On Oceanic rogue waves

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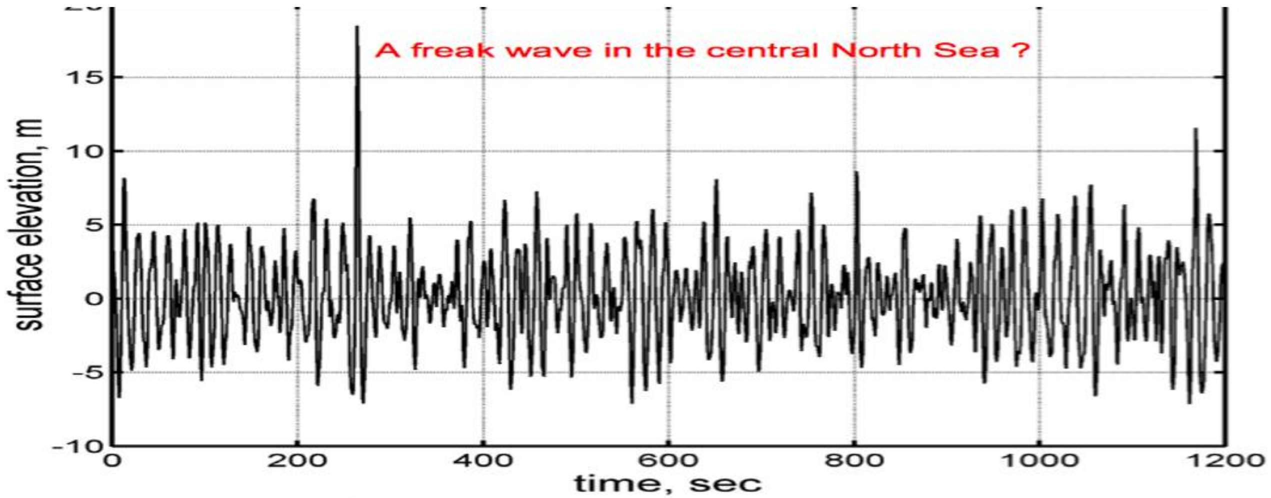
DRAUPNER EVENT JANUARY 1995

H_{max}=25.6 m

h/Hs~1.63Hs

H/Hs~2.15Hs



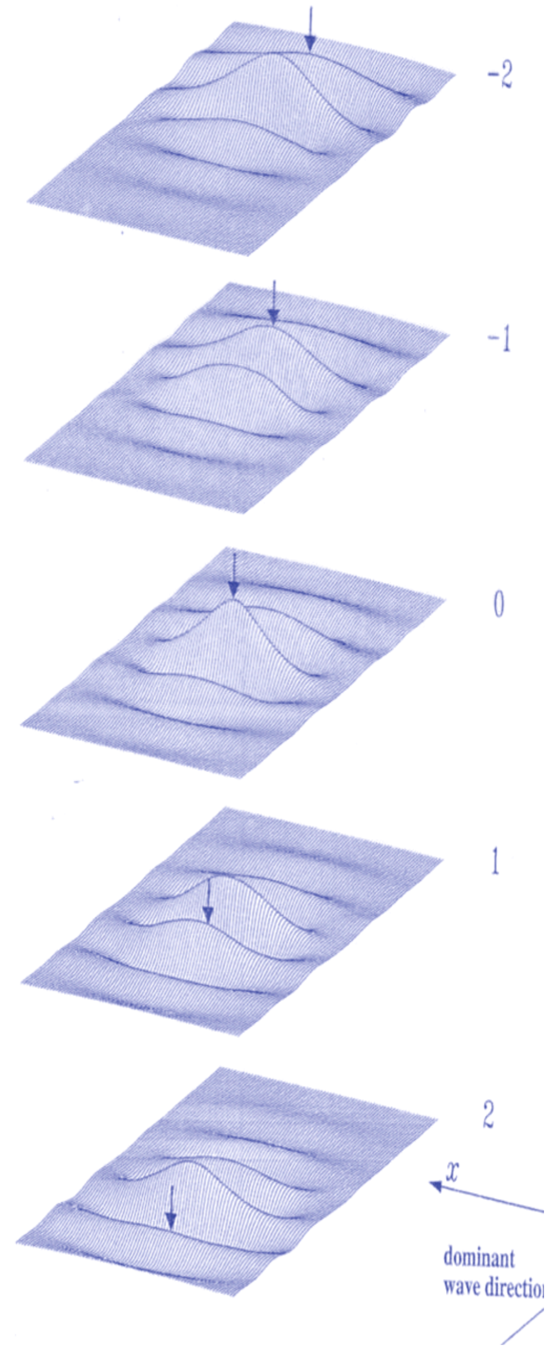
Possible physical mechanisms:

- **Linear focusing (Boccotti 1989,2000)**
- **Nonlinear focusing (Janssen 2003)**
- **Space-time extremes (Fedele 2012)**

What happens in the neighborhood of a point x_0 if a large crest followed by large trough are recorded in time at x_0 ?

$$\Psi(X,T) = \langle \eta(x_0, t_0)\eta(x_0 + X, t_0 + T) \rangle$$

SPACE-TIME Covariance Boccotti 1989,2000



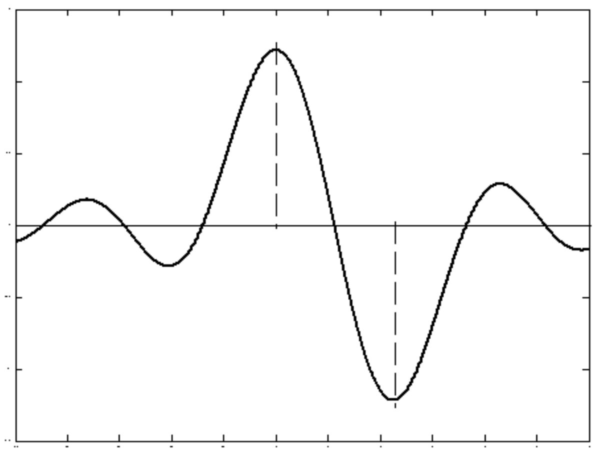
LINEAR FOCUSING

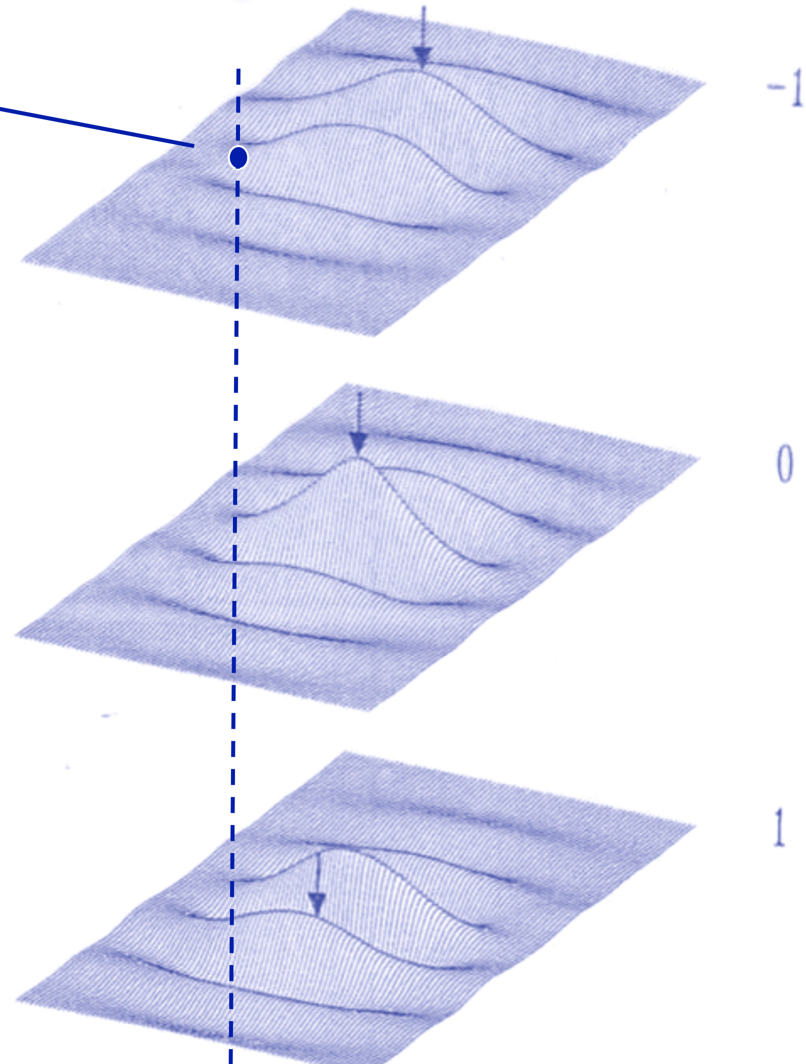
CONSTRUCTIVE INTEFERENCE OF MANY ELEMENTARY WAVES

BAD DAY AT THE TOWER

Slepian model: Lindgren 1972, Adler 1981, Boccotti 1989, Piterbarg 1995)

SPACE-TIME EXTREMES IN OCEAN SEAS
(Fedele 2012 JPO)





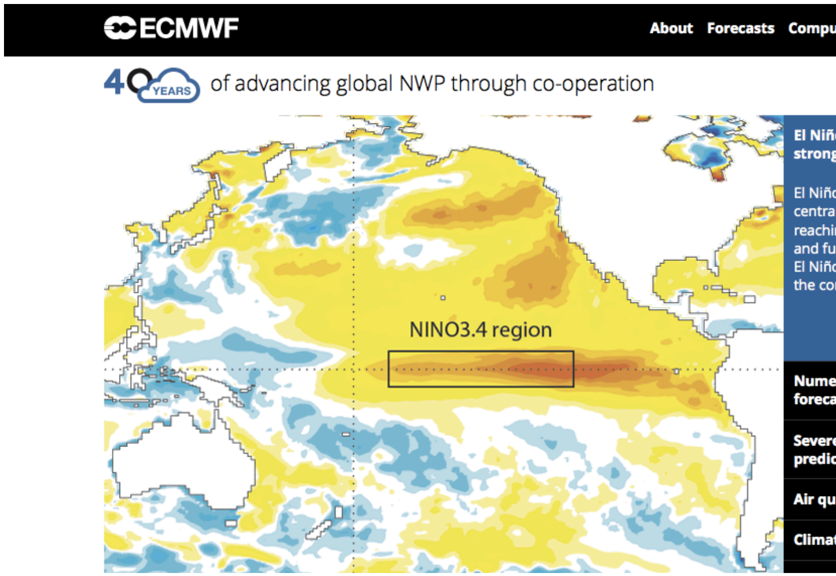
The crest height measured at a fixed probe IS NOT the largest amplitude of the wave group

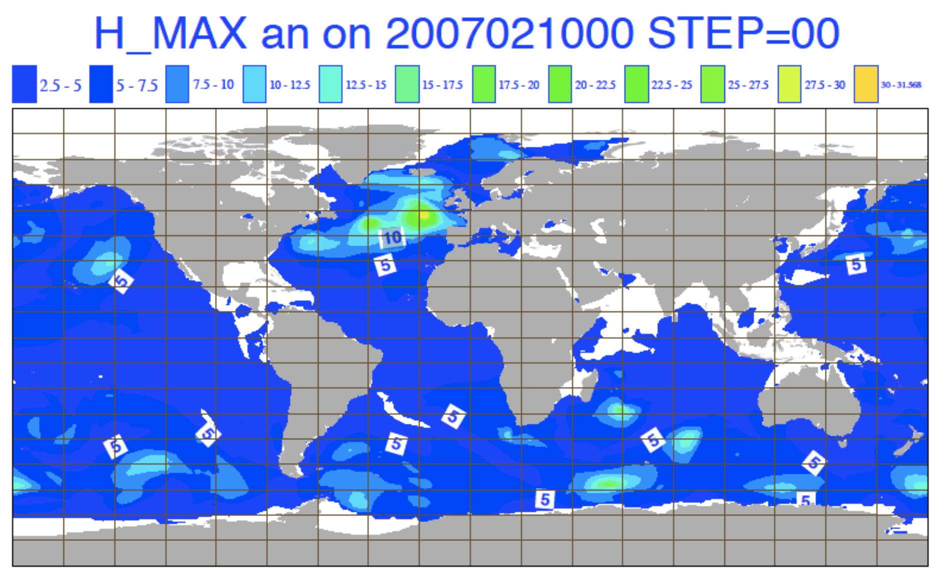
The probability that the wave group passes by probe at the apex is zero

Random fields, Euler Characteristics (Adler 1981, Adler and Taylor 2000, Piterbarg 1995)

NOAA WAVEWATCH III®

ECMWF freak wave warning system
(Janssen JPO 2003, Mori and Janssen JPO 2006, Janssen & Bidlot 2009 *tm588*, Mori, Onorato and Janssen JPO 2011)





Analytical large-time kurtosis for NLS turbulence

$$C_4^{dyn} = \frac{0.031}{\delta_\theta} \times \frac{\pi}{3\sqrt{3}} BFI^2,$$

$_\theta$ is seen to give a considerabl
aves the total kurtosis become:

$$C_4 = C_4^{dyn} + \alpha\varepsilon^2.$$

Benjamin-Feir Index

Steepness over spectral bandwidth

Analogous of Re in strong turbulence

On the kurtosis of deep-water gravity waves

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2. Dynamic excess kurtosis

Drawing on Janssen (2003) the dynamic excess kurtosis of weakly nonlinear sea states, initially homogenous and Gaussian, is given by

$$C_4^d = \frac{4g}{\sigma^2}\mathrm{Re}\int T_{12}^{34}\delta_{12}^{34}\sqrt{\frac{\omega_4}{\omega_1\omega_2\omega_3}}G(t)E_1E_2E_3\mathrm{d}\omega_{1,2,3}\mathrm{d}\theta_{1,2,3}, \tag{2.1}$$

where the resonant function

$$G(t) = \frac{1-\exp(-i\omega_{12}^{34}t)}{\omega_{12}^{34}}, \tag{2.2}$$

T_{12}^{34} is the Zakharov kernel (Zakharov (1968, 1999); Krasitskii (1994)) as a function of the wavenumber vectors $\mathbf{k}_j = (k_j\cos(\theta_j), k_j\sin(\theta_j))$ and $\mathrm{Re}(x)$ denotes the real part of x. The sixfold integral in Eq. (2.1) is defined over the manifold

$$\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4 = \mathbf{0}, \tag{2.3}$$

or equivalently $\delta_{12}^{34} = \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)$, where $\delta(\mathbf{k})$ is the Dirac delta. The frequency

50 x 11.00 in

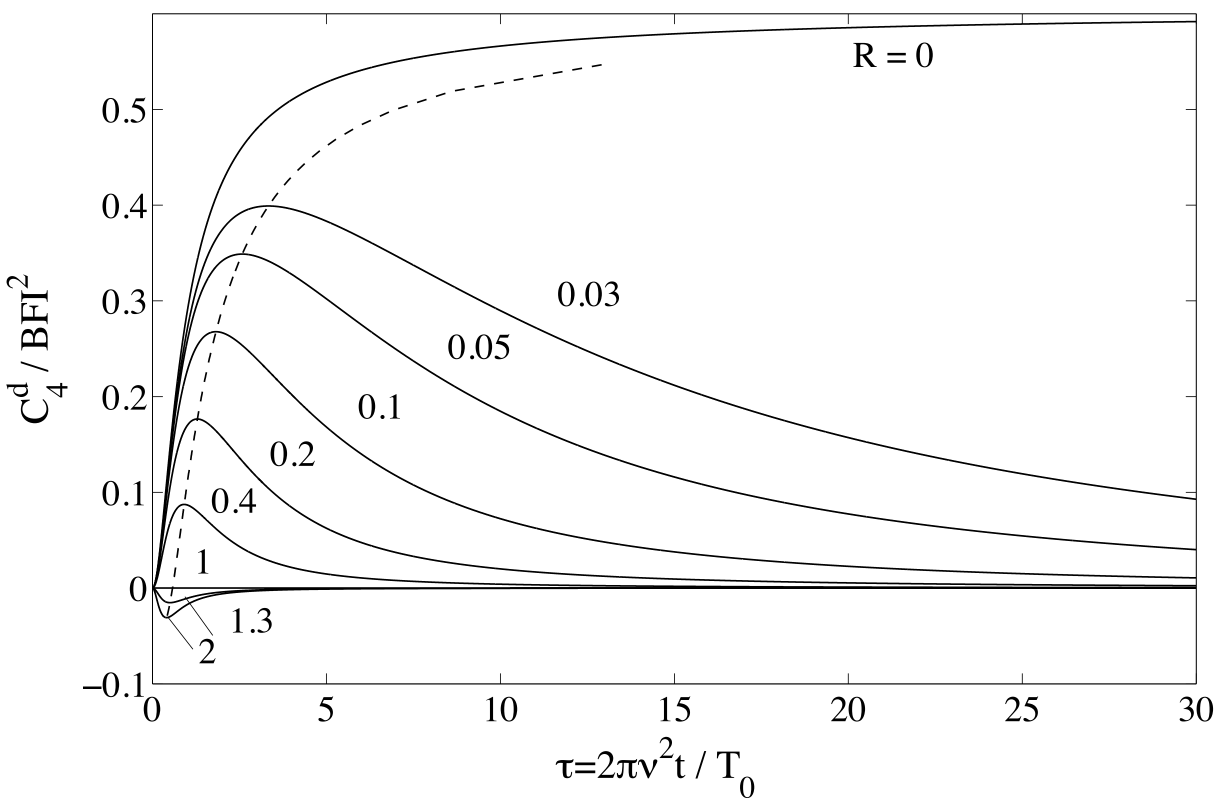
$$\frac{\mathrm{d}C_4^d}{\mathrm{d}\tau} = BFI^2 \frac{\mathrm{d}J}{\mathrm{d}\tau}, \tag{2.8}$$

and

$$\boxed{\frac{\mathrm{d}J}{\mathrm{d}\tau} = \frac{\mathrm{d}J_0(\tau;1,R)}{\mathrm{d}\tau} = 2\,\mathrm{Im}\left(\frac{1}{\sqrt{1-2i\tau+3\tau^2}\sqrt{1+2iR\tau+3R^2\tau^2}}\right),} \tag{2.9}$$

where the function $J_0(\tau;P,Q)$ is defined in appendix A and $\mathrm{Im}(x)$ denotes the imaginary part of x. On this basis, the factor J in Eq. (2.6) follows by quadrature as

$$J(\tau;R) = 2\,\mathrm{Im}\int_0^\tau \frac{1}{\sqrt{1-2i\alpha+3\alpha^2}\sqrt{1+2iR\alpha+3R^2\alpha^2}}d\alpha. \tag{2.10}$$



- My refinement of Janssen's (2003) theory implies that in typical multidirectional oceanic fields **third-order quasi-resonant interactions do not appear to play a significant role in the wave growth (however, they can affect wave phases)**

- The large excess dynamic kurtosis transient is a result of **the unrealistic assumption that the initial wave field is homogeneous Gaussian with random phases**. **The ocean does not have paddles**

- A random wave field forgets its initial conditions and adjusts to a **non-Gaussian state dominated by bound nonlinearities** (Annenkov and Shrira JFM 2013).

- In this regime, statistical predictions of rogue waves can be based on the Tayfun (1980) and Janssen (2009) models to account for both skewness and bound kurtosis (Tayfun & Fedele 2007,Fedele 2015, arxiv.org)

The Acqua Alta Project

Chevron

Georgia Institute of Technology

PROTECNO

ISMAR

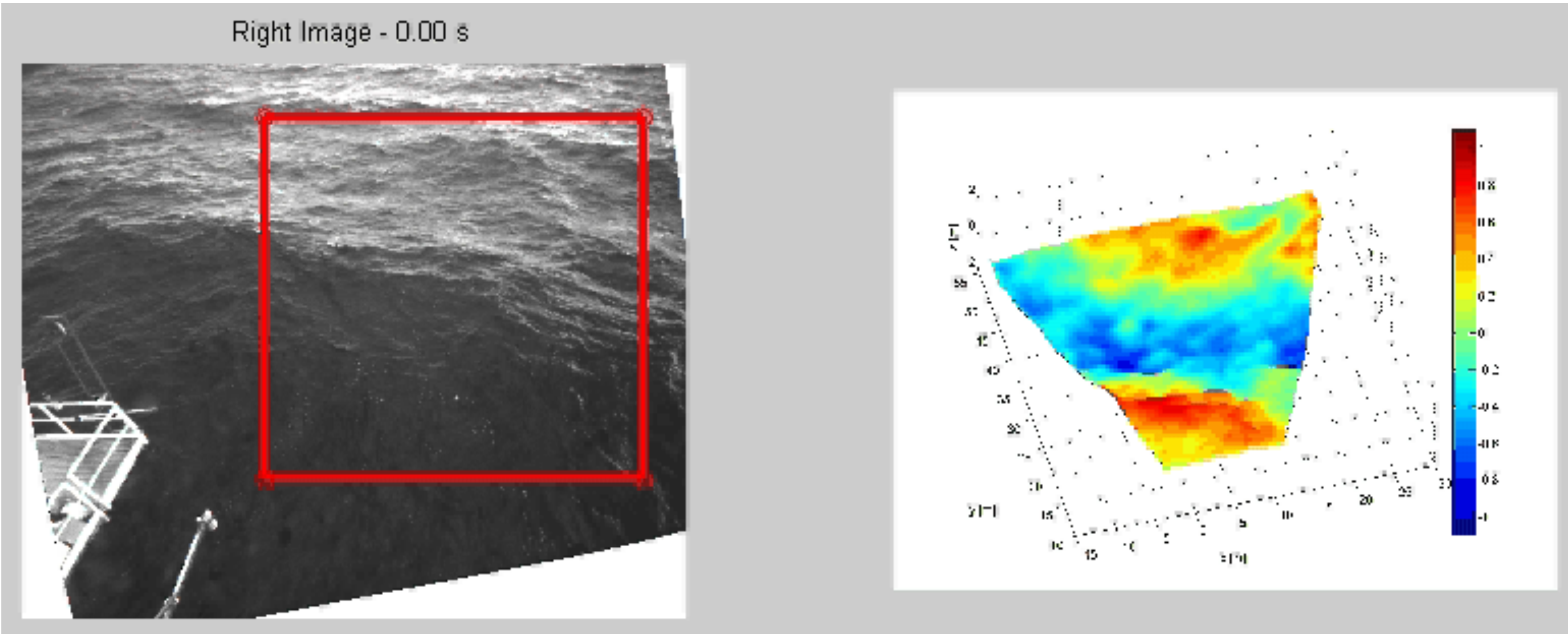
PROJECT SITE

Gigi Cavaleri's tower

'ACQUA ALTA'

Venice, ITALY

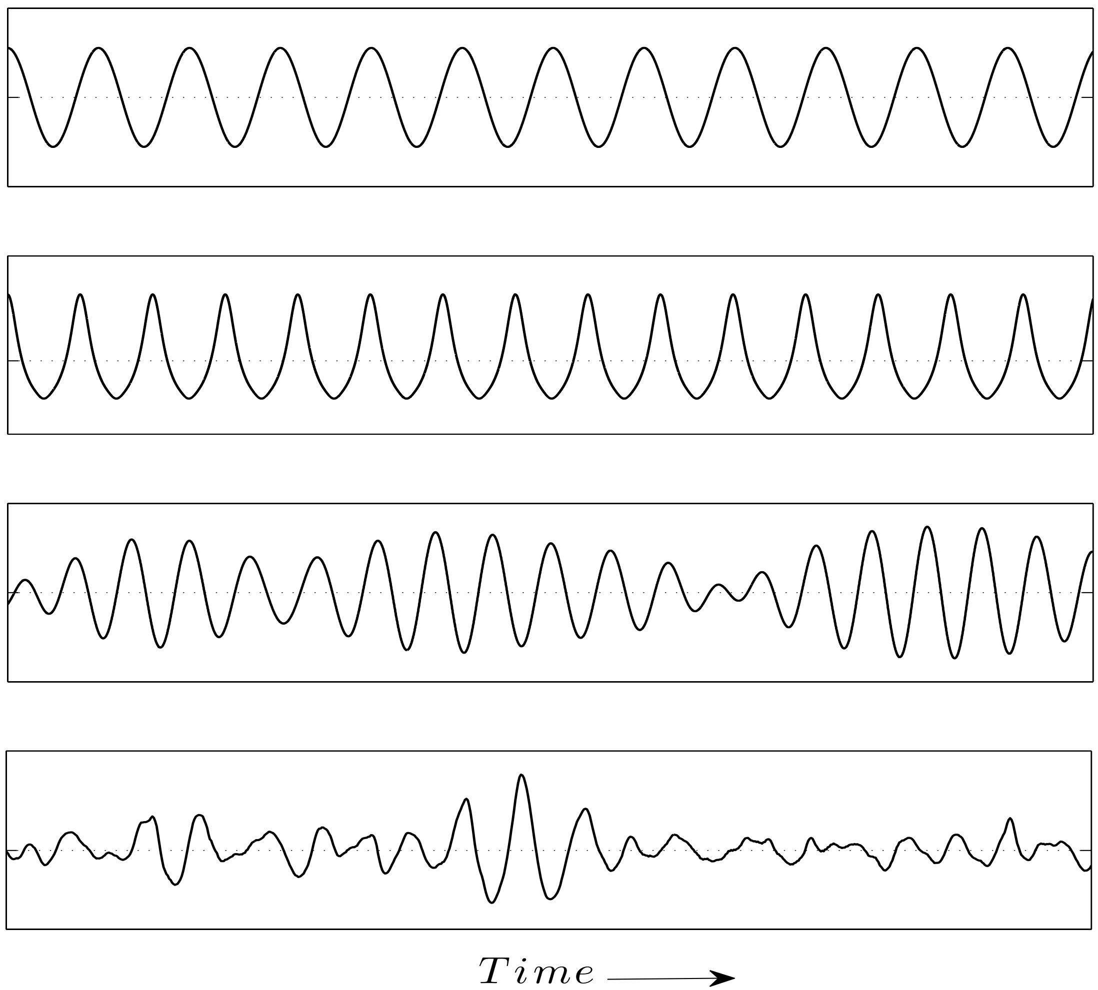
October 2009 event



- Duration 35′
- 21000 3D maps
- Reconstructed area ~ 80x80 m^2

Bora wind, Hs=1.3 m
Mean wind speed ~ 10 m/s

Steady (periodic) vs. unsteady waves



Linear Stokes wave

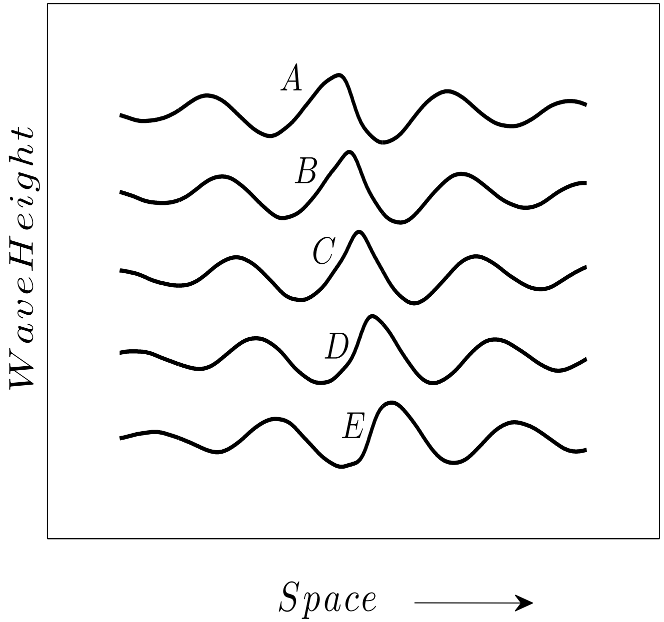
Nonlinear Stokes wave (Cnoidal waves)

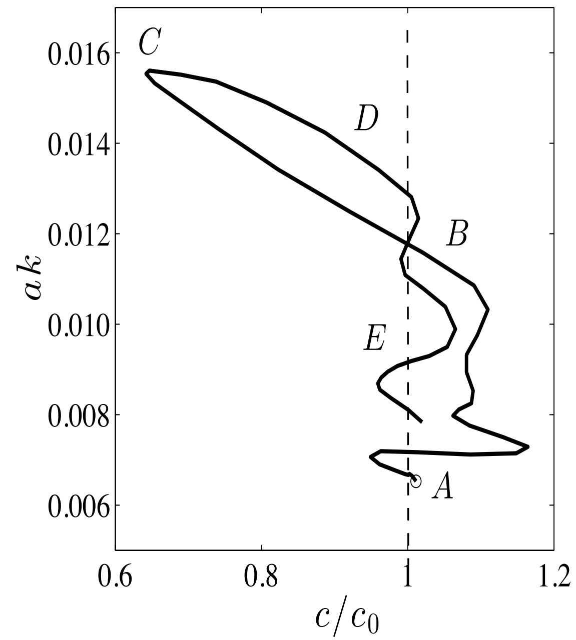
Narrowband waves

Realistic ocean waves

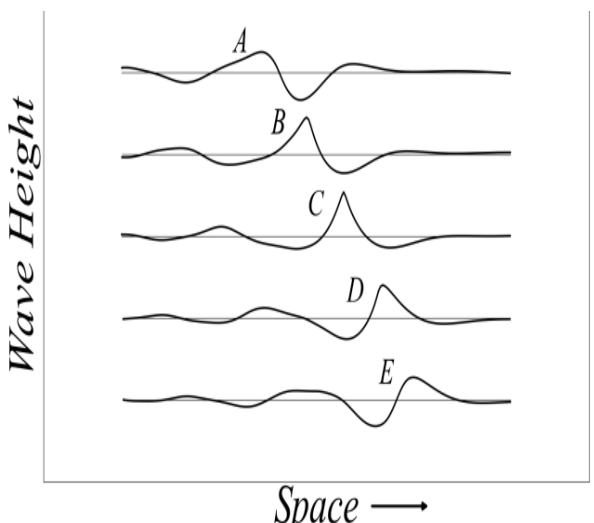
$C = c_0[1+(ak)^2]^{1/2}$ phase speed $c_0=(gk)^{0.5}$

Slowdown of crests in unsteady nonlinear wave groups explains reduced speed of breakers (Banner et al. 2014 PRL, Fedele 2015 EPL, Barthelemy et al. 2015, arxiv.org)





The crest slowdown provides an explanation and quantifies the puzzling generic (O(20%)) slowdown of breaking wave crests



Growth phase A->B->C
<u>crest leans backwards as it slows down</u>

Decay phase C->D->E
<u>crest leans forward as it speeds up</u>

Are there hidden physical mechanisms that delay the wave from breaking and lead to a rogue wave?

H1. (potential energy growth is inhibited) nonlinear dispersion reduction limits the crest slowdown of deep-water ocean waves leading to breaking (superharmonic instability Fedele JFM 2014)

H2. breaking and the associated kinetic energy growth are inhibited by enhancement of the crest slowdown, allowing waves to grow to larger amplitudes

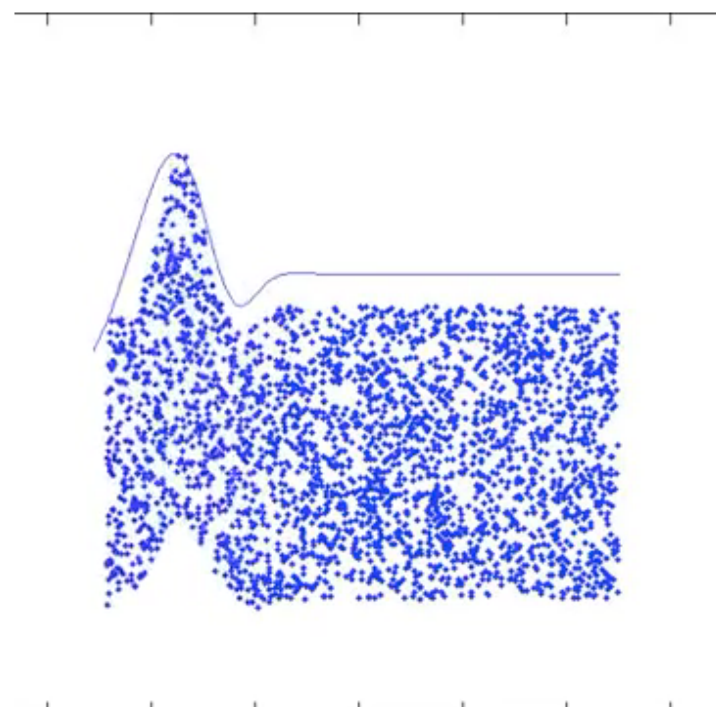
For me H1 ….. Time irreversibility …..

H2 is for NLS turbulence ……

Kinematic criterion for wave breaking

Particle speed U= crest speed

Energy flux on the surface= U ($\rho g \eta + 0.5 \rho U^2$)



Refined criterion:

Particle speed=0.84 x crest speed

Barthelemy et al. 2015
arxiv.org

Fluid particle kinematics on the ocean surface. Part 1. Hamiltonian theory

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[1]School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA

Hamiltonian structure of fluid particle kinematics

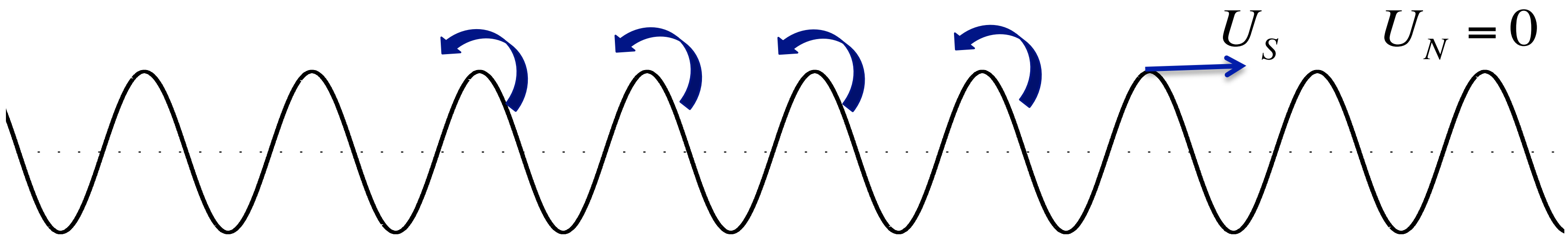
SYMPLECTICITY = VORTICITY

[Fedele et al. 2015, arxiv.org]

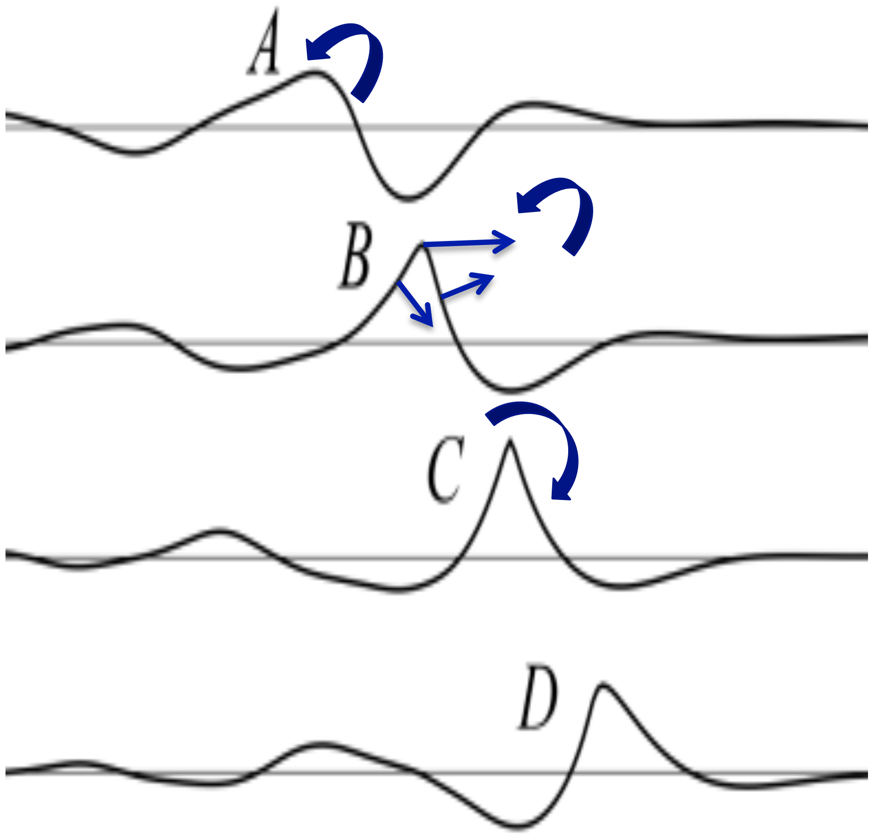
Different perspective drawing on differential geometry

$$vorticity = -2 \times K \times U_S$$

Positive vorticity (curvature <0 at crest)
(Longuet Higgings JFM 1992)
waves don't want to break



$$vort. = -2 \times K \times U_S - 2 \times \frac{dU_N}{ds}$$



Particle speed=crest speed

Vorticity at crest =0

Vorticity must change sign if breaking occurs

Fedele et al. (2015) arxiv.org

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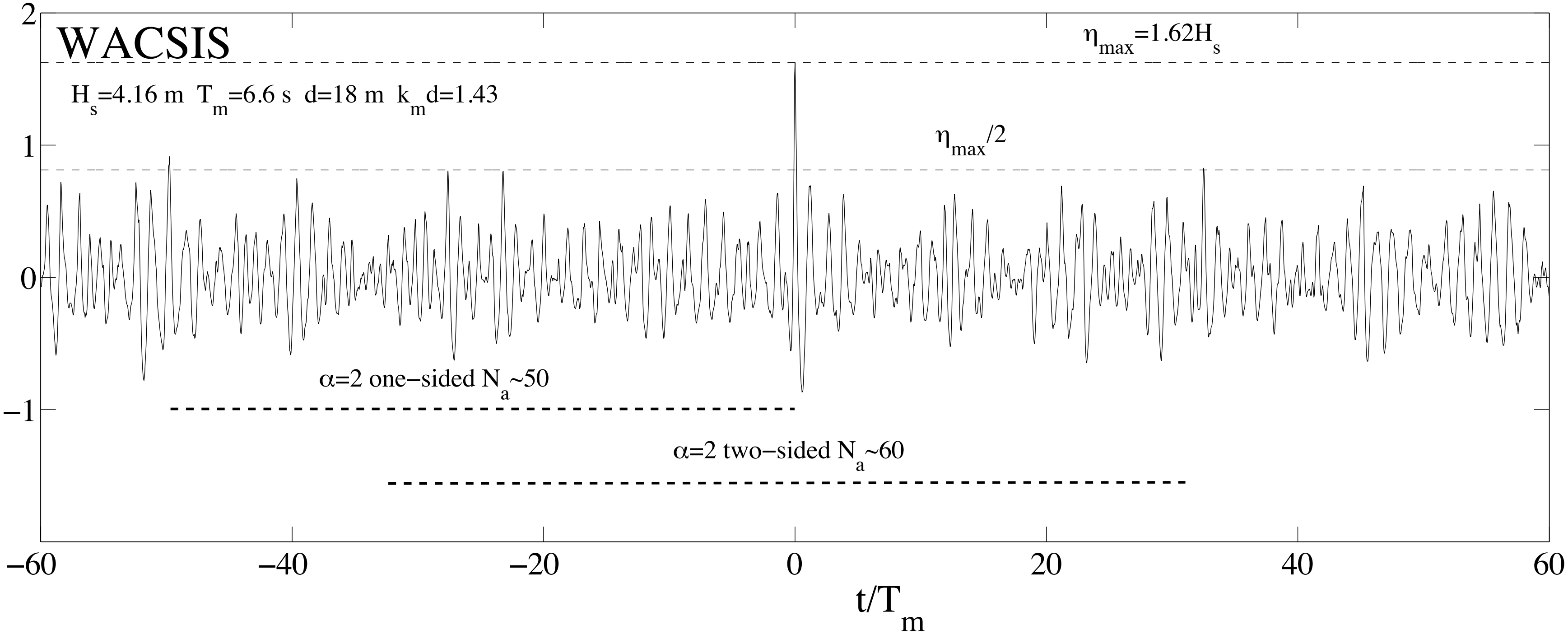
- The large excess dynamic kurtosis transient is a result of **the unrealistic assumption that the initial wave field is homogeneous Gaussian**. (**the ocean does not have paddles**)

- A random wave field forgets its initial conditions and adjusts to a **non-Gaussian state dominated by bound nonlinearities**

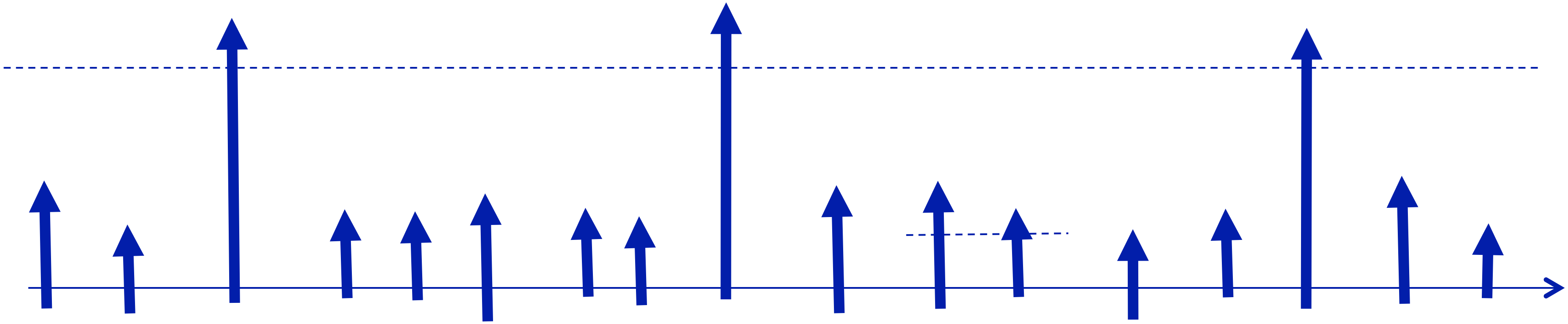
- **Breaking has to be accounted for to obtain realistic extreme wave predictions**

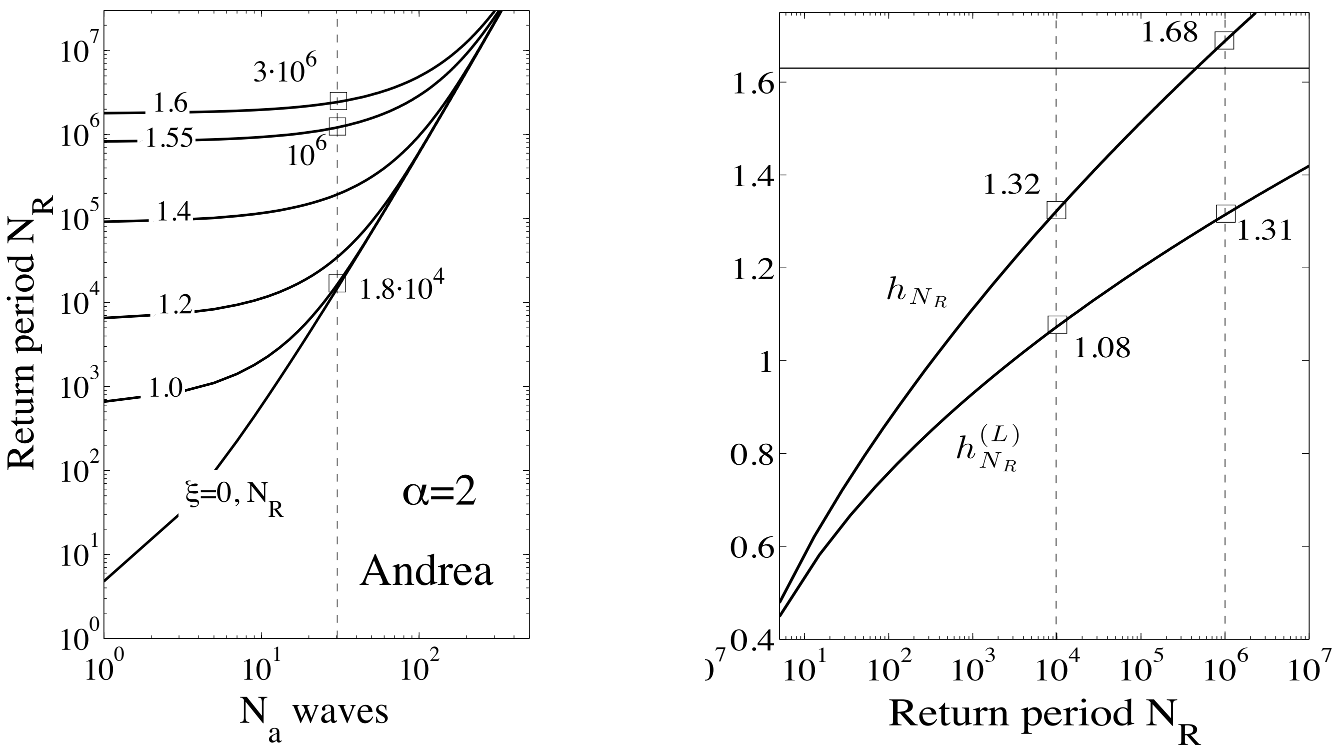
- **The onset of breaking may be the kinematic manifestation of vorticity created on a free surface and it depends on the space-time evolution of energy fluxes**

Unexpected waves (Gemmrich & Garrett 2008)



- A wave alpha-times larger than the surrounding Na waves
- Return period R(alpha,Na) = mean inter-time between two unexpected waves
- Unconditional period: harmonic mean over all unexpected waves of any amplitude



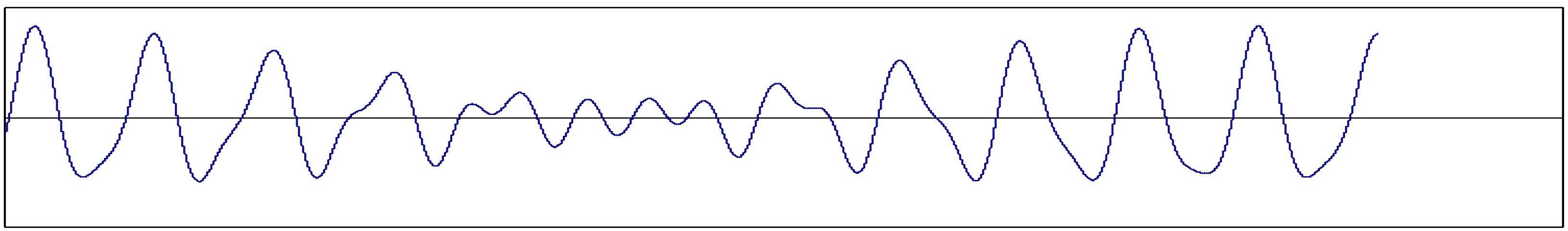


Fedele 2015, arxiv.org

The Rayleigh distribution

$$\Pr(crest\,height > Z) = \exp[-\frac{1}{2}Z^2]$$

$$P[H] = \frac{number\ of\ waves\ with\ height\ greater\ than\ H}{total\,number\,of\ waves}$$



H_{max}=25.6 m !
Crest height=16 m

L~300 m, T~13 s

Z=Crest_Height/standard deviation

observed extreme waves Z~6

Return Period R~T/P

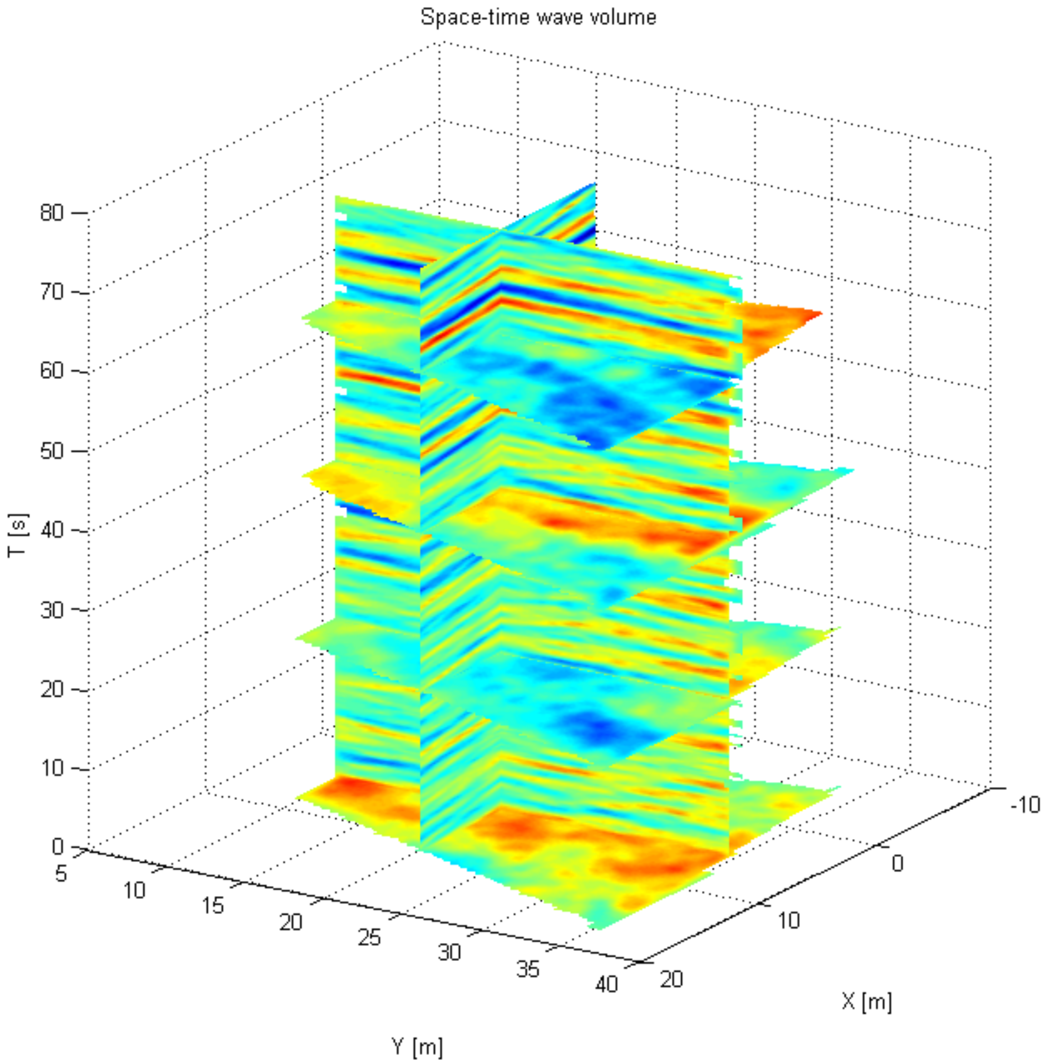
z=4 R=0.5 days !

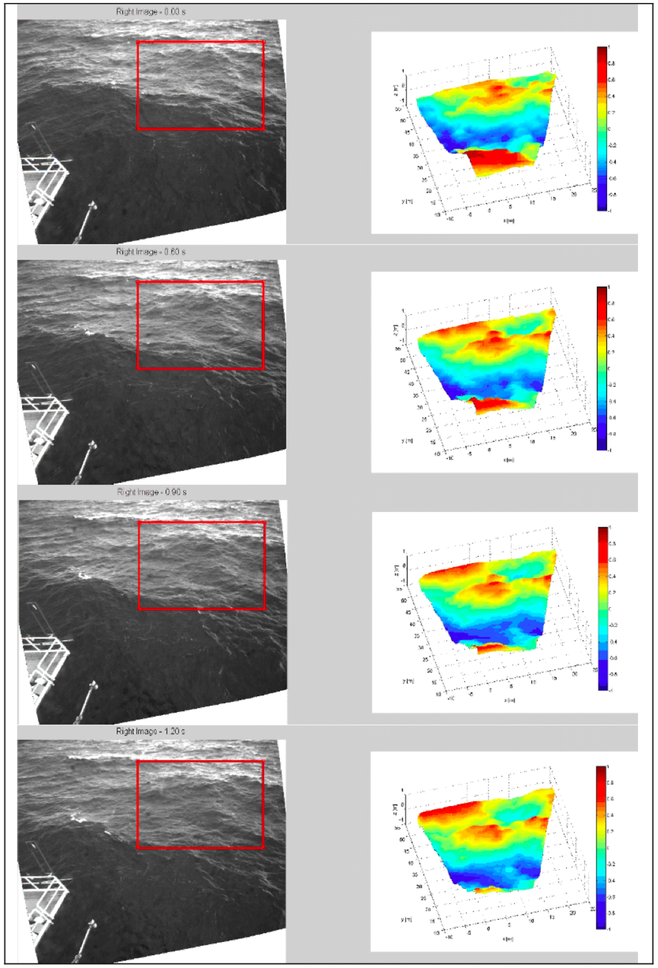
z=5 R=2 month

z=6 R=30 yrs

…. terabytes of data ….

Space-time wave manifold z-F(x,y,t)=0





Fedele 2012, JPO; Benetazzo, Fedele et al. 2012 Coastal Engineering; Fedele et al. 2011, OMAE; Gallego, Fedele et al. 2011, IEEE TGRS

Probability of exceedance for nonlinear crests :

The Tayfun-Fedele distribution

$$\Pr(crest\,height > Z) = \exp[-\frac{1}{2\,\mu^{*2}}\,(-1+\sqrt{1+2\,\mu^{*}Z}\,)^2\,]\,[1+\frac{\Lambda}{64}\,(Z^4-8Z^2+8)\,]$$

It requires skewness and kurtosis of the ocean field

The Rayleigh distribution

$$\Pr(crest\,height > Z) = \exp[-\frac{1}{2}Z^2\,]$$

Fedele 2005, 2008, 2009, 2012, JFM, JPO, Physica D

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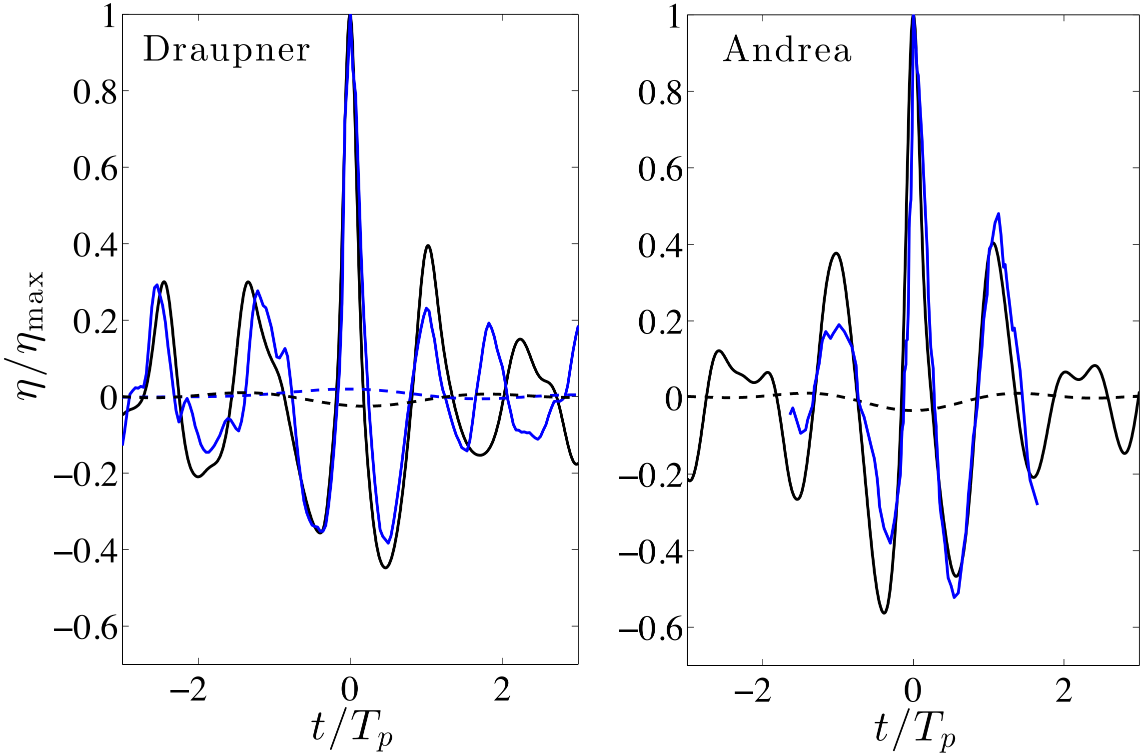
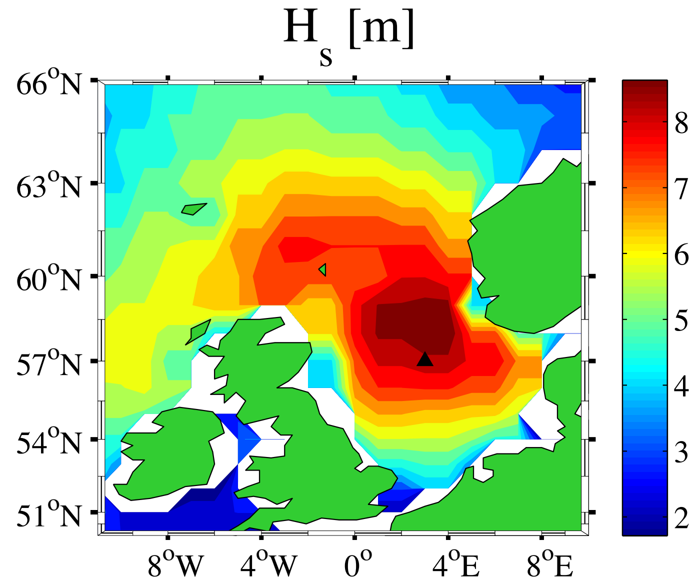
Fedele 2005, 2008, 2009, 2012, JFM, JPO, Physica D

1995

2007

ERA-Interim





ERA Hs map at the peak time of the Draupner event, North Sea