

# Two-Scale Approximation for Full Boltzmann Integral in Turning Winds

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## Abstract

We describe the implementation of TSA in WAVEWATCHIII<sup>TM</sup> and the modifications, comparisons, and test results that were done to get it to work reasonably well. Implementation involved construction of a diagonal matrix for implicit integration in the propagation of waves, following the approach used in the original DIA (discrete interaction approximation) implementation in the WAM model. Modifications involved consideration of complex spectra, typical of field data, with turning winds, or propagating storm-generated waves. Tests included hypothetical fetch- and duration-limited wave growth, and simple ocean geometry cases, with constant winds. Real data cases are simulations of hurricane Juan (2003). Results show that TSA implemented within WW3 can perform competitively, compared to DIA and WRT.

## Introduction

The Two-Scale Approximation (TSA) to the Full Boltzmann Integral (FBI) solution for wave-wave interactions in wind-driven seas decomposes directional spectra into two parts, a broad-scale form (parametric - with a limited number of degrees of freedom) and a superposed local-scale (non-parametric - which retains all of the degrees of freedom in a modeled directional spectrum). For the broad-scale portion of the spectrum, such an approximation utilizes a discrete set of parameter values in the approximation. How many parameters are needed for an accurate approximation for operational purposes? This becomes important for simulations of complicated spectra, for example, waves generated by a hurricane.

Previously, we considered parameterizations for directional spectra using analyses of observed directional spectra from selected field experiments, and found that much of the directionally-integrated characteristics as well as the directional characteristics can be represented well with a small number of dimensions. However, the case when winds are changing, or when winds rapidly shift direction, as when a front passes a given location, is different. We provide a detailed analysis of the passage of a front whereby winds blow for 48 hr in one direction and then suddenly turn by 90° for a following 48hr. In these simulations TSA is implemented within WAVEWATCHIII<sup>TM</sup> (version 3.14, hereafter WW3), and compared to WW3 implementations of DIA (discrete interaction approximation) and WRT (Webb-Resio-Tracy), the full Boltzmann integral. In these simulations we also use standard source terms for wind input and dissipation, for example as prescribed by WAMDIG (1988), and Tolman and Chalikov (1996).

In these tests, new windsea spectra are generated with dominant directions that may differ notably from the old windsea spectra, which then become swell. The standard TSA formulation of Resio and Perrie (2008) and Perrie and Resio (2009) cannot accommodate this situation. The problem is that the swell portion of the spectrum dominates the broad-scale term and thus the growing windsea has to be represented by the new local-scale, even as it becomes relatively quite large. Thus the TSA formulation fails to turn the wave spectrum rapidly, until the windsea finally exceeds the swell, leading to a very long lag in the model formulation's ability to turn the wave spectrum in the direction of the new windsea, whereas with DIA, or WRT, the waves would turn more rapidly with the wind. To solve this problem, we apply the broad-scale twice, once it is determined that a second distinct peak is present in the spectrum, and for each broad-scale term, we judiciously parameterize the high-frequency Phillips' – type coefficient so that it is consistent with Resio and Perrie (1989; 1991), and Long and Resio (2007). In this manner, we show that TSA can be generalized to provide accurate simulations, consistent with observations, and with WRT results.

As an additional test we use the new formulation of TSA (implemented in WW3) to simulate waves generated during hurricane Juan, which made landfall in Halifax in 2003 as a category 2 storm. At buoy 44258 at the mouth of Halifax Harbour, maximum significant waves were ~10m, with winds of ~27.5 m/s. Tests with observed data show that TSA performs well compared to DIA and WRT.

## 2. Model description

TSA has been given a detailed description by Resio and Perrie (2008) and Perrie and Resio (2009). Implementation of TSA in WW3 follows the approach given in WAMDIG (1988). As in WAM, a semi-implicit integration scheme is used. The change in action density  $N(k, \theta)$  is:

$$\Delta N(k, \theta) = \frac{S(k, \theta)}{1 - \varepsilon D(k, \theta) \Delta t} \quad (1)$$

where  $S(k, \theta)$  represents the nonlinear transfer due to wave-wave interactions, as approximated by TSA,  $t$  is time,  $\varepsilon$  is a parameter set to unity, and  $D(k, \theta)$  is the diagonal term, computed with the full Boltzmann integral, using the substitution  $N = \hat{n} + n'$  for broad-scale and local-scale terms for action densities of all the interacting wavenumbers, and neglecting  $n_2$  and  $n_4$  as suggested in the usual TSA formulation.

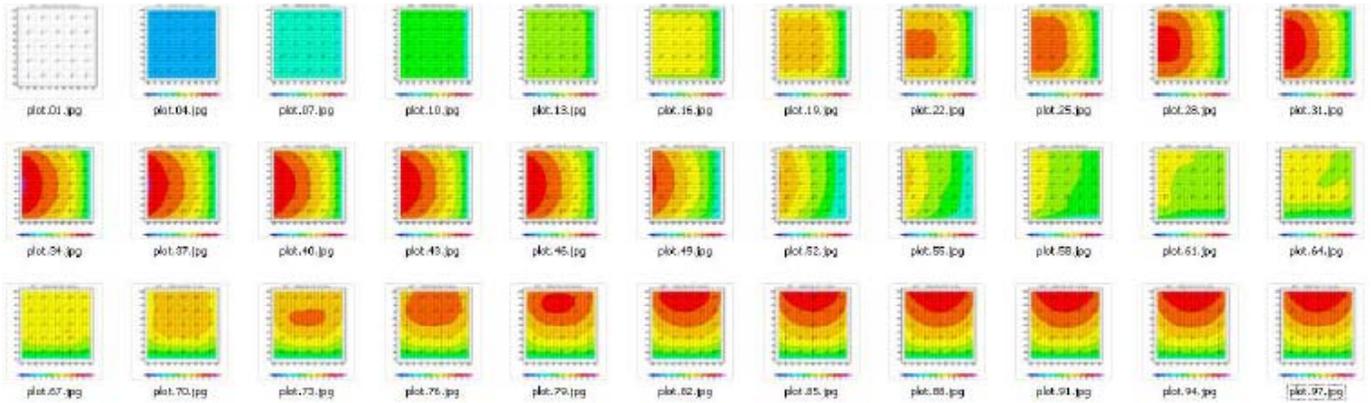
When the wind changes direction, new waves are generated in directions different from the dominant wave direction prior to the wind shift. Thus multiple spectral peaks may be generated. To accommodate this case, the TSA formulation of Resio and Perrie (2008) needs to be modified. The proposed methodology is that separation frequencies are defined between successive peaks, and multiple successive fittings of the broad-scale parameterization are made to the multiple spectral peaks. The Phillips-type coefficient, denoted  $\beta$  by Long and Resio (2007), is modified so that it can be reasonably estimated in constrained equilibrium ranges, in the case of multiple peaked spectra. Alternate parameterizations using a

JONSWAP-type broad-scale formulation were also explored to ensure accurate computations, and to keep the local-scale term, relatively small.

### 3 Experiment Tests

#### (a) Hypothetical tests

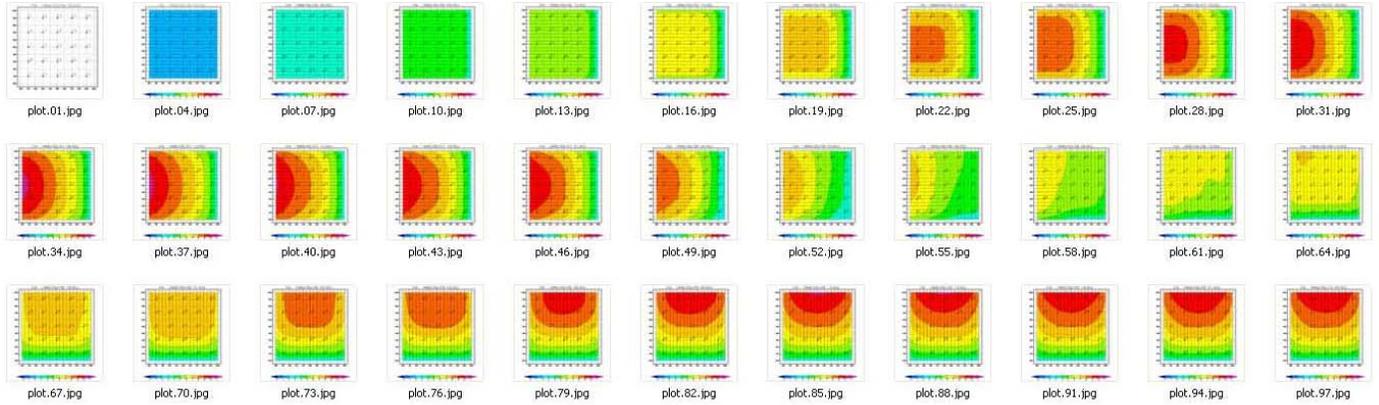
In these tests, we assume a simple square deep ocean with initial constant easterly  $U_{10}$  (towards *negative* x-axis) of 20 m/s for 48 hr, then suddenly shifted directions to become southerly winds for another 48 hr. As a baseline for comparisons, nonlinear wave-wave interactions are calculated using WRT. Usual approximations from WAMDIG (1988), and also from Tolman and Chalikov (1996), are used for wind input  $S_{in}$ , and wave dissipation,  $S_{ds}$ . Resulting significant heights (Hs), are shown in Figure 1. This shows that Hs reaches dynamic equilibrium before 48 hr. When the wind shifts direction by  $90^\circ$ , Hs decreases rapidly before again reaching equilibrium consistent with the southerly winds.



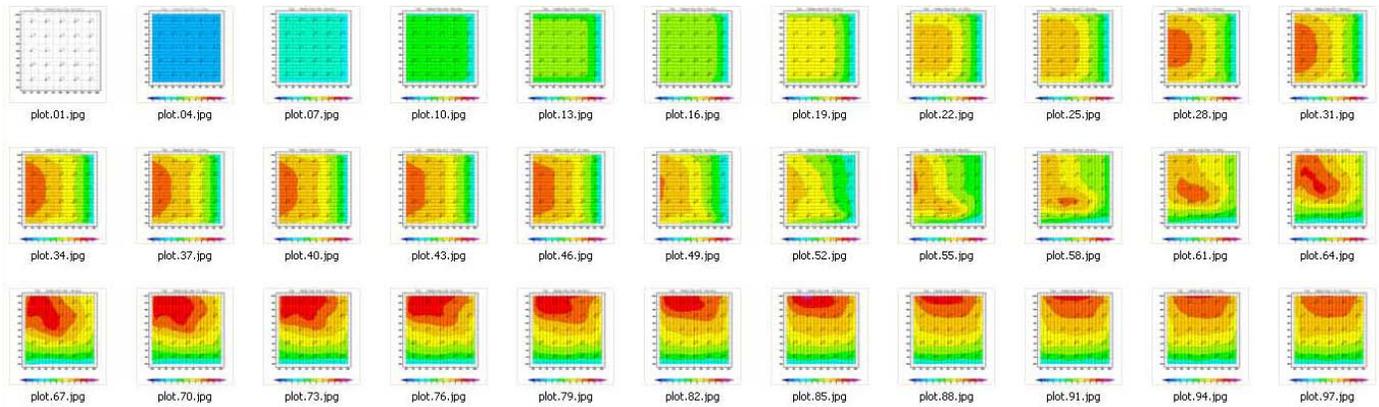
**Figure 1.** Significant wave height Hs, as a function of time, showing 3-hourly plots for square ( $10^3 \times 10^3$  km<sup>2</sup>) ocean using WRT in WW3 with initial constant easterly wind  $U_{10}$  (towards negative x-axis) of 20 m/s for 48 hr, then shifting to southerly (towards positive y-axis) winds for 48 hr.

For comparison, results using DIA are shown in Figure 2. Overall, DIA results are similar to those for WRT in Figure 1, showing that Hs reaches an equilibrium before 48 hr. When the wind shifts direction by  $90^\circ$ , Hs decreases rapidly before again reaching equilibrium, consistent with the southerly winds.

Results for the original TSA are given in Figure 3. What is different is the ridge of large Hs values that form soon after the wind direction shifts, in plots ~55 – 70, not seen in either DIA or WRT results in Figures 1-2. This suggests that the broad scale term of the original TSA formulation is not able to handle the evolving windsea – swell developments that occur in this case of  $90^\circ$  shift in wind directions. The broad-scale continues to parameterize the old portion of the spectrum, and thus the local-scale term is required to account for an increasing amount of spectral energy generated by winds in the new wind direction.

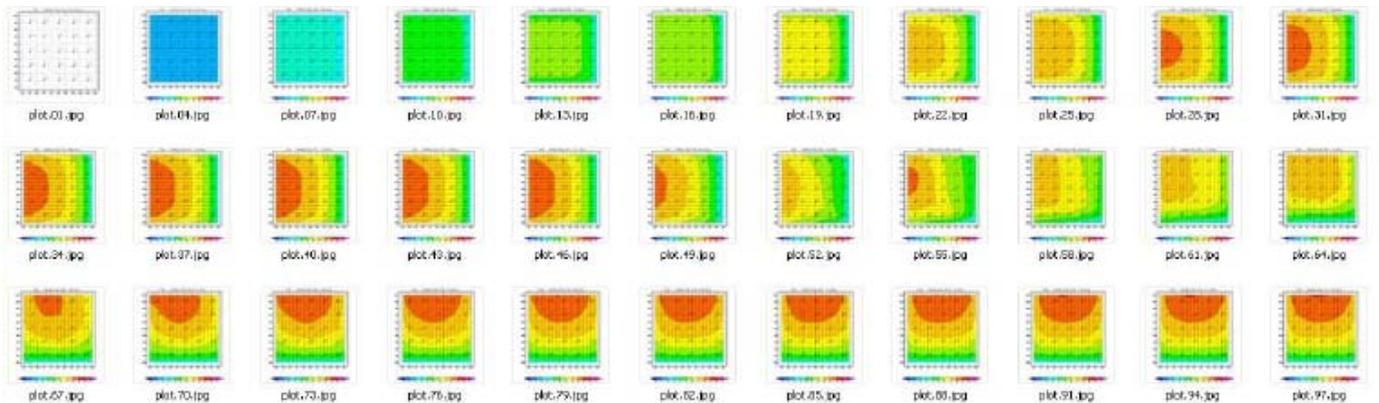


**Figure 2.** As in Figure 1 showing results using DIA implemented in WW3.



**Figure 3.** As in Figure 1 showing results using original TSA.

Results using the new TSA for multiple spectral peaks are given in Figure 4, following the methodology described in the previous section. This shows that we can correct the biases inherent in the standard TSA formulation results (Figure 3) and get Hs plots that have similarity to those of WRT in Figure 1.



**Figure 4.** As in Figure 2 showing results from new TSA formulation.

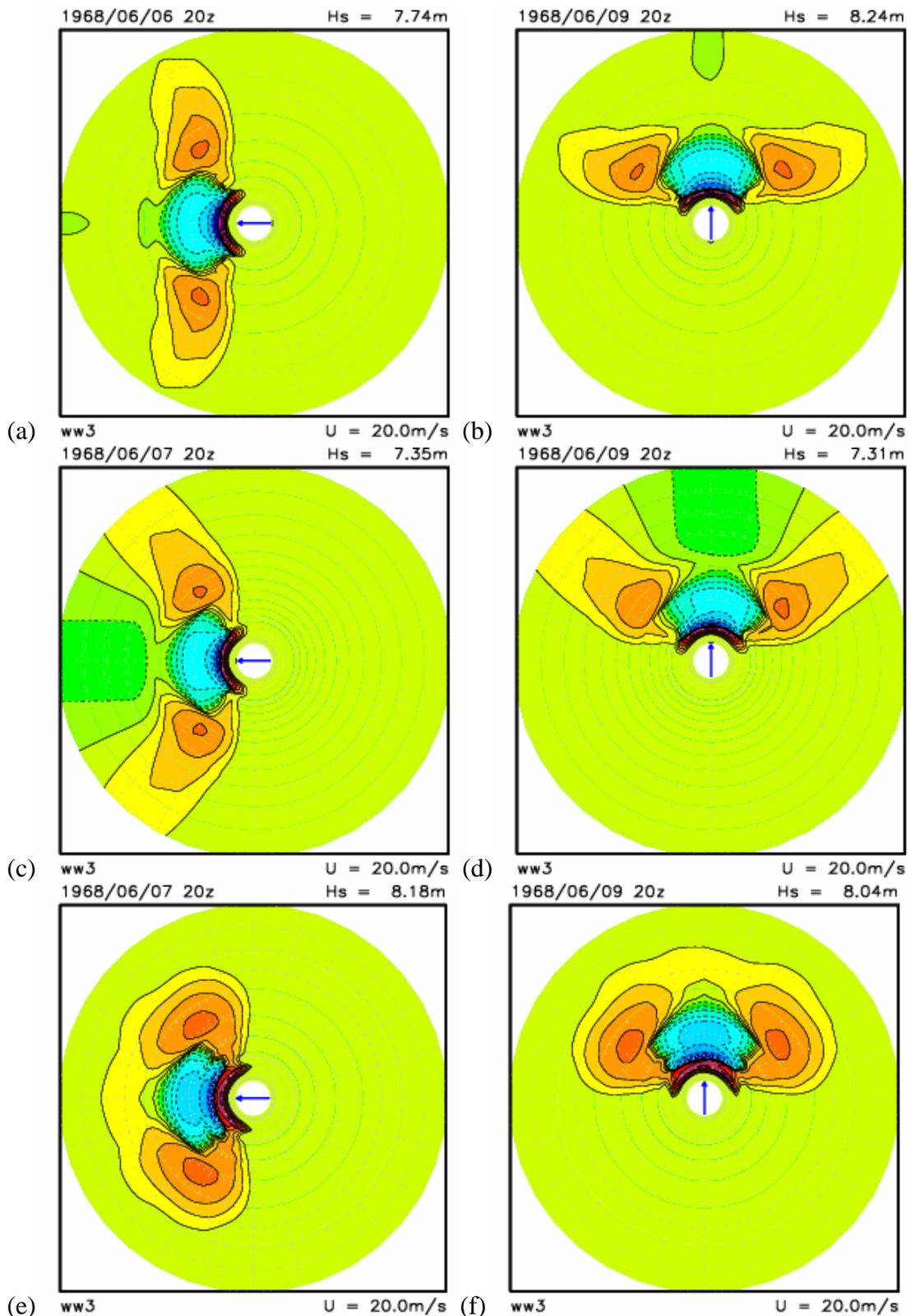
In terms of 2-d  $S_{nl}$ , we show the results for WRT, TSA and DIA in Figure 5, at an output point in the middle of the square ocean. The selected output times are 20 hr after the simulation began, before the wind has shifted direction and 20 hr after the shift in wind direction. Results for WRT and TSA appear similar, whereas DIA results appear to differ in terms of high-frequency region of the spectrum, side-lobes, and forward face regions of the spectrum.

#### 4. Hurricane Juan

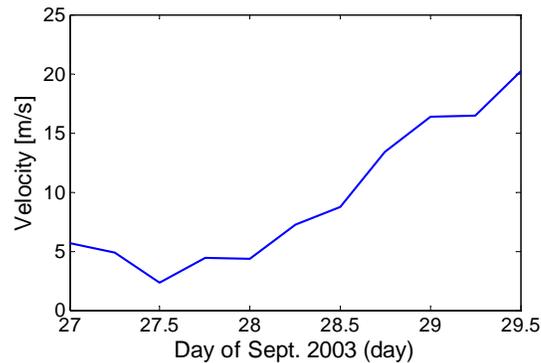
A discussion of hurricane Juan is given by Fogarty et al. (2006) and the Canadian Hurricane Center [http://www.novaweather.net/Hurricane\\_Juan.html](http://www.novaweather.net/Hurricane_Juan.html). Juan reached hurricane strength by 1200 UTC on 26 Sept. near Bermuda, and moved northward and then northwestward, as a subtropical ridge to the northeast of its location extended to the west. It reached maximum wind intensity of 90 knots at 1800 UTC on 27 Sept., and then turned northward towards Nova Scotia, with increasing propagation speed. By 1800 UTC on 28 Sept., Juan was north of the Gulf Stream, and its intensity began to weaken due to the cooler shelf waters south of Nova Scotia. However, because of its accelerating translational speed, Juan spent relatively little time over these cooler waters and therefore did not weaken significantly. Juan made landfall near Halifax (0300 UTC on 29), with sustained winds of 85 knots. Two distinctive ocean stages occurred after it reached its peak intensity. In the first of these, Juan's intensity was almost unchanged before it moved northward to cooler SSTs. In the second, it had a short transition stage leading to landfall near Halifax. Figure 6 shows that Juan's translation speed increased dramatically from  $2.28 \text{ ms}^{-1}$  at 1200 UTC on 27 Sept. to  $20 \text{ ms}^{-1}$  at 1200 UTC on 29 Sept. Preparation of wind fields is described by Xu et al., (2007), which involved blending COAMPS operational background forecast winds, with observed winds from buoys, assuming a typical vortex structure for the wind field shape, as prescribed by Moon et al., (2003).

Resulting winds were used in simulations of the waves generated during Juan's passage from Bermuda to landfall in Halifax. Figure 7 and 8 give the 2-d  $S_{nl}$  and energy  $E(f, \theta)$  as the hurricane passed over buoy 44258, at 03 UTC and 04 UTC on 29 Sept. 2003. No tuning has been done to  $S_{in}$  and  $S_{ds}$ , with respect to TSA. TSA is notable in that estimated  $H_s$  values are lower than those resulting from DIA in WW3, or WRT in WW3. Figure 7 suggests that WRT and DIA respond more quickly to a rapid change in wind direction than TSA, e.g. the "3<sup>rd</sup>" positive lobe in the plots for WRT and DIA, but not evident in the plot for TSA. There is also a suggestion of more symmetry in the TSA plots, than is evident in WRT and DIA plots, at the time of turning winds, at 04 UTC.

Figure 8 suggests that spectra results at the peak of the storm are somewhat more symmetric for simulations made with WRT and DIA, than the results from TSA. Again there is the evidence that WRT and DIA respond more quickly to turning winds than TSA. Directional wave data is not available to compare with simulation results. TSA spectra suggest waves in  $360^\circ$  which is not evident in results from WRT or DIA.



**Figure 5.** 2-d S<sub>NL</sub> from (a, b) WRT, (c, d) TSA and (e, f) DIA at 44 hr (20 hr in the 2<sup>nd</sup> day) and after wind shift 44hr, respectively. Arrow indicates wind direction.



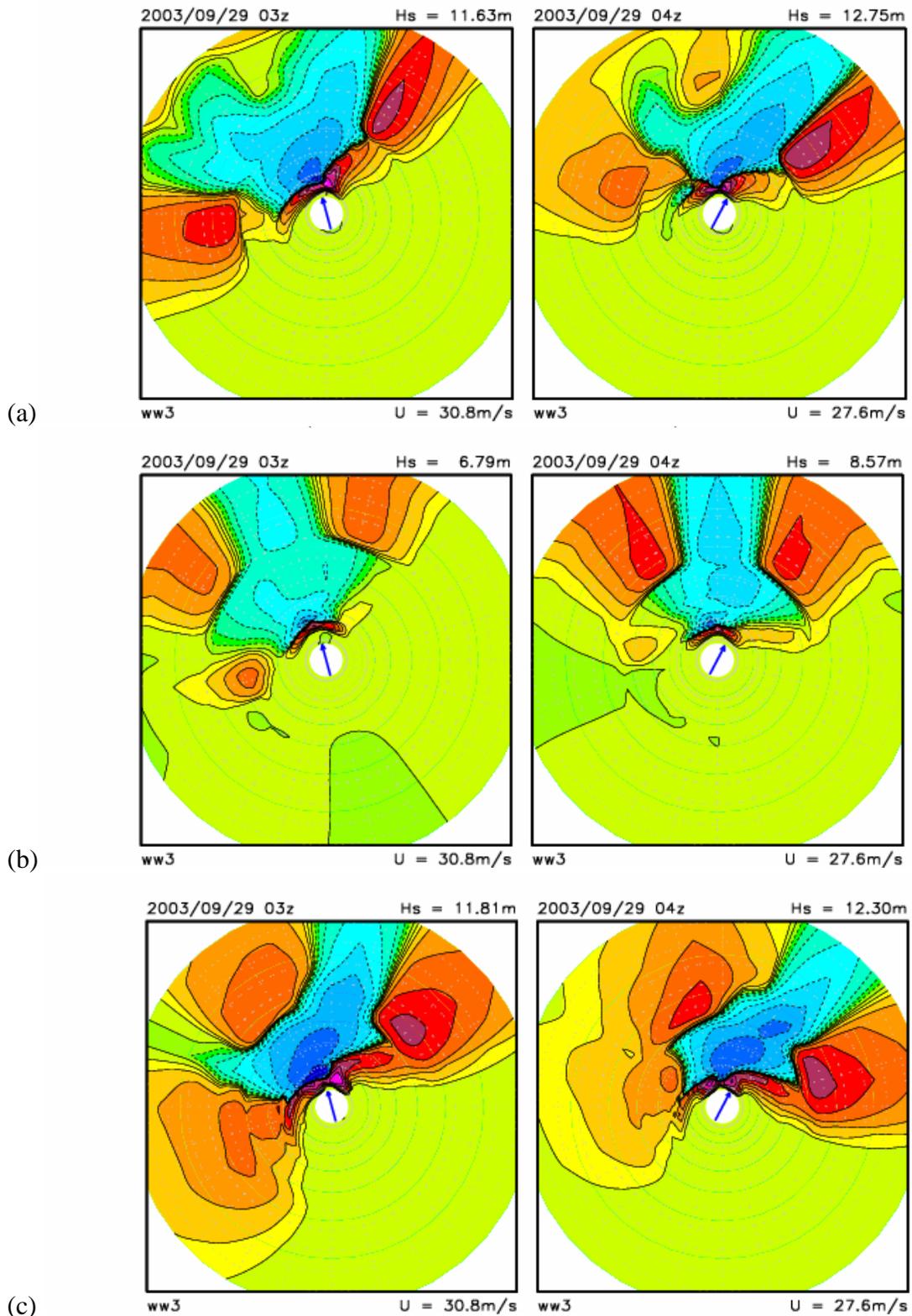
**Figure 6.** Variation of hurricane Juan’s translational speed, as a function of time as it moved from near Bermuda to landfall in Nova Scotia.

Figure 9 shows the distribution of  $H_s$  at the peak of the storm, before Juan made landfall in Nova Scotia, showing a large area of high waves. Results from WRT, DIA and TSA appear similar, in terms of overall directions of peak wave spectra, but magnitudes of  $H_s$  are larger for WRT and DIA results, compared to those of TSA. Figure 10 compares  $H_s$  time series from WW3 using TSA, DIA, WRT, and no nonlinear wave-wave interactions, to observed data at buoys 44258 at the mouth of Halifax Harbour, and 44142 near the edge of the continental shelf just to the right of Juan’s track, where largest waves were measured,  $H_s$  of ~12 m. Evidently, there was a lag in the wind fields, compared to observed wind data, because the  $H_s$  time series are slightly delayed compared to observed wave heights at the buoys. Overall, TSA underestimates waves at 44142, and appears to capture the peak wave heights at 44258, whereas WRT and DIA are too large in their  $H_s$  estimates. These results were obtained with standard formulations for WAMDIG (1988) versions of  $S_{ds}$  and  $S_{in}$  in WW3. Alternate formulations, such as those of Tolman and Chalikov (1996) give different results.

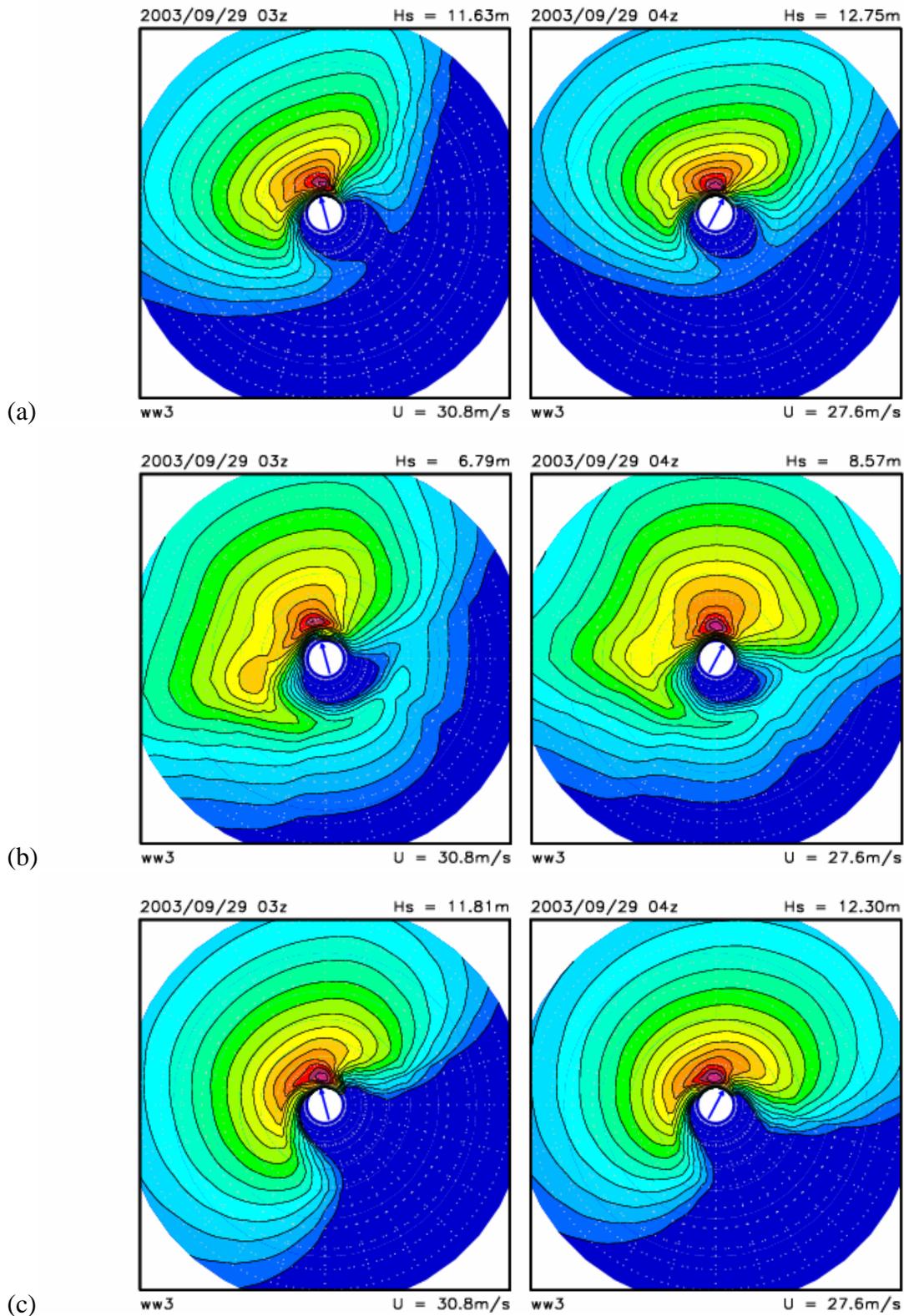
## 5. Conclusions

This paper reports the implementation of TSA in an operational wave forecast model, WW3. The implementation is straightforward, for the original TSA formulation of Resio and Perrie (2008), and fetch curves are reasonably similar to observed data.

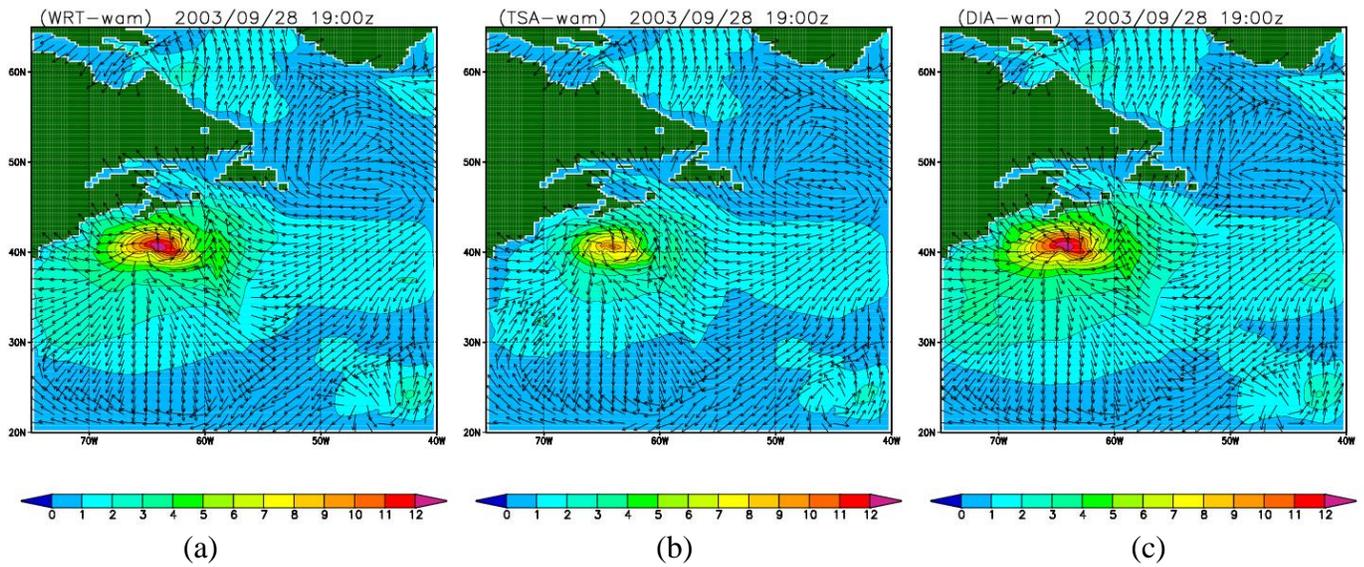
However, when the winds change direction, the original TSA formulation cannot respond quickly enough. The broad-scale term continues to fit to the old sea spectra, and the new waves growing at the new wind direction must become relatively quite large and must be accommodated by the local-scale term, before the broad-scale term will fit to the growing secondary spectral peak. When the original TSA formulation is generalized by allowing multiple broad-scale terms, we achieve a resultant formulation that appears to better handle turning winds. Results are shown for hypothetical tests as well as comparisons from field data collected during hurricane Juan.



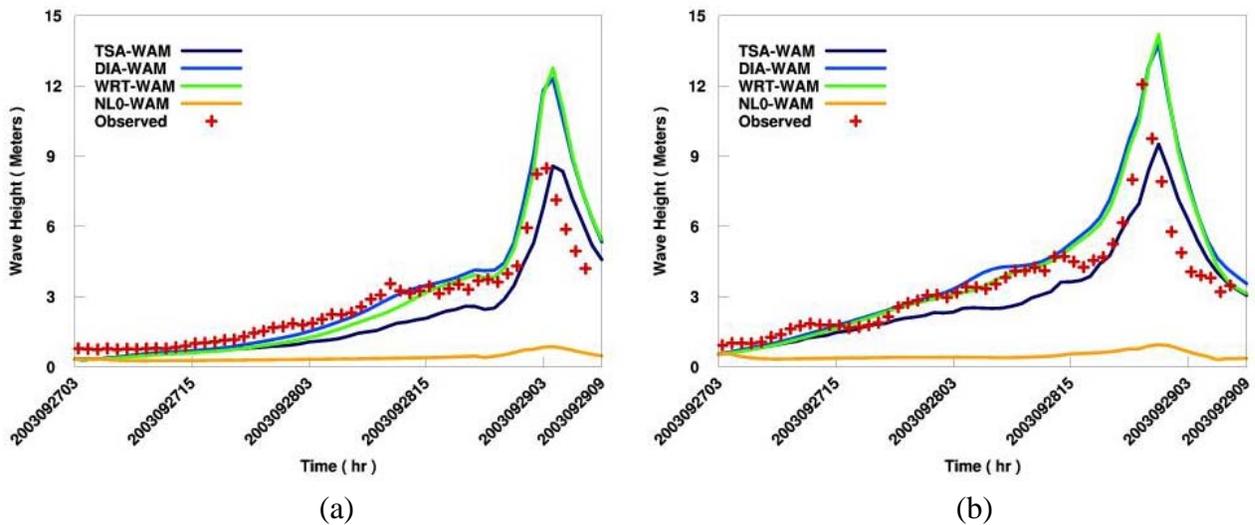
**Figure 7.** 2-d  $S_{nl}$  from WW3 with (a) WRT, (b) TSA, and (c) DIA, as the peak of the storm passed buoy 44258 at the mouth of Halifax Harbour.



**Figure 8.** 2-d spectra from WW3 with (a) WRT, (b) TSA, and (c) DIA as the peak of the storm passed buoy 44258 at the mouth of Halifax Harbour.



**Figure 9.** Distribution of  $H_s$  at 19 UTC on 28 September at the peak of the wave heights, before the storm made landfall in Halifax with WW3 and (a) WRT, (b) TSA, and (c) DIA implemented. Color bar indicates distribution of  $H_s$ , with directions of peak waves indicated by vectors.



**Figure 10.** Comparison of  $H_s$  time series from WW3 at buoys (a) 44258 and (b) 44143, using TSA, DIA, WRT, no nonlinear wave-wave interactions, and observed buoy data.

### Acknowledgements

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