A two-scale approximation for wave-wave Interactions in operational wave forecast models

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What are the objectives of the project?

1. New wave model using new formulations for efficient quadruplet (nonlinear) wave-wave interactions representation
2. Tests for issues regarding wave transitions: waves and swell, turning winds, shallow water…
3. Tests for real North Atlantic storms
1: New wave model using new efficient formulations for nonlinear wave physics.

- Two-Scale Approximation (TSA)
- fetch-limited growth and TSA characteristics
- example of hurricane Juan (2003)


Summary

1. Implemented TSA in WW3 and WWM
2. Tests with different source terms: ST1 ~ ST4
3. Reliable results for ‘academic’ JONSWAP tests
4. "fetch- and duration-limited growth"
5. Turning winds: ongoing test …
6. N Atlantic tests with hypothetical constant winds
7. Computational efficiency improved
8. Optimization of TSA code is ongoing.

Acknowledgements: Panel on Energy R & D, NOPP ONR.
Wave generation and growth…

a balance equation …

$$\frac{\partial E(f, \theta)}{\partial t} = -\vec{c}_g \cdot \vec{\nabla} E(f, \theta) + \sum_k S_k(k, \theta)$$

where

- \( \vec{c}_g \) = group velocity
- \( S_{in} \) = wind input
- \( S_{ds} \) = wave dissipation
- \( S_{nl} \) = nonlinear transfer due to wave-wave interactions
For internal transfer of wave action (or energy) in the spectrum at $n_1$ (e.g. at $k_1$) via wave-wave interactions by $k_2$, $k_3$, $k_4$ - Hasselmann (1962), Zakharov (1966)

$$\frac{\partial n_1}{\partial t} \equiv S_{nl} = \int \int T(k_1, k_3) \, dk_3$$

where

$$T(k_1, k_3) = 2\int [n_1 n_3 (n_4 - n_2) + n_2 n_4 (n_3 - n_1)] C(k_1, k_2, k_3, k_4) \theta(|k_1 - k_4| - |k_1 - k_3|) \frac{\partial W}{\partial n}^{-1} \, ds,$$

TSA – Two-Scale Approximation

$$n_i = n_i \text{[broad-scale]} + n_i \text{[local-scale]} ; \ i = 1, 2, 3, 4$$

Neglect $n_2 \text{[local-scale]}$ and $n_4 \text{[local-scale]}$

[Resio and Perrie 2008; Perrie & Resio 2009]
JONSWAP $\gamma=1$ with Hasselmann-Mitsuyasu directional:
(a) broad- and local-scale terms normalized by $f^{-4}$,  
(b) 1-d comparison of DIA, WRT and TSA, (c) 2-d action density $n_j$,  
(d) $S_{nl}(f,\theta)$ results from DIA (e) WRT, (f) TSA. $f_p=0.1$, $\alpha=0.0081$, $\sigma_A=0.07$, $\sigma_B=0.09$. 
JONSWAP sheared spectrum with Hasselmann-Mitsuyasu directional:
(a) broad- and local-scale terms normalized by $f^{-4}$,
(b) 1-d comparison of DIA, WRT and TSA,
(c) 2-d action density $n_i$,
(d) $S_{nl}(f,\theta)$ results from DIA (e) WRT (f) TSA.
$f_p=0.1$, $\alpha=0.0081$, $\sigma_A=0.07$, $\sigma_B=0.09$. 
WW3 – with ‘old’ ST1: fetch-limited growth

6 hr
WW3 – with ‘old’ ST1: fetch-limited growth

12 hr
WW3 – with ‘old’ ST1: fetch-limited growth

18 hr
WW3 – with ‘old’ ST1: fetch-limited growth

24 hr

DIA

FBI

TSA
WW3 – with ‘old’ ST1: fetch-limited growth

30 hr
WW3 – with ‘old’ ST1: fetch-limited growth

36 hr
WW3 – with ‘old’ ST1: fetch-limited growth

42 hr
WW3 – with ‘old’ ST1: fetch-limited growth
WW3 – with ‘old’ ST1: fetch-limited growth
1-point time integration

**WWM** (Roland et al., 2012) with **ST4**
From Ardhuin et al. (2010)

**WW3** (Tolman, 2009) with **ST2**
From Tolman + Chalikov (1996)
Ongoing issue: turning winds by 90° at 48 hr

WRT with ST1

original TSA with ST1
Multiple spectral peaks - mTSA

Broad-scale term parameterization…?

$$F(k)_{Norm} = F(k) \times k^{2.5} / \beta$$  
[Resio&Perrie, 1989; Resio et al. 2004…]

Should be $$\beta \sim 1/\Delta f \sum[F(k) \times k^{2.5}]_{\text{equilibrium range}}$$

**But** equilibrium range is hard to define when $$f_{p1}$$ and $$f_{p2}$$ are close…

So let $$\beta = F(k) \times k^{2.5} |_{f_s} \ldots$$  
for the first peak …
Ongoing issue: turning winds by 90° at 48 hr

WRT with ST1

New double-peak dTSA with ST1
Hurricane Juan (2003)

- Wave at 44258: $H_s \approx 9.2m$
- Winds at 44258: $\sim 27.5 \text{ m/s}$, at 2-3 AM UTC 29 Sept.
- Sea elevation at 44258: 1.9 m MSL
- 1.64 m at Point Pleasant Park
Hurricane Juan wave heights, $H_s$

DIA  
FBI  
*difference* DIA-FBI

~10%
Hurricane Juan wave heights, $H_s$

DIA: Sig. Wave Heights at 20030928 17

FBI: Sig. Wave Heights at 20030928 17

DIA - WRT: Sig. Wave Heights at 20030928 17

$\text{difference} \ DIA\text{-}FBI$

$\sim 15\%$
Hurricane Juan wave heights, $H_s$

DIA  

FBI  

difference DIA-FBI

$\sim 15\%$
Hurricane Juan wave heights, Hs

DIA

FBI

difference DIA-FBI

~15%
WW3 – constant U10 – 55 hr ‘old’ ST1

ST1 – wamCy3 physics
WW3 – constant U10 – 55 hr ‘new’ ST4

ST4 – ‘new’ Ardhuin et al. IFREMER physics
Computational time

- Alternate frequency, angles & interaction points can speed up TSA about 30-40 $\times$ or more
- Presently, best accurate results are 20 $\times$ DIA
- Optimization of TSA code is ongoing

DIA  WRT  dTSA,a
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