On Another Concept of Hasselmann Equation Source Terms

An Exploration of Tuning Free Models

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Abstract

The new ZRP wind input source term is examined for its consistency via numerical simulation of the Hasselmann equation. The results are compared to field experimental data, collected at different sites around the world, and theoretical predictions based on self-similarity analysis. Consistent results are obtained for both limited fetch and duration limited statements.

Motivation

We are trying to build S_{in} consistent with mathematical properties of Hasselmann equation and requiring minimal tuning of the model:

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss}$$

$$0 \;+\; 0 \;\;= S_{nl} + \;0\; + \;0 \;\;\Rightarrow\;\; \varepsilon \sim \frac{P^{1/3}}{\omega^4}$$

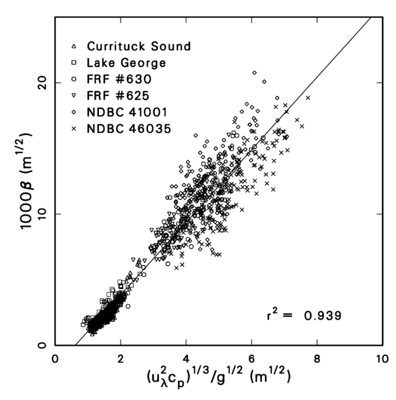
for deep water case $\omega = \sqrt{gk}$.

- S_{nl} – derived from free surface Euler equations
- S_{in} – multiple versions, differences up to $300 - 500\%$, see [2], [4]
- S_{diss} – multiple LF and HF versions, see [2]

Experimental Evidence

The analysis of multiple field experiments [5] showed that the averaged energy spectra can be approximated by linear regression:

$$\beta = \frac{1}{2}\alpha_4 \left[(u_\lambda^2 c_p)^{1/3} - u_0 \right] g^{-1/2}$$



Theory

Self-similarity analysis shows that:

Duration Limited Case	Limited Fetch Case
$\varepsilon = t^{p+q} F(\omega t^q)$	$\varepsilon = \chi^{p+q} G(\omega \chi^q)$
$\varepsilon = \varepsilon_0 t^p, \quad <\omega> = \omega_0 t^{-q}$	$\varepsilon = \varepsilon_0 \chi^p, \quad <\omega> = \omega_0 \chi^{-q}$
$9q - 2p = 1$	$10q - 2p = 1$
$p = 10/7 \quad q = 3/7 \quad s = 4/3$	$p = 1 \quad q = 3/10 \quad s = 4/3$

ZRP wind input term :

$$S_{in}(\omega, \theta) = \gamma(\omega, \theta) \cdot \varepsilon(\omega, \theta)$$

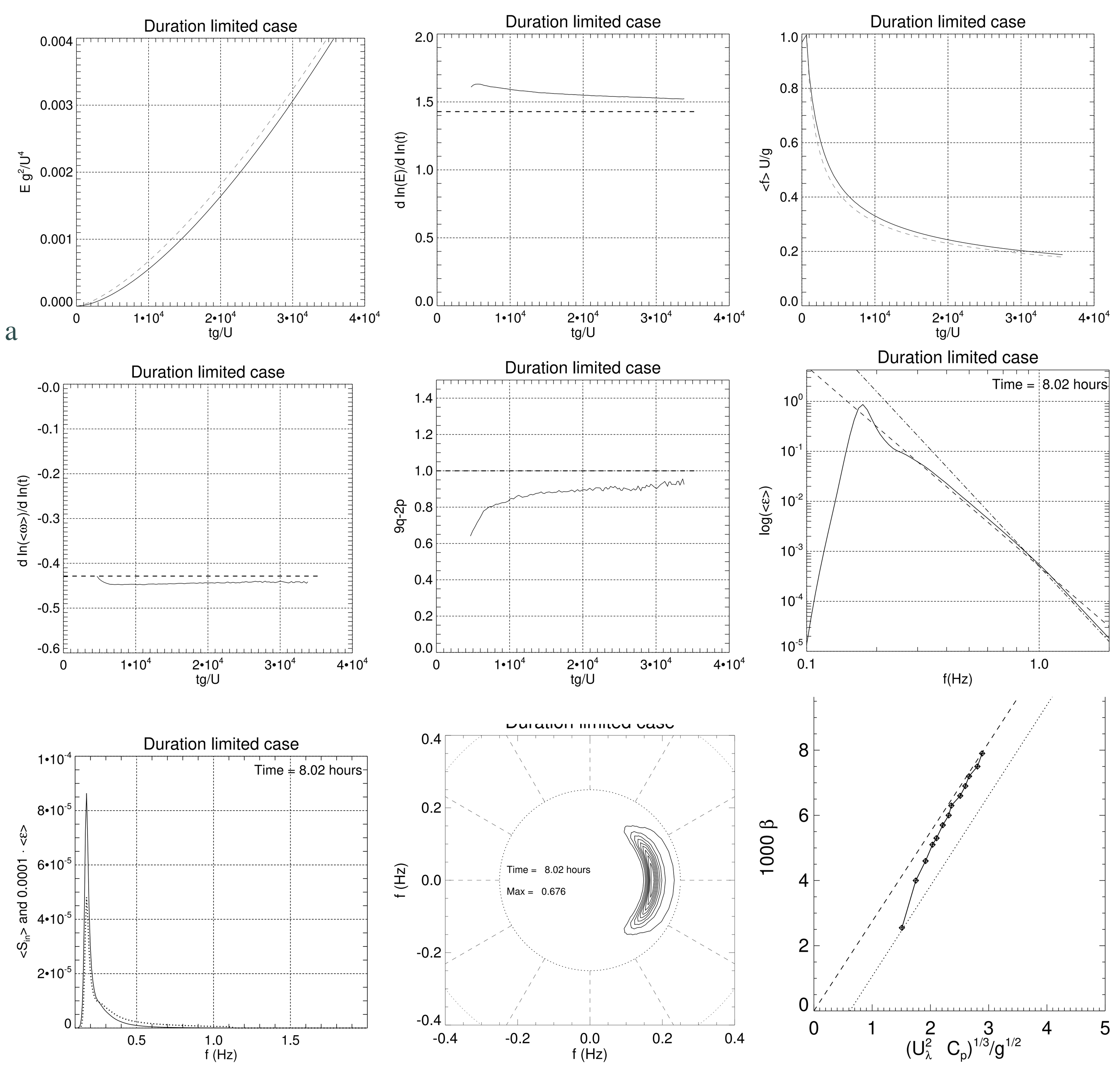
$$\gamma(\omega, \theta) = \begin{cases} 0.05 \frac{\rho_{air}}{\rho_{water}} \omega \left(\frac{\omega}{\omega_0}\right)^{4/3} q(\theta) & \text{for } f_{min} \le f \le f_d, \quad \omega = 2\pi f \\ 0 \;\; \text{otherwise} \end{cases}$$

$$q(\theta) = \begin{cases} \cos 2\theta & \text{for } -\pi/4 \le \theta \le \pi/4 \\ 0 \;\; \text{otherwise} \end{cases}$$

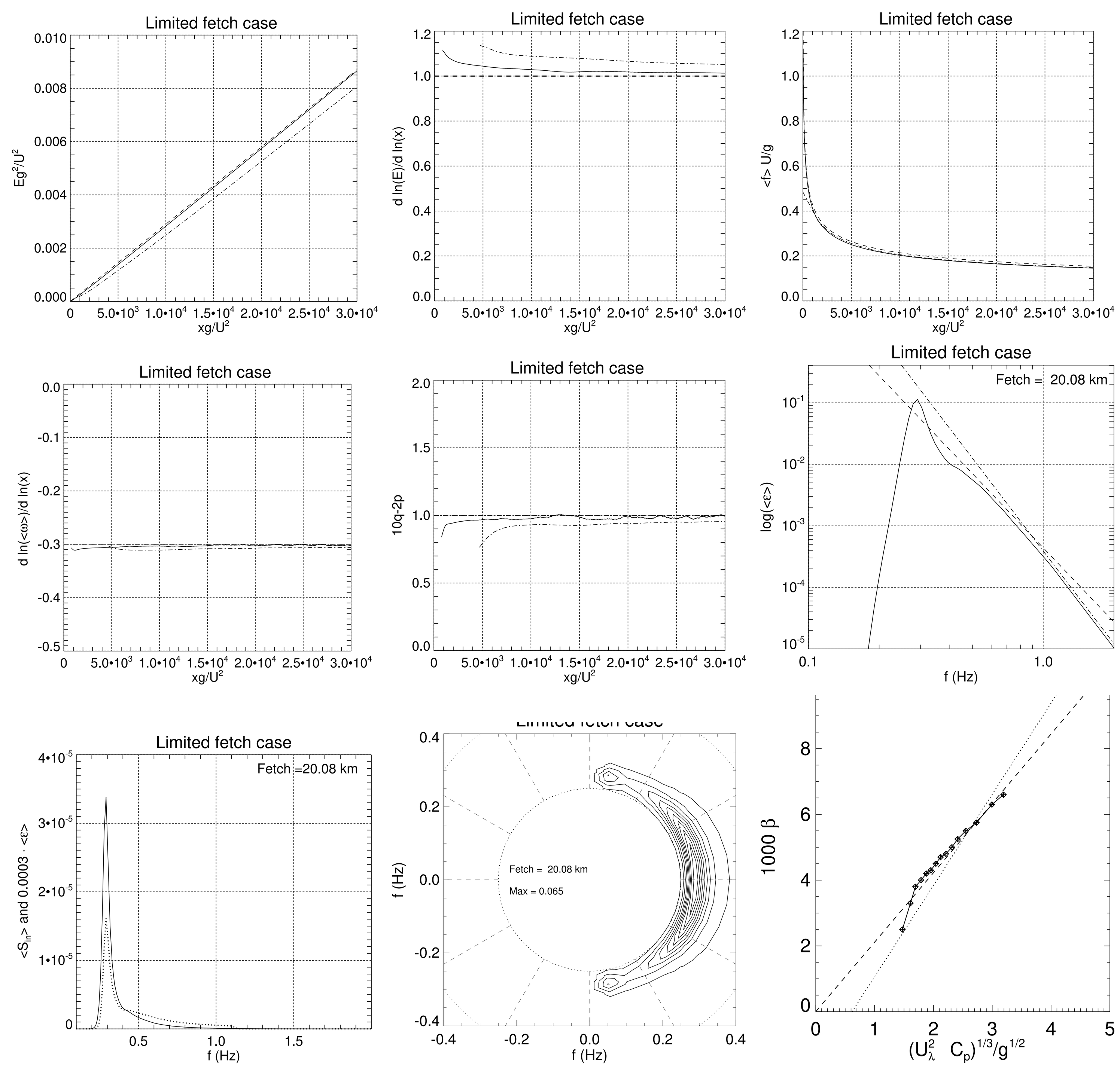
$$\omega_0 = \frac{g}{U}, \quad \frac{\rho_{air}}{\rho_{water}} = 1.3 \cdot 10^{-3}$$

"Implicit" dissipation in the form of Phillips tail [6] Af^{-5}, which starts at $f = 1.1$ Hz [5]

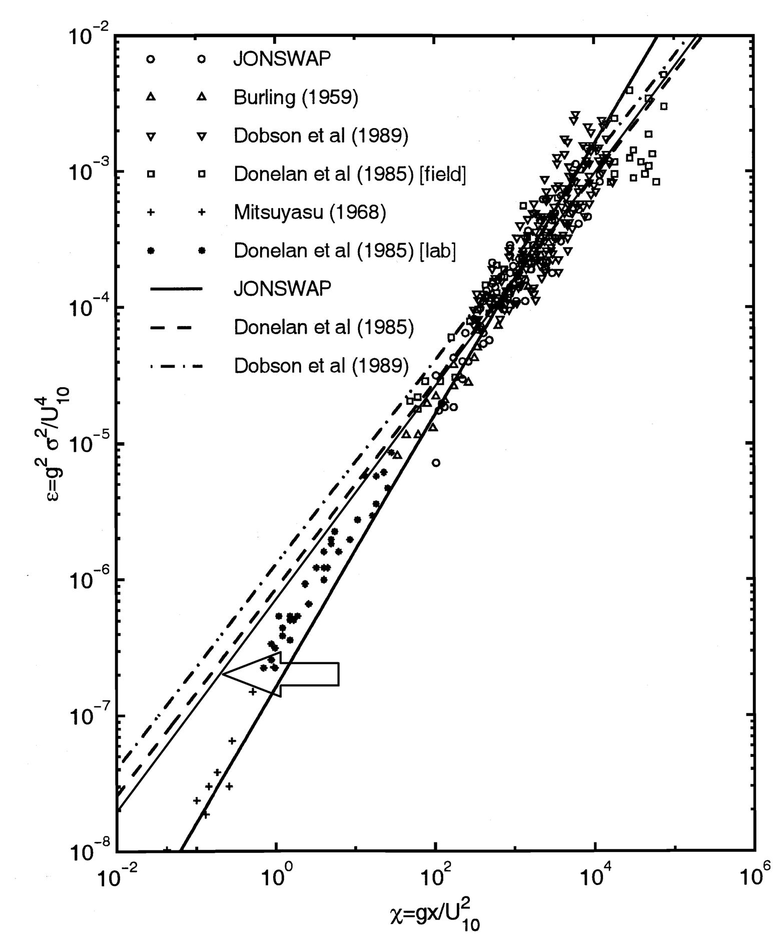
Duration limited simulation

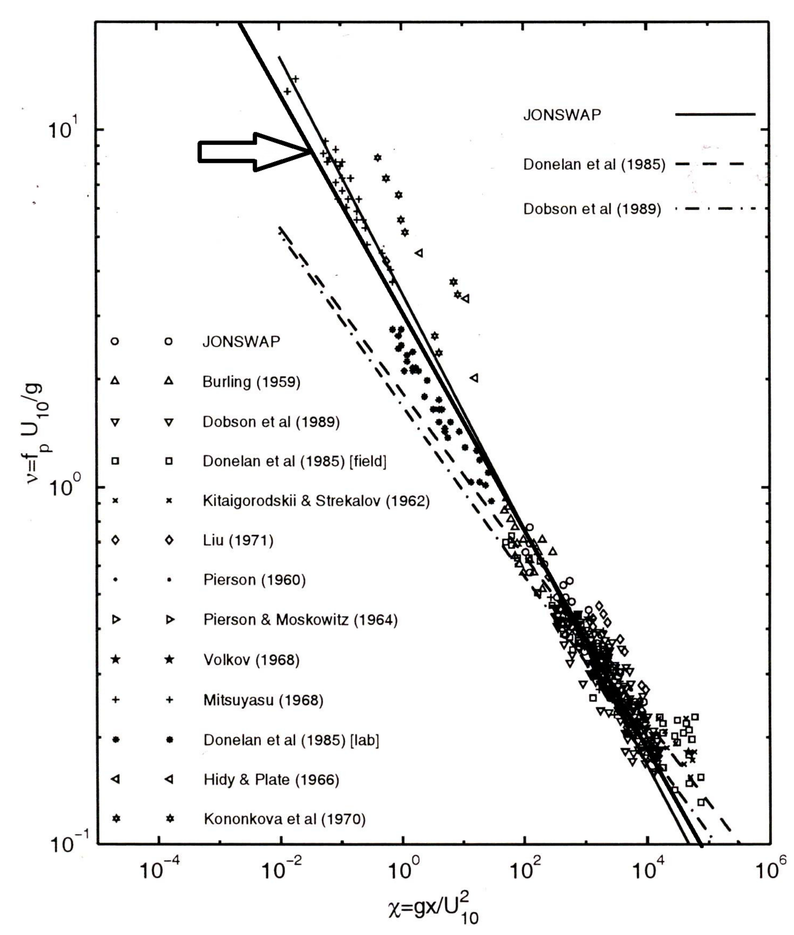


Limited Fetch Growth



Comparison with the experiments [3]





Conclusions

- New set of Hasselmann equation source terms has been introduced, based on XNL, self-similarity analysis and experimental observations
- ZRP S_{nl} is the same for fetch limited and duration limited statements
- The numerical simulation of HE, using new set of source terms, reproduces self-similar properties of Hasselmann equation and is close to field experiments data

Acknowledgements

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References

1. Zakharov V., Resio D., Pushkarev A. Balanced source terms for wave generation within the Hasselmann equation, , accepted for publication in NPG, 2017
2. Pushkarev, A., Zakharov, V. Limited fetch revisited: comparison of wind input terms, in surface wave modeling, Ocean Modeling, 103, 18-37, 2016
3. Young I.R., Wind Generated Ocean Waves, Elsevier, 1999
4. Badulin S., Pushkarev A., Resio D., Zakharov V. Self-similarity of wind-driven seas, NPG, 12, 891-945, 2005
5. Resio D., Long C., Vincent C.L. Equilibrium-range constant in wind-generated wave spectra, JGR, 109, C01018, 2004
6. Phillips O., Spectral and statistical properties of the equilibrium range in wind-generated gravity waves, JFM, 505-531, 1985