On Another Concept of Hasselmann Equation Source Terms

An Exploration of Tuning Free Models

Vladimir Zakharov¹,²,³,⁴, Donald Resio⁵ and Andrei Pushkarëv²,³,⁴

¹University of Arizona, USA; ²Lebedev Physical Institute, Russia; ³Novosibirsk State University, Russia; ⁴Waves and Solitons LLC, USA; ⁵University of North Florida, USA

Abstract
The new ZRP wind input source term is examined for its consistency via numerical simulation of the Hasselmann equation. The results are compared to field experimental data, collected at different sites around the world, and theoretical predictions based on self-similarity analysis. Consistent results are obtained for both limited fetch and duration limited statements.

Motivation
We are trying to build Sₚₐ consistent with mathematical properties of Hasselmann equation and requiring minimal tuning of the model:

\[
\frac{dt}{dt} + \frac{\partial S_{\scriptscriptstyle \text{HF}}}{\partial \omega} = \delta_{\scriptscriptstyle \text{HF}} + \delta_{\scriptscriptstyle \text{LF}} \quad \text{in} \quad \omega < \omega_{\text{sat}},
\]

\[
\frac{dt}{dt} = \delta_{\scriptscriptstyle \text{HF}} + \delta_{\scriptscriptstyle \text{LF}} \quad \text{in} \quad \omega = \omega_{\text{sat}},
\]

\[
\frac{dt}{dt} = \delta_{\scriptscriptstyle \text{HF}} + \delta_{\scriptscriptstyle \text{LF}} \quad \text{in} \quad \omega > \omega_{\text{sat}}.
\]

for deep water case \( \omega = \sqrt{\frac{(\lambda_1 C_p)}{g}} \).

• \( S_{\scriptscriptstyle \text{LF}} \) – derived from free surface Euler equations
• \( S_{\scriptscriptstyle \text{HF}} \) – multiple versions, differences up to \( 300 \sim 500\% \), see [2], [4]
• \( S_{\scriptscriptstyle \text{HF}} \) – multiple LF and HF versions, see [2]

Experimental Evidence
The analysis of multiple field experiments [5] showed that the averaged energy spectra can be approximated by linear regression:

\[
\beta = \frac{1}{2} \left\{ \left( \frac{\langle f \rangle}{\langle g \rangle} \right)^{1/3} \right\} \frac{1}{2} + \frac{1}{2} \langle w \rangle
\]

Theory
Self-similarity analysis shows that:

<table>
<thead>
<tr>
<th>Duration Limited Case</th>
<th>Limited Fetch Case</th>
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<td>( \delta = \frac{1}{\omega} \langle \omega \rangle \left( \frac{\langle f \rangle}{\langle g \rangle} \right)^{1/3} )</td>
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<td>( \frac{\omega}{\omega} ) \langle \omega \rangle \langle f \rangle \sim 3/\omega )</td>
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ZRP wind input term:

\[
S_{\scriptscriptstyle \text{HF}}(\omega, \theta) = \frac{\gamma(\omega, \theta) \cdot \langle \omega \rangle}{\left( \frac{\langle f \rangle}{\langle g \rangle} \right)^{1/3} \left( \frac{\langle f \rangle}{\langle g \rangle} \right)^{1/3}}
\]

\[
\gamma(\omega, \theta) = \begin{cases} \frac{1}{(1 + \theta \omega)^{1/3}} & \text{for } \omega \leq \omega_{\text{sat}} \text{ and } f_{\text{min}} \leq f \leq f_{\text{sat}} \text{, } \omega = 2 \pi f \text{, } \theta = 1 \text{, } \omega_{\text{sat}} = 0 \text{, } f_{\text{sat}} = f_{\text{max}} \\ 0 & \text{otherwise} \end{cases}
\]

\[
\langle \omega \rangle = \omega_{s} = \sqrt{\frac{(\lambda_1 C_p)}{g}} \left( \frac{\langle f \rangle}{\langle g \rangle} \right)^{1/3} \left( \frac{\langle f \rangle}{\langle g \rangle} \right)^{1/3} \text{ for } \omega \leq \omega_{\text{sat}} \text{ and } f_{\text{min}} \leq f \leq f_{\text{sat}}
\]

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\]

"Implicit" dissipation in the form of Phillips tail [6]. \( A_{\text{FF}}^{-1/3} \), which starts at \( f = 1.1 \text{ Hz}[5] \)

Conclusions

• New set of Hasselmann equation source terms has been introduced, based on XNL, self-similarity analysis and experimental observations
• ZRP \( S_{\scriptscriptstyle \text{HF}} \) is the same for fetch limited and duration limited statements
• The numerical simulation of HF, using new set of source terms, reproduces self-similar properties of Hasselmann equation and is close to field experiments data

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References
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