A Hierarchical Bayesian Spatiodirectional Model for Wave Heights and Structural Response

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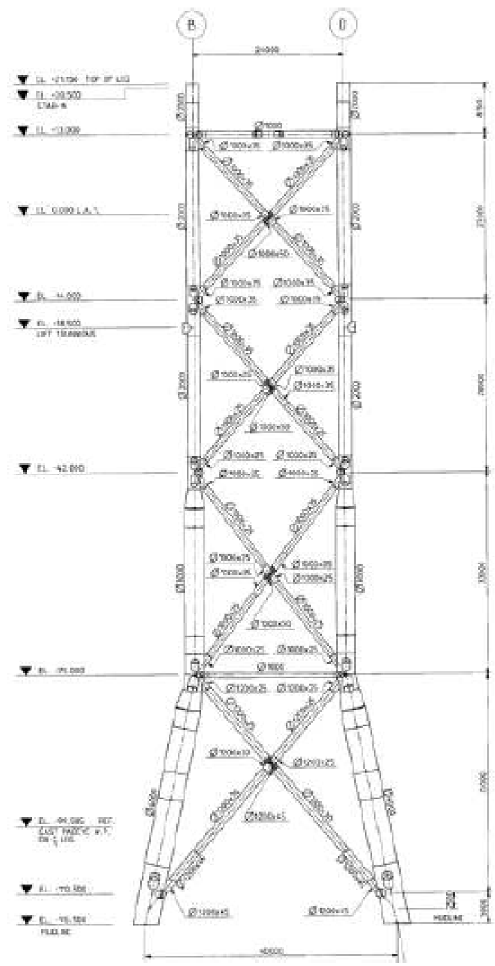
Motivation

- Metocean conditions for the design of offshore structures.
  - Wave height and associated wave periods, currents and wind speeds.
- Smooth estimates across a region.
  - Jonathan and Ewans 2011.
- Robust estimates of covariates such as direction.
- Use both hindcast and measured data.
- Estimates of uncertainty for determining safety factors.

Method

- Extrapolation of structural response.
  - Tromans and Vanderschuren 1995.
- Bayesian analysis
  - Prior distribution determined from hindcast data.
  - Parameters of the extreme value distribution function of latitude and longitude.
    - Coles and Powell 1996.
  - Data at a point from a hindcast or from measurements.



Conclusions

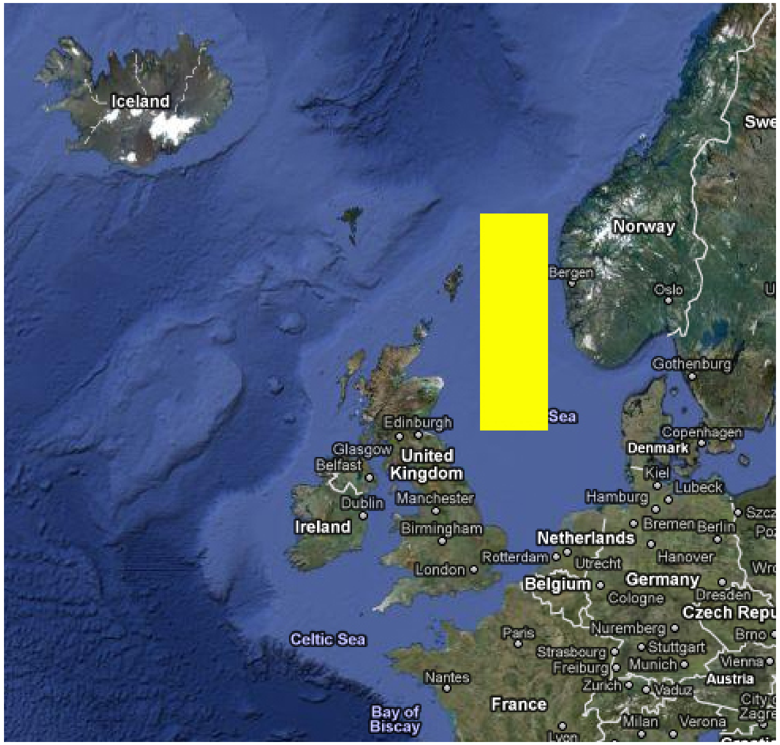
- Bayesian approach to extreme value analysis of structural response provides robust estimates of design values.
  - Can incorporate uncertainty in load factor studies.
- Possible to use short data sets in conjunction with a hindcast.
- Possible to improve the spatial model.
- Interesting to consider using measurements to derive the prior in order to correct for bias in hindcasts.

Overview

- Motivation, Method and Conclusions.
- Data sources and Area of Interest.
- Method
  - Structural Model
  - Extreme Value Analysis
  - Bayesian Approach
- Results
  - Wave Height
  - Structural Response
  - Bias and COV
- Conclusions

Data Sources and Area of Interest

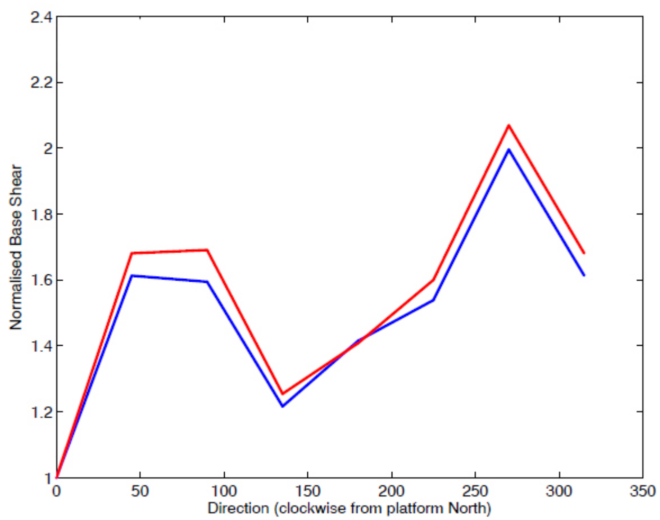
- North Sea
  - 56^o - 62^oN
  - 0^o – 4^oE
- Nextra wave hindcast and DHI currents.

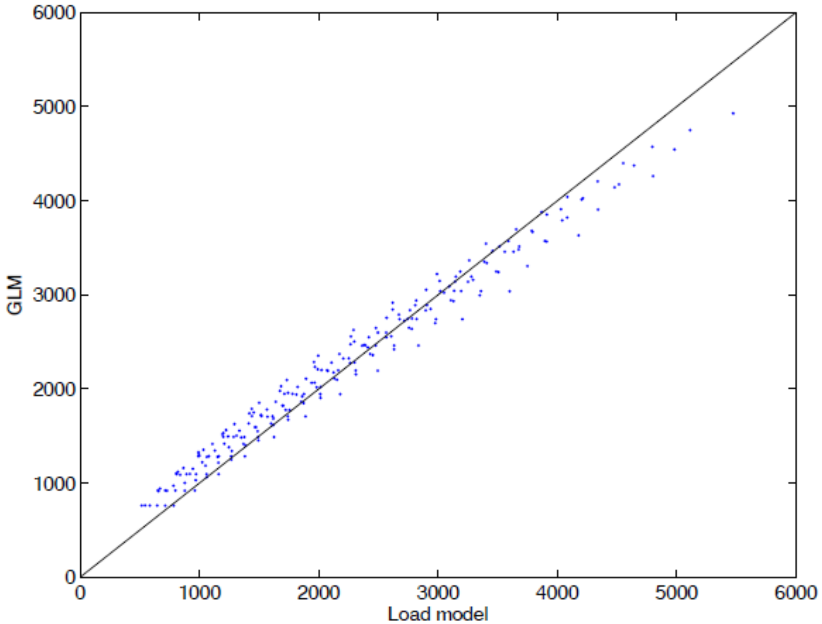


Method: Structural Response

- Time series of metocean parameters converted to response associated with storm events.
- Generic load model
  - Stick representation of jacket structure
  - Second-order irregular wave theory.

$$
\begin{aligned}
X &= A_1 u^2 + A_2 u a T \Phi \cos\theta \\
&+ A_3 \Phi^2 a^2 + A_4 u \Phi a^2 \cos\theta / T \\
&+ A_5 \Phi^2 a^3 / T^2 + A_6 \Phi^2 a^2 T^2 \\
&+ A_7 W^2 ,
\end{aligned}
$$





Method: Extreme Value Analysis

- Peak over threshold analysis.
  - Generalised Pareto distribution.
- Directional distribution
  - Robinson and Tawn 1997.
  - Jonathan and Ewans 2007.
- Long and short term distributions of loading.

$$P(X_{mp}) = 1 - (1 + \epsilon(X_{mp} - \gamma)/\sigma)^{(-1/\epsilon)}$$

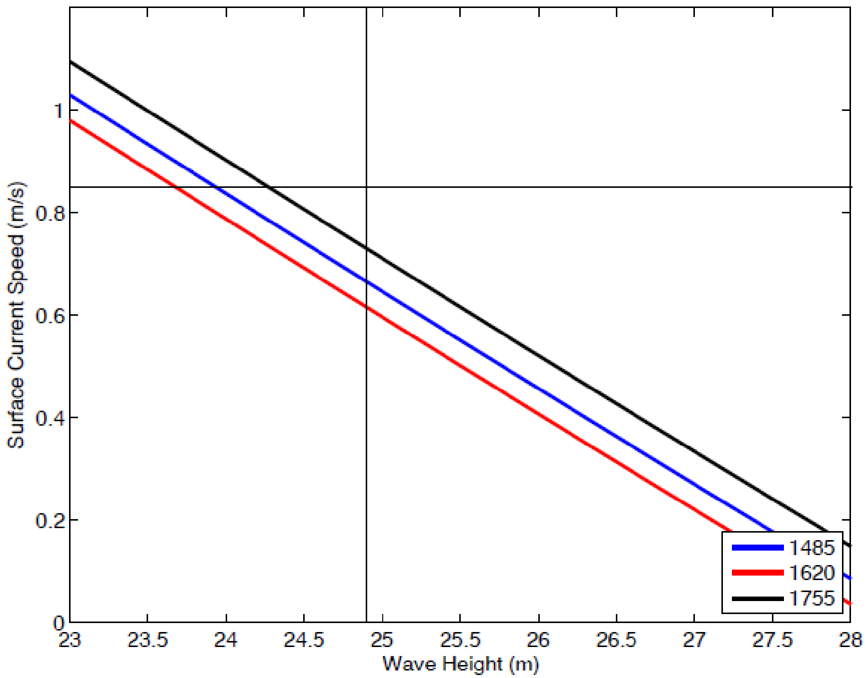
$$\sigma(\theta) = a_0 + \sum_{i=1}^{N}(a_i \cos(i\theta) + b_i \sin(i\theta))$$

$$\epsilon(\theta) = c_0 + \sum_{i=1}^{N}(c_i \cos(i\theta) + d_i \sin(i\theta))$$

$$P(X) = \int P(X|X_{mp})p(X_{mp})dX_{mp}$$

Associated Criteria

Direction	H (m)	T (s)	$\overline{C}$ (m/s)
Omni	24.6	16.5	0.52
N	26.6	17.2	0.61
NE	23.2	16.1	0.44
E	25.4	16.8	0.33
SE	24.4	16.5	0.51
S	21.4	15.4	0.62
SW	20.5	15.1	0.55
W	22.3	15.7	0.46
NW	26.1	17.0	0.59



Method: Bayesian analysis

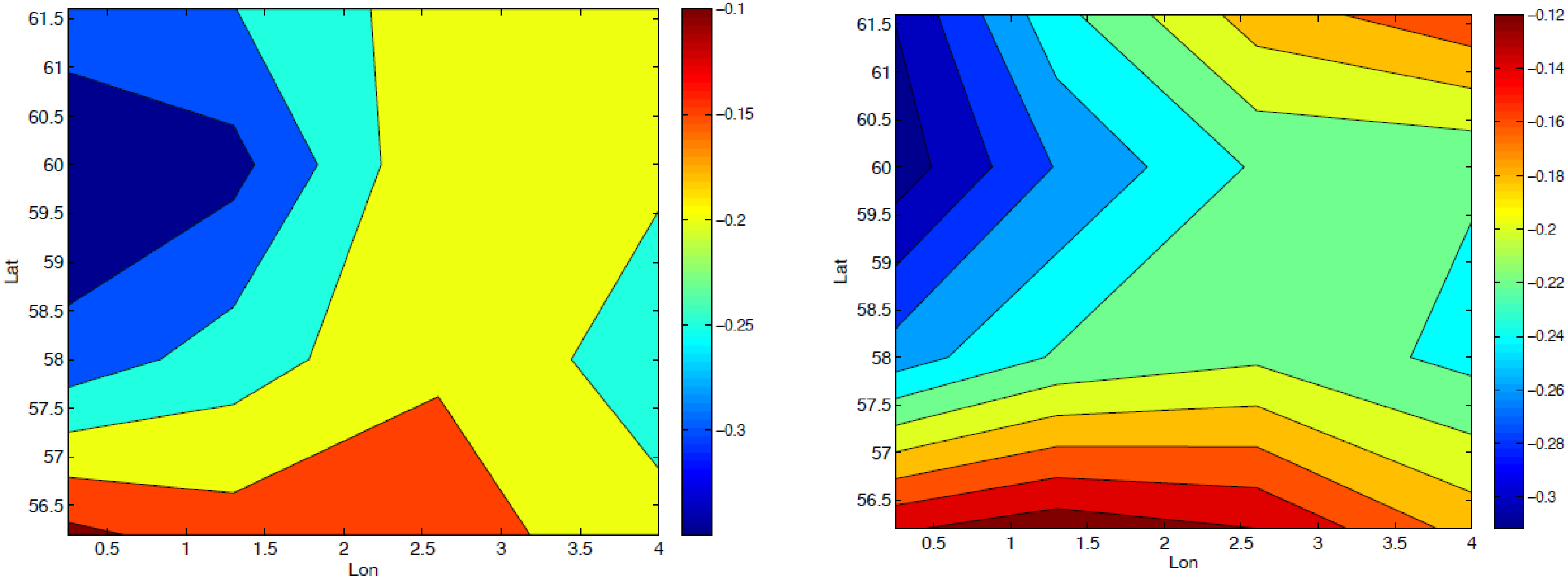
- Bayesian
  - Prior distribution.
  - Data.
  - Posterior distribution.
- Parameters of the prior distribution a function of latitude and longitude.

$$p(\theta|x) = \frac{p(\theta)p(x|\theta)}{\int_\theta p(\theta)p(x|\theta)d\theta}$$

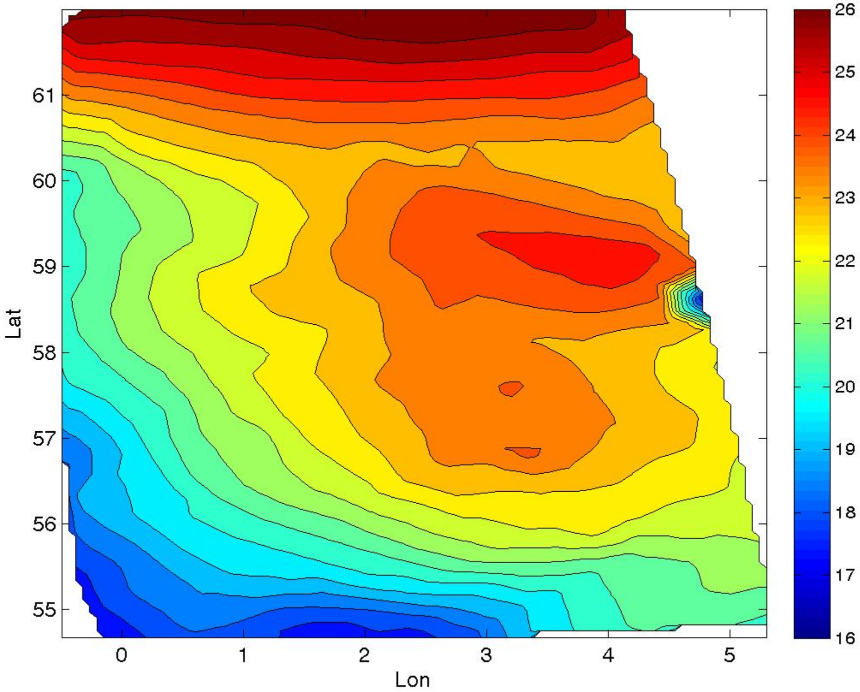
$$p(X|x) = \int_\theta p(X|\theta)p(\theta|x)d\theta$$

$$\upsilon(X,Y) = \upsilon_a + \upsilon_b X + \upsilon_c Y + \upsilon_d XY + \upsilon_e X^2 + \upsilon_f Y^2 + \varepsilon_\upsilon$$
$$\epsilon(X,Y) = \epsilon_a + \epsilon_b X + \epsilon_c Y + \epsilon_d XY + \epsilon_e X^2 + \epsilon_f Y^2 + \varepsilon_\epsilon\ ,$$

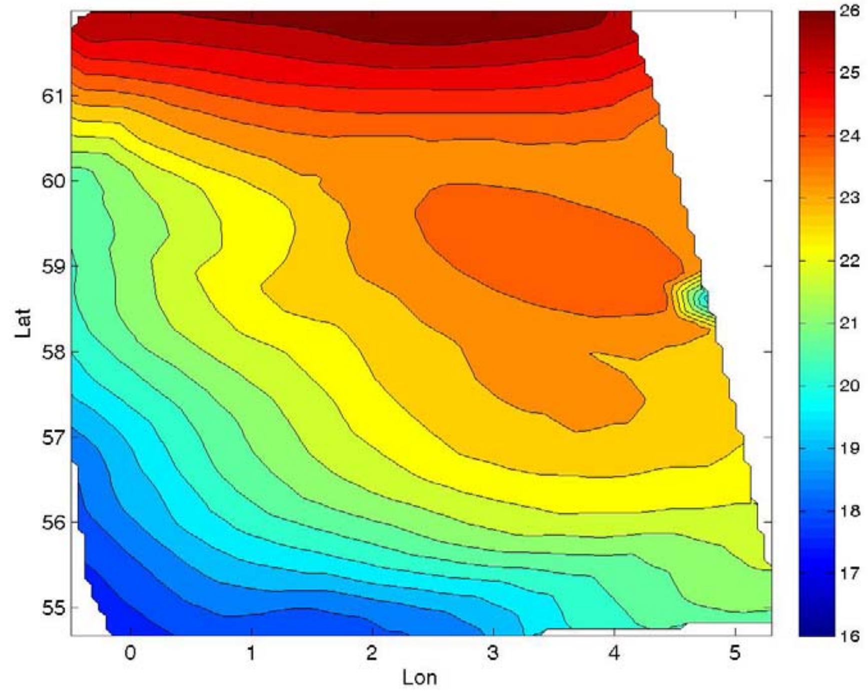
Shape parameter



Wave height



Lat
Lon
61
60
59
58
57
56
55
0
1
2
3
4
5
26
25
24
23
22
21
20
19
18
17
16



Lat
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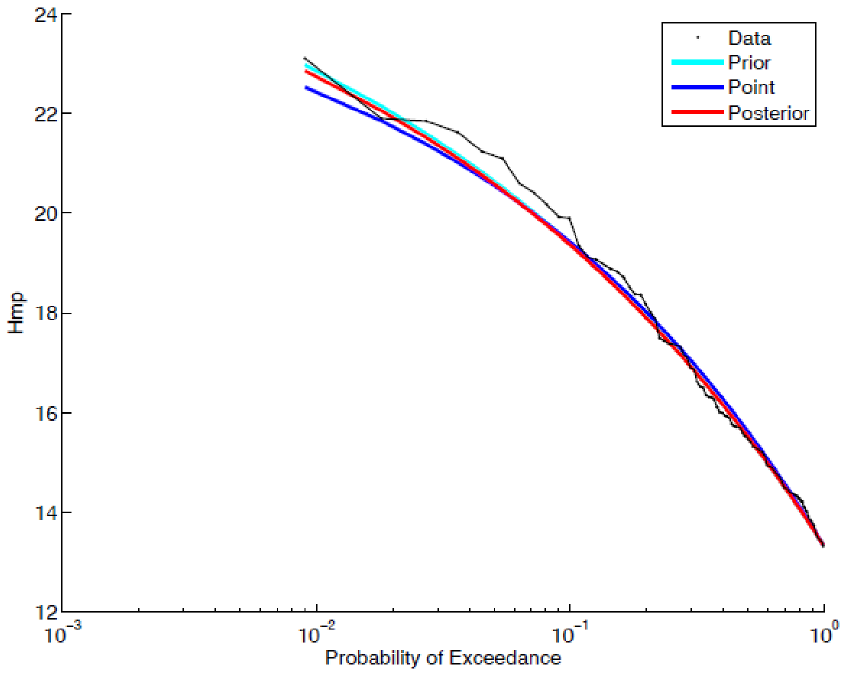
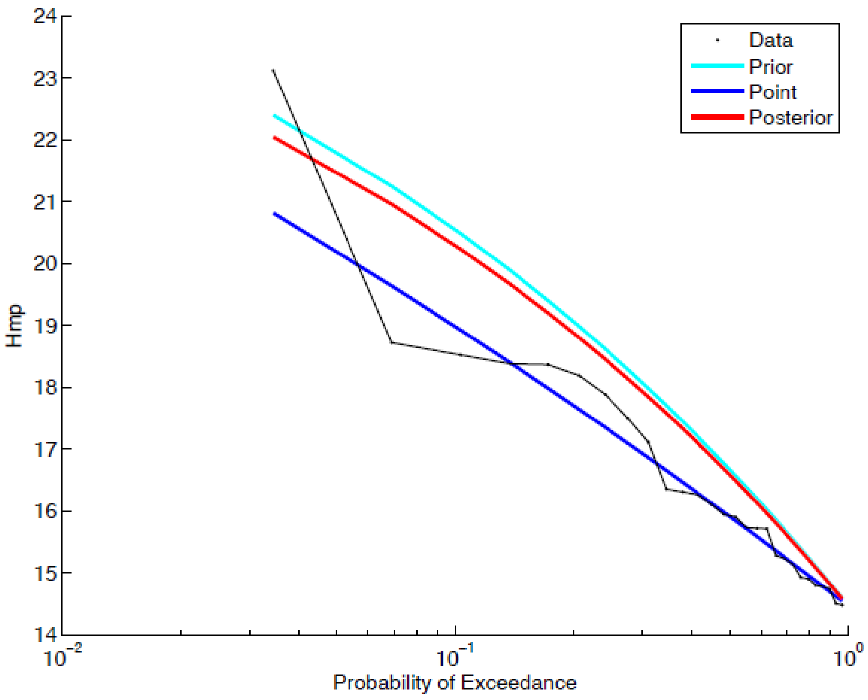
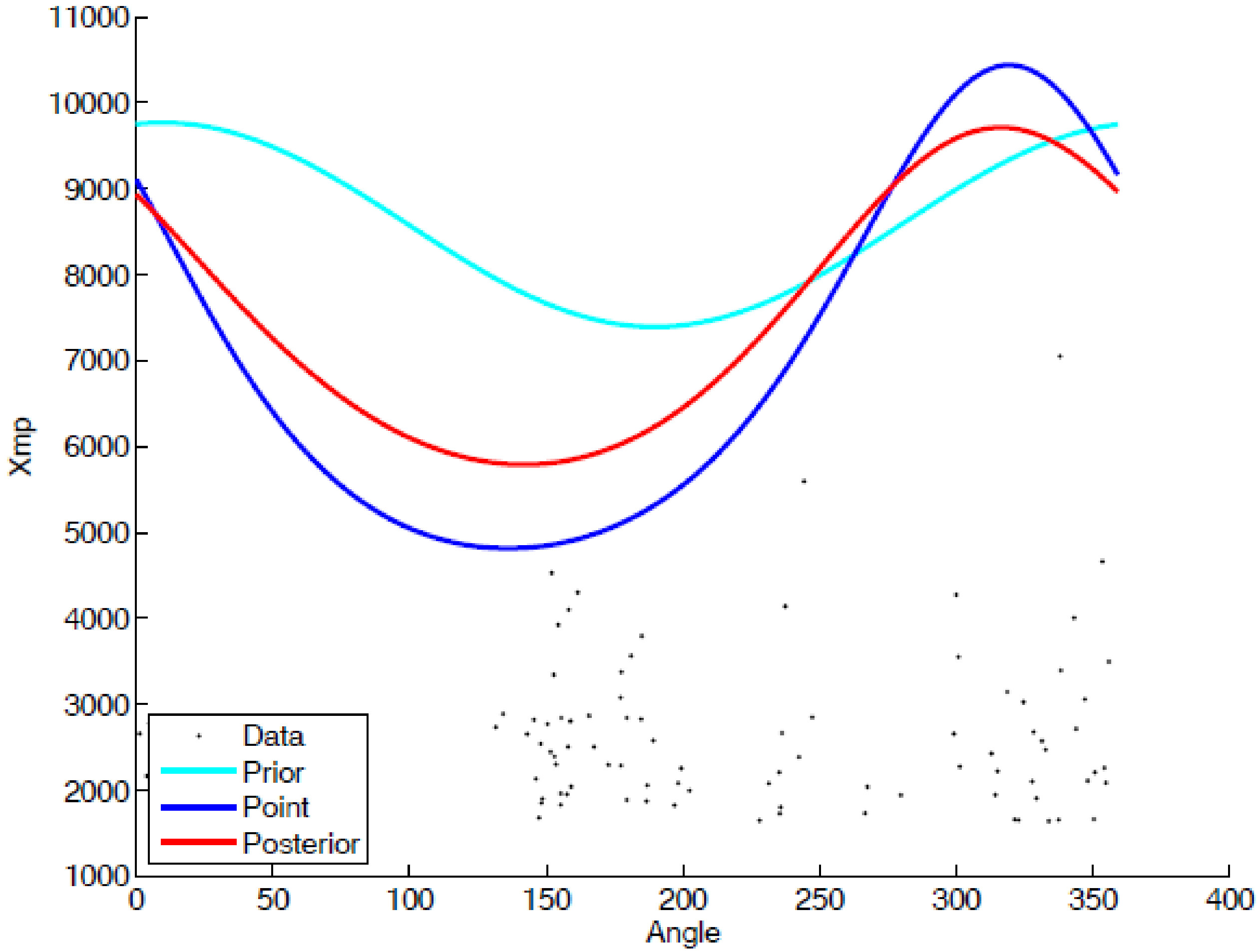
Omnidirectional Wave Height

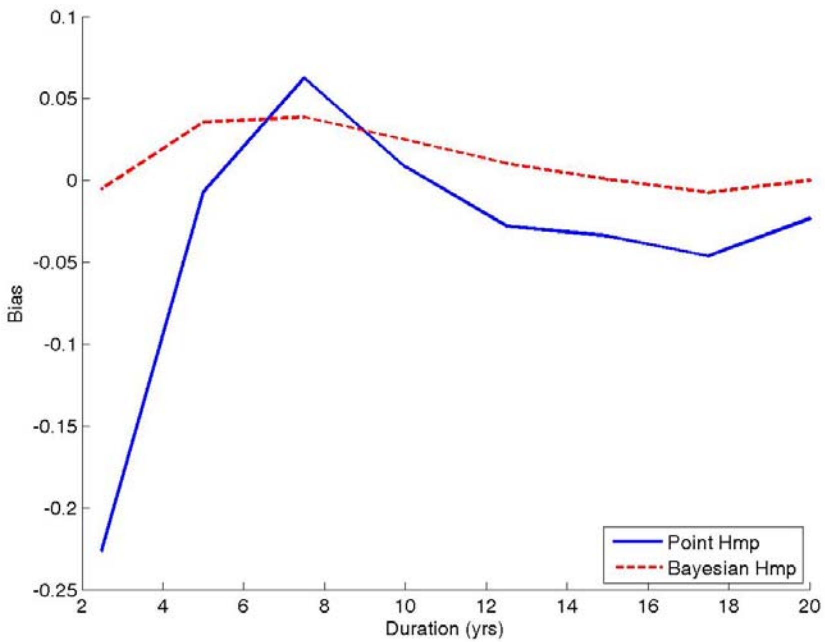




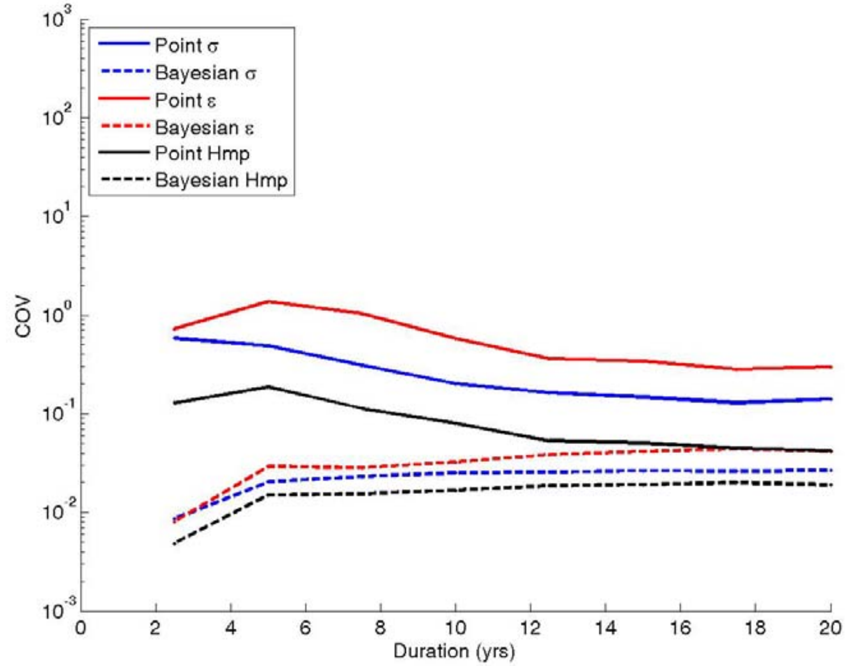
Directional structural response



Bias and COV



0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
Bias
2
4
6
8
10
12
14
16
18
20
Duration (yrs)
Point Hmp
Bayesian Hmp



10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
COV
0
2
4
6
8
10
12
14
16
18
20
Duration (yrs)
Point σ
Bayesian σ
Point ε
Bayesian ε
Point Hmp
Bayesian Hmp

Conclusions and Further Work

- Bayesian approach to extreme value analysis of structural response provides robust estimates of design values.
  - Can incorporate uncertainty in load factor studies.
- Possible to use short data sets in conjunction with a hindcast.

- Possible to improve the spatial model.
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