Environmental contour method: An approximate method for obtaining characteristic response extremes for design purposes

Sverre Haver & Kjersti Bruserud, Statoil ASA
Gro Sagli Baarholm, Det Norske Veritas
Rule requirements for characteristic design response

- Characteristic response, $x_c$, are specified by requirements regarding the annual probability, $q$, of exceeding the characteristic value.

- Ultimate limit state (ULS): $q \leq 10^{-2}$ (per year)

- Accidental limit state (ALS): $q \leq 10^{-4}$ (per year)
Sources of inherent randomness

• Long term variability of slowly varying weather characteristics, e.g. significant wave height, $H_s$, and spectral peak period, $T_p$.

Possible description: 

$$f_{H_s T_p}(h, t) = f_{H_s}(h) f_{T_p|H_s}(t|h)$$
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• Short term variability of 3-hour (or 30 minute) maximum given the weather condition, i.e.:

$$ F_{X_{3h}|H_sT_p}(x|h, t) $$
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\[ F_{X_{3h}|HsT_p}(x|h,t) \]

- Long term distribution of \( X_{3h} \):

\[
F_{X_{3h}}(x) = \int_h \int_t F_{X_{3h}|HsT_p}(x|h,t) f_{HsT_p}(h,t) dt dh
\]

Target response:

\[
x_q = F_{X_{3h}}^{-1} \left( 1 - \frac{q}{2920} \right)
\]
Environmental contour method

1. Determine contours from $f_{H_sT_p}(h, t)$.
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2. Find worst sea state along e.g. $10^{-2}$ - annual probability contour for response under consideration.

![Graph showing Hs versus Tp contour lines with data points and lines for different probability levels.]
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3. Establish distribution function for $X_{3h}$ for the worst sea state, i.e. design sea state (DSS):
   $$F_{X_{3h}|DSS}(x|DSS).$$
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4. Estimate $x_{0.01}$ by:

$$x_{0.01} = F_{X_{3h}|DSS}^{-1}(\alpha)$$

where typically $\alpha = 0.85 - 0.90$
Why should it work?

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\[ x_{0.0001} = 400 \] (we can think of this as the median response in a vary narrow extreme value distribution). Design sea state (DSS) is shown on contour.
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- \( x_{0.0001} = 400 \) (we can think of this as the median response in a very narrow extreme value distribution).

- In reality, the 3-hour extreme will be of an inherent random nature. The median will be too small. We have to go to a higher percentile. How high depends on the relative importance of the short term variability. Experiences indicate that this is rather similar for a broad range of problems. Good estimates are often obtained selecting the 0.90-0.95 fractile (for \( q = 10^{-4} \)).
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• For a typical response problem \( \text{cov} \) for \( X_{3h} \) is in the 0.1 – 0.25. \( \alpha = 0.85 - 0.9 \) often ok when \( q = 10^{-2} \).

For loads from breaking wave impacts, the \( \text{cov} \) of \( X_{3h} \) is 0.5 – 1.0 !!! Method may work – but one will most proably have to adopt high fractiles.

\( \Rightarrow \) A long term analysis is possibly to be preferred?
Challenge: Modelling $T_p$ conditionally on $H_s$

- $T_p$ given $H_s$ is assumed to follow a log-normal model, parameters are $\mu = E(\ln T_p | H_s)$ and $\sigma^2 = \text{Var}(\ln T_p | H_s)$.
Challenge: Modelling $T_p$ conditionally on $H_s$

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- Estimating $\mu$ is not to critical, but uncertainties are introduced.
Challenge: Modelling $T_p$ conditionally on $H_s$

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• Estimating $\mu$ is not to critical, but uncertainties are introduced.

• Estimating $\sigma^2$ outside range of data is a challenge!

\[
\bar{t}_p = \exp\{\mu + 0.5\sigma^2\}
\]

\[
\sigma_{T_p} = \bar{t}_p \sqrt{\exp\{\sigma^2\} - 1} \approx \bar{t}_p \sigma
\]
Uncertainties in standard deviation of $T_p$ given $H_s$
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We need:

* More data of extreme sea states (not so easy).

* Better understanding of accuracy of hindcast $T_p$. 
Consequence of spreading uncertainty
Part II: Can contour method be used in a GoM hurricane climate?

- We consider all hurricanes exceeding some threshold.
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• The basic response variable is hurricane maximum response, $Y$. This variable is carrying the short term variability, i.e.:

$$F_{Y \mid hurricane \ characteristics}(y \mid hurricane \ characteristics)$$
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\[ F_Y \mid \text{hurricane characteristics}(y \mid \text{hurricane characteristics}) \]

• When applying the environmental contour method we would characterize a hurricane (for the purpose of a analysis of wave induced response) by three parameters:

\[ H_{sp} = \text{maximum significant wave height of the storm}, \quad T_{pp} = \text{spectral peak period associated with } H_{sp} \quad \text{and } D_p = \text{duration of the most severe part of hurricane}. \]

These carry the long term variability:

\[ f_{H_{sp}T_{pp}D_p}(h, t, d) \quad \text{(In long term analysis these are replaced by } \bar{Y} \quad [\text{mpm of } Y].) \]
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\]

• If the two sources of inherent randomness have the same relative contribution to total variability for a broad range of response cases, the contour method may well be a useful approximate method for hurricane governed areas also.
Modelling of joint distribution of $H_{sp}$ and $T_{pp}$
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**Challenge:**

Limited amount of independent hurricane data within an area of say $1^\circ \times 1^\circ$
Contour & example results

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- If we artificially increase duration of peak event to 3 hours, target percentiles reduces to 0.75 – 0.80 about.

→ Short term variability is of somewhat less importance in GoM than in North Sea (as expected).
Background IV

(And it is clear why we need a percentile of $X_{3h} > 0.5$ ??)

1. 0.01 annual prob sphere
   $\beta = -\Phi(0.01/2920) = 4.5$
   $\Phi(4.5) = 0.9999966$

2. 100-year design point for response

3. 0.01 annual prob. contour for $U_1$ and $U_2$ combinations.

4. Projection of design point in $U_1$-$U_2$ plane

5. 0.01 annual prob. design sea state
Conclusions

Most important:

A too small amount of data of extreme weather conditions is the largest challenge!
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Sverre Haver
svha@statoil.com
Tel: +4748072026
www.statoil.com
Introduction to background got the environmental contour method I
Problem is transformed to u-space
(u-space consists of independent, standard Gaussian variables)
Background II

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\[ (h_s) \]
\[ (x_{3h}) \]
Background III

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