Spectral Energy Dissipation due to Surface-Wave Breaking

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Motivation

- Quantify and characterize the strength of deep-water wave breaking from field observations

Methodology

- Close the energy balance equation using wave observations of fetch-limted waves to calculate the energy dissipation

- The semi-empirical energy dissipation is combined with the measured statistics of wave breaking (Kleiss and Melville, 2010) to calculate the strength of breaking.

- Guided by available laboratory measurements of the strength of breaking from focusing packets propose and fit a spectral model of the strength of breaking for field conditions.

Summary

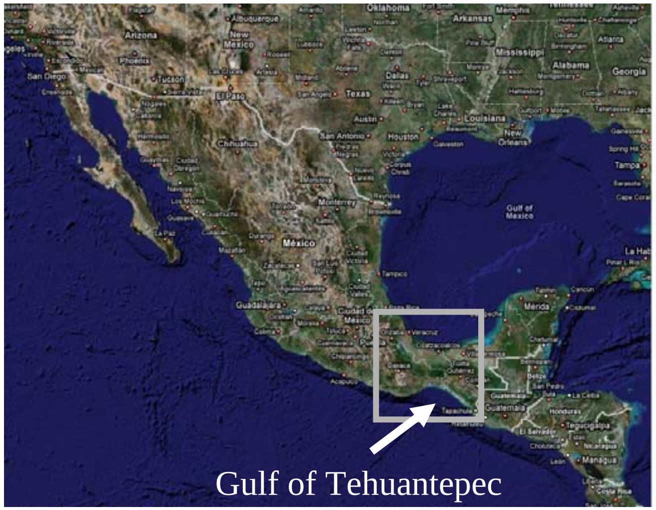
- First quantification of the spectral strength of breaking in the field
- The breaking strength $b(k)$ is not a constant, it varies by up to one order of magnitude between the spectral peak and high wavenumbers.
- Based on laboratory measurements and the inertial scaling by Drazen et al. (2008) a spectral model of $b(k)$ with a saturation threshold behavior was fitted to the data.

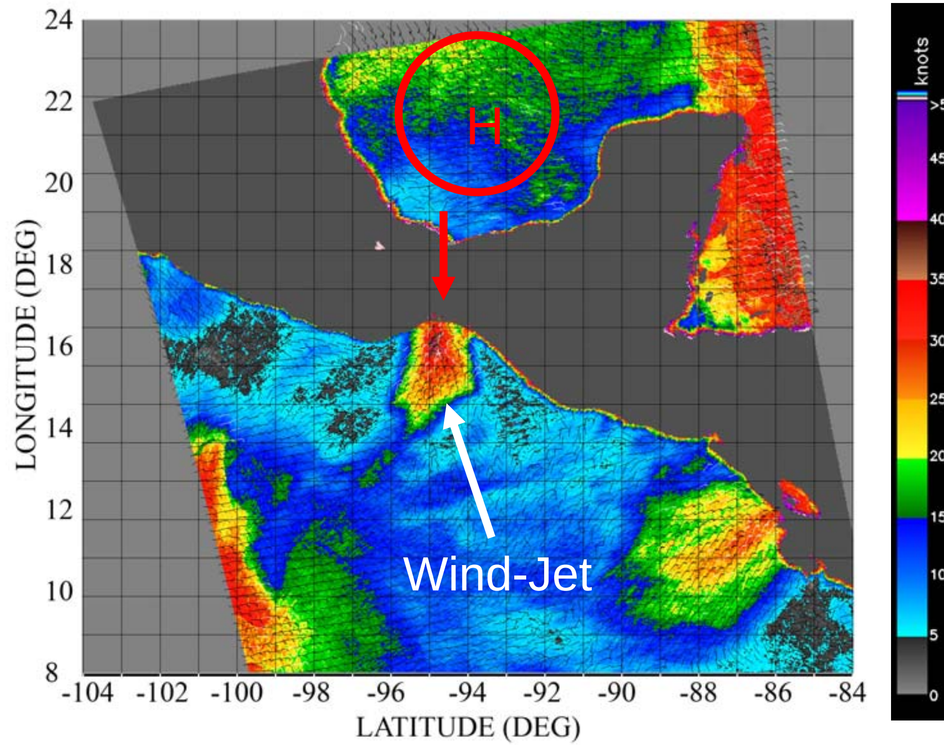
Semi-empirical Dissipation S_{ds}

Energy Balance of deep-waver waves: $\frac{\partial F(k,\theta)}{\partial t} + \bar{c}_g \cdot \nabla F(k,\theta) = S_{in} + S_{ds} + S_{nl}$

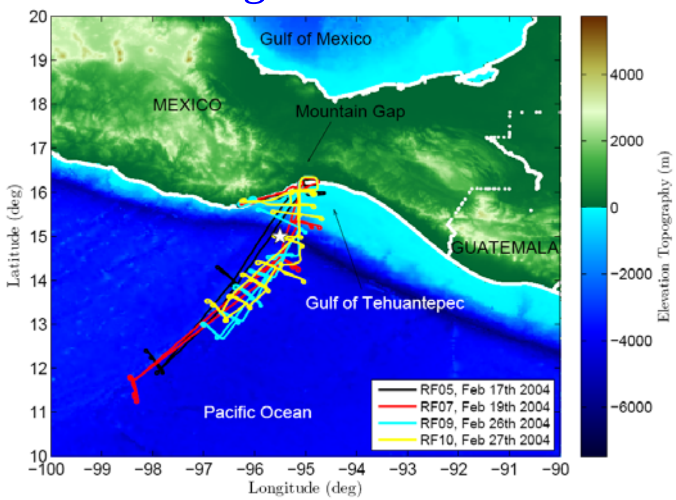
- Wave energy advection $\bar{c}_g \cdot \nabla F \approx c_{g_1} \frac{\partial F}{\partial x_1}$ (calculated from measured spectra)
- Wind input S_{in} : calculated from measured spectra, friction velocity, and the forcing functions by Snyder (1981), Janssen (1991), and Janssen (1991) with sheltering
- Nonlinear energy fluxes due to four-wave resonant interactions S_{nl} : calculated from the measured spectra and the “exact” algorithm WRT (Webb, 1978; Resio and Tracy, 1982; van Vledder, 2006; WW3)
- Semi-empirical dissipation: $S_{ds} = c_{g_1} \frac{\partial F}{\partial x_1} - S_{in} - S_{nl}$

Gulf of Tehuantepec Experiment





Flight Tracks



http://winds.jpl.nasa.gov/

-Romero and Melville 2010a,b, 2011
Characterization and modeling of the directional spectrum and wave statistics

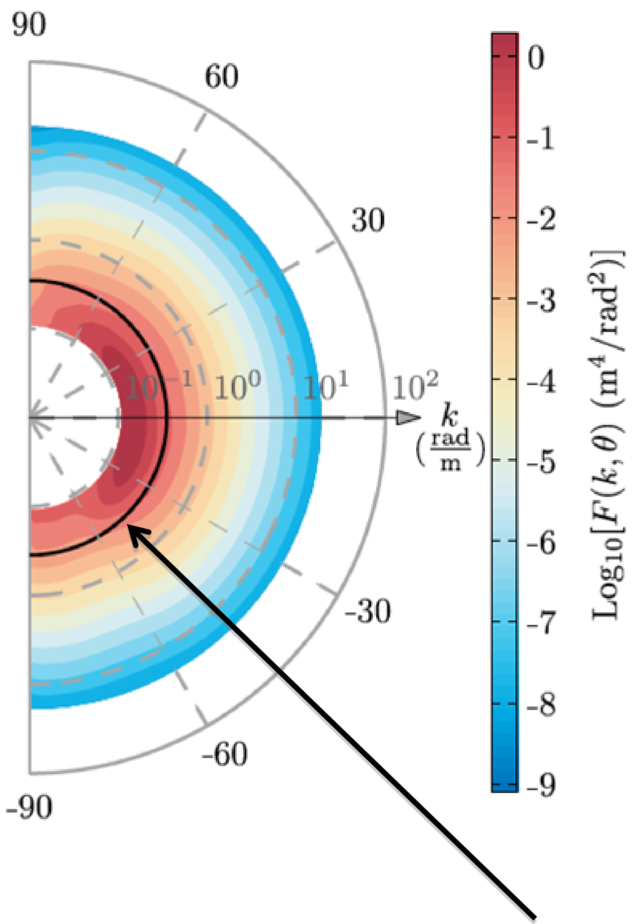
-Kleiss and Melville 2010,2011
Quantification and characterization of the wave breaking statistics.

Main Airborne Instrumentation

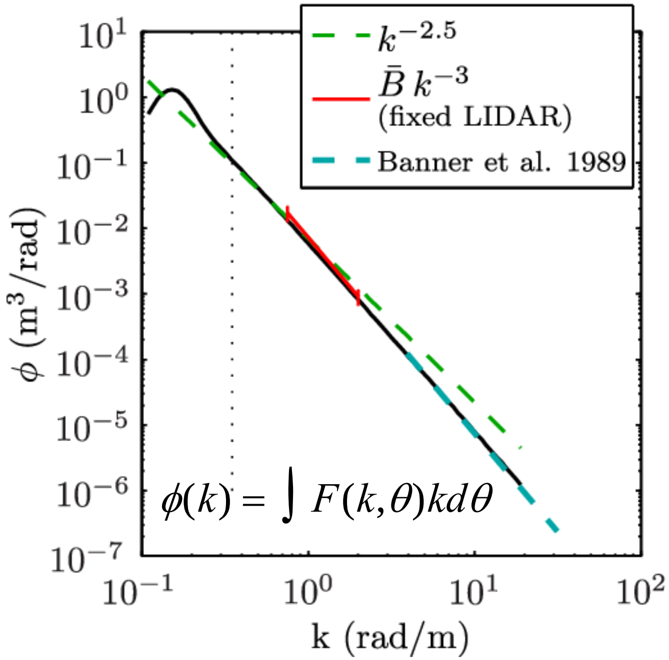
- Wind measurements including turbulent fluxes and standard atmospheric variables: temperature, humidity, and pressure
- GPS / Inertial Navigation Unit
- LIDARs – surface waves
- Nadir looking camera – wave breaking

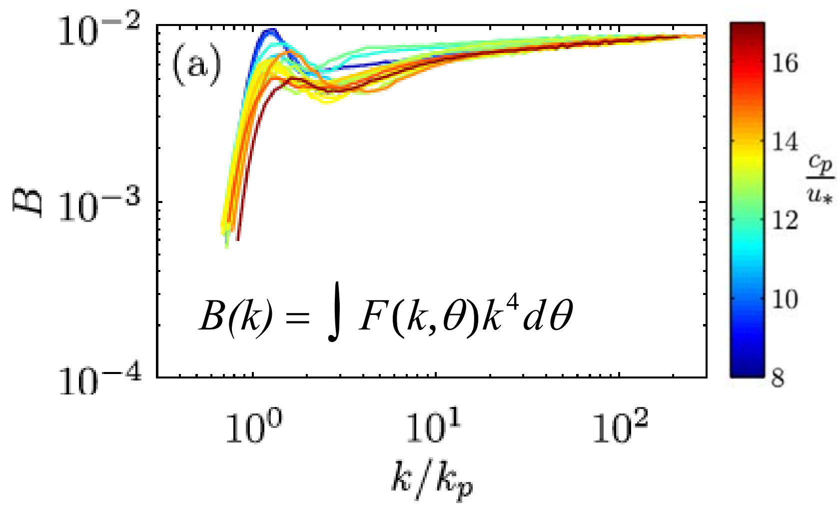
Extrapolation of Spectrum to Large Wavenumbers

- Calculation of S_{nl} requires a broad bandwidth

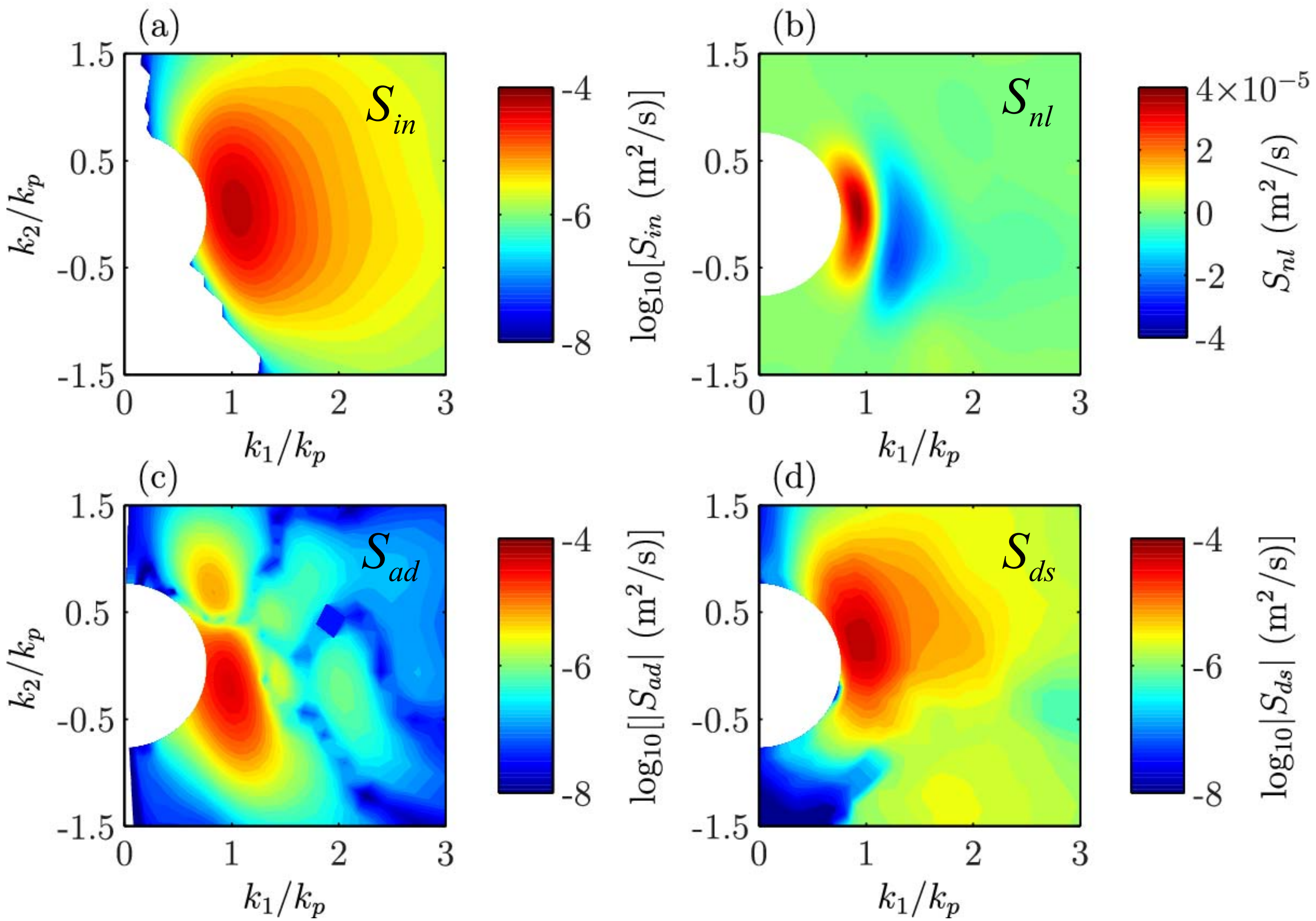


Limit of scanning lidar data

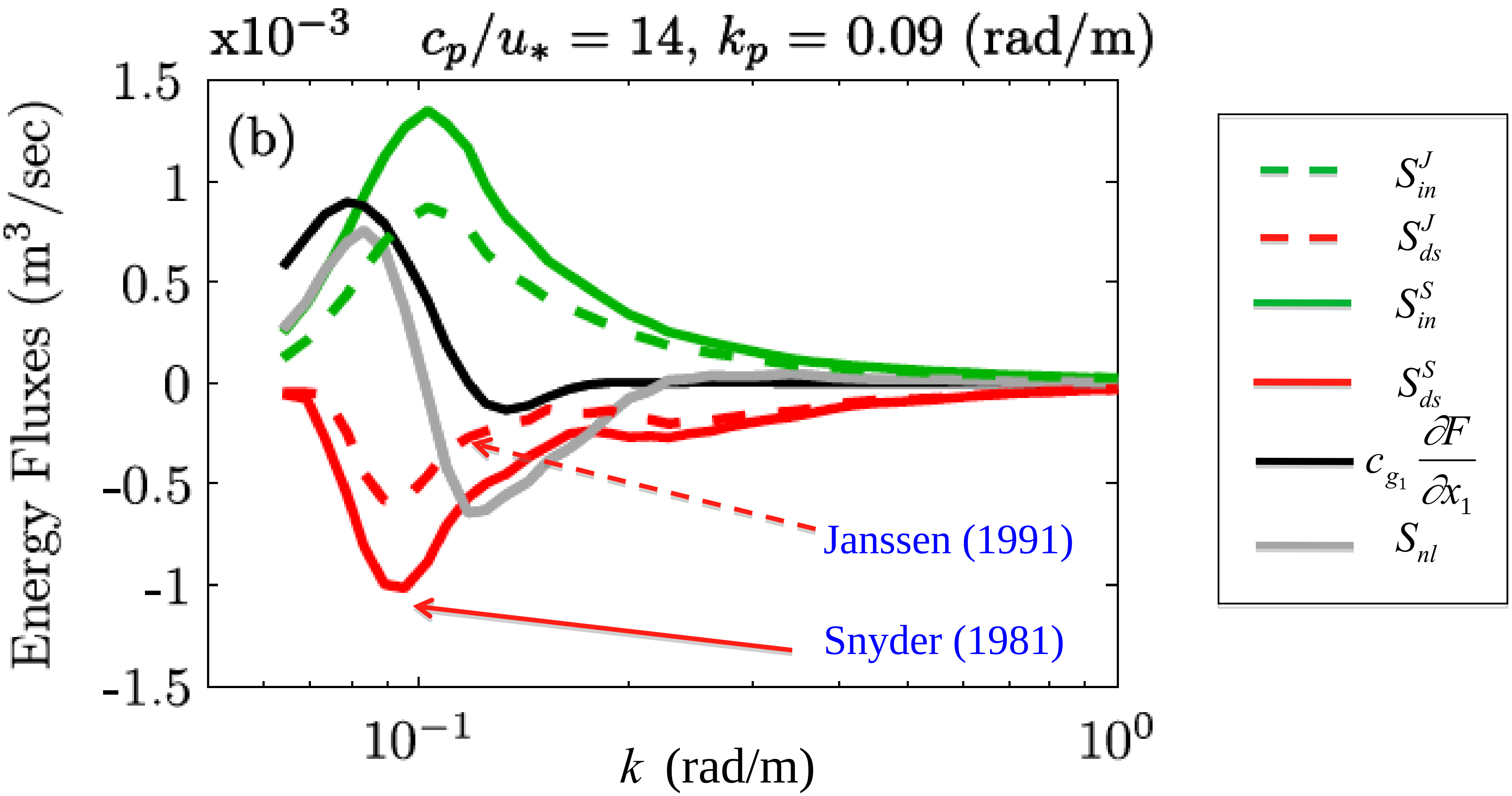




Directional Source Terms



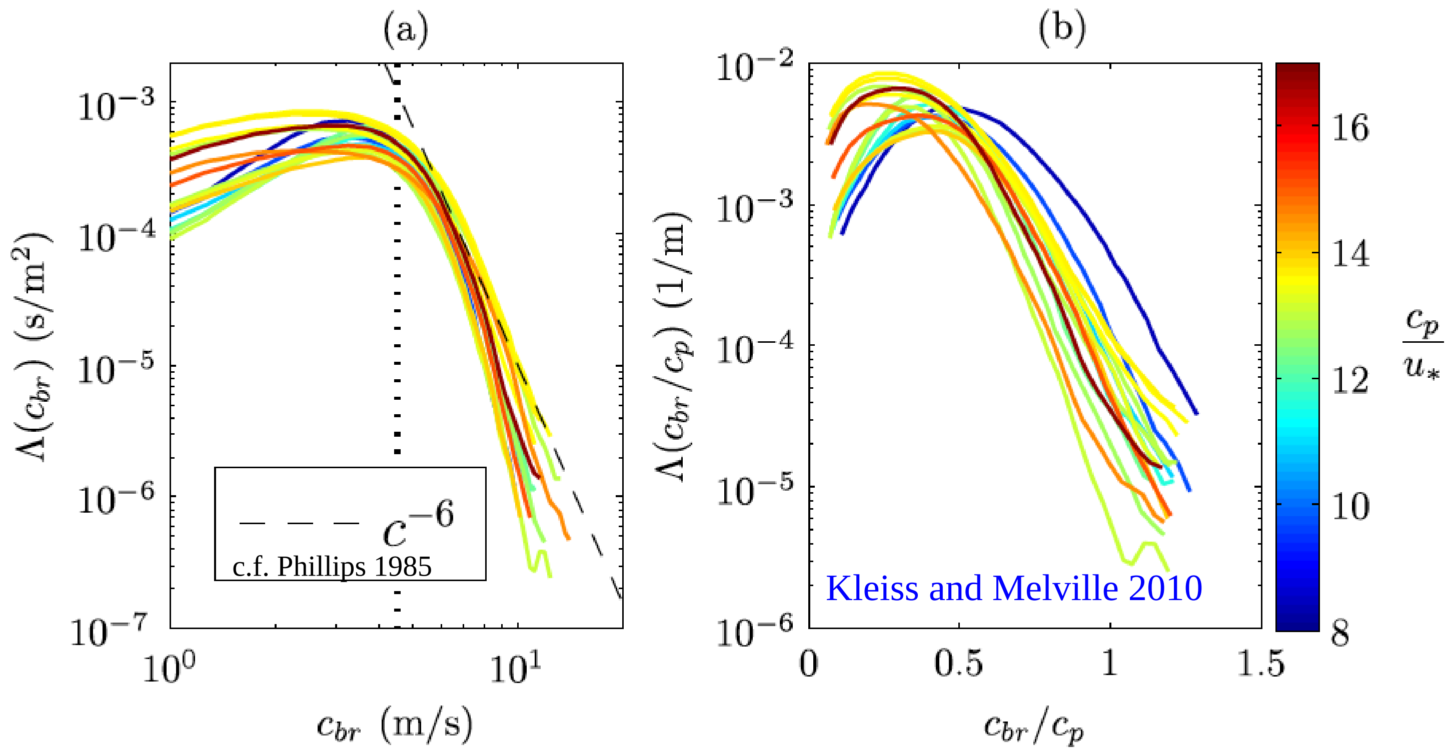
Spectral Energy Balance



S_{in}^S Snyder (1981)

S_{in}^J Janssen (1991)

Statistics of Breaking Waves



Breaking speed c_{br} and wave phase speed c are linearly related by: $c_{br} = \alpha c$, where $\alpha \approx 0.7$-0.95 is an empirical constant measured in laboratory studies of breaking waves (Rapp and Melville 1990;Stansell and MacFarlane 2002; Banner and Peirson 2007)

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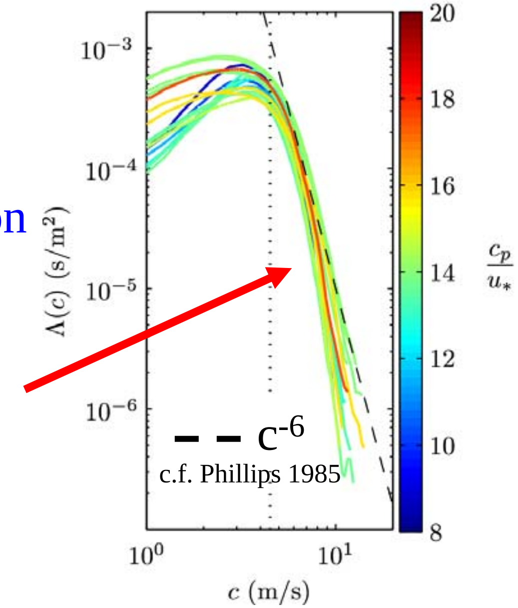
Spectral Breaking Strength Function *b(k)*

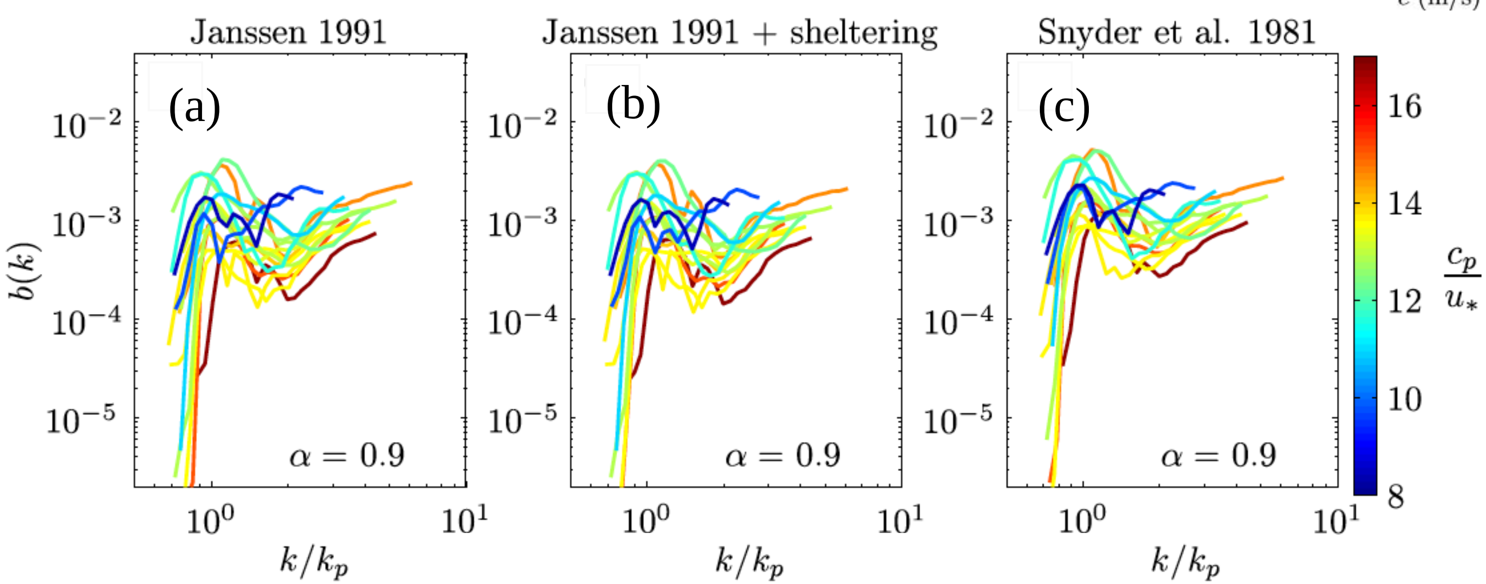
$$b(c) = \frac{g^2 S_{ds}(c)}{\Lambda(c) c^5} = \frac{g^2 S_{ds}(k) \partial k / \partial c}{\Lambda(c) c^5}$$

$S_{ds}(k) = \int S_{ds}(k,\theta) k \, d\theta$: omnidimensional semi-empirical dissipation

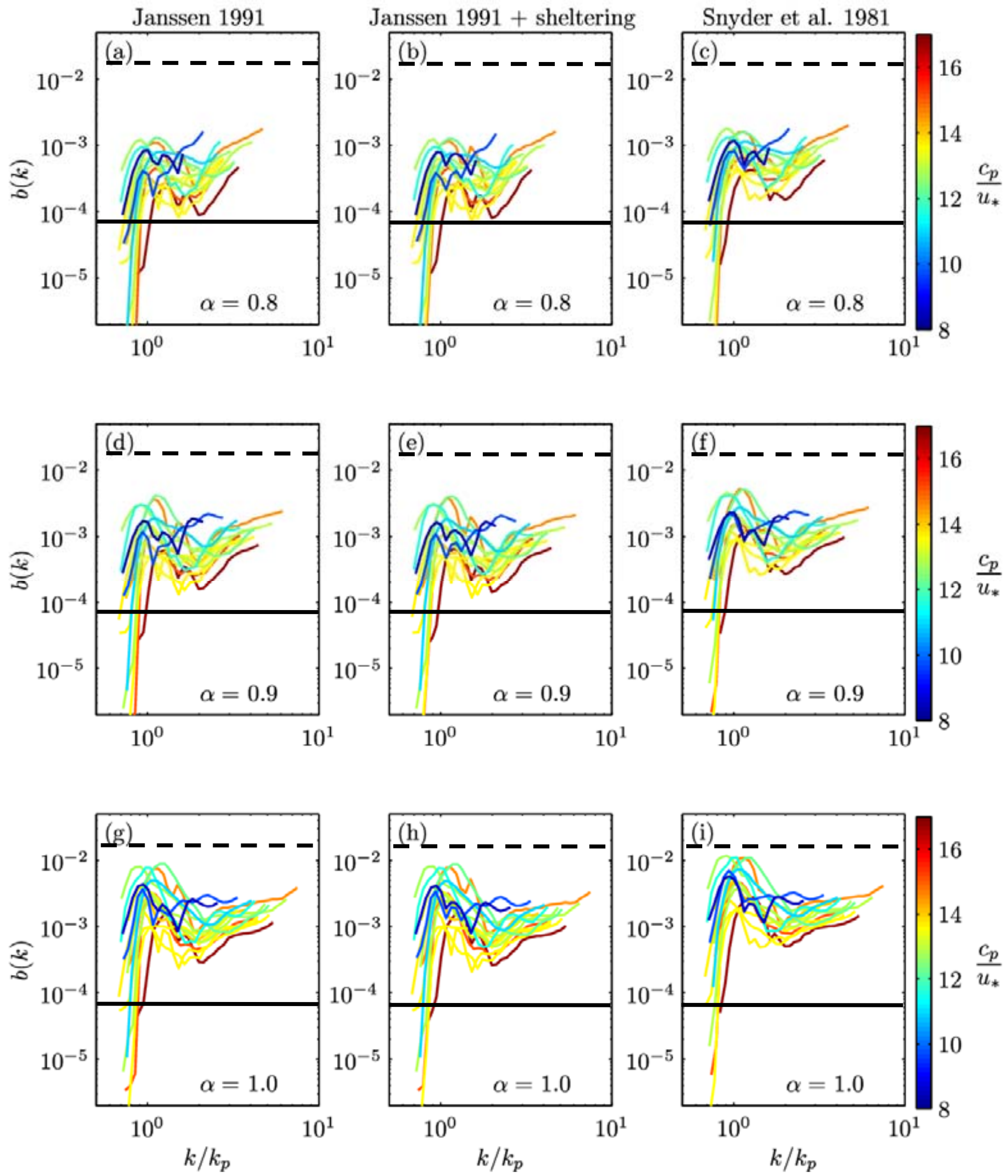
$\Lambda(c) = \int \Lambda(c,\theta) c \, d\theta$: omnidimensional distribution of breaking statistics (Kleiss and Melville, 2011)



$c = \sqrt{g/k}$: phase speed (linear dispersion relationship)



Sensitivity of $b(k)$ to the value of α



Average values of b obtained by:

——— Gemmrich, et al. (2008)

- - - - Thomson et al. (2009)

$$c_{br} = \alpha c$$

Independent Model of $b(k)$

$$b = A_l (S - S_T)^{5/2}$$

Inertial scaling from laboratory experiments

$$B(k) = \int F(k,\theta) k^4 d\theta$$

: one-dimensional spectral saturation, which is a bandwidth-independent measure of the spectral steepness squared (Banner et al. 2000; Banner et al. 2002).

$$\text{mss} = \int B(k) \frac{dk}{k}$$

Mean Square Slope

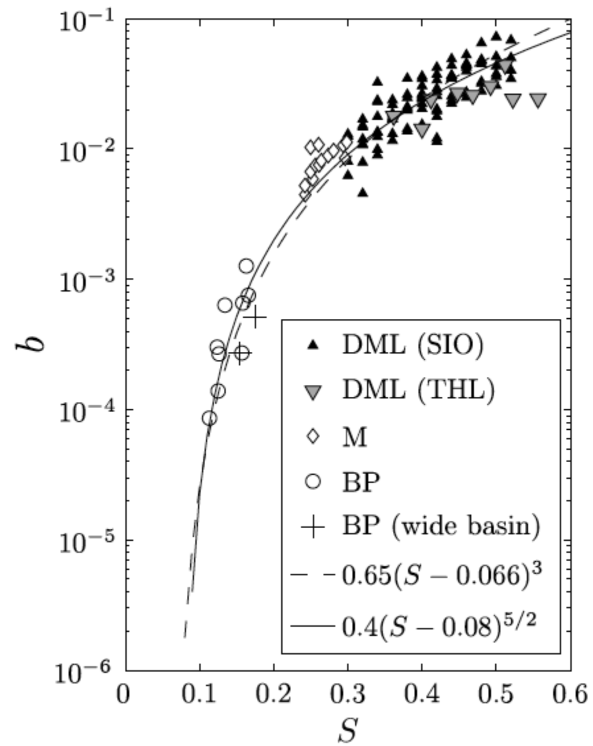
$$b(k) = A\,(B^{1/2} - B_T^{1/2})^{5/2}$$

: Model of b(k)

$$A \approx 3.9 - 5.5$$

$$B_T \approx 0.8 \times 10^{-3} - 1.6 \times 10^{-3}$$

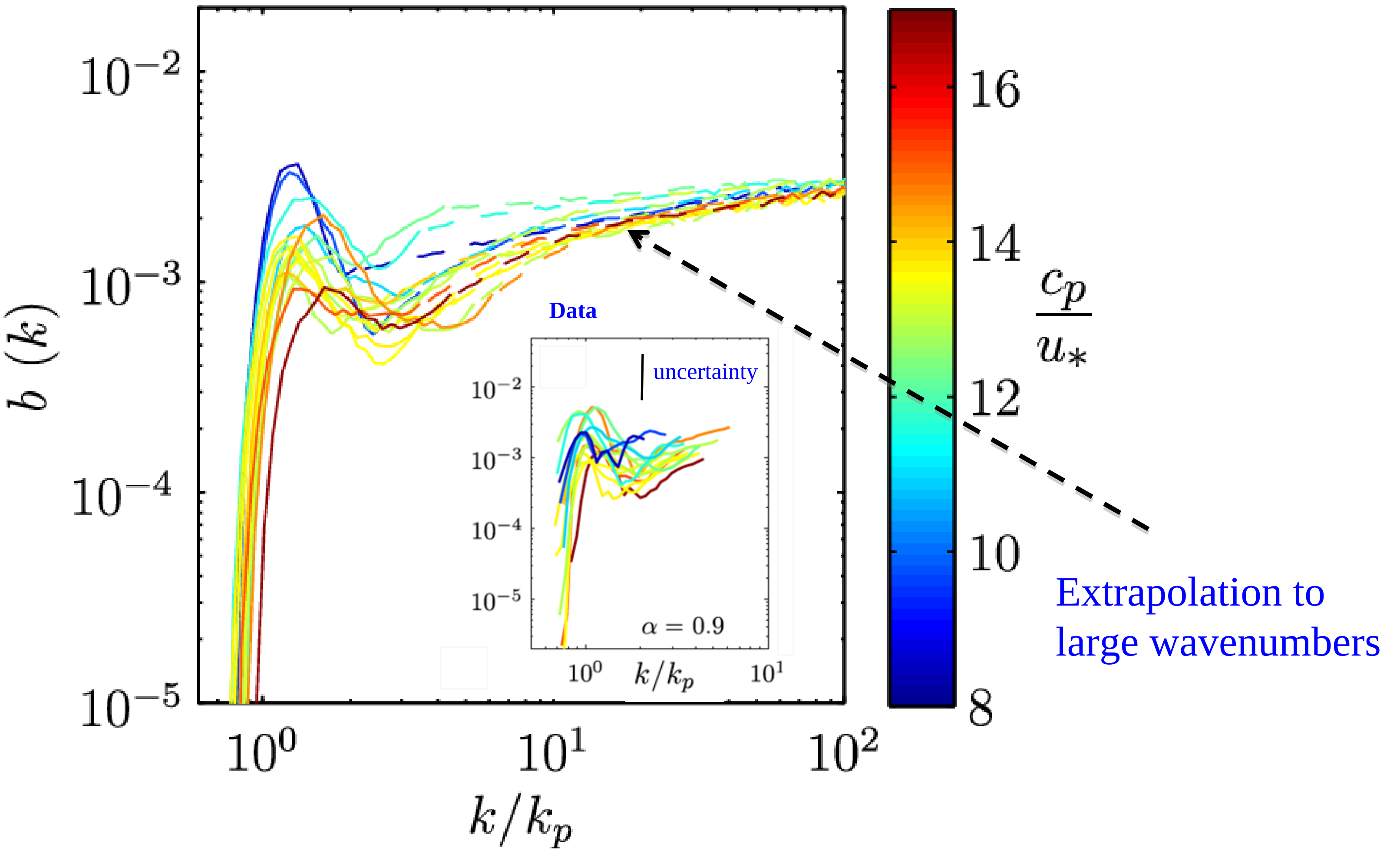
Fits to the field data



-BP : Banner and Peirson 2007
-M: Melville 1994
-DML: Drazen, Melville, and Lenain 2008

Model of *b(k)*

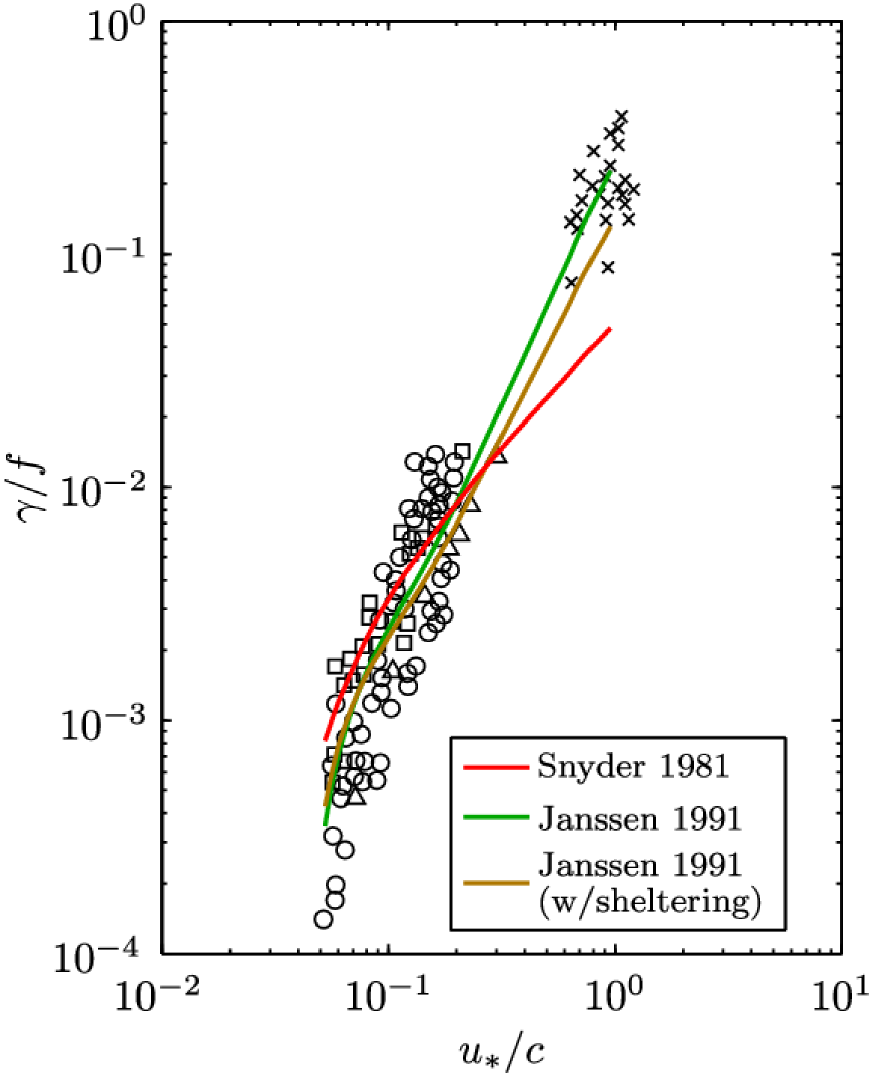
$\alpha = 0.9$, S_{in}: Snyder 1981



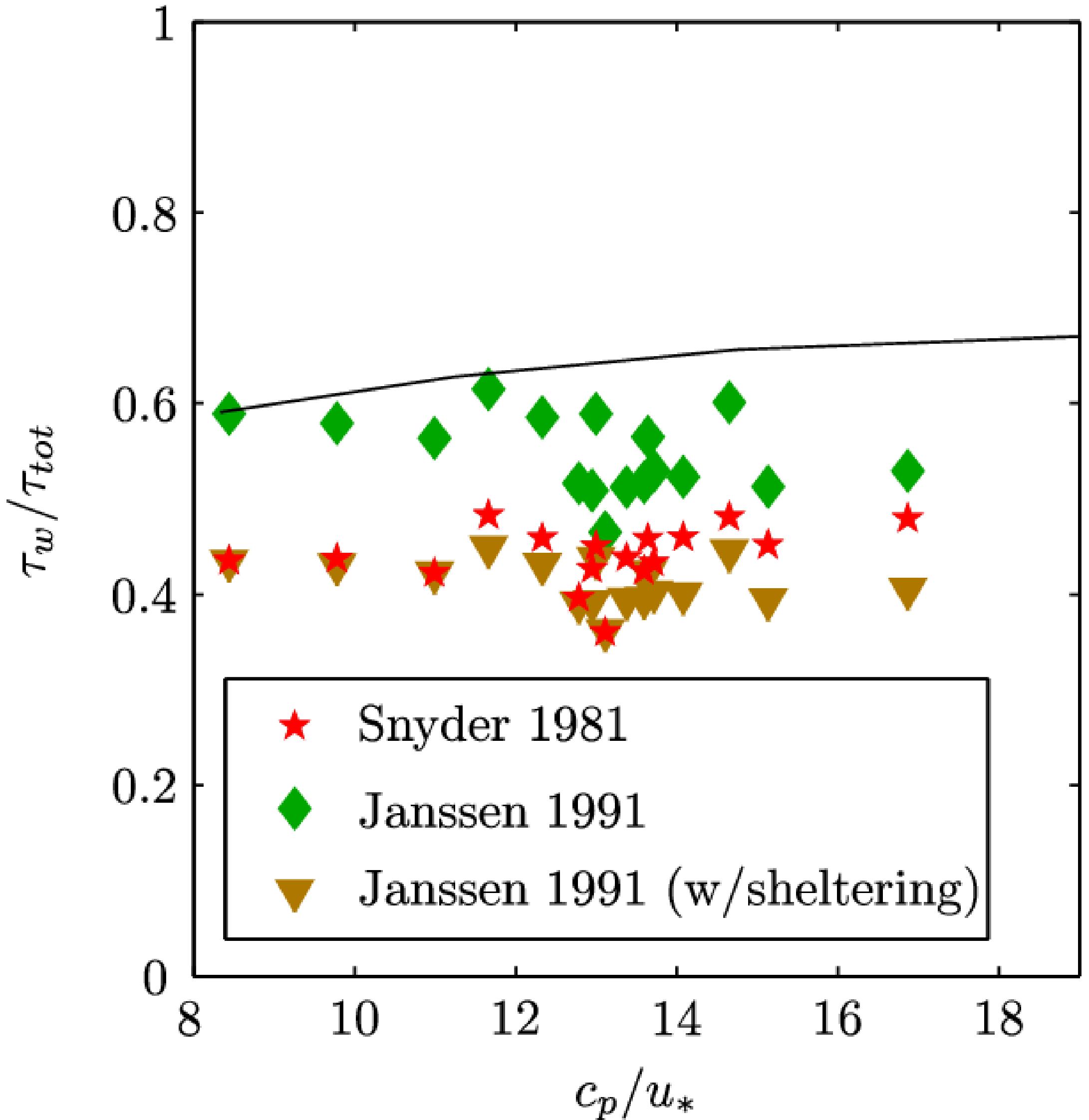
Concluding Remarks

- First quantification of the spectral strength of breaking in the field
- *b(k)* is not a constant, it varies by up to an order of magnitude between the spectral peak and high wavenumbers.
- Based on laboratory measurements and the inertial scaling by Drazen et al. (2008) a spectral model of *b(k)* with a saturation threshold behavior was fitted to the data.
- This work is currently in review for publication in JPO.

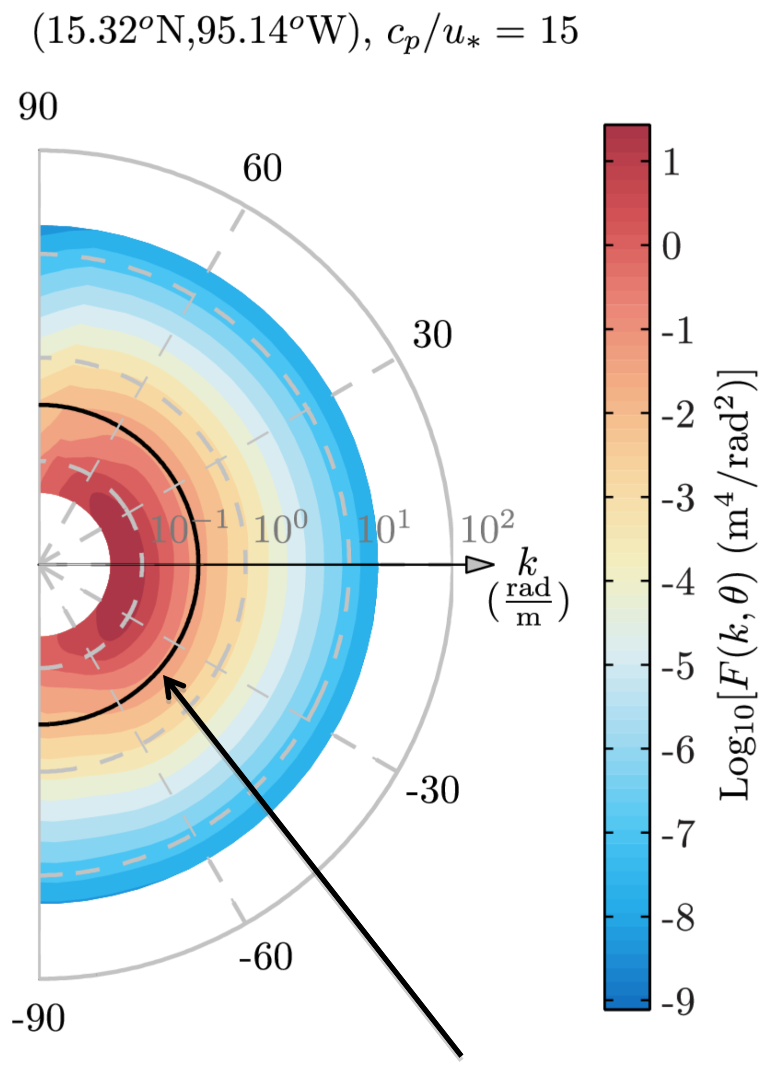
Dimensionless Growth Rate

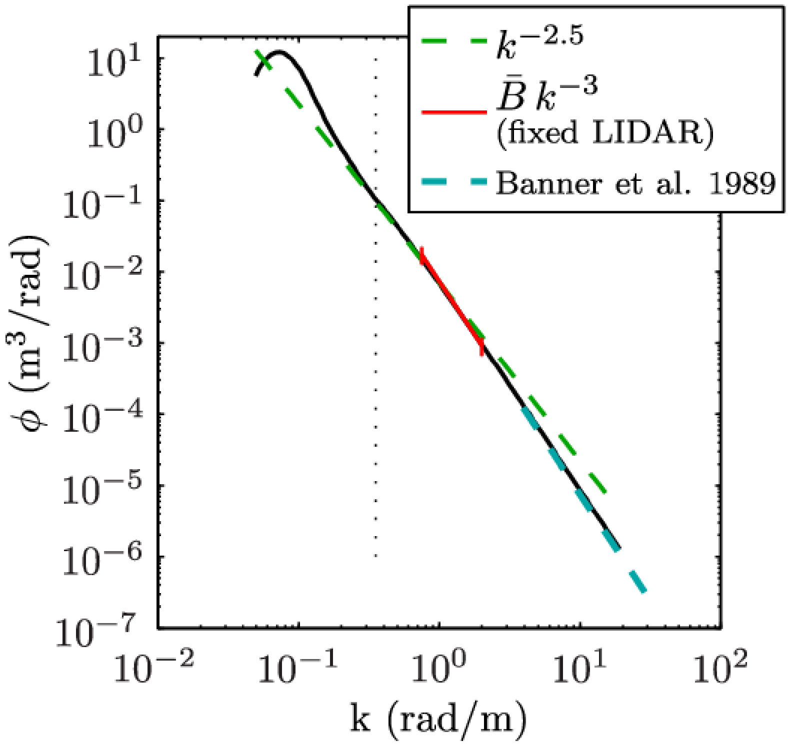


Wave Induced Momentum



Extrapolation of Spectrum to Large Wavenumbers





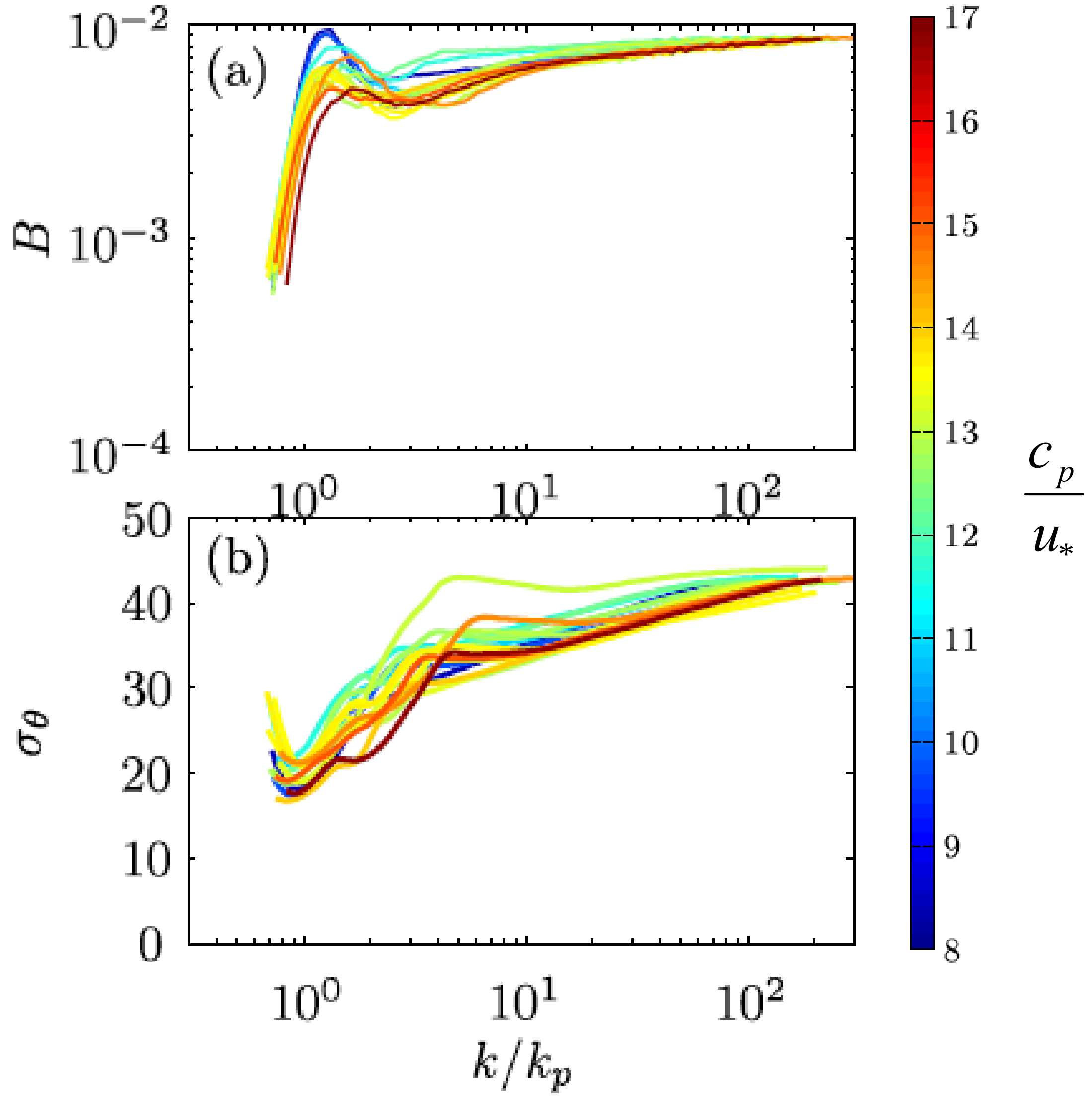
- Calculation of S_{nl} require a broad bandwidth

Limit of scanning lidar data

One-dimensional Saturation and Directional Spreading

$$B(k) = \int F(k,\theta) k^4 d\theta$$

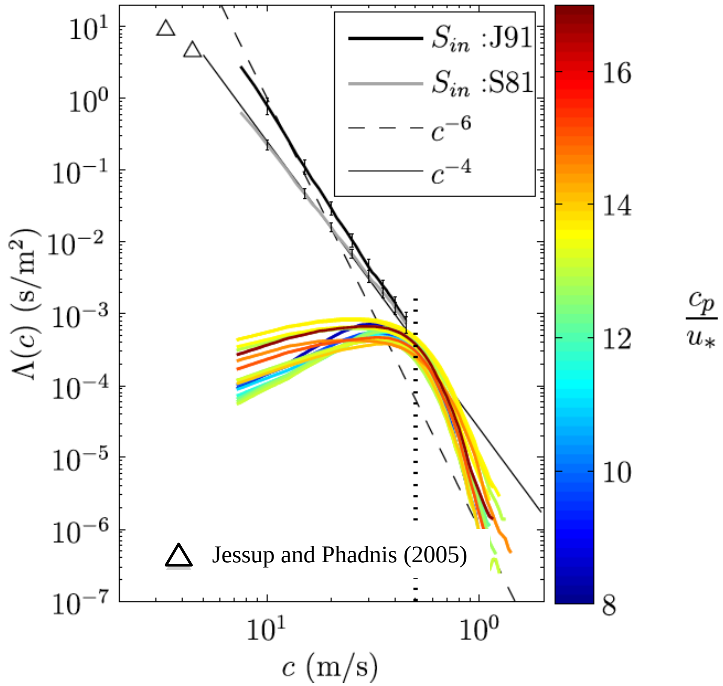
$$\sigma_\theta(k) = \frac{\int F(k,\theta)\,|\theta|\,d\theta}{\int F(k,\theta) d\theta}$$



Future Work

- Need to extend this work to lower values of c.

$$\rho g S_{ds}(c)dc = b\frac{\Lambda(c)c^5}{g}dc$$



- Other regimes:
  - -Extreme wind forcing
  - - Broader range of wave age
  - - Areas with significant wave current interaction.

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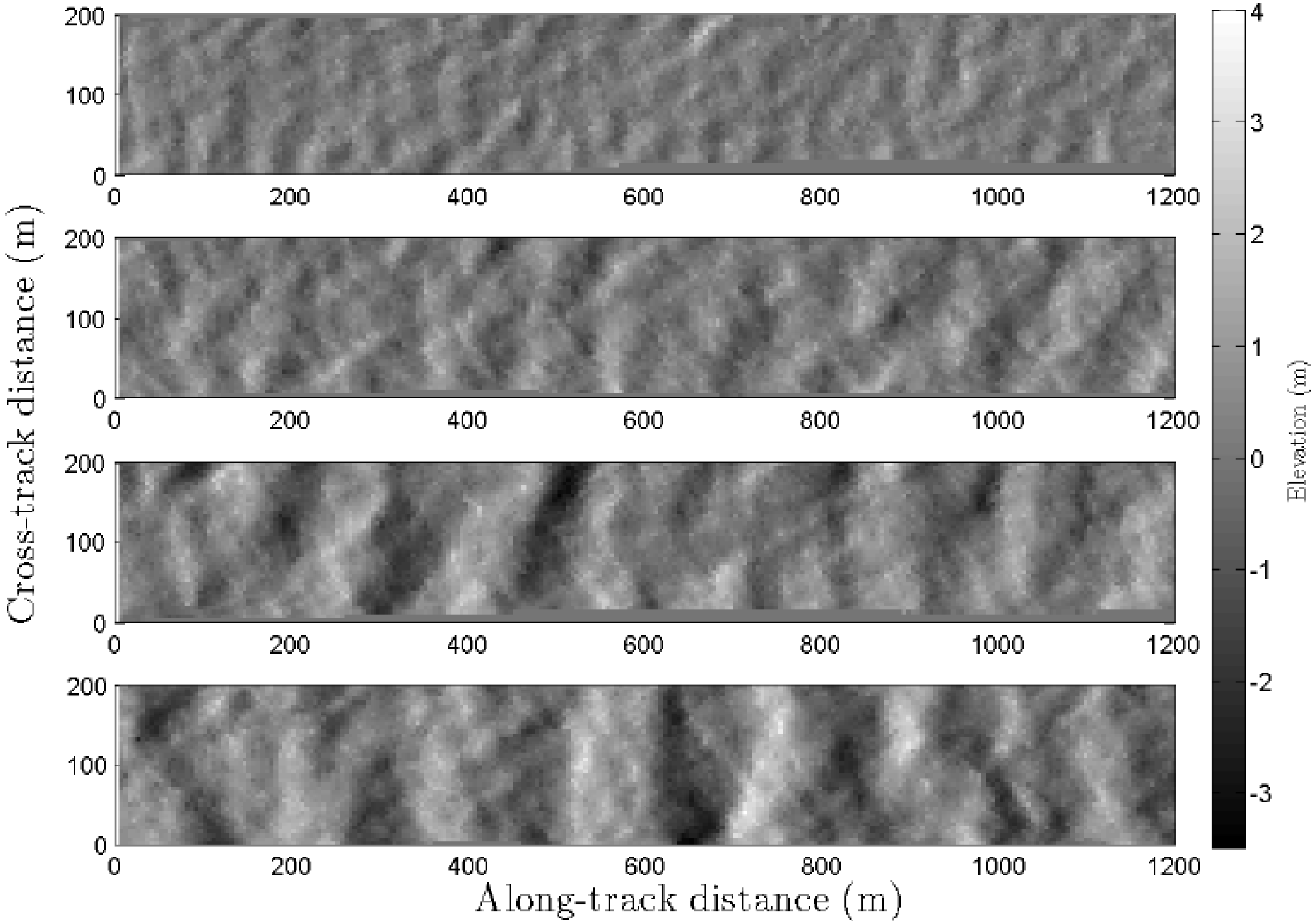
Surface Topography Data



Offshore Distance (km)

25

65

200

356

Motivation

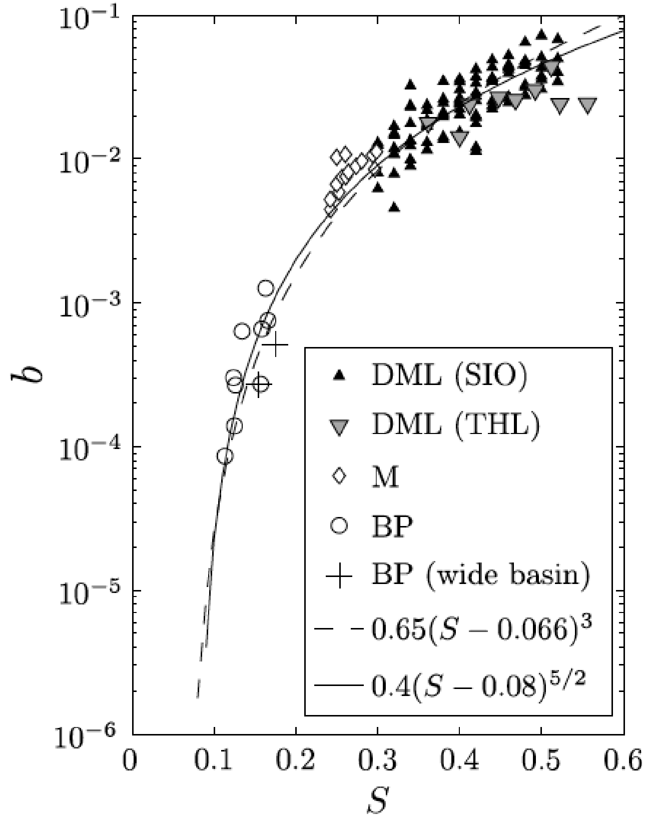
*Duncan, 1981

$$\varepsilon_l = \frac{b\rho_w c^5}{g}$$

ε_l : energy dissipation per unit length

b : empirical parameter characterizing the **strength of breaking**

c : wave phase speed



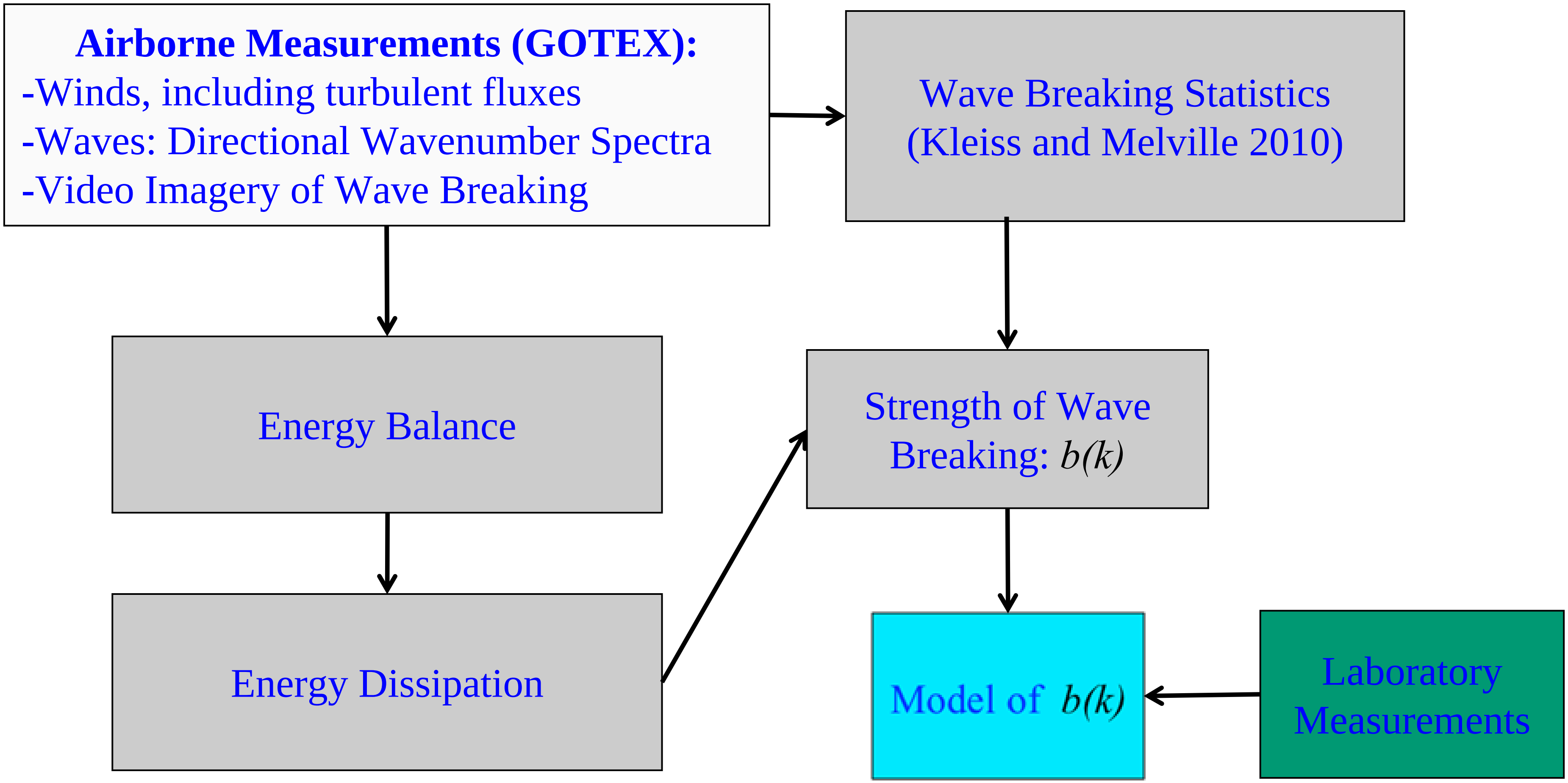
S : predicted linear wave slope of the focusing wave packets

-BP : Banner and Peirson 2007

-DML: Drazen, Melville, and Lenain 2008

- b is not a constant, spans over three orders of magnitude
- The data shows a threshold behavior $b=A\,(S-S_T)^N$, with $N=5/2$ which is consistent with the inertial scaling by DML.
- **The goal is to determine and model *b(k)* from field observations**

Quantifying the Strength of Wave Breaking in the Field *b(k)*



*The model of *b* would allow the prediction of the statistics of breaking waves in the field

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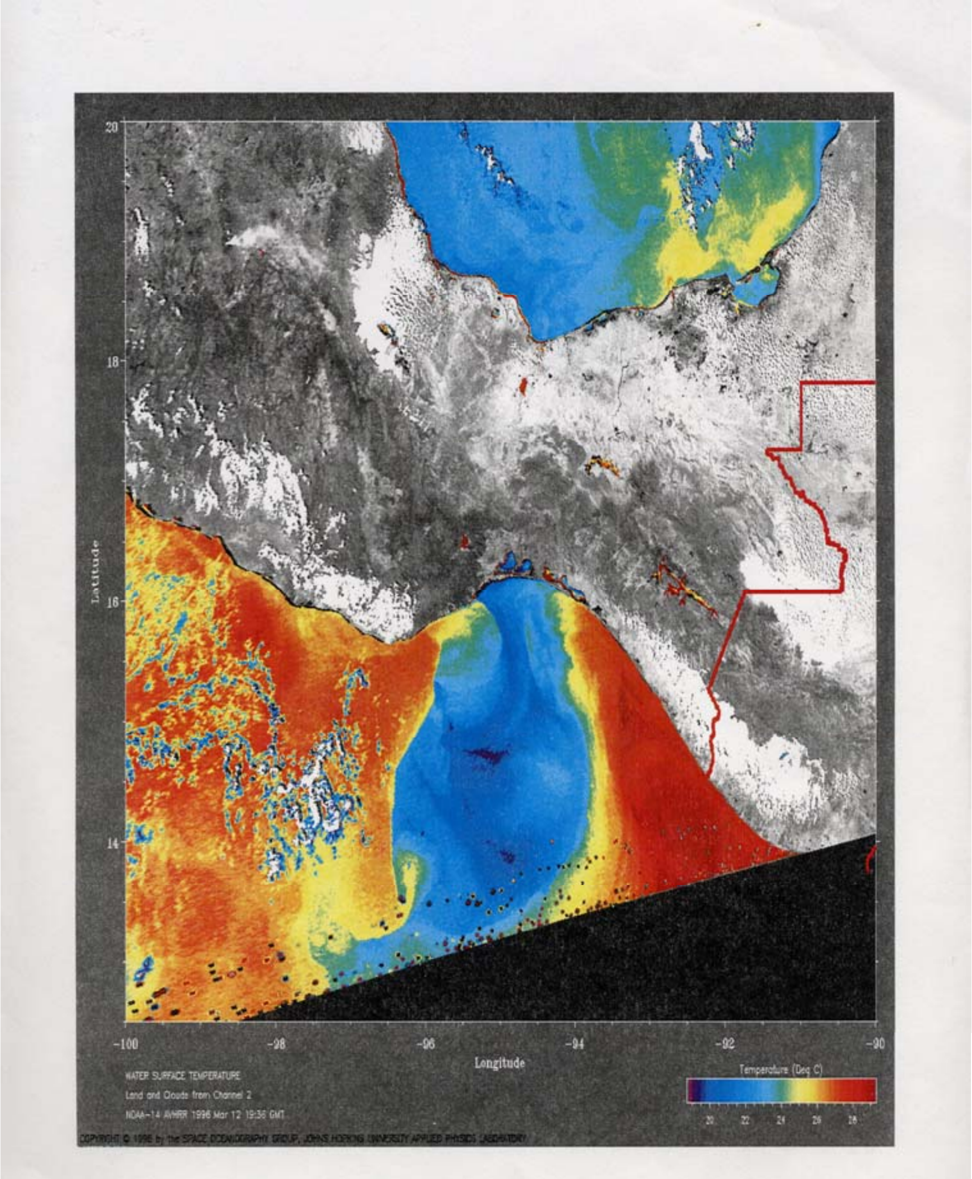
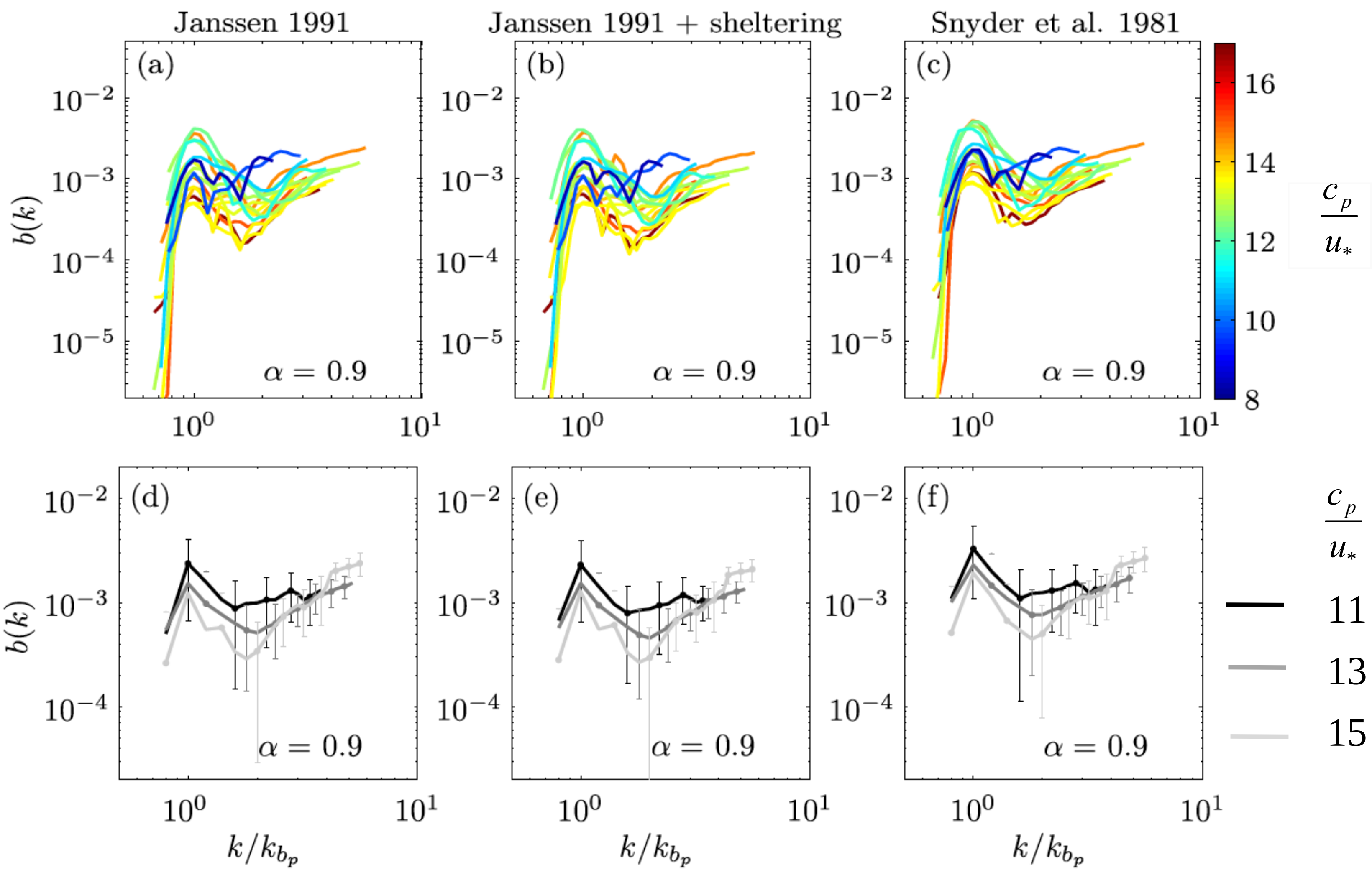
SST associated with upwelling due to gap winds in the Gulf of Tehuantepec



NSF/NCAR C-130

Bin-Averaging and Error Propagation



k_{b_p} is the wavenumber at the peak of $b(k)$

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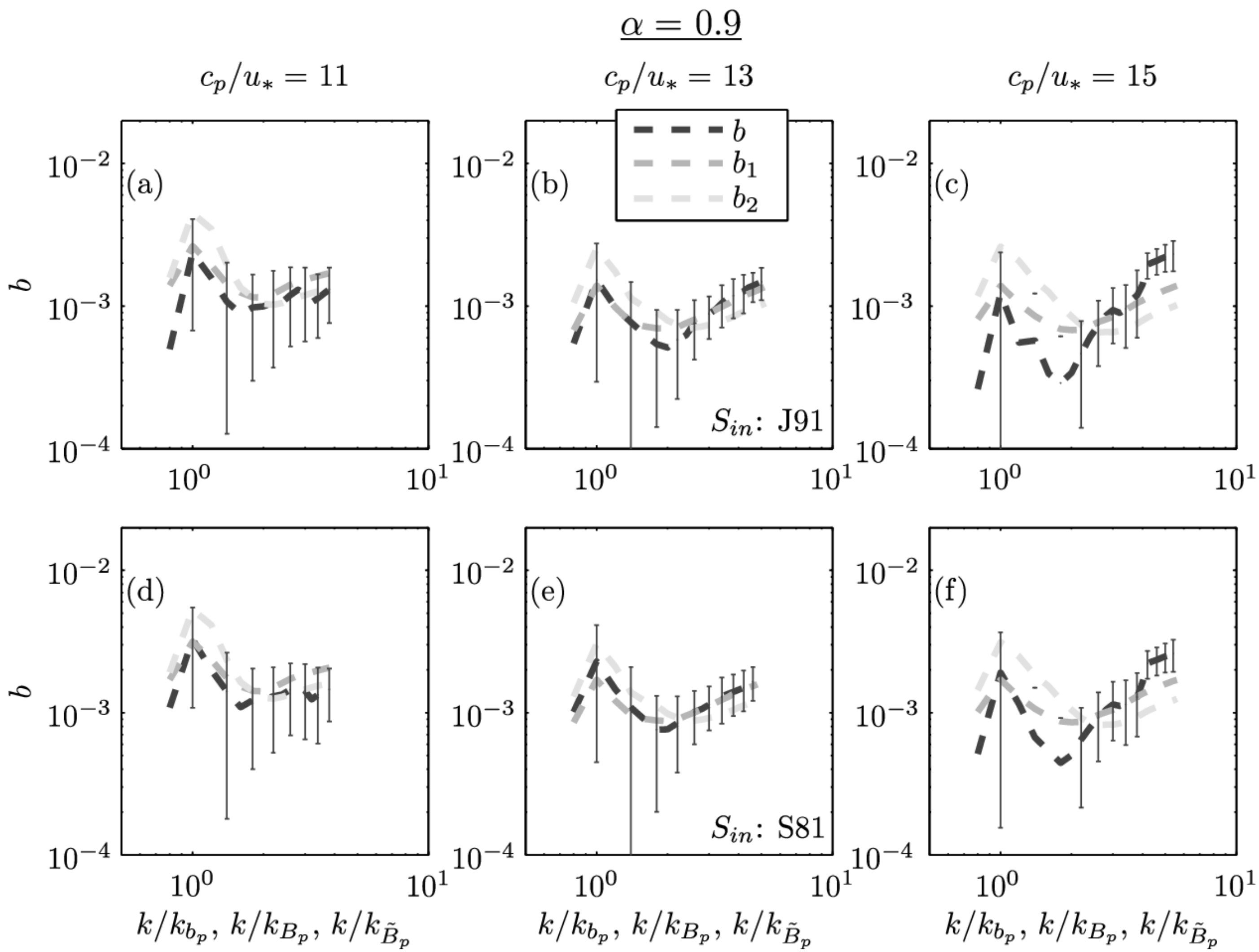
b(k) Models and Field Data



Models of *b(k)*

$$b_1(k) = A_1(B(k)^{1/2} - B_T(k)^{1/2})^{5/2}$$

$$b_2(k) = A_2(\bar{B}(k)^{1/2} - \bar{B}_T(k)^{1/2})^{5/2}$$

TABLE 1. Average and standard deviation of the parameters of models $b_1(k)$ and $b_2(k)$ in equations (26) and (27) fitted against the bin-averaged data of $b(k)$. The mean and standard deviation values given correspond to the mean and standard deviation of the parameters fitted for each bin-averaged distribution of $b(k)$

.

Parameter	Average	Std. Dev.	Average	Std. Dev.
	$\alpha = 0.9$		$\alpha = 1.0$	
		Janssen 1991		
A_1	3.9	0.8	4.9	0.8
B_T	10.6×10^{-4}	2.0×10^{-4}	8.8×10^{-4}	1.2×10^{-4}
N_{w_1}	6.1	1.1	7.4	0.9
A_2	1.7	0.4	2.2	0.4
$\bar{B}_T$	2.1×10^{-3}	0.4×10^{-3}	1.6×10^{-3}	0.2×10^{-3}
N_{w_2}	3.2	0.5	4.0	0.5
		Snyder 1981		
A_1	4.4	0.9	5.5	0.8
B_T	9.7×10^{-4}	1.8×10^{-4}	8.0×10^{-4}	1.0×10^{-4}
N_{w_1}	6.7	1.1	8.1	1.0
A_2	1.9	0.4	2.5	0.4
$\bar{B}_T$	1.9×10^{-3}	0.3×10^{-3}	1.5×10^{-3}	0.2×10^{-3}
N_{w_2}	3.5	0.6	4.4	0.6

Phillips' (1985) Equilibrium Range Model

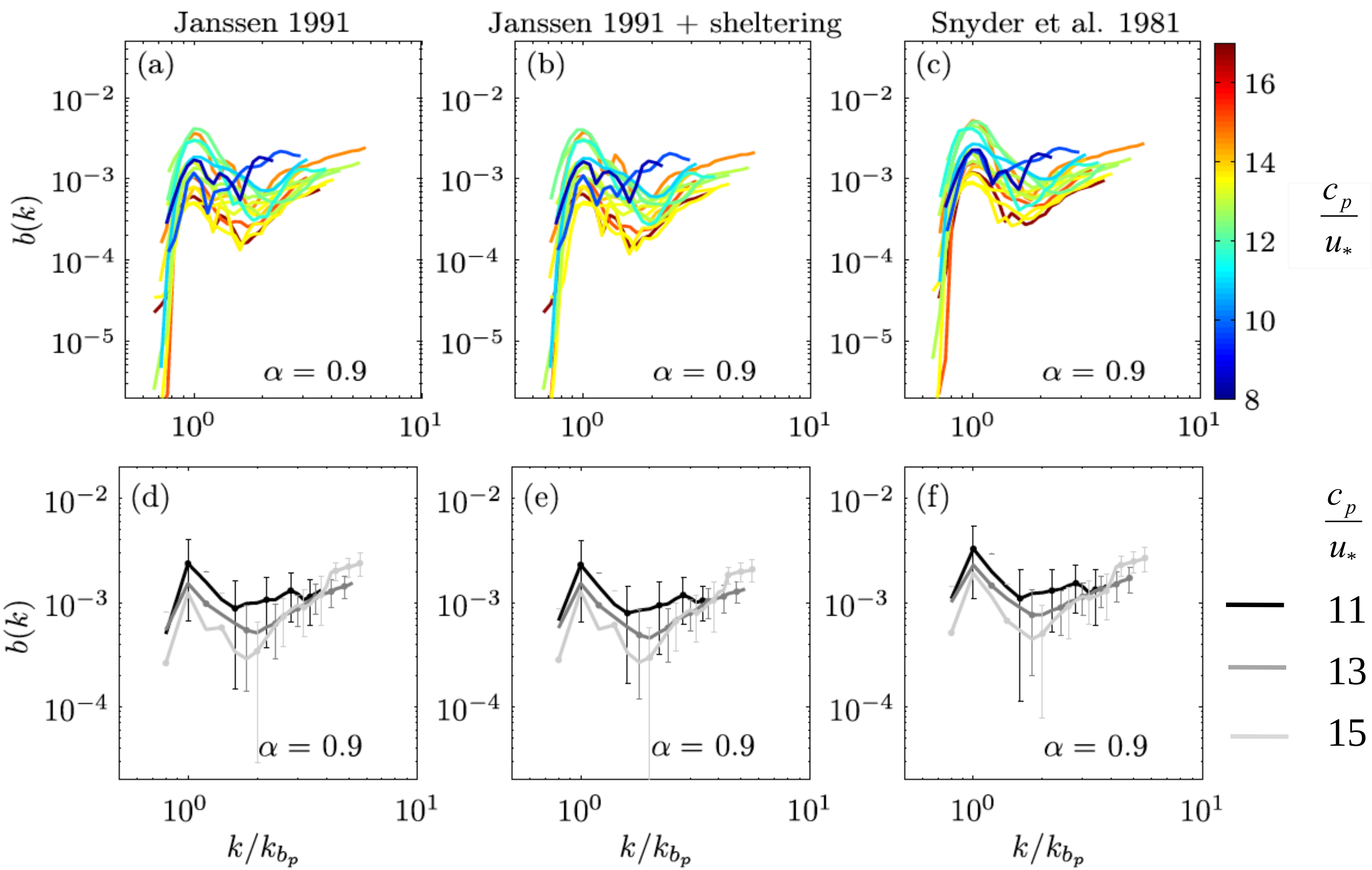
$$S_{in} + S_{ds} + S_{nl} = 0$$

$$|S_{in}| \sim |S_{ds}| \sim |S_{nl}|$$

- Based on empirical knowledge of the spectrum, wind input by Plant (1982), and scaling of $S_{nl} \rightarrow S_{ds}$
- Prediction of the spectral statistics of breaking front per unit surface area $\Lambda(c)$ using a constant value of the breaking parameter b.

$$\rho g S_{ds}(c)dc = b\frac{\Lambda(c)c^5}{g}dc \quad \longrightarrow \quad \Lambda(c) \propto c^{-6}$$

Bin-Averaging and Error Propagation



k_{b_p} is the wavenumber at the peak of $b(k)$

Energy dissipation

$$\rho_w g\, S_{ds}(c)\, dc = \rho_w \frac{b(c)}{g} \Lambda(c) c^5 dc$$

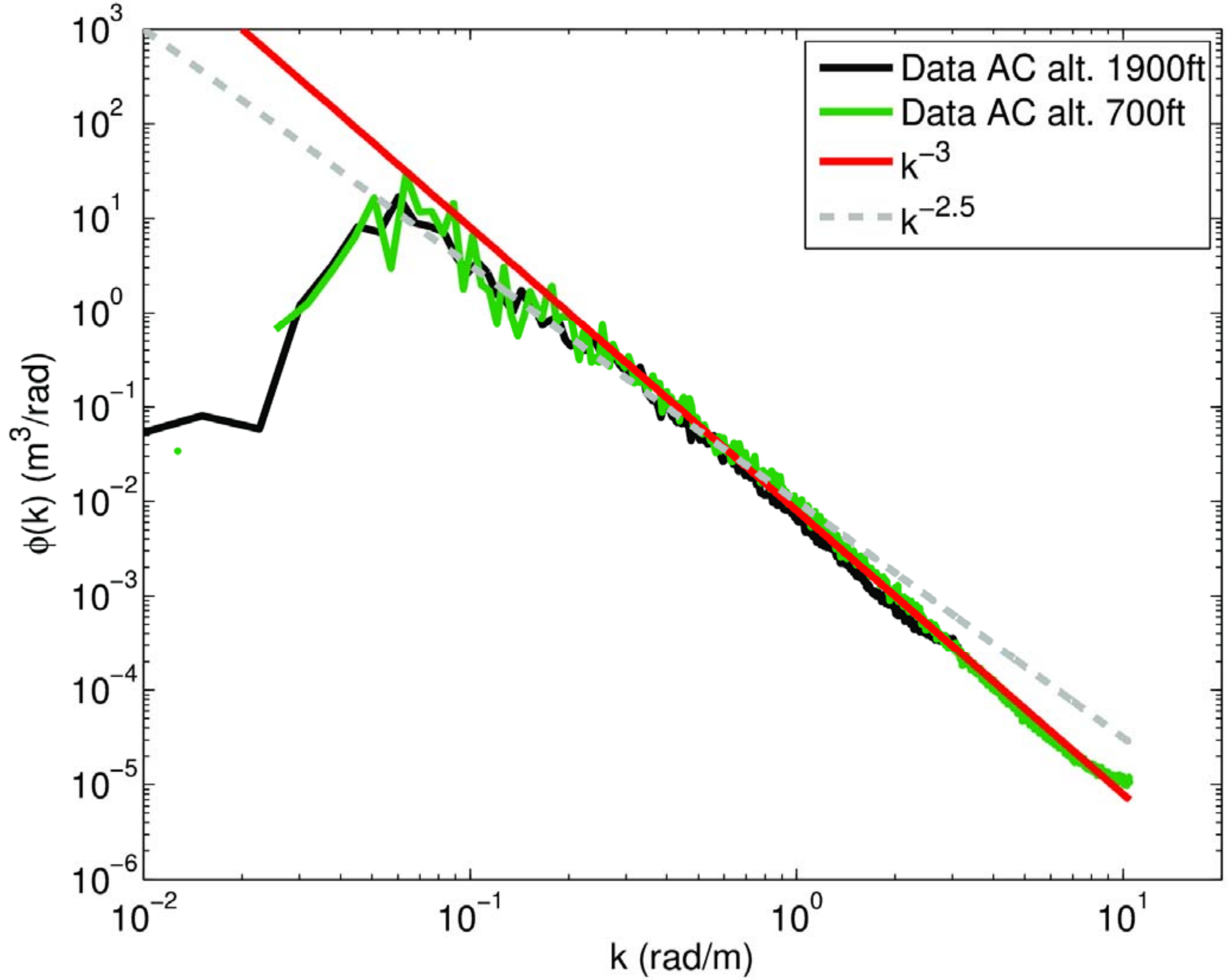
- $b(c)$ - strength of wave breaking

Momentum loss due to wave breaking

$$\rho_w g \frac{S_{ds}(c)}{c} dc = \rho_w \frac{b(c)}{g} \Lambda(c) c^4 dc$$

- $\Lambda(c_{br})$ quantified from the GOTEX video observations in Kleiss and Melville 2011.

Gulf of Mexico
(October 2011)



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Gulf of Mexico
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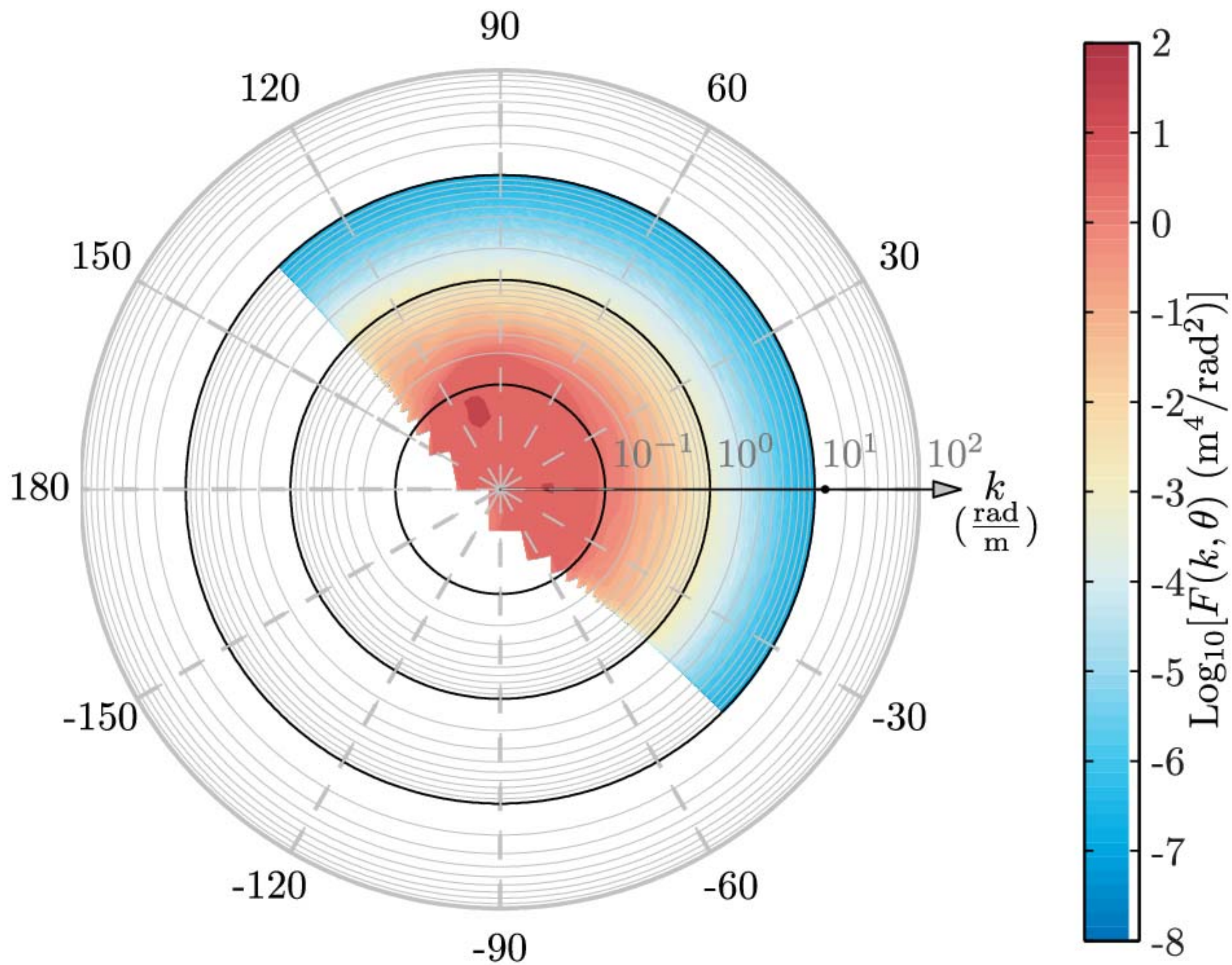
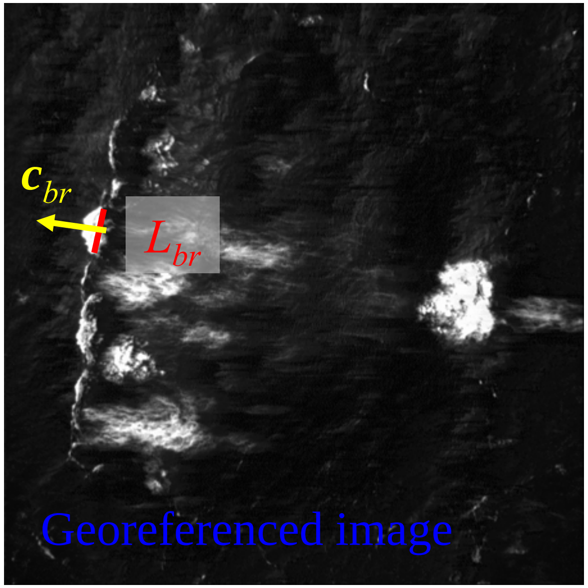


Wave Breaking Statistics



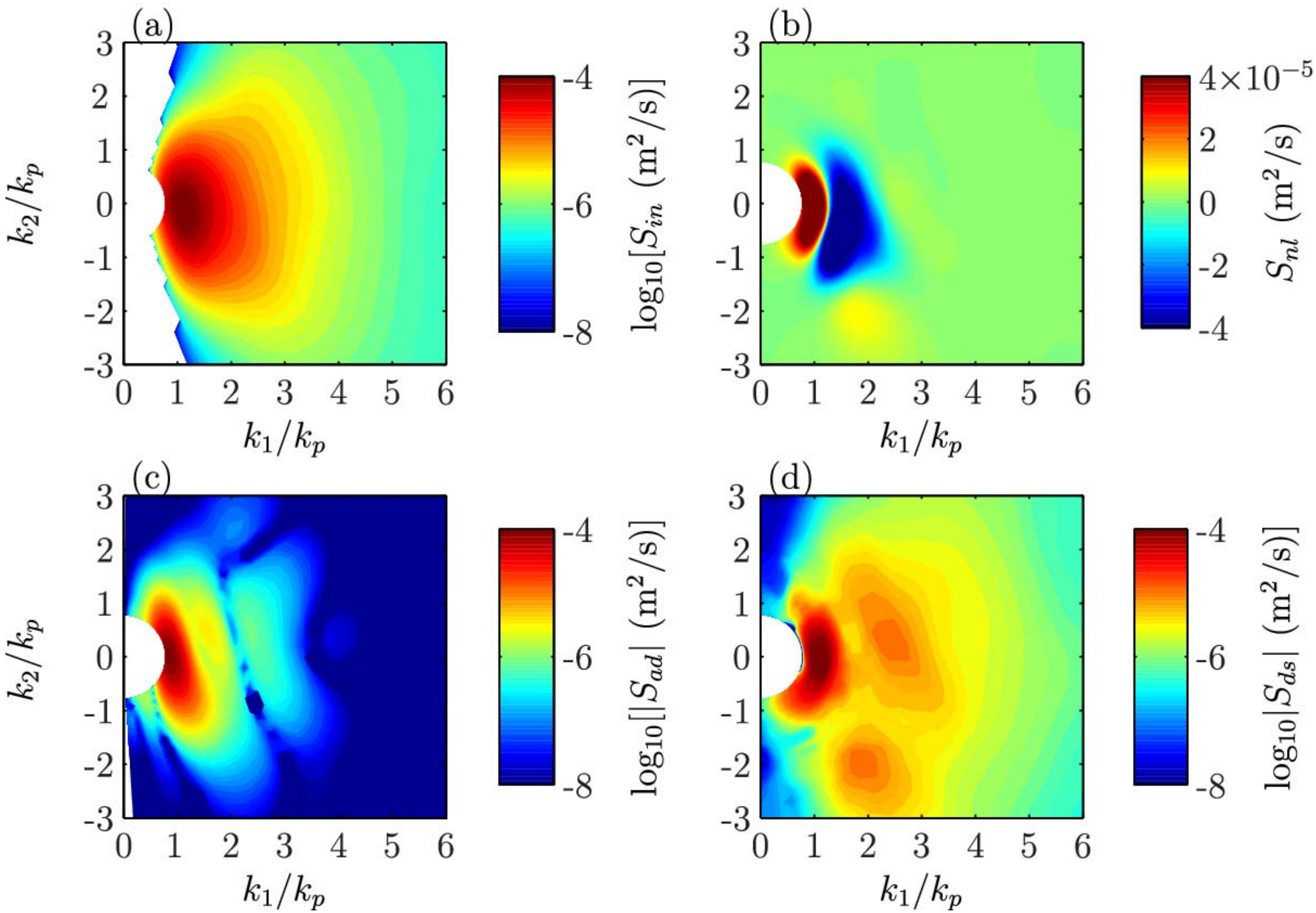
$\Lambda(c_{br})$: length of breaking fronts with velocity in the range $(c_{br}, c_{br}+\Delta c_{br})$ per unit surface area.

$$\Lambda(c_{br}) = \frac{\sum L_{br}(c_{br} - \frac{\Delta c_{br}}{2} < c < c + \frac{\Delta c_{br}}{2})}{A_{tot}\,\Delta c_{br}}$$

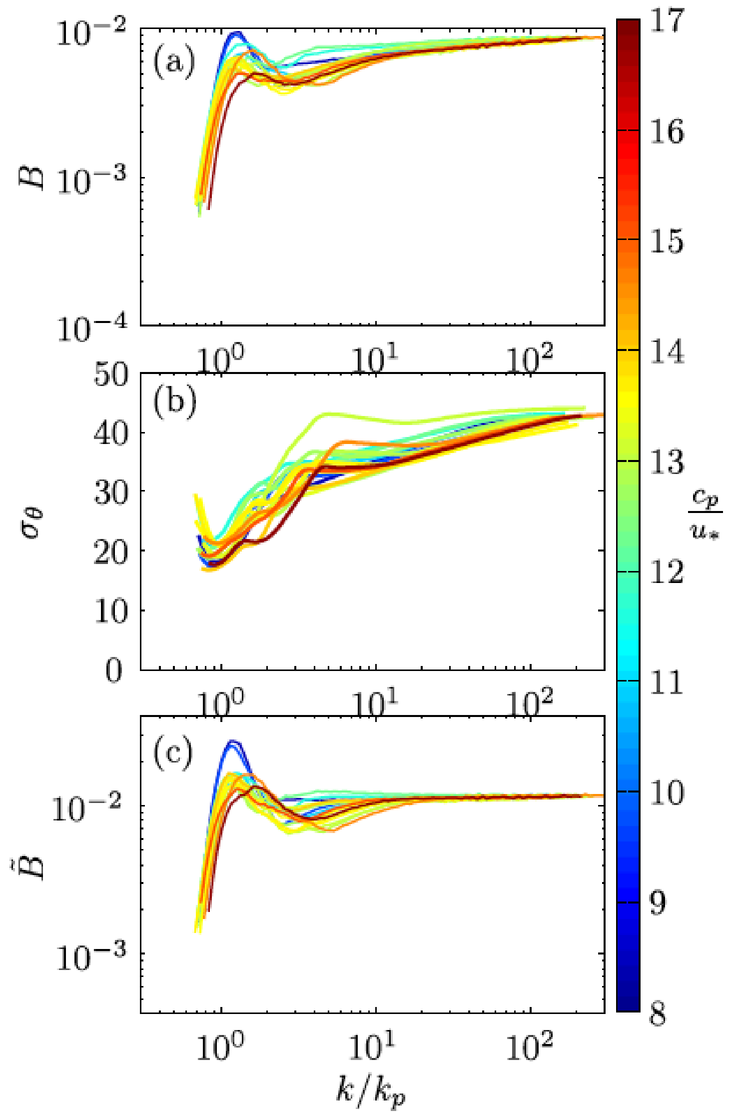
L_{br} is the length of each breaking crest moving with velocity $(c_{br}, c_{br}+\Delta c_{br})$
A_{tot} the total surface area

- $\Lambda(c_{br})$ and its moments provide a framework to relate the kinematics and dynamics of breaking waves through a dimensionless function $b(c_{br})$ (Phillips 1985)

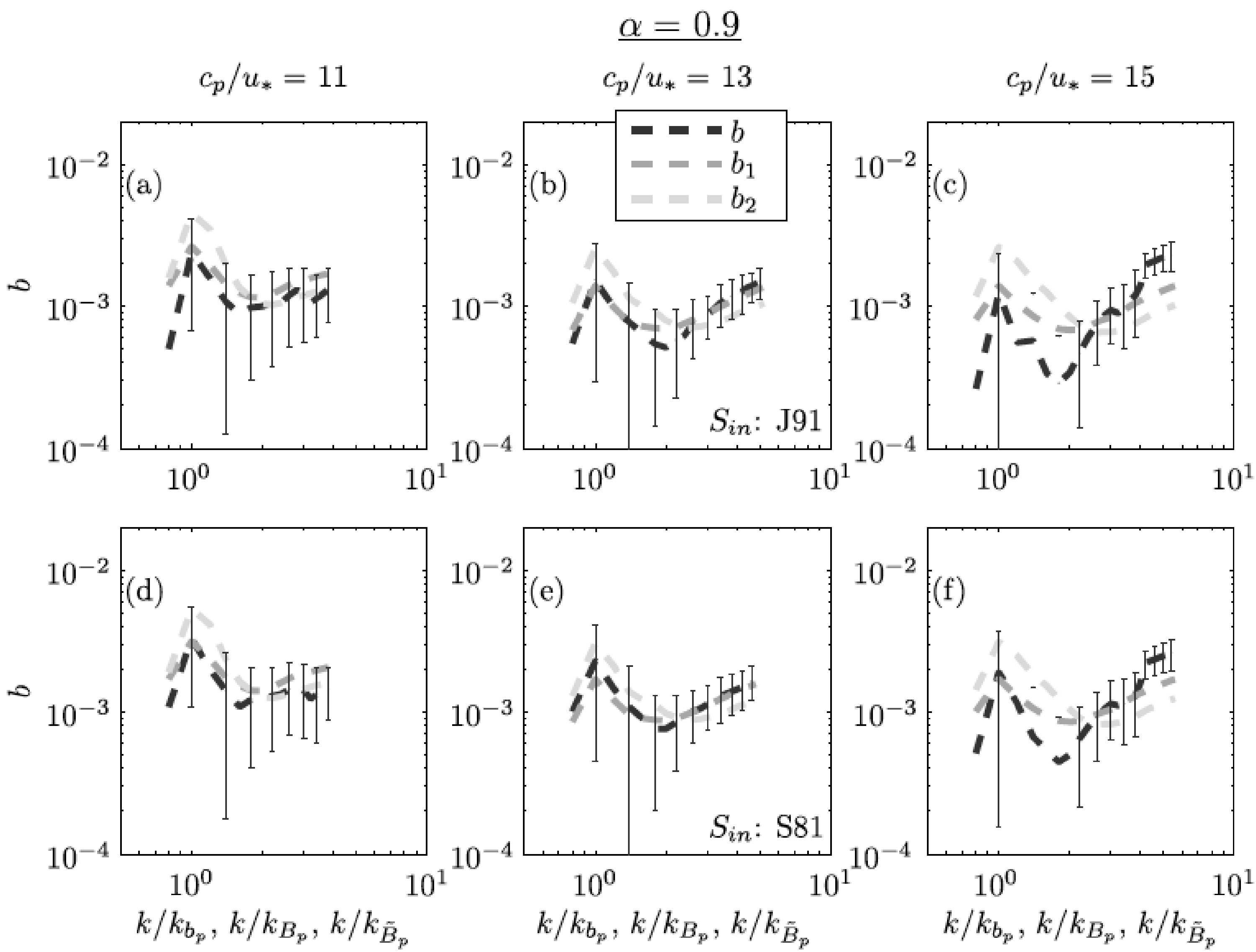
Directional Source Terms



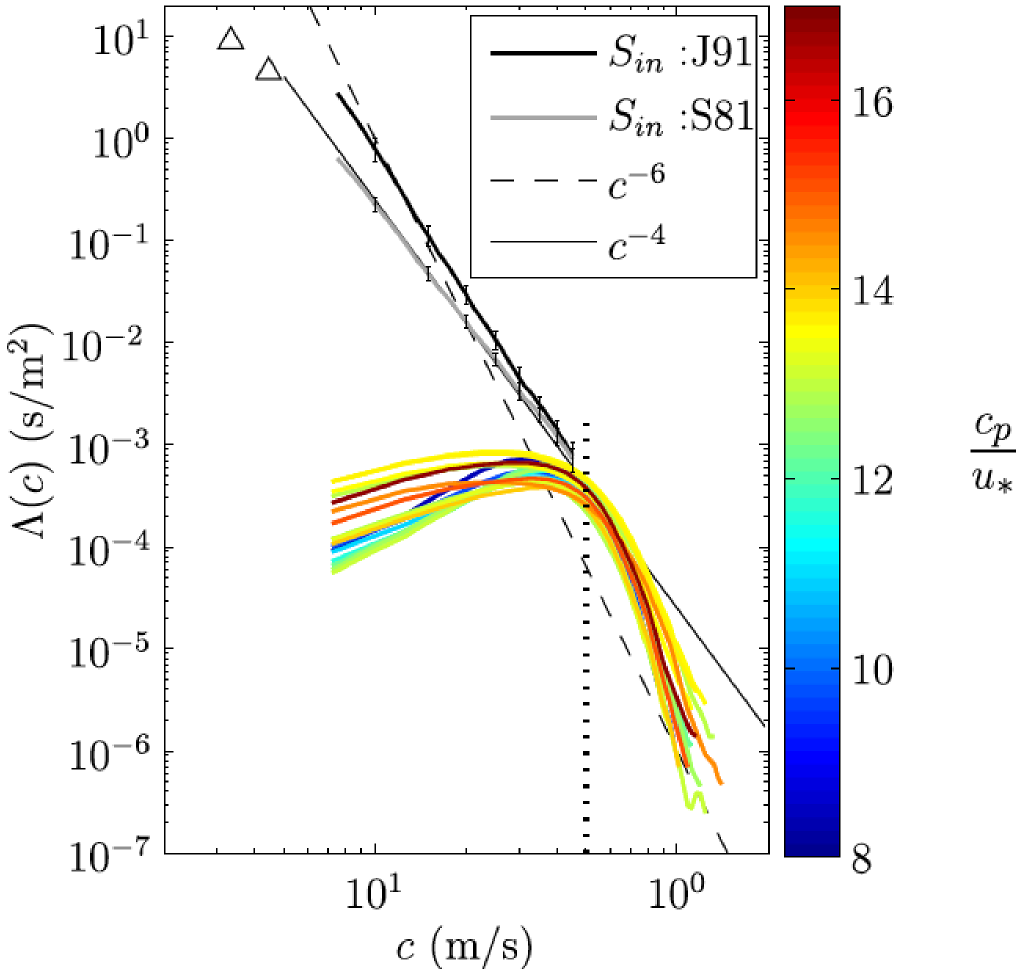
One-dimensional Saturation and Directional Spreading



Strength of Breaking Model Fitting



Predictions of $\Lambda(c)$ Based on the Model of b at larger values of c



$\alpha = 0.9$

Gulf of Tehuantepec Experiment (GOTEX)

*February, 2004

Goals:

• Processes of **wave development** and **breaking**

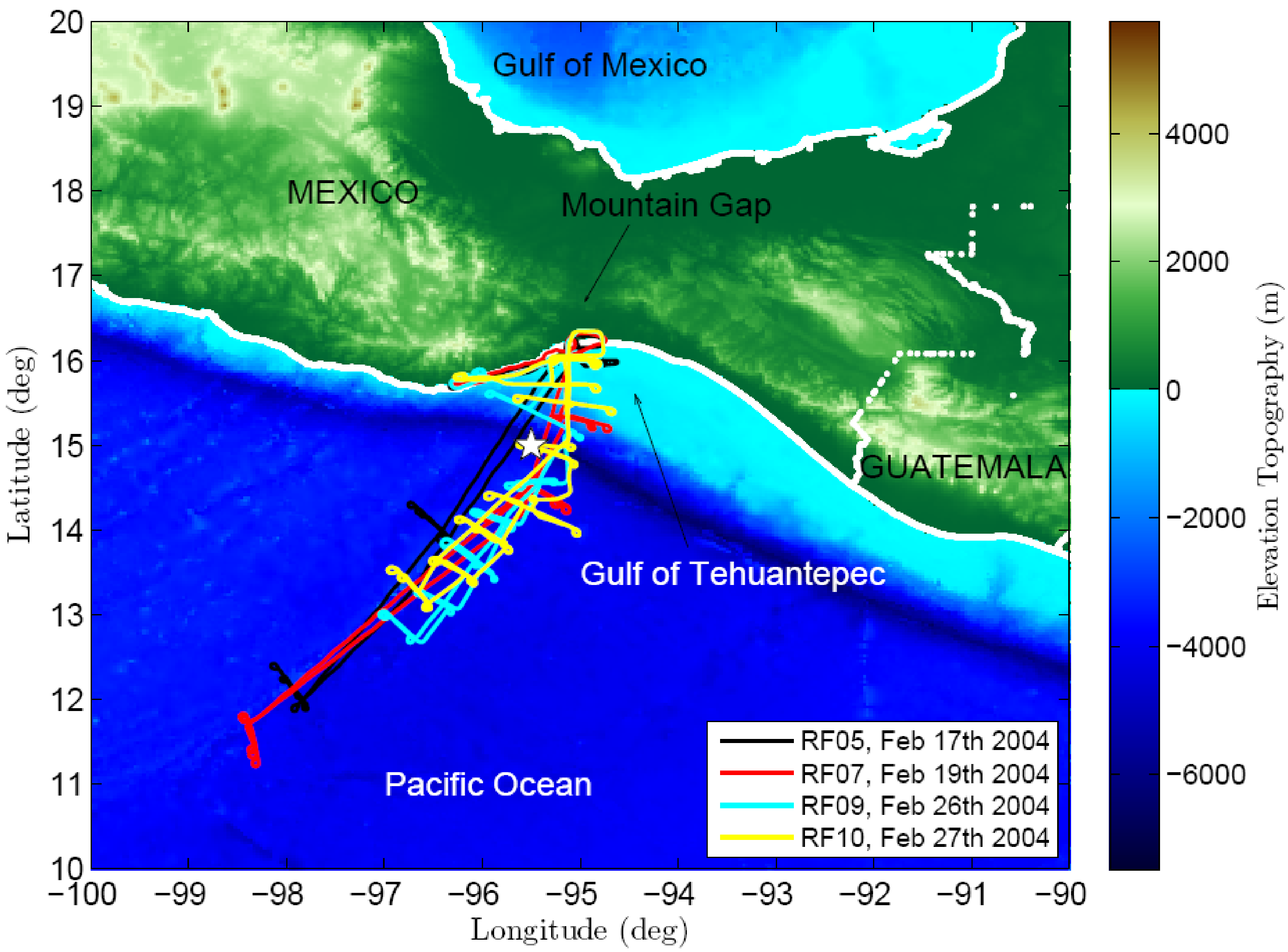
Setting:

• Offshore, gale-force winds
• Predictable wind events with high probability
• Essentially open airspace

Platform:

• C-130 Aircraft based in Huatulco, Mexico

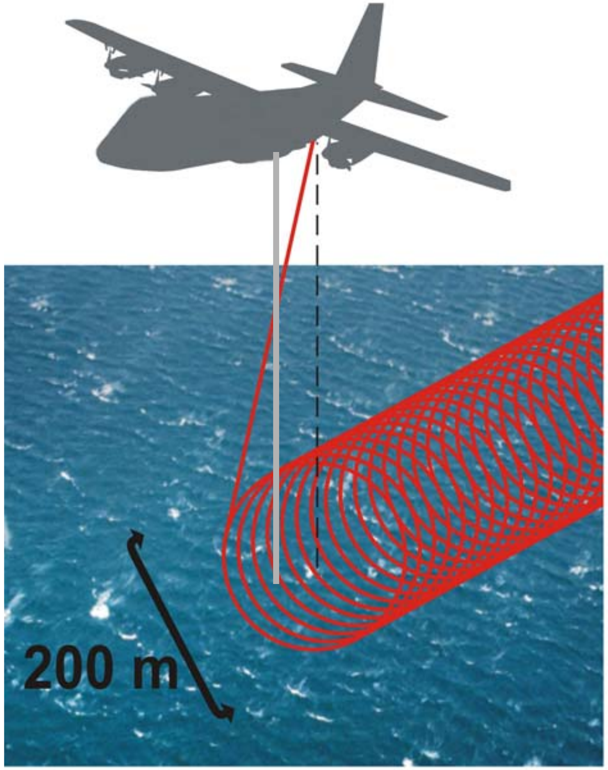
Flight Tracks



Airborne LIDARS

ATM

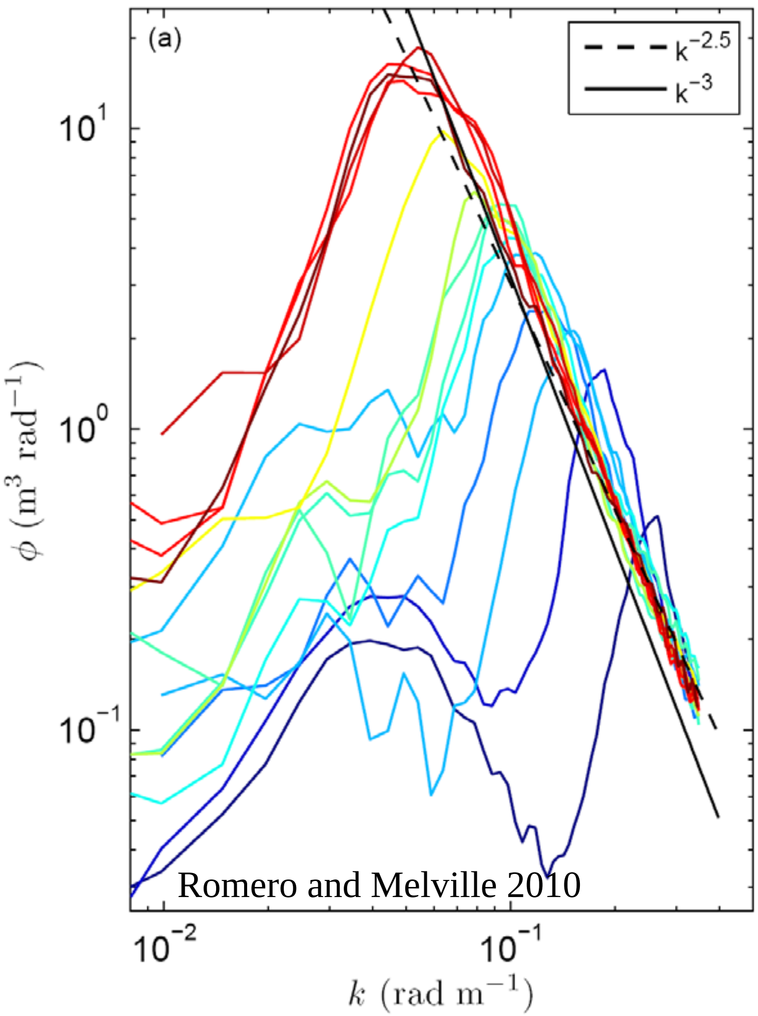
- Pulse rate: 5 khz
- Conical scanning rate: 20 hz
- Along track resolution: 5 m (at 100 m/s)
- Cross track resolution: 2.5 m
- Nominal altitude: 400 m
- Swath Width: 200m
- Calibrated error of elevation per pulse of approx 8 cm (Krabill & Martin, 1987)



RIEGL

- Pulse Rate: 5khz
- Nominal altitude: 30 and 200m
- Net rms elevation error ~ 11cm

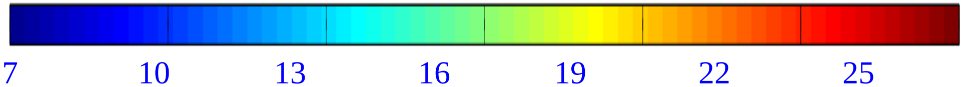
Omnidirectional Spectra



$$\phi(k) = \int_{-\pi/2}^{\pi/2} F(k,\theta) k d\theta$$

- *F(k, θ)* : directional wavenumber spectrum
- Peak is enhanced at early stages of development.
- A $k^{-2.5}$ power-law is consistent with the equilibrium models by Phillips 1985, and Kitaiigorodski, 1983.

Young seas



Old seas

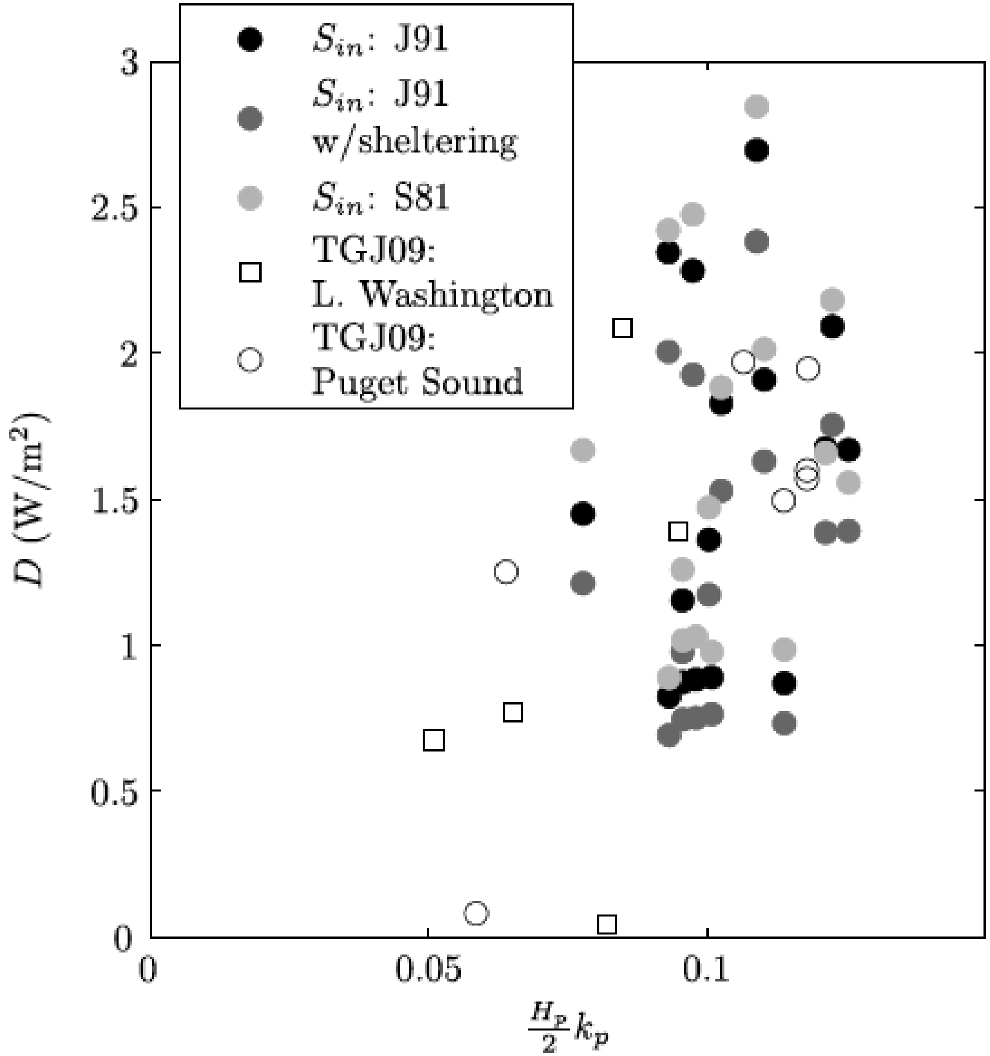
wave age: c_p / u_*

Implications for Wave – Current Interactions

- The model of $b(k)$ provides a rational framework to predict the statistics of breaking fronts in the field
  - Opens a way to include wave breaking effects in models of the upper ocean circulation (e.g. Sullivan et al. 2007, Restrepo 2007, and Restrepo et al. 2010)

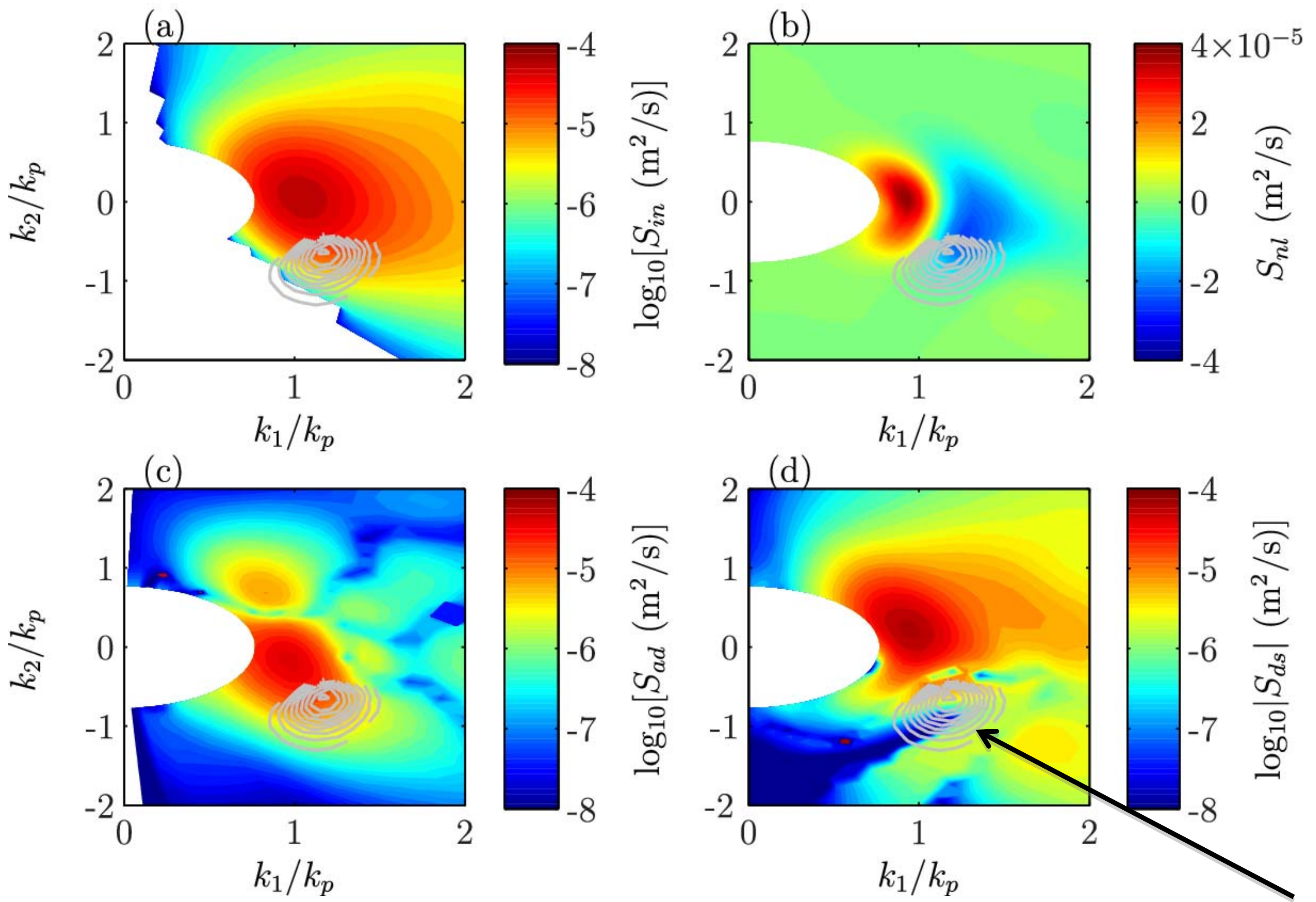
Other Applications

- Remote sensing
- Air-sea interaction (momentum flux, whitecapping coverage, aerosol productions, gas exchange)



S_{in}: J91
S_{in}: J91 w/sheltering
S_{in}: S81
TGJ09: L. Washington
TGJ09: Puget Sound
D (W/m^2)
$\frac{H_p}{2}k_p$
0
0.5
1
1.5
2
2.5
3
0
0.05
0.1

Directional Source Terms



$S_{ds} > 0$