Simulating nearshore wave transformation with non-hydrostatic wave-flow modelling

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1 Introduction

Ever since the first successes, two decades ago, in reproducing the laboratory data on phase-resolved wave properties of interest in coastal regions and harbours with a set of evolution equations and empirical formulations, Boussinesq-type wave models have rapidly gained in popularity. The corresponding equations are vertically integrated equations for wave propagation in two horizontal dimensions with different assumptions made for the variation of fluid motion over the water depth. As such, they can be interpreted as extended shallow water equations including the lowest order effects of frequency dispersion and nonlinearity. In addition, they can resolve rapid variations that occur at scales of one wave length or lesser. This type of phase-resolved modelling emerged as a mature discipline and, in conjunction with suitable numerical techniques, became the most widely employed predictive tools in coastal engineering and in morphodynamics. Most well-known and well-established Boussinesq-type wave models are FUNWAVE (Kirby et al., 1998), BOUSS-2D (Nwogu and Demirbilek, 2001), Mike 21 BW (DHI Group, 2008) and COULWAVE (Lynett and Liu, 2002). Numerous researchers and users all over the world contributed to the testing, development and refinement of these tools.

In spite of a wide recognition of the general achievements in Boussinesq modelling of wave transformation over the past twenty years, there are some physical effects the modelling of which has not yet resulted in a comparable feeling of satisfaction in the coastal engineering community. Wave breaking and moving shoreline are perhaps the most prominent examples. Yet various attempts to incorporate these processes have produced only partial successes, usually at the expense of physical clarity and computing economy. For instance, energy dissipation due to wave breaking is modelled either by introducing an artificial viscosity term into the Boussinesq equations; see Zelt (1991), Karambas and Koutitas (1992) and Kennedy et al. (2000) or by using the concept of the surface rollers as described in Schäffer et al. (1993) and Madsen et al. (1997). For the calculation of wave runup on the beach, use of moving boundary conditions is required. Several numerical strategies have been proposed for a proper representation of the interface of water and land. We refer to Brocchini et al. (2002) for an overview on this subject. Permeable-bed techniques, like the one described in Madsen et al. (1997), are a very popular approach for simulating the moving shoreline. They employ the notion of artificial porosity to allow a more gradual transition between dry and wet areas.

Yet their modelling still poses difficulties because of an apparent need to employ empirical formulations and numerical schemes of much greater complexity than had been used so far to model other wave processes such as dispersion, shoaling, refraction and diffraction. These difficulties include uncertainty on choice of empirical parameters, complexity of implementation, reduced numerical stability and robustness, high computational costs, and the need for greater physical accuracy. For instance, the principal constraint inherent in Boussinesq-type wave models is the unability to conserve momentum because of the physically unclear higher order approximations due to frequency dispersion and nonlinearity involved. As

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a consequence, it may not be guaranteed that the wave properties during breaking are modelled correctly. Yet these models contain a trigger that provide the onset of breaking and some memory effects during post-breaking, by which a calibration of some tunable parameters inherent in the artificial viscosity and roller concepts is required. Because of these parameters involved are almost invariably determined in the context of relatively simple wave settings, e.g. breaking of regular waves on uniformly sloping bed, these wave breaking models cannot be applicable to a wide range of wave conditions. These models are thus extrapolated to conditions far removed from the original configuration, with consequent and inevitable loss of realism and accuracy. Not surprisingly, their performances are of varying quality; see e.g. Madsen et al. (1997), Kennedy et al. (2000), Veeramony and Svendsen (2000), Musumeci et al. (2005), Lynett (2006), Roeber et al. (2010) and Wenneker et al. (2011). Also, the permeable-bed approach introduces some numerical stability problems of which the loss of mass is often the cause (Kennedy et al., 2000; Brocchini et al., 2002; Lynett et al., 2002).

Despite the recognition that both artificial viscosity and surface roller concepts have been shown to display serious weaknesses, in terms of physics and numerics, the reality is that the large majority of Boussinesq-type model computations have been and are still being performed with these popular wave breaking concepts. A reason for this apparent contradiction is the fact that numerous engineers and front-end users have conceded to the charm of an acceptable predictive ability of these concepts. Indeed, some of their results look very impressive with predicted details of both mean wave parameters and wave statistics. Less tangible but of equal importance is the perception that inadequate modelling merely degrades the quantitative value of the solution but does not prevent a solution from being obtained. Nevertheless, researchers and scientists feel that a dose of caution is still often advisable and that not all of the computed results are to be fully trusted.

Still the question remains as to whether there is an alternative, economically tolerable, modelling strategy which does account wave breaking and wave runup merely based on some fundamentally physical principles. The present paper focuses on the use of a different approach for wave transformation in coastal areas, namely non-hydrostatic wave-flow modelling. Basically, the simulation of broken waves and wave runup amounts to the solution of the nonlinear shallow water (NLSW hereinafter) equations for free-surface flow in a depth-integrated form (Hibberd and Peregrine, 1979; Brocchini and Peregrine, 1996). The NLSW equations are mathematically equivalent to the Euler equations for compressible flows. Discontinuities are admitted through the weak form of these equations and can take the form of bores which are the hydraulic equivalent of shock waves in aerodynamics. The conservation of energy does not hold across the discontinuities but the conservation of mass and momentum remains valid. By considering the similarity between broken waves and steady bores, energy dissipation due to turbulence generated by wave breaking is inherently accounted for. In the pre-breaking region, however, the NLSW equations do not hold as they assume a hydrostatic pressure distribution, and thus prohibit a correct modelling of dispersive waves. However, by extending the NLSW equations to include the effect of vertical acceleration, the propagation of those waves can be simulated. Basically, such a non-hydrostatic wave-flow model represents a good balance between nonlinearity (enables wave shoaling) and frequency dispersion (corrects celerity of shoaling wave) so that the process of incipient wave breaking and the associated energy losses can be described adequately. However, at the front face of the breaking wave, the effects of vertical acceleration must be neglected so that the front face will steepen continuously until the front becomes vertical. In any case, it is evident that the numerical schemes involved must treat shock propagation adequately in order to capture the propagation bores.

Non-hydrostatic wave-flow models are gaining recognition as to be evolved out of a wish to achieve a compromise between the capabilities of the Boussinesq-type wave models and operational-based requirements for numerical robustness, simplicity, ease of use and economy. These models are still being explored, refined and validated but are likely to remain the most appropriate route to simulate surf zone dynamics for some years to come.

Over the past ten years, strong efforts have been made at Delft University to advance the state of wave modelling and flooding simulations for coastal engineering applications. These efforts have focused on developing and validating the newly developed non-hydrostatic model SWASH (an acronym of Simulating WAves till SHore) (Zijlema et al., 2011). This open source code (http://swash.sourceforge.net) is intended to be used for predicting transformation of surface waves and rapidly varied shallow water flows in coastal waters.

In what follows, Section 2 summarises essential model aspects and some numerical issues featuring in SWASH. In Section 3, a few application examples are then presented, reflecting both capabilities and performances. Finally, Section 4 concludes the paper with some closing remarks.

2 SWASH: a non-hydrostatic wave-flow model

SWASH takes as its starting point the incompresssible Navier-Stokes equations or Euler equations for the computation of the surface elevation and currents. In fact, these equations can be regarded as NLSW equations including the effect of vertical acceleration. For the present purpose of outlining the principles adopted, the precise form of the governing equations is irrelevant. However, one is refer to Zijlema and Stelling (2005) and Zijlema et al. (2011) for details. Also, details on the imposition of the boundary conditions can be found in Zijlema et al. (2011).

The numerical framework of SWASH has been extensively presented and discussed in Zijlema and Stelling (2005) and Zijlema et al. (2011). In this section, a brief outline of some numerical procedures relevant to the surf zone applications is given.

SWASH employs an explicit, second order finite difference method for staggered grids. This framework is the most natural and advantageous basis for advanced wave modelling in coastal areas. In addition, a discretized form of the NLSW equations can automatically be shock-capturing *if* the momentum conservation is retained in the finite difference scheme. The principle of this approach, as well as its underlying rationale are documented in Stelling and Duinmeijer (2003), Zijlema and Stelling (2008) and Zijlema et al. (2011).

In the vertical, the computational domain can be divided into a fixed number of terrain-following layers, the so-called multi-layered case. Space discretization in the vertical direction is carried out in a finite volume fashion. For details, see Zijlema and Stelling (2005).

With respect to time integration of the continuity and horizontal momentum equations, the second order leapfrog scheme is adopted so that the wave amplitude will not altered. This scheme requires less storage compared to the classical leapfrog scheme and makes the algorithm easy to implement. The MacCormack predictor-corrector technique is employed in order to retain second order accuracy in time for the advection terms in the horizontal momentum equations.

Local mass continuity is enforced by solving a Poisson equation for the pressure correction which steers the non-hydrostatic pressure towards a state at which all mass residuals in the active grid cells become negligible small, reflecting a satisfaction of local mass conservation. Global mass conservation is obtained by solving a depth-averaged continuity equation for the solution of the surface elevation.

In order to resolve the frequency dispersion up to an acceptable level of accuracy using as few layers as possible, a technique as proposed in Stelling and Zijlema (2003) is used that is tailored to coastal wave propagation. It is based on a compact Keller-box difference scheme for the approximation of vertical gradient of the non-hydrostatic pressure. A main advantage of this scheme is to provide a much better representation of the shorter length scales of solution compared to the classical finite difference schemes. This better representation can be attributed to the implicit nature of the box scheme which makes such a scheme more accurate than standard schemes on relatively coarse grids even with the same order of accuracy. As a result, accurate propagation of *progressive* waves for $kd \leq 3$ (k is the wave number, d is the still water depth) can be obtained with just two equidistant layers.

The adopted momentum-conservative scheme is able to track the actual location of incipient wave breaking accurately, but generally underestimates the subsequent dissipation of wave energy. This is due, principally, to the excessive levels of non-hydrostatic pressure gradient at the front of the breaking wave, preventing the wave from transitioning into the characteristic saw-tooth shape often observed in the surf zone. This defect is rooted in the model's representation of the free surface as a streamline. It requires that particles at the free surface undergo rapid vertical acceleration as the wave front passes, and in turn necessitates increasingly large pressure gradient in the vertical. A simple approach is to locally impose a hydrostatic pressure distribution under the wave front. By doing this, the bore-like wave front steepens continuously until it becomes vertical. Subsequently, the broken wave propagates with a correct gradual change of form and resembles a steady bore in a final stage. This leads to a correct amount of energy dissipation on the front face of the breaking wave. Moreover, intra-phase properties such as asymmetry and skewness are preserved as well. In practice, this very simple treatment of wave breaking does not require any additional calibration.

Finally, a very simple wet-dry approach as treated in Stelling and Duinmeijer (2003) is adopted. This method tracks the motion of the shoreline very accurately without posing numerical instabilities by ensuring non-negative water depths and using the upwind water depths in the momentum flux approximations.

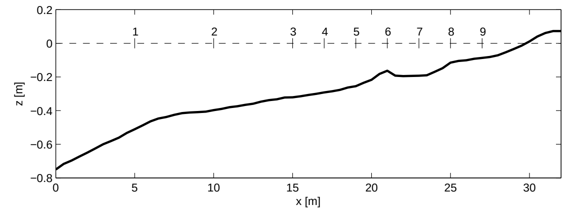

Figure 1: Bottom topography and location of wave gauges of the experiment of Boers (1996).

3 Applications

This section presents results for two cases, one with cross-shore motions of irregular breaking waves and the other with nearshore circulations induced by breaking waves. The selection of the two cases was motivated, on the one hand, by the wish to investigate the phenomenon of wave breaking. On the other hand, the selected cases are amongst very few realistic conditions which there are extensive data, allowing a quantitatively reliable assessment of the performance of wave breaker models.

3.1 Irregular wave breaking in a laboratory barred surf zone

Often, breaking of regular waves on planar beaches has been validated using the many available Boussinesq-type wave models with varying success; see Madsen et al. (1997), Kennedy et al. (2000), Musumeci et al. (2005), Lynett (2006), Dimas and Dimakapoulos (2009), Tonelli and Petti (2009), Roeber et al. (2010) and Cienfuegos et al. (2010). In this regard, the physical configuration of Ting and Kirby (1994) is a very popular benchmark test for studying the performance of the associated wave breaker models.

To our knowledge, no Boussinesq-type wave model has been validated against irregular waves propagating over a barred beach profile, except the one as discussed in Wenneker et al. (2011). They have carried out a validation study of the laboratory flume test of Boers (1996) in which random, unidirectional waves propagate towards a bar-trough beach profile that was adopted from a barred sandy beach; see Figure 1. The Boussinesq-type wave model of Wenneker et al. (2011) employs a semi-empirical wave breaker model which is a combination of the eddy viscosity and roller concepts. The latter concept is similar to that of Schäffer et al. (1993). This wave breaker model appears to be rather complicated and has no less than 5 calibration parameters. For instance, the manner in which the breaker delay has been implemented seems to be improvised and does not rely on physical grounds. These related defects including *ad hoc* modifications are all rooted in the model's fundamental inability to account for momentum conservation. This fact is probably responsible for the poor prediction of both the wave height and wave setup in the surf zone as demonstrated in Figure 9 of Wenneker et al. (2011). Indeed, their model underpredicted the extent of the surf zone resulting in modest energy dissipation and excessive overestimation of the wave height at the onset near the first bar. It should be noted that the wave heights in the shoaling region were represented reasonably well. In addition, the Boussinesq model did not reproduce the major features of the spectral evolution through the flume. In particular, the bound infragravity wave peak was underestimated while, from the breaker bar and further, the high-frequency energy was substantially overestimated. This latter suggests that the used wave breaker model affected negatively the overall surf zone budget.

Results obtained with SWASH will be presented for the related flume test of Boers (1996). At the offshore boundary, an irregular wave was imposed with the significant wave height of 0.206 m and the peak period of 2.03 s. This wave field is energetic and has a relatively high mean steepness. The grid size was set to 0.02 m and only one layer was chosen. The time step was initially taken as 0.001 s, while the maximum Courant number was set to 0.5. The simulation time was set to 1700 s. No calibration nor tuning has been carried out in the course of simulation.

In Figure 2, spectral comparisons with the numerical and laboratory data are made. The spatial evolution of the wave spectra is characterized by an amplification of spectral levels at both sub- and superharmonic ranges, consistent with three-wave interaction rules, followed by a transformation toward a broad spectral shape in the surf zone, attributed to the nonlinear couplings and dissipation. The model captures the dominant features of the

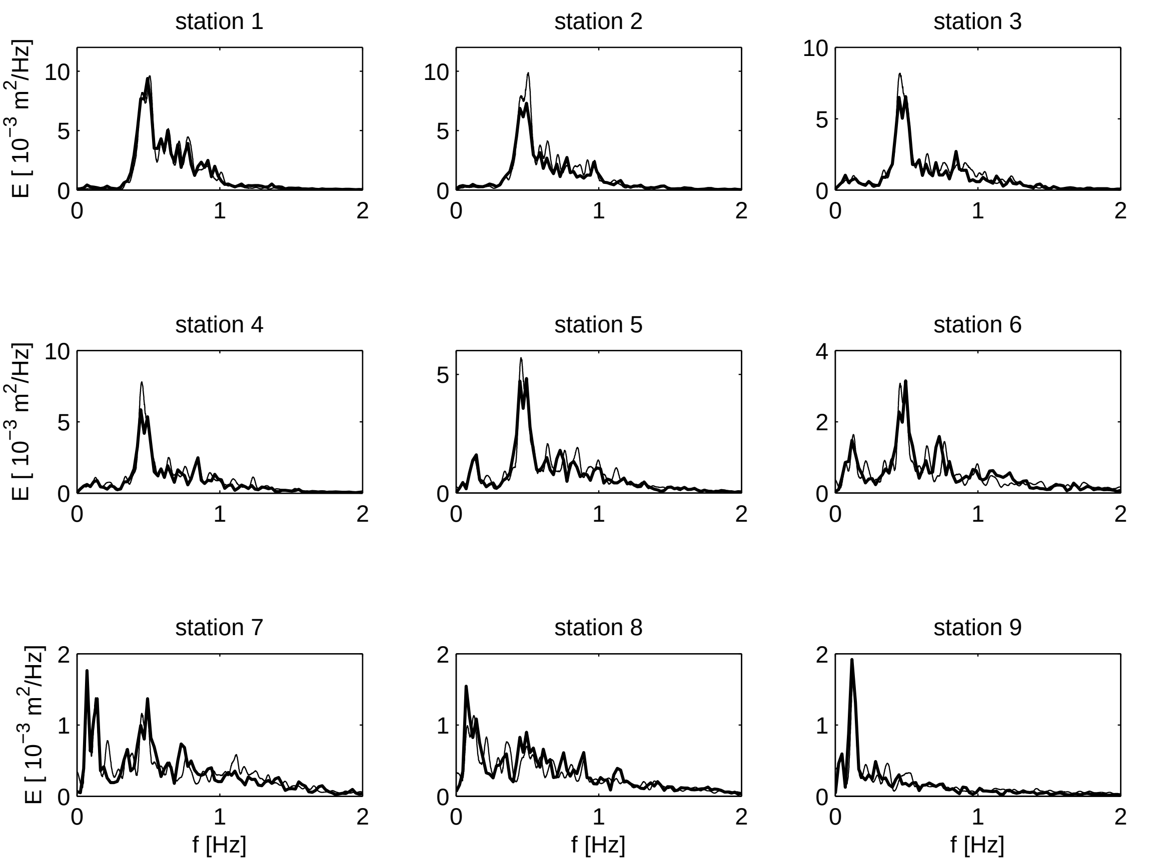

Figure 2: Observed (thick line) and predicted (thin line) energy density spectra of shoreward propagating waves for the Boers (1996) case.

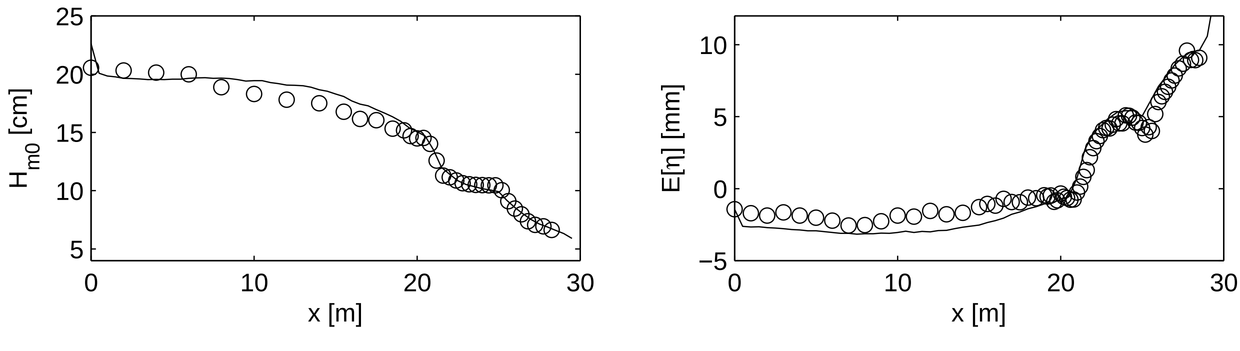

Figure 3: Computed and measured significant wave height (left panel) and wave setup (right panel) along the flume for the case of Boers (1996). Circles: laboratory data; solid line: model.

attendant spectral evolution, both in the shoaling region and the surf zone. In terms of the wave height and wave setup, Figure 3 shows that the quantitative trends are much well resolved by the model. Clearly, a strong and localized dissipation of energy has been taken place around the bars. The model prediction of both the onset of the breaking process and the amount of energy dissipation is excellent.

At the shoreline the wetting and drying algorithm has been applied to mimick the runup of short waves and reflection of infragravity waves at the beach. An in-depth analysis (not shown in this paper) revealed that the transformation of the infragravity waves is modelled in good agreement with the measurements. An incoming infragravity wave is generated, which is out of phase with the forcing wave groups. This incoming bound infragravity wave propagates in shoreward direction with the group velocity and is reflected at the shoreline. This reflected free infragravity wave propagates with the phase velocity in seaward direction. These results are in line with the generation and transformation of the infragravity waves according to the time-varying breakpoint location mechanism (Symonds et al., 1982). Also, the reflection of the incoming infragravity waves appears to be frequency dependent, i.e. the wave height of relative long infragravity waves is larger than that of shorter infragravity waves, in conformity with the measurements.

3.2 Multi-directional waves propagating through a barred basin

Few experiments related to breaking of short-crested waves in the surf zone under laboratory and field conditions have been carried out and, in addition, there is a few available numerical studies on this matter. Examples can be found in Sørensen et al. (1998) and Chen et al. (2000). There might be a good reason for supposing that both artificial viscosity and roller concepts are ill suited to two-dimensional hydrodynamic conditions which are characterized by nearshore circulations induced by wave breaking. This brings us to the question of how well the performance of SWASH might be with respect to this type of applications.

The second case considered concerns the propagation of directionally spread waves propagated over a submerged semi-cylindrical bar in a basin. Extensive and very detailed laboratory data for wave parameters and wave spectra have been obtained by Dingemans et al. (1986). The directional wave

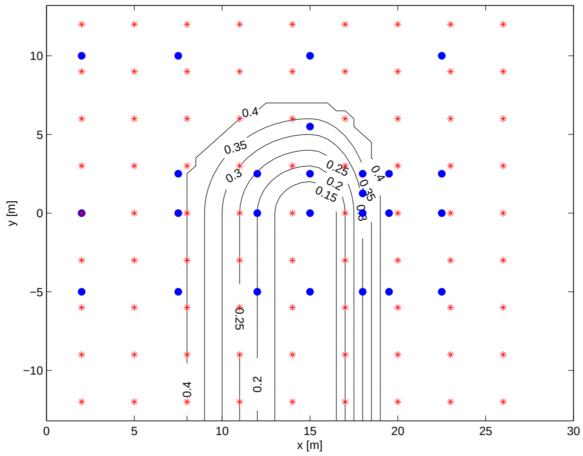

Figure 4: Contour plot of the bathymetry of directional wave basin (Dingemans et al., 1986), wave gauge locations indicated by blue filled circles and current meter locations indicated by red stars.

basin is 26.4 m wide, along which a wave maker is placed, and 35 m long with onshore a beach starting at x =30 m with a slope of 1:7. The lateral boundaries are closed. The bed of the basin is flat and a submerged, semi-cylindrical bar of 20.2 m long and 11 m wide is placed parallel to the wave board. The center of the tip of the bar is located at $(x, y) = (15, 0)$. The still water depth is 0.4 m at the deepest part and 0.1 m at the top of the bar. The bathymetry is depicted in Figure 4.

The wave input condition is characterized by a Jonswap spectrum with significant wave height of 0.1 m, peak period of 1.24 s and directional spread with a $\cos^4(\theta)$ distribution. The mean wave direction is more or less perpendicular to the seaward boundary.

The grid spacing was $\Delta x = 0.05$ m and $\Delta y = 0.03$ m, the initial time step was 0.005 s with a maximum Courant number of 0.7 and the simulation period was 1920 s, which is long enough to get a steady-state condition. To optimise wave dispersion, two equidistant layers were taken. To get a realistic wave-induced flow pattern, the classical Smagorinsky-type subgrid model was utilized.

Figure 5 shows the variation of the significant wave height along four cross-shore transects as indicated in Figure 4. Obviously, model-predicted heights agree very well with the measurements and wave breaking is adequately captured by the model. The less performance at section $y = 10$ m is probably due to some reflection of the spread waves against the lateral boundary, $y = 13.2$ m, in the measure-

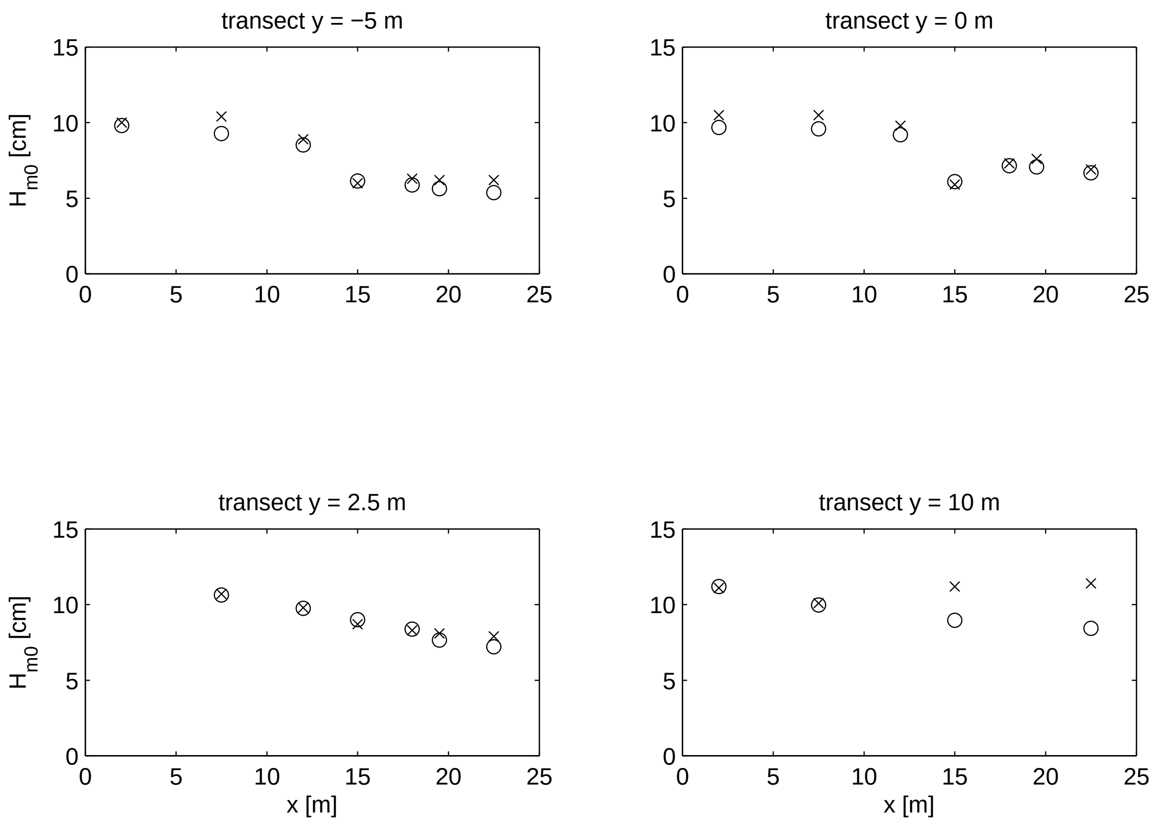

Figure 5: Comparison between the computed (circles) and measured (crosses) cross-shore variation of the significant wave height along different transects for the case of Dingemans et al. (1986).

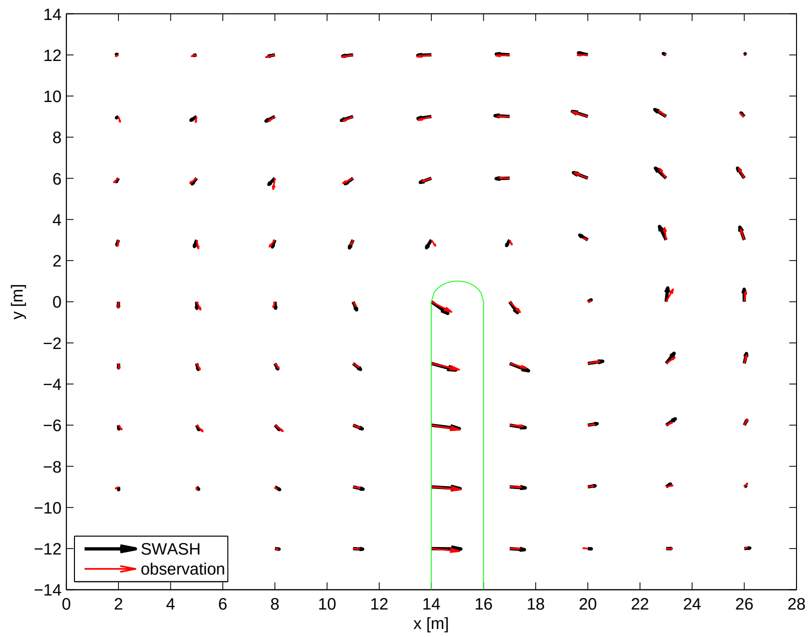

Figure 6: Comparison between the computed and measured depth-averaged mean current field around the bar.

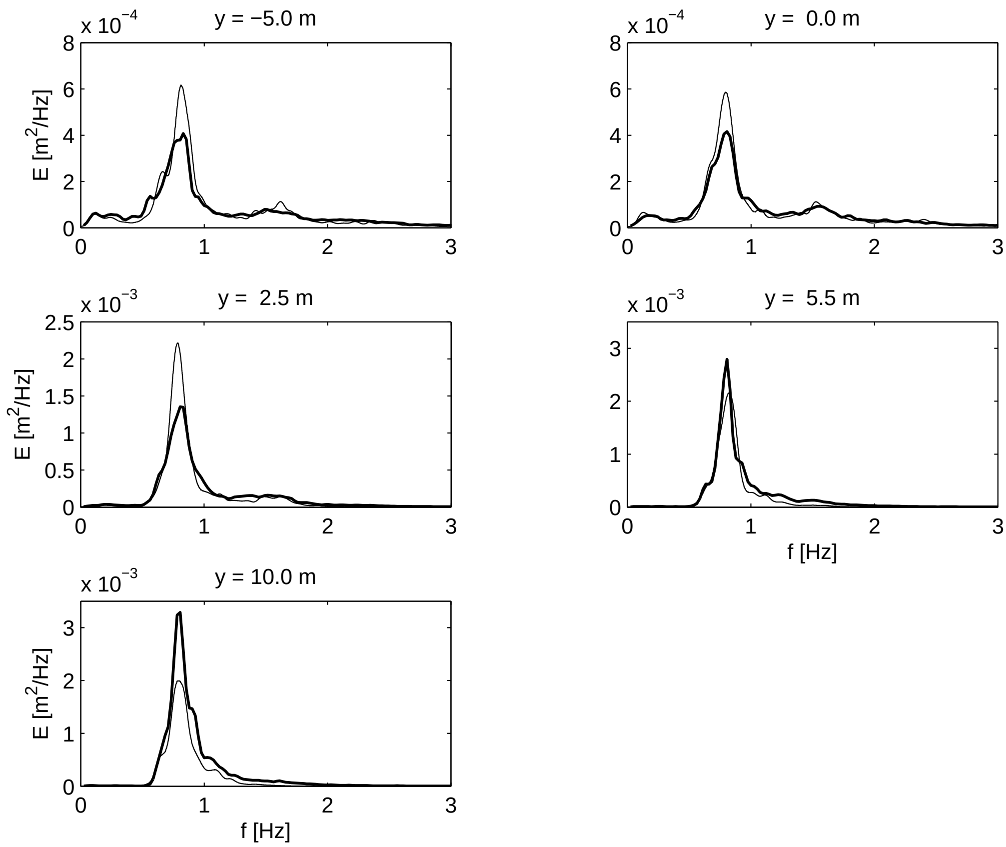

Figure 7: Computed (thin line) and measured (thick line) energy density spectra at different sites along transect $x = 15$ m for the case of Dingemans et al. (1986).

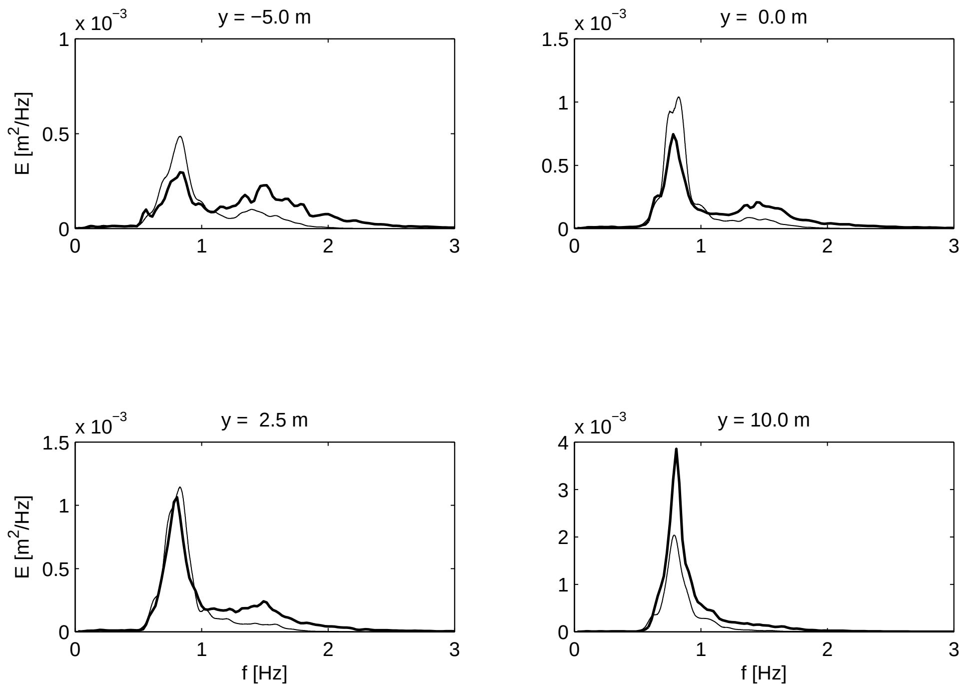

Figure 8: Computed (thin line) and measured (thick line) energy density spectra at different sites along transect $x = 22.5$ m for the case of Dingemans et al. (1986).

ments as indicated by Dingemans et al. (1986).

A vector plot of the steady state depth-averaged mean velocity pattern is displayed in Figure 6. The current measurements of Dingemans et al. (1986) at locations indicated in Figure 4 are included for comparison and the agreement is excellent. As expected, a strong current is seen along the top of the bar.

Figures 7 and 8 present the comparison of computed and measured wave spectra at different sites along the centerline of the bar ($x = 15$ m) and a long-shore transect behind the bar ($x = 22.5$ m), respectively. The model-predicted spectral evolution is generally in line with the observations. For instance, the generation of both sub- and super-harmonics at the top of the bar (sites $y = -5$ m and $y = 0$ m) is distinctly observed. However, some discrepancies between computed and measured energy density are clearly visible, in particular at those sites lying close to the beach. This might be due to some imperfections in the supposedly fully absorbing beach and so, some reflection effects occured in the computed results near the beach. This is also the case for observed data as reported in Dingemans et al. (1986). Hence, the differences observed herein reflect model schematization and are not contaminated by modelling errors. A similar conclusion can be drawn for the sites located near the northern boundary ($y = 10$ m) where the influence of reflection and spreading effects is evident.

4 Concluding remarks

At present, the two primarily model approaches for the simulation of wave transformation in coastal areas at any time and space scale are Boussinesq-type wave and non-hydrostatic wave-flow models. Although Boussinesq-type wave models have become being widely accepted as standard tools for a long time, this paper argued the preference for the latter, especially in respect of capturing effects arising from wave breaking. For instance, the intuitive nature of both artificial viscosity and roller concepts for modelling wave breaking does not only make them weak parts in the Boussinesq-type wave models, but also makes it difficult to introduce rationally and physically sound modifications to cure some defects. One such defect is the lack of momentum conservation, which is a prerequisite for a reliable representation of wave breaking in the surf zone. Non-hydrostatic wave-flow models, on the other hand, offer a sounder framework for which momentum conservation can be properly accounted for. Indeed, the mathematical modelling of propagation of broken waves and steady bores and its related numerical issues have provided a powerful argument for employing non-hydrostatic models, and this route is likely to be followed in the near future. For this reason alone, they deserve continuing study, improvement, validation and application.

At Delft University, a non-hydrostatic wave-flow model named SWASH has been developed for predicting transformation of surface waves in coastal waters. The principal aim of this model is to provide a route for computing the surface elevation and currents to a satisfactory degree of accuracy over a broad spectrum of wave processes in both surf and swash zones. What constitutes *satisfactory accuracy* or a *broad* spectrum of applicability differs, of course, from one field of application to another. Nevertheless, there is intrinsic to non-hydrostatic wave-flow modelling the idea of a set of equations incorporating simple but fundamentally physical principles, applicable to a diversity of wave phenomena without *ad hoc* intervention by the model developer. As such, SWASH is a practical engineering tool suitable for a wide range of simulation scenarios in shallow and intermediate water.

Two realistic cases have been chosen for demonstration and validation purposes. Of particular interest is the second case, directionally spread waves propagating through a barred basin, in which detailed measurements, including wave-driven currents, are available. This permits the performance of wave breaking model to be assessed in considerable depth. SWASH has been found to be particularly appropriate and beneficial when the coastal environment is dominated by wave breaking while wave-induced circulation is involved. The encouraging point is that the physically accurate results are obtained without the aid of empirical formulations and do not require any additional calibration. SWASH is inherently able to account for wave breaking, which leads automatically to a correct amount of dissipation of wave energy. The results reported for the first test case serve to illustrate that SWASH indeed provides a superior representation of wave breaking to that returned by the Boussinesq-type wave models.

Further developments of inclusion of some physics, like the vertical motion of undertow, and further refinements of numerical techniques aimed at improving the computing efficiency (see e.g. Bai and

Cheung, 2011) will create in the coming years a favorable environment for further advancements in non-hydrostatic wave-flow modelling, open new prospects and give new impetus for a wider application of this type of modelling in coastal engineering and morphodynamics. The open source SWASH model (http://swash.sf.net) provides the opportunity to make this possible.

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