the distribution and varying the threshold of admittance of storm peaks until the fit was achieved which maximized the correlation coefficient of the best-fit regression line. Sensitivity analysis on the effect of the distribution (the Borgman versus GUMBEL distribution was also tried) and the fitting method (i.e., maximum likelihood versus method of moment) were carried out before the final scheme was adopted. The sensitivity of the final extremes, however, to distribution, fitting scheme and threshold were small in general.

6. RESULTS

The results of the extremal analysis constitute the principal study product. At 51 points which are shown in Figure 2, we provided estimates of extreme $H_S$ (best fit and upper 90% confidence level) for return periods between 2 and 100 years, and for each ice edge dependent subpopulation, i.e., real ice edge, 98% probability, 50% probability and 30% probability, and all three climatological ice edges combined. Estimates of the corresponding extreme wind speed for some return periods were also given.

The results are graphically presented in Figure 3 (for 100 year significant wave height), Figure 4 (100 year maximum individual wave height, $H_{max}$ and Figure 5 (for the corresponding 100 year wind speed). Summary of the results are also given in Table 2. The table
shows estimated design significant wave height for 5, 10, 25, 50, and 100 year return periods, for the case of joint probability ice edge.

For the population of hindcast peaks using the actual ice-edge, extreme 100-year $H_s$ varied from about 2m at the shallowest depths modelled (about 7.5m depth) to 5.7m in deep water. These extremes turned out to be slightly higher than the extremes derived from hindcasts made with the median (50%) ice edge. As expected, extremes derived from the hindcast peaks with the 98% ice edge (which represents maximum open water) were higher than those based on actual ice edge, ranging between 5% and 60% with an average of 20% higher, or as great as 2.25m in $H_s$. The results of the joint ice edge probability analysis provided extremes lower than those obtained using 98% ice edge and higher than those using the real ice edge. These results (i.e. from ice edge probability analysis) are the recommended extremes for design, i.e. the 100 year extreme $H_s$ of 6.2+/- 0.8m for 90% confidence limits. Comparison of these new results with existing estimates indicate that our extremes are at the lower end of the wide range of extremes provided by previous studies.

7. REFERENCES


Report submitted to Environmental Studies Research Funds (ESRF), Calgary, Alberta.


8. ACKNOWLEDGEMENTS

This study was funded by Environmental Studies Research Funds (ESRF). Additional funding was provided by Canada Oil, Gas Lands Administration, NEB on behalf of Panel on Energy Research and Development (PERD). The study review committee consists of Mr. Val Swail, AES; Ken Sato, NEB; and Mr. Brian Wright, Gulf Canada Resources Limited. The authors acknowledge with gratitude the contributions from the above organizations and individuals.
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JOINT PROBABILITY (98% + 50% + 30% ICE EDGE)

Latitude (degrees N)

Longitude (degrees W)
BEAUFORT SEA 100 YR MAXIMUM WAVE HEIGHT (m)

JOINT PROBABILITY (98% + 50% + 30% ICE EDGE)

Figure 4.
Figure 5. Estimated 100-year wind speed corresponding to 100-year wave height.
DEVELOPMENT OF WIND AND WAVE CLIMATE ATLAS FOR
THE EAST COAST OF CANADA AND THE GREAT LAKES

Bassem M. Eid, Cindy Morton, and Ewa Dunlap

MacLaren Plansearch (1991) Limited
A Division of SNC\Lavalin Inc.
Halifax, Nova Scotia

and

Thomas Pierce
Transportation Development Centre
Transport Canada
Montreal, Quebec

1.0 INTRODUCTION

Offshore exploration and shipping activities are affected by the climate conditions in the area. With an accurate description of the operating environment, ships and other marine structures can operate safely. This study was initiated by Transportation Development Centre on behalf of the Canadian Coast Guard, Transport Canada to develop a wind and wave Climate Atlas for the East Coast of Canada including the Gulf of St. Lawrence and the Great Lakes.

The study objective is to collect, consolidate and summarize, in a useable form, existing wind and wave data for use by naval architects, ship owners, offshore marine operators, and classification societies when assessing strength and operational requirements of vessels and other marine structures for operation in Canadian waters.

It is a well known fact that the quality and the usefulness of the present Atlas are directly related to the quality of the data put into it. There have been a number of similar climate atlases for the study areas. However, the main difference between these atlases and the present one ties in the careful selection of the most suitable, high quality data utilized (particularly wave data including wave spectral information), which are required for the design and operations of ships and other marine structures in the study areas.

The design and evaluation of ships to ensure their structural integrity and tolerable dynamic response require a reliable description of the sea-state climatology for the intended areas of operation and a complete description of the vessel response characteristics. The current trend towards utilization of a dynamic and probabilistic approach in lieu of the statistical deterministic approach has led to increasing requirements for a comprehensive and more reliable data base which contains a full description of the
sea-state, including directional wave spectra and long-term variations of spectral shape, joint probability distribution of wave height and period, reliable design extreme parameters, etc.

The main thrust of this study is to provide the above information in a form of an Atlas suitable as a tool to the naval architects and engineering design, as well as to operators and regulatory bodies to assess the suitability and safety of a vessel when operating in the study area.

A review of user requirements from classification societies, regulatory organizations, ship owners, offshore operators, and research institutions was carried out to identify the type of information to be included in this atlas and its presentation format. The review included a survey of key user groups, design considerations and techniques (e.g. deterministic, probabilistic), and applications. Various data uses were outlined for areas of design, operation and seakeeping.

The review of design requirements, classification society requirements, current practices and future trends in the structural design, and data source identification has revealed the need for a comprehensive, more reliable data base.

Data sources, both measured and hindcast, were identified and their quality and coverage were assessed for each of the three regions. Data gaps were identified in the measured or observed data. Possible sources to fill the gaps were examined. Data from the selected sources were then acquired and checked for quality, quantity, and spatial and temporal coverages. Additional model hindcasts were carried out to complete the data sets used in the development of the Atlas.

2.0 STUDY AREA

The study area covered in the Atlas consists of three regions:

1. The East Coast of Canada extending from the Bay of Fundy through Gulf of Maine, George’s Bank, Scotian Shelf to the Grand Banks of Newfoundland and Labrador Sea (within the 200 miles offshore limits);

2. The Gulf of St. Lawrence; and

3. The Great Lakes.

The above regions were divided into a number of sub-areas (or sites). The selection of the sub-areas was affected by such parameters as: the size, climatologically similar conditions, shipping routes, fishing grounds, and data coverage. The East Coast region was divided into seventeen sub-areas comprising the Bay of Fundy, Gulf of Maine, George’s Banks, Scotian Shelf, The Grand Banks, and Labrador Sea
The Gulf of St. Lawrence was divided into five sub-areas including Anticosti, North and South Magdalen Islands, Belle Isle, and Cabot Strait (Figure 2). Eight areas were chosen to represent the Great Lakes which include three in Lake Superior, two in Lake Huron, and one in each of Lakes Ontario, Erie and Michigan (Figure 3).

3.0 DATA BASE ASSEMBLY

An extensive data base was compiled from all relevant data sources. Historical period considered in this study extends from 1957 to 1989. The data sources from earlier years are much less extensive and less accurate. The data sources reviewed include various marine observation or measured data archived at Atmospheric Environment Service (AES), Marine Environmental Data Service (MEDS), and the U.S. National Oceanic and Atmospheric Administration (NOAA). In addition, various hindcast studies were reviewed for possible use to fill the data gaps. Only good quality data are to be used in the production of the Atlas, with high weights given to measured and hindcast data from well calibrated and tested models.

Examination of available measured data has shown that, although the list of measured data sources is quite lengthy, there is a severe limitation in data coverage both spatially and temporally in many sub-areas within the study area. The review of previous hindcast studies in the study areas have shown that these are sufficient gaps in measured wave data to warrant the use of hindcast models. In spite of uncertainties of hindcasting methods, the immediate need for wave data within the study area makes an approach via hindcasting is the only viable alternative.

The data sources used in the present atlas are presented below.

3.1 Data Sources – East Coast of Canada/Gulf of St. Lawrence

The data sets used to produce the climatological statistics presented herein were:

- "ship of-opportunity" wind data from COADS (Comprehensive Ocean-Atmospheric Data Set) and real-time buoy and rig data sets archived at Atmospheric Environment Service (AES), Downsview, Ontario, covering a time period from 1957-1988.

- Waverider buoy and the U.S. National Oceanic and Atmospheric Administration (NOAA) wave buoy data archived at Marine Environmental Data Service (MEDS), Ottawa, covering a time period from 1970-1989. Measured directional wave data is very scarce or non-existent in the study areas.

- Three-year ODGP wind/wave model hindcast database, covering a time period from 1983-1986. This continuous three-year database
provides a reliable description of the full directional wave spectra for the East Coast and the Gulf of St. Lawrence. The model accuracy was evaluated in several studies and under different environmental conditions (e.g., Eid et al. (1989)). The model is a deep water, discrete directional spectral wave model.

- Extreme storm hindcasts using the ODGP model. The top 30 storms in each area were used to estimate the design values for a given recurrence interval or return period (e.g., 100 year or probability of exceedance of 0.01). See Canadian Climate Centre (1991) and Swail et al. (1989).

For each sub-area, the maximum wind and wave values during three-hour time intervals were selected from each data set. The “ship-of-opportunity” wind data were used for providing wind statistics, and the waverider/NOAA buoy wave data were used for providing wave statistics. The three-year ODGP hindcast data were used when data from the other two data sets were insufficient. Also for duration statistics, (e.g., persistence analysis) a continuous time series of the parameters considered is needed. The ODGP data set was used for this purpose and also for providing wave direction statistics.

### 3.2 Data Source – The Great Lakes

The data set used to provide the climatological statistics presented in the Atlas was obtained from the U.S. Army Engineers Waterways Experiment Station (WES) wind and wave hindcast model; a 32 year hindcast covering a time period from 1956–1987 (Hubertz, 1989). An extensive evaluation of the WES hindcast model predictions in each lake was carried out as part of this study and model accuracy was documented in MacLaren Plansearch Limited, (1992).

A database was compiled from the WES wind and wave hindcast model. Preliminary statistical analysis was carried out on the model hindcast, at a number of representative grid points in each lake. These statistics (e.g., average, maximum, 95% upper limit, prevailing direction) and the known climate characteristics of different parts of each lake were used to chose a single WES model grid point, for each sub-area, to be used to represent the entire sub-area. The grid points, which represent more severe conditions was selected to represent the entire sub-area under consideration.

### 4.0 DATA PROCESSING AND ANALYSIS

#### 4.1 Winds

For the East Coast and Gulf of St. Lawrence, the three-hourly marine wind data were used in the analysis presented in this Atlas. It presents one-hour mean winds.
In the marine observation data set, the anemometer height of the measured wind speed varies with each ship, buoy or rig. For some ship call-signs, the anemometer height was known and was used to adjust the wind speed to a height of 20 in (65 ft.) above mean sea level (which is similar to the ODGP model wind-reference height) using the marine planetary boundary layer (MPBL) approach (Cardone, 1978). Most of the anemometer heights of the rigs and buoys were known and were used to adjust the wind to a standard 20 m level. However, not all ship anemometer heights were known. Since the average height of ship anemometers is approximately 20 m, most ship-reported wind speeds should be compatible with the adjusted wind speeds. All wind speed statistics were compiled using the “ship-of-opportunity” data except for the persistence analysis. Since the persistence analysis requires a continuous time series of data with no gaps, the “ship-of-opportunity” data were not appropriate. Therefore, the ODGP data base was used for this purpose.

For the Great Lakes, the WES winds were used directly. The WES wind filed was calculated using a number of coastal stations encompassing the lakes and the NOAA buoys present in four of the lakes. For the land stations, an over land to over water wind conversion was applied, which included correction for the difference of surface roughness, anemometer height and atmospheric stability, Resio and Vincent (1977). The wind data used in the analysis for the Great Lakes represent one-hour mean winds at 10 in above mean sea level.

### 4.2 Waves

For the East Coast and the Gulf of St. Lawrence, waverider and NOAA buoy data were used for non-directional wave spectral analysis and significant wave height and wave period statistics in areas where a sufficient number of measurements were available. The three-year ODGP hindcast data base was used for the remaining areas, and for directional statistics such as wave roses and percent frequency of occurrence by direction tables.

For the Great Lakes, the 32-year WES hindcast data, which is based on the Wave Information Study (WIS) deep water wave model developed by Resio (1989), was used. The WIS model is a discrete directional spectral model which simulated wave growth dissipation and propagation in deep water. Spectra are represented by energy in discrete bands of frequency and direction (20 frequency bands x 16 directions). The model is driven by a wind source term which is determined using the wind fields described above. A wave-wave interaction source term controls the transfer of energy across frequency bands.

### 4.3 Extreme Statistics

Extreme value statistics were calculated by using the method of moments (MOM) to fit a Gumbel distribution to peakstorm wind and wave values (i.e., peak over threshold (POT) method).
For the East Coast and Gulf of St. Lawrence, the criteria for selecting wind storms were the winds surpassing a threshold of 35 knots for a minimum duration of 12 hours and storm peaks at least 24 hours apart. From this, a preliminary list of potential storms was obtained and verified. The peak wind speed for each storm was then identified and used in the extreme analysis. The significant wave height extreme statistics were based on the hindcast of the top severe wave-generating storms in the study area using the ODGP model. See Canadian Climate Centre (1991) and Swail et al. (1989).

For the Great Lakes, the criteria for selecting wind and wave storms were: wind speeds, significant wave heights surpassing a threshold of 30 knots, 2.5 metres, respectively, for a minimum duration of 24 hours. From this, preliminary list of potential storms was obtained and verified. The peak wind speed or wave height for each storm was then identified and used in the extreme analysis.

The top severe storms in each area were used to estimate the design values for a given recurrence interval or return period (e.g. 100-year return period or probability of exceedance of 0.01).

4.4 Sea Ice Treatment

The presence of sea ice in the Labrador Sea and the Gulf of St. Lawrence during January through May affects the wave statistics. This is noted on the affected items in the Atlas. It should be noted that the actual ice edge was incorporated into the ODGP model hindcasts using 5/10 concentration to represent the ice edge.

For the Great Lakes, the WES wave hindcast assumed a median ice coverage for the period mid-December through mid-April. The median ice cover was defined by ice concentration of 5/10 or greater, from 20 years of ice observation (1960-1979).

4.5 Spectral Family

Many marine engineering applications require knowledge of the shape of the wave spectrum. The analysis used in this atlas is based on the six-parameter model of Ochi and Hubble (1976). To generate the representative spectrum for a given sea-state value, the model function was fitted to the measured or hindcast data. Statistical analysis of the fitted parameters leads to a family of spectra, i.e., most probable spectrum and a set of 95% confidence spectra. Six sea-state classes were considered: $H_s=0.5-2$ m, 2-3 m, 4-5 m, 5-6 m, and greater than 6 m.

5.0 RESULTS

Based on the optimum requirements for deterministic or stochastic design analyses, and operational, planning and seakeeping analyses,
the data format and presentation is produced for the present Atlas, while keeping in mind future requirements. In summary, the following statistics were included in the Atlas (in a form of monthly or annual statistics):

- wind/wave roses;
- wind speed/wave height exceedance diagrams;
- wind speed/wave height/wave period frequency of occurrence diagrams;
- wave height vs. peak period and wind speed vs. wave height scatter diagrams;
- wind speed/wave height and period extreme analysis (return period) curves;
- wind speed/wave height/wave period persistence diagrams;
- wave spectrum;
- statistical summary tables of wind speed and direction, and wave height, period and direction.


6.0 RECOMMENDATION

Ultimately, the data base produced in this study should be put in an online “electronic atlas” which can be accessed from remote terminals. This arrangement will allow continuous update of the data base should future data, both measured and hindcast, become available.

In addition, the electronic atlas should have the capabilities of including design formulae for strength analysis, ship performance, and response in different environmental conditions.

7.0 REFERENCES


8.0 ACKNOWLEDGEMENTS

This study was funded by Transportation Development Centre, Transport Canada and Panel for Energy Research and Development (PERD). The authors wish to acknowledge with gratitude the technical support and contributions from Mr. Val Swail, AES, and Dr. Ron Wilson, MEDS, and also for providing access to their computer facilities and marine data bases.

Special thanks to the Canadian Wave Committee members and Lloyd’s Register of Shipping for their review of this study.
1. Bay of Fundy
2. Lurcher/Brown's Bank
3. Georges Bank
4. West Scotian Slope
5. Nova Scotian Shore
6. Sable
7. East Scotian Slope
8. Banquereau/Laurentian Fan
9. South West Coast
10. South East Coast
11. South Western Grand Banks
12. South Eastern Grand Banks
13. Northern Grand Banks
14. North East Coast
15. Belle Isle/Funk Island Banks
16. Labrador Coast
17. Labrador Sea

Figure 1. East Coast Study Sub-Areas
Figure 2. Gulf of St. Lawrence Study Sub-Areas

Figure 3. The Great Lakes Study Sub-Areas
### ANNUAL WIND STATISTICS
#### EAST COAST AREA 6 – SABLE

#### Frequency of Wind Speed by Direction (from)

#### Annual Wind Speed Statistics

#### Frequency of Occurrence by Direction

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<td>30.0 - &lt;35.0 kts</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.9</td>
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<td>7.1</td>
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<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
<td>1.0</td>
<td>0.6</td>
<td>4.0</td>
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<tr>
<td>40.0 - &lt;45.0 kts</td>
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<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
<td>0.3</td>
<td>2.0</td>
<td>887</td>
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<tr>
<td>45.0 - &lt;50.0 kts</td>
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<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.8</td>
<td>323</td>
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<tr>
<td>50.0 - &lt;55.0 kts</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.3</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>55.0+ kts</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.3</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11.8</td>
<td>7.4</td>
<td>8.7</td>
<td>7.3</td>
<td>12.1</td>
<td>18.6</td>
<td>22.8</td>
<td>125.8</td>
<td>1000</td>
<td>45044</td>
</tr>
</tbody>
</table>

#### Monthly Data Statistics

| Month | Mean | Std | Min | Max | | Lower | Upper | Obs |
|-------|------|-----|-----|-----| |       |       |     |
| Jan   | 22.8 | 10.0| 22.0| 85.0| | 0.0  | 42.3  | 6.0  | 3465  |
| Feb   | 22.4 | 10.8| 21.6| 86.4| | 0.0  | 40.9  | 6.0  | 3512  |
| Mar   | 21.8 | 10.1| 21.0| 92.4| | 0.0  | 40.1  | 5.0  | 3451  |
| Apr   | 19.8 | 8.8  | 18.8| 76.0| | 0.0  | 37.0  | 5.0  | 3448  |
| May   | 19.8 | 8.7  | 18.8| 76.0| | 0.0  | 36.0  | 5.0  | 3940  |
| Jun   | 19.8 | 8.8  | 18.8| 78.0| | 0.0  | 35.0  | 4.0  | 3940  |
| Jul   | 19.8 | 8.8  | 18.8| 78.0| | 0.0  | 34.0  | 3.0  | 3940  |
| Aug   | 19.8 | 8.8  | 18.8| 78.0| | 0.0  | 33.0  | 3.0  | 3940  |
| Sept  | 19.8 | 8.8  | 18.8| 78.0| | 0.0  | 32.0  | 2.0  | 3940  |
| Oct   | 19.8 | 8.8  | 18.8| 78.0| | 0.0  | 31.0  | 2.0  | 3940  |
| Nov   | 19.8 | 8.8  | 18.8| 78.0| | 0.0  | 30.0  | 0.0  | 3940  |
| Dec   | 19.8 | 8.8  | 18.8| 78.0| | 0.0  | 29.0  | 0.0  | 3940  |
| Total | 19.8 | 8.9  | 18.9| 90.4| | 0.0  | 27.0  | 5.0  | 45155 |

---

**Figure 4. Examples of the Atlas’ Products**

- **Per Cent Occurrence**
  - X-axis: Wind Speed (kts)
  - Y-axis: Percent Occurrence

- **Probability of Exceeding**
  - X-axis: Wind Speed (kts)
  - Y-axis: Probability of Exceeding

- **Annual Wind Speed Statistics**
  - Table showing frequency of occurrence by direction

- **Monthly Data Statistics**
  - Table showing monthly data statistics with mean, standard deviation, minimum, maximum, and more.
Figure 4. Examples of the Atlas’ Products (Cont’d)
### Directory
- Table of Contents
- List of Tables
- Figures

---

**MOST PROBABLE SPECTRA**

EAST COAST AREA 6 – SABLE

**Waverider**

---

**WAVE SPECTRAL COEFFICIENTS**

EAST COAST AREA 6 – SABLE

**FITTED PARAMETERS FOR THE REPRESENTATIVE WAVE SPECTRUM**

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>$a_1$ (rad/s)</th>
<th>$b_1$ (m)</th>
<th>$c_1$</th>
<th>$a_2$ (rad/s)</th>
<th>$b_2$ (m)</th>
<th>$c_2$</th>
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<tbody>
<tr>
<td>0.5 ≤ $H_s$ &lt; 2 m</td>
<td>0.7271</td>
<td>0.8634</td>
<td>1.3692</td>
<td>1.2087</td>
<td>0.7605</td>
<td>1.6085</td>
</tr>
<tr>
<td>2 ≤ $H_s$ &lt; 3 m</td>
<td>0.8034</td>
<td>2.0686</td>
<td>1.8426</td>
<td>1.1708</td>
<td>1.1787</td>
<td>1.7431</td>
</tr>
<tr>
<td>3 ≤ $H_s$ &lt; 4 m</td>
<td>0.8237</td>
<td>3.0742</td>
<td>1.5525</td>
<td>1.1125</td>
<td>1.4711</td>
<td>1.7755</td>
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<tr>
<td>4 ≤ $H_s$ &lt; 5 m</td>
<td>0.5740</td>
<td>2.9924</td>
<td>1.4639</td>
<td>1.0699</td>
<td>1.8739</td>
<td>1.7428</td>
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<tr>
<td>5 ≤ $H_s$ &lt; 6 m</td>
<td>0.5415</td>
<td>4.8687</td>
<td>1.3052</td>
<td>1.0683</td>
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<tr>
<td>$H_s$ ≥ 6 m</td>
<td>0.5176</td>
<td>6.0960</td>
<td>1.4324</td>
<td>1.0631</td>
<td>1.9278</td>
<td>2.0986</td>
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**SPECTRAL COEFFICIENTS AS A FUNCTION OF SIGNIFICANT WAVE HEIGHT $H_s$ FOR MOST PROBABLE SPECTRUM (NO. 1) AND TWELVE 95% CONFIDENCE SPECTRA (NO. 2 THROUGH 13)**

- $a_1 = A_1 \exp(A_2H_s)$
- $b_1 = B_1 + B_2H_s + B_3H_s^2$
- $c_1 = C_1 \exp(C_2H_s)$

<table>
<thead>
<tr>
<th>NO.</th>
<th>$A_{11}$</th>
<th>$A_{12}$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$A_{21}$</th>
<th>$A_{22}$</th>
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<th>$B_{22}$</th>
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<th>$C_{21}$</th>
<th>$C_{22}$</th>
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<tbody>
<tr>
<td>1</td>
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<td>-0.20</td>
<td>0.93</td>
<td>0.01</td>
<td>1.50</td>
<td>-0.004</td>
<td>1.23</td>
<td>-0.027</td>
<td>0.23</td>
<td>0.48</td>
<td>-0.03</td>
<td>1.71</td>
<td>0.025</td>
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<tr>
<td>2</td>
<td>0.48 -0.023</td>
<td>-0.28</td>
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<td>0.01</td>
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<td>0.25</td>
<td>0.68</td>
<td>0.00</td>
<td>0.95</td>
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<td>3</td>
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<td>0.038</td>
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<td>1.88</td>
<td>-0.059</td>
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<td>0.64</td>
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Figure 4. Examples of the Atlas’ Products (Cont’d)
COMPARISON OF WIND SPEEDS RETRIEVED FROM 
SSM/I DATA WITH HF RADAR OBSERVATIONS

1I.G. Rubinstein, 2R. Khan and 2J. Walsh

1Institute for Space and Terrestrial Science  
North York, Ontario

2C-CORE  
Memorial University of Newfoundland  
St. John’s, Newfoundland

ABSTRACT

The multi-spectral measurements of microwave emission from the ocean surface can be translated into surface wind speeds. The increase in the emission can be correlated with the wind-induced surface roughness (capillary waves) and whitecaps coverage. The retrieval algorithms were developed for Special Scanning Microwave Imager (SSM/I) observations at 19 and 37 GHz. The attenuating effects of the atmosphere at these wavelengths were taken into consideration in this retrieval procedure. The expected accuracy of the retrievals was calculated to be less than 2.5 m/s, for a non-precipitating atmosphere. The effects of the wind stress on the ocean surface can also be observed with microwave sensors at much longer (3 to 30 MHz radar) wavelengths. High frequency radar waves interact selectively with ocean gravity waves. This mechanism has been extensively studied at Memorial University and algorithms were developed to extract ocean wave information. The use of the Pierson-Moskovitz model allows the extraction of wind speed information. Surface wind speeds inferred from these two sensors were compared for several satellite passes. A ground wave radar system operating at 6.75 MHz from Cape Race provided the HF radar data. This comparison study was initiated in order to gain more understanding of the air-sea interactive processes. The results look very promising.
Comparision of Measured Directional Wave Spectra
in the ERS-1 CAL/VAL Experiment

J. R. Buckley†, M. Allingham‡, F. W. Dobson‡ and P. W. Vachon*

†Department of Physics,
Royal Roads Military College
FMO Victoria, BC, V0S 1B0

‡Bedford Institute of Oceanography
PO Box 1006, Dartmouth, NS, B2Y 1A2

*Canada Centre for Remote Sensing
588 Booth St., Ottawa, ON, K1A 0E4

1 The ERS-1 Wave Spectral Calibration/Validation Experiment

The ERS-1 Calibration-Validation Experiment was carried out on the Grand Banks of Newfoundland from Nov 10-27 1991. It was organized primarily to provide in situ, aircraft and numerical forecast model validation of the Synthetic Aperture Radar (SAR) on the "ERS-1" satellite launched by the European Space Agency in July 1991. Other goals included the open-ocean determination of the relation between the wind stress and the sea state, validation of the algorithms used to invert SAR images and assimilate them in numerical wave models, testing of the wave-imaging capabilities of shipboard marine radars, intercalibration of the meteorological sensors on buoys and ships, and the relation of SAR image features to near-surface currents in the ocean.

To achieve these goals a sharply focused cooperative experiment was organized and carried out. The ERS-1 "Commissioning phase" orbit produced a "crossover node" on the Grand Banks of SAR swaths from descending and ascending passes within 11 hours of each other every 3 days. This crossover node formed the principal validation site for the experiment. At this site, surface measurements were made from an array of two ships: CSS Hudson and the Soviet RV Georgi Ushakov, four meteorological buoys and three wave buoys (two directional, one wave height only), all of which were deployed at the grid points of the AES "CSOWM" operational wave forecast model located in the node. The validation site was overflown at ERS-1 overpass times by two aircraft: the CCRS Convair-580 with C-band SAR and the NASA P-3 with Radar Ocean Wave Spectrometer or Surface Contour Radar and Radar Altimeter. The crossover node also lay within the swath of a high-frequency radar at Cape Race, which provided winds and waves at overpass times on a 1km grid.

On Hudson were a bow-mounted wind stress measurement system and an acoustic doppler current profiler (ADCP) operated by the Bedford
Institute of Oceanography (BIO) and two X-band marine radar systems dedicated to wave measurement, one of which was a prototype operational unit operated by McLaren-Plansearch Ltd, and the other a research unit from Royal Roads Military College (RRMC). On the Ushakov, at the site from Nov 19-21, were a radiosonde system and standard meteorological sensors. Figure 1 shows the experimental region and the locations of the measurement systems.

Wave conditions throughout the experiment (Figure 2) ranged from almost glassy calm on November 11 to over 5m significant wave height November 19, about standard for the region at this time of year. In this paper we describe three different directional wave measurement systems: the ERS-1 SAR, the RRMC marine radar and the MEDS Wavec buoy, N46 and then compare them under different wave conditions.

2 Experimental Equipment and Wave Spectral Analysis Techniques

2.1 ERS-1 Synthetic Aperture Radar

The ERS-1 C-band SAR system has been described in detail by Attema (1991). For this experiment SAR imagery were collected and processed to standard image products (each sixlook image scene was comprised of 8000 x 8000 pixels with 12.5m spacing in azimuth and ground range) at the Gatineau receiving station during our field program. SAR image spectra were calculated for each overpass based upon a 1024 x 1024 pixel subscene chosen in the vicinity of the Hudson. The subscene was further broken down into nine overlapping 512 x 512 pixel regions which were detrended, windowed and 2-D Fourier transformed. The periodograms from the nine were averaged to create a preliminary spectrum. Subsequently, the spectrum transfer function and speckle bias were estimated and removed and the spectrum was high pass filtered, smoothed and decimated to a regular grid representing roughly twice the Nyquist wavelength of the original data.
2.2 RRMC Marine Radar

The RRMC X-band radar system combined a standard Racal-Decca BT-362 25KW ship's navigation radar with an Integrad Technologies RSC-20 20MHz radar video digitizer and scan converter. This eight-bit digitizer was attached to the output of the logarithmic IF stage of the radar receiver where it was able to acquire data before the radar processor began preparing the signal for optimized navigational display. The output of the digitizer was therefore directly proportional to the actual strength of the reflected radar signal. The digitizer sampled 1024 bins along each of 1024 radials for a single radar sweep. The data were converted in real-time onto a 512 x 512 pixel grid and stored on a SCSI streamer tape for transfer to an archiving computer system. Capture and transfer of each image took 7 seconds.
Images were collected in groups of 16 every half hour during the three hours around an ERS-1 overpass time, and on other occasions as wave conditions or other operations warranted. In all, about 3300 images were collected and stored for further processing.

Preliminary directional spectral estimates were formed from each group of 16 images by the following methodology: the Slow Time Constant (STC) curve (the radial dependence of reflected radar amplitude) was removed empirically from each image by calculating a best fit $r^{-x}$ curve and subtracting it from the data, seven regions were selected, each 64 pixels square spaced evenly around the radar location at equal radial distances from the site, a least-squares plane was removed from each region. Regions at identical locations were processed in each of the 16 images in a set. Data in each region were cosine tapered and a 3-D Fourier transform of each region was calculated. The third dimension, gained from the time sequence of images, assisted in resolving the 180 degree directional ambiguity inherent in a single 2-D transform, as described in Young et al. (1985). The temporal sampling was considerably complicated by the interaction of the radar sweep, which renewed the display every 2.5 seconds, and the writing of each sample to tape, which took 7 seconds. Thus, the sample interval was sometimes two radar sweeps, and sometimes three, yielding an indeterminate Nyquist period in the 10 to 15 second range. This technique only truly resolved the longest period waves. For wave periods much shorter than the Nyquist period, use of the dispersion
relation allowed selection of the correct frequency for the waves and hence determination of the correct direction of propagation. For waves of intermediate period however, results of both techniques are inconclusive, and the direction of these waves cannot be unambiguously determined. Spectra from all seven regions were averaged together to minimize the effect of the azimuthal dependence of the modulation transfer function for the reflected microwave energy.

2.3 Directional Wave Buoys

The Marine Environmental Data Service (MEDS) Datawell Wavec surface following buoy was located at CSOWM grid point 2519. It recorded internally 4096 half-second samples of heave, pitch and roll once per hour. The resulting time-waves were Fourier transformed and cross-spectra of the three series were calculated by standard means. Spectra of wave energy by frequency and direction were calculated using an Iterated Maximum Likelihood Method technique.

3 Sample Cases

For the four cases described here, the raw radar image is displayed along with the resulting radar spectrum, the spectrum from the MEDS Wavec buoy and the spectrum from the ERS-1 SAR. In all the spectral plots, the data have been normalized, and contours drawn at intervals of 10% of the plot maximum value. Wave direction on the plots is direction to, in the oceanographic tradition. Spectra from the two radar instruments are shown as wavenumber spectra, and that from the buoy as frequency spectra. The scales of the axes of the plots are chosen so that the location of a 10 second wave (f=0.10Hz, k=0.04 rad m\(^{-1}\)) appears to be the same on both frequency and wavenumber spectra.

3.1 November 14, 1200Z

This overpass of ERS-1 found a very strong southward propagating swell. This swell is evident in the radar image, and in spectra from all three sensors. The Wavec isolated a weaker southeast propagating wave peak, that is also present in the both of the radar spectra, but is less obviously separate. The Wavec also found a northward propagating wave at 10 second period which is not evident in the radar spectra. This wave may be hidden in the radar spectra by the directional ambiguity of the spectra. It may be due to a low pressure region approaching the experimental area at this time from the southwest that had a large area of southerly winds to the south of the site.
3.2 November 20, 1600Z

In these spectra there are three wave trains visible, propagating northeast, southeast and south. Visually, the southeast wave is the most obvious, and in the marine radar spectrum, this wave is the strongest. The Wavec spectrum however shows that the northeast propagating wave contains the most energy. The SAR spectrum shows the southeast wave weakly. The northeast wave is not seen in this spectrum, since the overpass is a descending one, that is, the satellite is moving to the southeast, and therefore the wave is
propagating azimuthally to the sensor. The inherent azimuth cut-off for polar orbiting SARs is known to inhibit the accurate imaging of azimuth travelling wind seas. The weather map indicates the wind-driven sea is the southeast propagating component; the northeast component is a residual from a small low to the north west, and the southerly component a swell from a weak low to the northeast.

3.3 November 22, 0100Z

At this time there was no ERS-1 overpass, however the data illustrate a wave condition not well measured by traditional directional wave

Figure 4. The same as figure 3, but for November 20, 1600Z.
buoy data processing. The sea surface radar image for this case shows two wave trains, of approximately the same wavelength travelling to the northeast with about a 25 degree separation. The radar spectrum shows two peaks at the same wavenumber separated by this angular displacement. A 3-D surface plot of the same spectrum shows the two peaks more clearly. A similar plot of the buoy spectrum shows energy at the same location, but overresolves the two peaks into several spurious ones. The weather map indicates a wind-driven sea travelling to the southeast at map time (0Z November 22) with the winds directly offshore from the Avalon Peninsula. The two separate wave trains may be wind-driven sea from the larger fetches to the northwest and west southwest.

Figure 5. November 22, 0100Z. a) Sea surface image from the RRMC radar. b) Spectrum from the RRMC radar. c) 3-D surface plot of the contour plot in b). d) 3-D plot of the spectrum from the MEDS Wavec.
3.4 November 23, 1130Z

This situation is similar to that on November 20, with three wave trains visible. The weather map shows a strong flow to the south southeast (long fetch) and east (short fetch). The spectra show similar results, with the marine radar and the Wavec finding all three waves but the SAR missing the northeast propagating one. In this case however, the three spectra agree on the relative strengths of the peaks.

Figure 6. The same as Figure 3, but for November 23, 1200Z.
4 Summary and Conclusions

The ERS-1 Calibration and Validation Experiment on the Grand Banks, Nov 10-27 1991, has produced an excellent and extensive data set covering the sea states and meteorological conditions on the Grand Banks in winter. The data set is particularly rich in radar images of the sea surface, and attached to these images is a high-quality set of calibrated in situ wave and sea surface meteorological data.

The principal characteristic of the wave spectra investigated so far is their extraordinary complexity. This will challenge the analysis and interpretation techniques we use to their limits and beyond.

The SAR and marine radar representations of the long-wavelength components of the wave field show excellent agreement. Due to the azimuthal asymmetry in the modulation transfer function for the marine radar, two dimensional spectra calculated at any particular azimuth do not reproduce the full 2-D spectrum very well. It is necessary to compute an azimuthal average of spectra to produce a meaningful result. The Wavec analysis technique (IMLM) appears to overresolve spectral peaks relative to the radar.

The marine radar and Wavec buoy spectra agree well on the wavenumber and direction of most of the peaks in energy found on a given day. In some cases the ambiguity in the radar spectra may mask real wave peaks travelling in the opposite direction. The relative energy in the various wave peaks in a marine radar spectrum is not always the same as that indicated by the wave buoy analysis.

5 Acknowledgements

We thank B. Toulany of BIO for processing the Wavec spectra and A.S. Bhogal of CCRS for doing the same with the SAR spectra. This experiment would not have been possible without the expertise of the Master, officers and crew of the CSS Hudson and the technical support from BIO and the Marine Environmental Data Service and R. Pajunen.

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6 References


ENVIRONMENTAL DATA ACQUISITION/TRANSMISSION SYSTEM

1David White, 1Savi Narayanan and 2Stuart Porter

1Department of Fisheries and Oceans
St. John’s, Newfoundland

2Atmospheric Environment Service
St. John’s, Newfoundland

ABSTRACT

The Department of Fisheries and Oceans (DFO) and Environment Canada’s Atmospheric Environment Service are co-operating in the development of a real-time Environmental Data Acquisition/Transmission System (EDATS). This system, designed for deployment on vessels of opportunity, collects, processes and transmits meteorological data automatically. Manually collected bathythermograph data is also automatically processed and sent. The data, transmitted over HF radio or satellite links, are placed in their corresponding data base. This paper provides an overview of the EDATS system.
THE HALLOWEEN STORM: DATA OBSERVATIONS FROM NDBC STATIONS

David Wei-Chi Wang and Theodore Mettlach
Computer Sciences Corporation
Stennis Space Center, Mississippi 39529-6000

ABSTRACT

In late October 1991, an extratropical storm developed off the Atlantic coast of North America. The strong, northeasterly winds from this storm persisted for two continuous days across a fetch of more than 1800 kilometers and generated high-height, long-period waves that caused widespread damage along the U.S. east coast from Maine to Florida. The National Data Buoy Center operates moored buoy and land stations located along and offshore the U.S. east coast that reported hourly waves and other marine environmental data during the storm. This study documents the storm data from 16 of these stations to provide a very extensive field observation of the storm-generated severe seas. This data set will also be valuable for the development of wave hindcast and forecast models for years to come.

1. INTRODUCTION

Northeasters, extratropical storms that occur off the northeast coast of the United States, are a major threat to marine navigation, offshore operations, beaches, and coastal structures. Although extratropical storms are generally less powerful than hurricanes with respect to wind strength, the longer duration of such storms over a longer fetch can generate waves that exceed those from hurricanes. In March 1962, the severe seas generated by the Ash Wednesday Storm pounded the U.S. east coast for 5 days and caused tremendous damage to coastal communities.

In the last 20 years, efforts have been made to develop numerical wave models for use in storm watch/waving systems that can forecast storm-generated severe seas. Extreme waves by past severe storms were also simulated by wave hindcast models to provide design criteria for offshore and coastal structures. One of the important factors in the successful development of numerical models is verification and calibration using field measurement data. But, extensive field measurements during severe seas are rare due to the difficulties in data collection. Often, even when the data were available, they came from an insufficient number of stations that could not adequately cover the entire wave field. Extensive data collection through a network of reliable automated stations is an essential element for advancing the development of wave hindcast and forecast models.

Since 1975, the National Data Buoy Center (NDBC) has expended much effort toward long-term and regular marine environmental data
acquisition. Currently, there are more than 100 stations located along the east and west coasts of the United States, in the Gulf of Mexico, and in the Great Lakes. Environmental data from these stations are sampled hourly and distributed to users in near real time. Long-term, regular data collection from the NDBC network of stations has provided essential information about severe storms. This information has been used in many studies (Wang et al. (1989), Wang and Carolan (1991), and Graber et al. (1991)).

In late October 1991, the most powerful northeaster in the last 50 years developed off the Atlantic coast of North America. Severe seas generated by the storm pounded the east coast from Nova Scotia to Florida for a period of 72 hours. The maximum reported significant wave height reached 12 meters. Widespread beach erosion, street floodings, and the destruction of several coastal structures and ocean front properties (including the summer house of the President of the United States) gained much public attention. The severity of the storm and the extremely severe seas generated by the storm present a very interesting and important case for the verification and calibration of wave hindcast and forecast models.

This paper documents the hourly wave and meteorological data collected from 16 NDBC stations located along and offshore the U.S. east coast. This data set provides a detailed and complete field observation of the storm and its impact on the ocean.

2. THE EVOLUTION OF THE STORM

Detailed descriptions of the evolution of the October 1991 storm are given by Dolan and Davis (1992) and Pusch and Avila (1992). On October 28, a cold front extended from a weak 1012-hPa low, located 300 kilometers east of Nova Scotia, southwestward to the Carolinas. There was a massive anticyclone over northern Labrador generating north winds, with speeds of 5 to 10 m/sec along the coast from Maine to the Carolinas, pushing the cold front into the Atlantic. Hurricane Grace was west of Bermuda and was moving north-northeastward. Figures 1 (a) through 1 (d) show four mean sea level pressure analyses by the U.S. National Weather Service on October 28, 29, 30, and 31, respectively.

By the next day, the Nova Scotia low had deepened from 1012 hPa to 988 hPa and had moved southeastward to near 40°N., 55°W., a position very near the axis of the Gulf Stream. The anticyclone had moved eastward across Labrador, and Hurricane Grace had merged with the cold front well north of Bermuda. A pressure gradient between the Labrador high and the west Atlantic cold front produced gale- to storm-force winds over a continuous, 1800-km fetch from Newfoundland to the Florida Straits.
By October 30, Hurricane Grace had merged with the other low at a location 750 km south of Halifax, Nova Scotia. The combined energy of the two systems produced a vigorous, 972-hPa, storm-force low with maximum sustained winds of 30 m/sec. As the Canadian high moved southeast, strong winds persisted from Nova Scotia to Florida for the second day.

After reaching maximum intensity, the low moved southwestward, then southward, and then weakened. As it moved over the warm waters of the Gulf Stream convection increased, and the system began taking on the characteristics of a subtropical cyclone. The movement of this low is shown in Figure 2. On November 1, the storm became a subtropical storm, and on November 2 it was observed by Air Force Reserve Unit aircraft to have all the characteristics of a hurricane; but, by this time, most of the coastal damage from severe seas had already occurred.

3. DATA MEASUREMENT SYSTEM

Table 1 lists information about the NDBC network in place during the storm. There were both moored buoys and automated headland stations. The headland stations are called Coastal-Marine Automated Network (C-MAN) stations. Table 1 identifies moored buoy and headland stations located along and offshore the entire stretch of the U.S. east coast, while Figure 2 is a location map of all these stations and the track of the storm.

Stations 44007, 44013, 44025, 44012, 44009, 41008, and 41009 are located nearshore along the coast, stations 44011, 44008, and 44014 are located offshore on the edge of continental shelf; and stations 41001, 41002, and 41010 are located offshore in deep water. The three C-MAN stations are located at light stations: Diamond Shoals lighthouse, North Carolina (Station DSLN7); Ambrose lighthouse, New York (Station ALSN6); and Chesapeake Bay lighthouse, Virginia (Station CHLV2).

Each station was equipped with a wave measurement system and a meteorological measurement system. Data were collected hourly and then relayed through the Geostationary Operational Environmental Satellite (GOES) to NDBC for further data processing and quality control.

The wave measurement system on the moored buoys used an accelerometer to record buoy heave motion. An NDBC onboard Wave Data Analyzer computes the wave spectral data from the time series of buoy motion. The details of the NDBC wave measurement system and data processing technique are described by Steele et al. (1990). Two stations (44014 and 44025), sponsored by the U.S. Army, Coastal Engineering Research Center (CERC), provided the directional wave data. Directional wave data are estimated from records of the buoy’s heave, pitch, and roll.
motions based on the approach proposed by Longuet-Higgins et al. (1963). The details can be found in Steele et al. (1990). Wave measurements at the three C-MAN stations were carried out by using the Infrared Laser Wave Height Sensor. The sensor is mounted on the platform in a fixed position above the ocean surface and measures the surface displacement. The details can be found in Brown and Gustavson (1990).

Table 1. NDBC station information.

<table>
<thead>
<tr>
<th>Station</th>
<th>WD</th>
<th>Lat</th>
<th>Long</th>
<th>Hull Type</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>44007</td>
<td>47</td>
<td>43°31’48”</td>
<td>70°05’24”</td>
<td>LNB09</td>
<td>Portland</td>
</tr>
<tr>
<td>44013</td>
<td>30</td>
<td>42°22’48”</td>
<td>70°46’48”</td>
<td>LNB11</td>
<td>Boston Harbor</td>
</tr>
<tr>
<td>44011</td>
<td>88</td>
<td>41°04’55”</td>
<td>66°34’49”</td>
<td>6N16</td>
<td>Georges Bank</td>
</tr>
<tr>
<td>44008</td>
<td>60</td>
<td>40°30’00”</td>
<td>69°25’39”</td>
<td>LNB07</td>
<td>Nantucket</td>
</tr>
<tr>
<td>ASLN6</td>
<td>25</td>
<td>40°27’30”</td>
<td>73°49’54”</td>
<td>C-MAN</td>
<td>Ambrose Light</td>
</tr>
<tr>
<td>44025</td>
<td>40</td>
<td>40°15’01”</td>
<td>73°10’00”</td>
<td>3D21</td>
<td>Long Island</td>
</tr>
<tr>
<td>44012</td>
<td>24</td>
<td>38°47’24”</td>
<td>74°34’48”</td>
<td>LNB01</td>
<td>Five Fathom</td>
</tr>
<tr>
<td>44009</td>
<td>28</td>
<td>38°27’00”</td>
<td>74°42’00”</td>
<td>LNB06</td>
<td>Delaware Bay</td>
</tr>
<tr>
<td>CHLV2</td>
<td>12</td>
<td>36°54’18”</td>
<td>75°42’48”</td>
<td>C-MAN</td>
<td>Chesapeake Light</td>
</tr>
<tr>
<td>44014</td>
<td>48</td>
<td>36°34’59”</td>
<td>74°50’01”</td>
<td>3D31</td>
<td>Virginia Beach</td>
</tr>
<tr>
<td>DSLN7</td>
<td>16</td>
<td>35°09’12”</td>
<td>75°17’18”</td>
<td>C-MAN</td>
<td>Diamond Shoals Light</td>
</tr>
<tr>
<td>41001</td>
<td>4206</td>
<td>34°55’30”</td>
<td>72°57’05”</td>
<td>6N07</td>
<td>E. Cape Hatteras</td>
</tr>
<tr>
<td>41002</td>
<td>3658</td>
<td>32°17’42”</td>
<td>75°14’26”</td>
<td>6N23</td>
<td>S. Cape Hatteras</td>
</tr>
<tr>
<td>41008</td>
<td>18</td>
<td>30°43’48”</td>
<td>81°04’48”</td>
<td>3D16</td>
<td>E. Jacksonville</td>
</tr>
<tr>
<td>41010</td>
<td>833</td>
<td>28°52’48”</td>
<td>78°31’59”</td>
<td>10D08</td>
<td>E. Cape Canaveral</td>
</tr>
<tr>
<td>41009</td>
<td>41</td>
<td>28°29’59”</td>
<td>80°10’48”</td>
<td>3D17</td>
<td>Cape Canaveral</td>
</tr>
</tbody>
</table>

WD: Water Depth in meters    Hull Type: 6N: 6-meter NOMAD 3D: 3-meter discus 10D: 10-meter discus LNB: USCG Large Navigational Buoy

Wind direction, wind speed, barometric pressure, air temperature, and water temperature were also collected hourly. Wind was measured by dual aerovane wind sensors installed on each buoy and at each C-MAN station. Hourly wind speed and wind direction are the mean values from an 8-minute ensemble of instantaneous measurements sampled at a rate of 1 Hz. The sensor heights on the buoy and C-MAN stations are different, depending on the type of hull and station location. In the study, the wind speed was converted to the wind speed at a 10-meter
height. More details about the design and evaluation of the meteorological and oceanographic sensors are provided by Michelen et al. (1986) and Gilhousen (1987).

4. DATA OBSERVATION AND ANALYSIS

Table 2 lists the maximum reported significant wave height at each station during the storm. Also listed are the wave period associated with the peak of wave spectrum (peak wave period), wind speed, wind direction, air temperature, water temperature, and barometric pressure at the time of maximum significant wave heights. The approximate water depth at each station is also given. As seen in the table, the largest significant wave height during the storm from all the stations was 12 meters at station 44011, which is located about 280 kilometers east of Cape Cod, Massachusetts, in the continental shelf waters of Georges Bank. The maximum reported significant wave heights from those offshore stations exceeded 5 meters with the peak wave periods ranging from 16.7 seconds to 25 seconds. The relatively low pressures at stations 44011 and 44008 were 996.6 hPa and 994.4 hPa, respectively, which indicate a close proximity to the storm center.
Table 2. Observed wave and meteorological data by NDBC stations during the storm.

<table>
<thead>
<tr>
<th>Station</th>
<th>Hs</th>
<th>Tp</th>
<th>Wspd</th>
<th>Wdir</th>
<th>Atmp</th>
<th>Wtmp</th>
<th>Barp</th>
<th>Time</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>44007</td>
<td>6.92</td>
<td>14.29</td>
<td>14.67</td>
<td>28</td>
<td>7.7</td>
<td>9.8</td>
<td>1021.0</td>
<td>31/0200</td>
<td>47</td>
</tr>
<tr>
<td>44013</td>
<td>9.06</td>
<td>16.67</td>
<td>21.99</td>
<td>29</td>
<td>8.6</td>
<td>11.0</td>
<td>1014.9</td>
<td>31/0200</td>
<td>30</td>
</tr>
<tr>
<td>44011</td>
<td>12.00</td>
<td>16.67</td>
<td>26.26</td>
<td>26</td>
<td>9.8</td>
<td>13.6</td>
<td>996.6</td>
<td>30/1600</td>
<td>88</td>
</tr>
<tr>
<td>44008</td>
<td>9.56</td>
<td>12.50</td>
<td>25.34</td>
<td>291</td>
<td>11.0</td>
<td>12.8</td>
<td>994.4</td>
<td>30/2300</td>
<td>60</td>
</tr>
<tr>
<td>ASLN6</td>
<td>3.11</td>
<td>12.50</td>
<td>15.54</td>
<td>51</td>
<td>11.1</td>
<td>14.6</td>
<td>1012.8</td>
<td>31/0900</td>
<td>25</td>
</tr>
<tr>
<td>44025</td>
<td>5.05</td>
<td>11.11</td>
<td>17.78</td>
<td>25</td>
<td>12.4</td>
<td>15.2</td>
<td>1010.1</td>
<td>31/1000</td>
<td>40</td>
</tr>
<tr>
<td>44012</td>
<td>4.72</td>
<td>25.00</td>
<td>12.31</td>
<td>14</td>
<td>15.6</td>
<td>15.6</td>
<td>1012.5</td>
<td>31/0002</td>
<td>24</td>
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<tr>
<td>44009</td>
<td>4.81</td>
<td>14.29</td>
<td>14.40</td>
<td>20</td>
<td>12.2</td>
<td>15.7</td>
<td>1011.0</td>
<td>31/1200</td>
<td>18</td>
</tr>
<tr>
<td>CHLV2</td>
<td>4.03</td>
<td>20.00</td>
<td>9.43</td>
<td>339</td>
<td>16.0</td>
<td>17.1</td>
<td>1013.5</td>
<td>31/0500</td>
<td>12</td>
</tr>
<tr>
<td>44014</td>
<td>8.05</td>
<td>16.67</td>
<td>10.51</td>
<td>328</td>
<td>15.7</td>
<td>14.8</td>
<td>1012.7</td>
<td>31/0300</td>
<td>48</td>
</tr>
<tr>
<td>DSLN7</td>
<td>6.82</td>
<td>16.67</td>
<td>8.57</td>
<td>320</td>
<td>14.3</td>
<td>*</td>
<td>1012.0</td>
<td>31/1000</td>
<td>16</td>
</tr>
<tr>
<td>41001</td>
<td>8.13</td>
<td>20.00</td>
<td>12.50</td>
<td>336</td>
<td>17.3</td>
<td>23.7</td>
<td>1009.7</td>
<td>31/0000</td>
<td>4206</td>
</tr>
<tr>
<td>41002</td>
<td>7.94</td>
<td>20.00</td>
<td>8.07</td>
<td>340</td>
<td>20.5</td>
<td>24.1</td>
<td>1012.4</td>
<td>31/0900</td>
<td>3658</td>
</tr>
<tr>
<td>41008</td>
<td>2.86</td>
<td>7.69</td>
<td>12.40</td>
<td>60</td>
<td>21.9</td>
<td>22.7</td>
<td>1023.6</td>
<td>30/0500</td>
<td>18</td>
</tr>
<tr>
<td>41010</td>
<td>5.18</td>
<td>20.00</td>
<td>3.60</td>
<td>305</td>
<td>23.4</td>
<td>26.1</td>
<td>1016.9</td>
<td>31/1830</td>
<td>833</td>
</tr>
<tr>
<td>41009</td>
<td>5.56</td>
<td>20.00</td>
<td>2.36</td>
<td>318</td>
<td>21.5</td>
<td>25.8</td>
<td>1018.0</td>
<td>31/1830</td>
<td>41</td>
</tr>
</tbody>
</table>

Hs: maximum significant wave height in meters  
Tp: peak wave period in seconds  
Wspd: wind speed at 10-meter height in m/sec  
Wdir: wind direction in degrees  
Atmp: sea surface air temperature in °C  
Wtmp: sea surface water temperature in °C  
Barp: sea surface barometric pressure in hPa  
Time: day and UTC hour of the Hs  
Depth: water depth in meters  
*: sensor failure

In the present study, the data from four stations are selected for further analysis. Data from the two offshore stations represent conditions in the northern (station 44011) and southern (station 41010) portions of the NDBC network. Data from the two nearshore stations (stations 44013 and 44014) give the representative wave field in the nearshore area where significant coastal damage was reported.

4.1 STATION 44011

Figures 3(a) through 3(f) show the data measured at station 44011 from October 27 to November 3, 1991. This station was moored about 280 kilometers east of Cape Cod, Massachusetts, in a water depth of about 88 meters. As seen in Figure 3(b), as the cold front passed on October 28, the wind speed increased rapidly from about 2 m/sec to about 20 m/sec, with the wind direction (see Figure 3(e)) shifting from about 320 degrees (northwest) to about 10 degrees (north). In the
meantime, the significant wave height (see Figure 3(a)) increased from less than 2 meters to about 6 meters. In the next 24 hours (October 29), the significant wave height gradually increased to about 8 meters, while wind speed and wind direction remained nearly constant at about 20 m/sec and around 10 degrees, respectively. Also, during this period the air temperature (see Figure 3(f)) gradually dropped from about 15°C to about 6°C. In the early hours of October 30, the wind gradually shifted to the northeast with the wind speeds increasing to greater than 25 m/sec and the barometric pressure (see Figure 3(c)) gradually decreasing to 990.50 hPa. Strong northeasterly winds further increased the significant wave height to the maximum of 12 meters at 1600 LITC, October 30, with a peak wave period of 16.7 seconds. During that hour, the wind speed was 26.4 m/sec and the wind direction was 24 degrees. The 12-meter significant wave height was the largest significant wave height reported from all the stations during the storm. After the peak, the wind speed decreased significantly, while the wind direction shifted to east. During the next 24 hours, the significant wave height gradually decreased to about 4 meters. Through the course of these 8 days, the water temperature decreased gradually from approximately 14°C to 11°C.

4.2 STATION 41010

Figures 4(a) through 4(f) show the data measurements from station 41010, which is located 290 kilometers east of Cape Canaveral, Florida, in a water depth of 833 meters. The data from this station provide representative observations of the effects of the storm along the southern portion of the U.S. east coast. As can be seen in Figure 4(a), before the easterly wind increased to 13 m/sec on October 29, the waves were predominantly long-period swell with a significant wave height around 2.7 meters and a peak wave period of about 12.5 seconds. On October 29, the significant wave height gradually increased from about 2.7 meters to 4 meters. The wind then started to decrease in the early hours of October 30 as the wind direction shifted counterclockwise from east to northwest. However, the significant wave height increased to 5 meters with a 20-second peak period. The increase in sea state was due to the arrival of long-period swells, which were observed earlier at other stations to the north (stations 44011, 44008, 41001, and 41002).

4.3 STATION 44013

Figures 5(a) through 5(f) show the data measurements from station 44013, which is located outside Boston Harbor in a water depth of 30 meters. Data reported from this station provided a field observation of sea state in this nearshore region during the storm. As seen in Figure 5(a), on October 28 the significant wave height started to
increase from less than 1 meter to about 3.5 meters in about 15 hours. The dramatic increase of significant wave height was due to the increase of wind (see Figures 5(b) and 5(e)) which shifted from southwest at 5 m/sec to northeast at about 15 m/sec, as the cold front passed. A similar shift in wind direction occurred along the entire northeast coast, setting up long fetch for wind-wave growth. For the next 96 hours the wind direction remained between 10 to 30 degrees. The wind speed gradually increased to 22 m/sec in the early hours of October 31, as the significant wave height increased to 9 meters with a 20-second peak period. These high-height, long-period waves caused significant beach erosion and damage to ocean-front properties.

Air temperature (see Figure 5(f)) started to drop from 19.1°C at 2100 UTC of October 27 down to 3.5°C at 1200 UTC of October 29, as the barometric pressure (see Figure 5(c)) increased from 1017 hPa to 1034 hPa.

4.4 STATION 44014

Figures 6(a) through 6(f) show the data measurements from station 44014, located about 80 kilometers east of Virginia Beach, Virginia, in a water depth of 48 meters. In addition to the nondirectional wave measurements, this station also provided directional wave measurements.

The wind speed (see Figure 6(b)) started to increase significantly on October 28 from 8 m/sec to 15 m/sec over a 12-hour period, while the wind direction remained from the north (see Figure 6(e)). In the meantime, the significant wave height (see Figure 6(a)) gradually increased from 2.5 meters to 4.3 meters. It is noted that the changes in wind speed on October 28 were also observed by the three stations discussed above and are related to the development of a cold front as described in Section 2. The significant wave height remained at about 4.5 meters for the next 36 hours as the wind speed varied from 12 m/sec to 14 m/sec and the wind direction shifted to northwest. At 1000 UTC of October 30, 1991, the significant wave height began to increase from 4.5 meters to 8 meters by the early hours of October 31, 1991. It is noted that as the significant wave height reached 8 meters the wind speed decreased to 10 m/sec with the wind direction shift to the more fetch-limited direction of northwest. It is apparent that the severe seas were due to the arrival of long-period swell observed earlier at stations 44011, 44013, and 41010 as discussed previously.

Figures 7(a) and 7(b) show the directional wave data at 0200 and 0300 UTC, October 31, when the significant wave height reached 8.05 meters and the peak period exceeded 17 seconds. The directional wave data show that the wave directions at higher frequencies aligned with the local wind directions (about 330 degrees). The direction of lower...
frequency waves was about 75 degrees. This direction is inconsistent with the placement of the storm but can be explained by refraction of long-period waves in shallow water. Before reaching this station, 20-second wave energy may have been significantly refracted as it traveled over the continental shelf.

5. DISCUSSION

This large data base consists of wave and meteorological data produced under storm conditions. Several interesting observational results are presented for further discussion.

5.1 THE HIGH-HEIGHT, LONG-PERIOD SWELL

As shown in Table 2, high-height, long-period swell significantly raised the sea state along the Florida coast (stations 41009 and 41010), while the local winds were very mild. These swells were generated by nearly continuous northeasterly gale- to storm-force winds that blew from west of the storm center to Cape Hatteras. These swells propagated southwestward and caused serious beach erosion and property damage along the U.S. east coast.

Figures 8(a) and 8(b) show time series plots of the wave energy at frequencies of 0.05 and 0.06 Hz (20 seconds and 16.7 seconds) from stations 44011, 44008, 41001, 41002, and 41010. These offshore stations covered the area from Georges Bank to Cape Canaveral, Florida. As can be seen, the wave energy appeared at station 44011 with a high level of energy (the energy peak at 0.06 Hz was about 200 m²/Hz), which was at least 250 percent of that value measured from other stations. Station 44011 is located about 1700 km northeast of station 41010. It takes about 31 hours for wave energy at 0.05 Hz to travel between these two stations with a traveling speed of about 56 kilometers/hour. This 31-hour time period agrees well with the time lag shown on the time series plot of 0.05 Hz wave energy at stations 44011 and 41010. This indicates that the high-height, long-period swell was probably from the northeast, and is consistent with the weather condition shown from the surface synoptic chart (Figure 2). It is noted that the wave direction of 0.05-Hz energy at station 44014 (Figure 7) was about 75 degrees, but the waves may have been significantly refracted due to the rather shallow water depth (48 meters).

5.2 GROWTH OF WIND-WAVE IN THE PRESENCE OF SWELL

Due to the effects of Hurricane Grace, the wave field at station 44011 was predominated by the long-period, southeasterly swells generated from the northern and northeastern sides of the hurricane before the rapid increase of local wind on October 28. As the local wind speed
began to rapidly increase in the direction of about 10 degrees, an interesting case of wind-wave evolution in the presence of swell became evident.

Figures 9(a) through 9(c) show the time series of significant wave height, wind speed, and wind direction at station 44011 from 0300 UTC to 2300 UTC, October 28, 1991. In the 10 hours (1100 UTC to 2100 UTC), the wind speed rapidly increased from 2 to 20 m/sec, the wind direction shifted to the north, and the significant wave height increased from 2 meters to 6 meters.

Figure 9(d) shows the evolution of hourly wave spectrum from 1100 UTC to 2100 UTC of October 28. The wave energy increase began at the high-frequency end of the spectrum and moved to lower frequencies in time, due to the combined effects of the input of wind energy, the resonant nonlinear interactions, and wave breaking.

Before the wind-generated energy appeared at the high frequency end of the spectrum, the slope in the higher frequency ranges of swell-dominated spectra (0.10 to 0.35 Hz) was about -5. The wind-generated energy started to increase at the high frequency end at a much higher level than those of swell-dominated spectra and gradually moved into the lower frequency portion of the wave spectrum, while a slope of -5 at high frequency end generally remained. The effect of the presence of swell on the wave evolution process has been demonstrated from laboratory data (Donelan, 1987). This data set provides a field observation about wave spectrum evolution under the influence of strong swells, which could be an interesting subject for further study.

5.3 EFFECTS OF BOTTOM FRICTION

Due to the wavelength of the long-period swell, energy dissipation due to bottom friction affected the waves at most of the buoy and C-MAN stations. A 20-second wave will "feel" the bottom at a water depth less than 300 meters, which is greater than the depth of water at all but the three deep-water stations: 41001, 41002, and 41010. Hence, the proper estimation of the energy dissipation due to bottom friction plays a key role for successfully modeling waves observed from NDBC nearshore stations during the storm. Station CHLV2 (water depth of 12 meters) was located 86 km west of station 44014 (water depth of 48 meters). The long-period swell passed by station 44014 before arriving at station CHLV2. Figures 10(a) and 10(b) show the time series of wave energy at 0.05 Hz and 0.06 Hz for stations 44014 and CHLV2. As seen in the figures, energy dissipation due to bottom friction caused the wave energy of the long-period swell measured at station CHLV2 to be much less than that measured at station 44014. Figure 10(c) shows the wave spectra from station 44014 and CHLV2 at 0300 UTC on October
31. The significant wave heights were 8.05 meters and 3.90 meters for stations 44014 and CHLV2, respectively. The significant energy dissipation was evident in the frequencies ranging from 0.04 Hz to 0.11 Hz.

5.4 EFFECT OF THE GULF STREAM

The Gulf Stream played an important role in several aspects of the storm. Figure 11 depicts the position of the Gulf Stream on October 30. As a large body of warm water it supplied the energy for the transition of the storm into a subtropical cyclone on November 1. In addition, the strong cur- rents of the Stream interacted with storm swell that can affect the wave environment both in offshore and in nearshore regions (Lai and Bales, 1986, and Holthuihsen and Tolman, 1991).

Three stations (41001, 41002 and 41010) were located east of the Gulf Stream. The remaining stations were located between the Gulf Stream and the U.S. east coast. Based on the time series of wave energy at 0.06 Hz at stations 44008, 44014, and 41001 (see Figure 12(a)), wave energy peak at 0.06 Hz measured at station 41001 (October 31) south of the Gulf Stream was about 80 percent of those measured at station 44008 and 44014 located north of the Gulf Stream. It is noted that wave energy at 0.06 Hz measured at stations 41001 on October 27 and 29 was much higher than that at stations 44008 and 44014. Figure 12(b) shows the time series of wave energy at 0.06 Hz for stations 41010 (located outside the Gulf Stream in deep water) and 41009 (located between the Gulf Stream and Florida’s coast). As seen in the figure, the differences between the wave energy at stations 41010 and 41009 are small. From the above two examples, the effects of the Gulf Stream on wave energy change seem to be insignificant.

5.5 COASTLINE SHELTERING EFFECT

Station 41008 is located nearshore east of Jacksonville, Florida, in a water depth of 18 meters. The high-height, long-period swells that significantly affected two stations to the south (stations 41009 and 41010) did not significantly affect station 41008 (Table 2). Figure 13 shows the time series of 0.05 Hz wave energy from stations 41008 and 41009. As can be seen, the 20-second swell arrived at station 41009 with a magnitude 20 times larger than those at station 41008. The significant difference could well be due to coastline sheltering provided by Cape Hatteras, North Carolina. It was also reported that, due to coastline sheltering provided by Cape Canaveral, beach erosion and coastal structure damage in Cocoa Beach, Florida, were not severe. However, further south severe beach erosions and structure damages were reported. The NDBC C-MAN station located on a pier at Lake Worth, Florida, was destroyed by wave action.
The above discussions illustrate that to model storm waves in the nearshore area along the US. east coast, there are various factors that must be properly considered.

6. SUMMARY

Wave and marine environmental data were collected from 16 NDBC stations during the strongest northeaster in 50 years. This data set provides an extensive field observation of the storm-generated severe seas along the U.S. east coast from Maine to Florida. The study documents the data and presents a preliminary analysis of the wave conditions present during the storm.

Several interesting observations were briefly presented to show the propagation of long-period swell, the bottom friction effect, the wind-wave evolution under the effect of swell, the effect of the Gulf Stream, and coastline sheltering.

This data set presents a good effort by NDBC to provide field observations from a network of stations during a severe storm. The field verification and calibration of numerical wave models using this data set will undoubtedly advance the development of wave hindcast and forecast models for years to come.

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8. REFERENCES


Figure 1. Surface synoptic weather chart.
Figure 2. Storm track and locations of East Coast NDBC stations.
Figure 3. Time series of measured data from station 44011: (a) significant wave height, (b) wind speed, (c) barometric pressure, (d) peak wave period (solid line) and average wave period (dotted line), (e) wind direction, (f) air temperature (solid line) and water temperature (dotted line).
Figure 4. Time series of measured data from station 41010: (a) significant wave height, (b) wind speed, (c) barometric pressure, (d) peak wave period (solid line) and average wave period (dotted line), (e) wind direction, (f) air temperature (solid line) and water temperature (dotted line).
Figure 5. Time series of measured data from station 44013: (a) significant wave height, (b) wind speed, (c) barometric pressure, (d) peak wave period (solid line) and average wave period (dotted line), (e) wind direction, (f) air temperature (solid line) and water temperature (dotted line).
Figure 6. Time series of measured data from station 44014: (a) significant wave height, (b) wind speed, (c) barometric pressure, (d) peak wave period (solid line) and average wave period (dotted line), (e) wind direction, (f) air temperature (solid line) and water temperature (dotted line).
Figure 7. Directional Wave Spectrum from station 44014: (a) at 0200 UTC, Oct. 31, (b) at 0300 UTC, Oct. 31.

Figure 8. Time series of wave energy from stations 44011, 44008, 41001, 41002, and 41010 at (a) 0.05 Hz, (b) 0.06 Hz.
Figure 9. Time series of measured data: (a) significant wave height, (b) wind speed, (c) wind direction; (d) wave spectrum.

Figure 10. Time series of wave energy and wave spectrum from stations 44014 and CHLV2: (a) at 0.05 Hz, (b) at 0.06 Hz, (c) Wave Spectra at stations 44014 and CHLV2.
Figure 11. The position of the Gulf Stream on October 30, 1991.

Figure 12. The time series of wave energy at 0.06 Hz: (a) stations 44008, 44014, and 41001, and (b) stations 41009 and 41010.
Figure 13. The time series of wave energy at 0.05 Hz from stations 41008 and 41009.