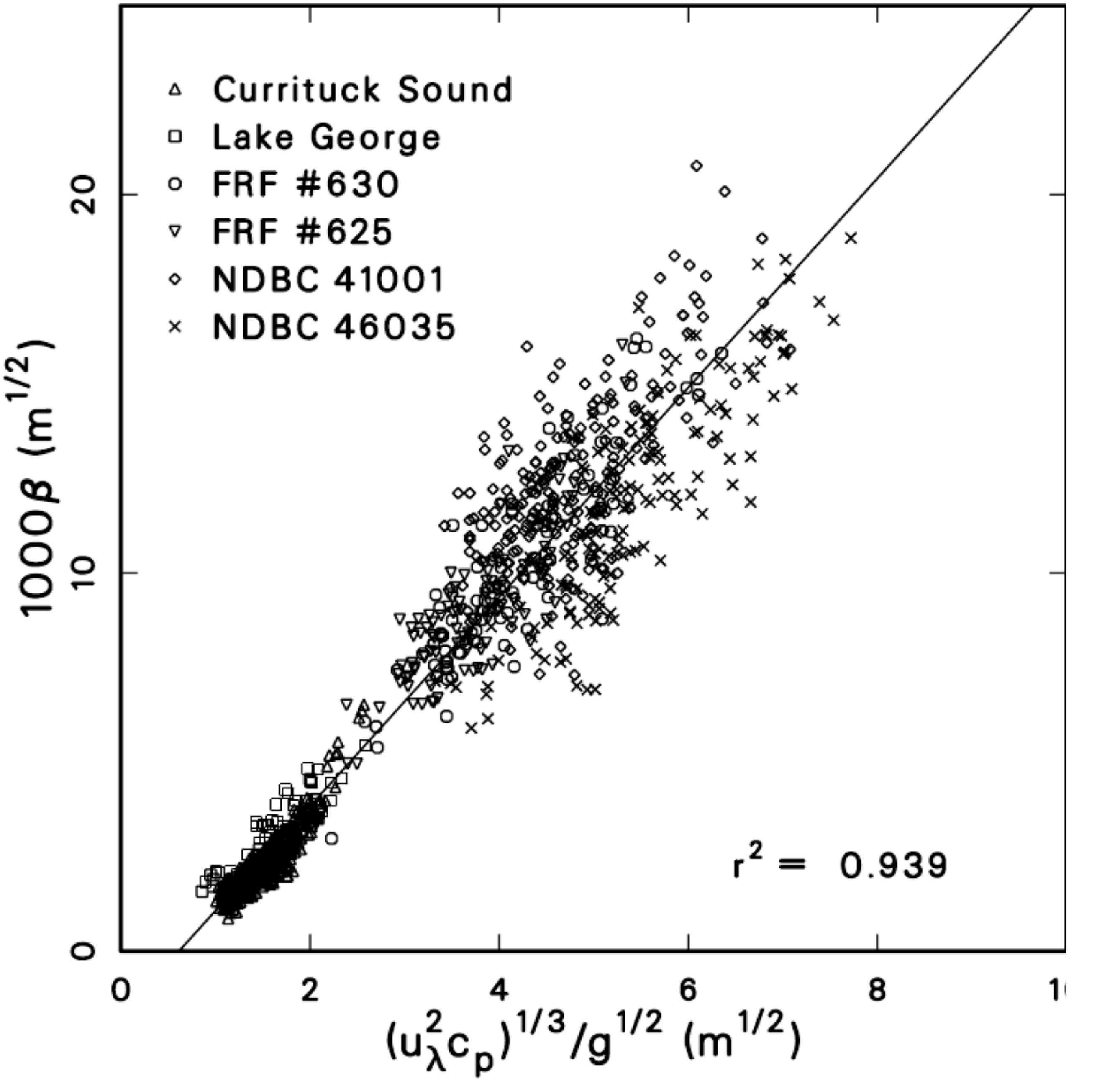
New Wind Input Term Through Experimental, Theoretical and Numerical Approaches

Vladimir Zakharov, Don Resio and Andrei Pushkarev

Waves and Solitons LLC
ERDC, US Army Corps of Engineers
Novosibirsk State University
Lebedev Physical Instutute RAS

$$F(k) = \beta k^{-5/2}$$

Resio and Long 2004-2007



Correlation of equilibrium range coefficient β *with* $(u_\lambda^2 c_p)^{1/3}/g^{1/2}$

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial \omega}{\partial \vec{k}} \frac{\partial \epsilon}{\partial \vec{r}} = S_{nl}(\epsilon) + S_{wind} + S_{diss}$$

$$\epsilon = \epsilon(\omega, \phi, t)$$

$$S_{nl} = \omega \left(\frac{\omega^5 \epsilon}{g^2} \right) \epsilon$$

$$S_{wind} = \alpha \omega^{s+1} f(\phi) \epsilon$$

$$S_{diss} = \begin{cases} C(\epsilon)\omega^{-5}\epsilon & \text{if } \omega > \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_0 = \frac{\omega_0}{2\pi} = 1.1$$

Time Limited Case

$$\frac{\partial \epsilon}{\partial t} = S_{nl}(\epsilon) + S_{wind} + S_{diss}$$

$$\epsilon = t^{p+q} F(\omega t^q)$$

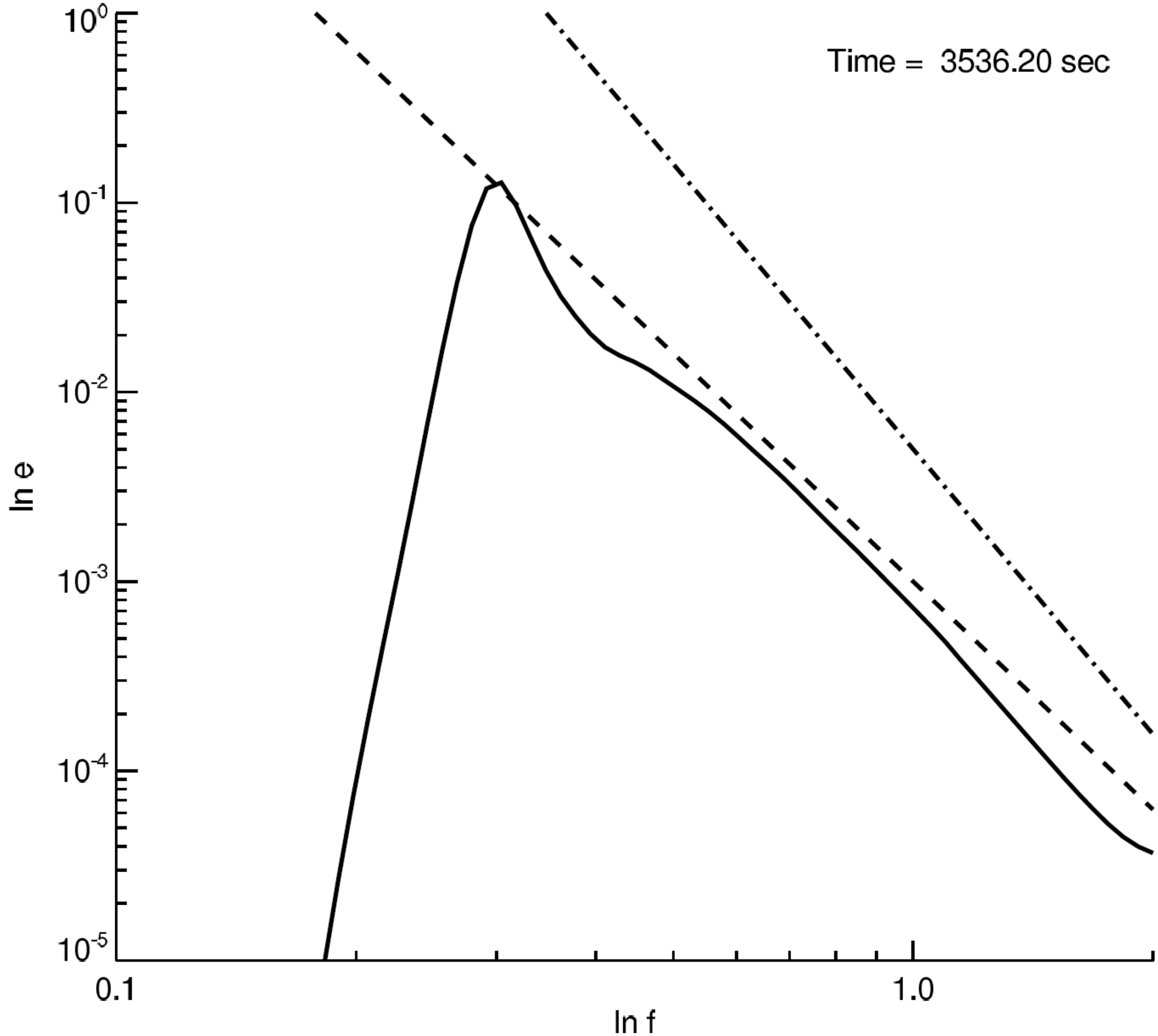
$$q = \frac{3}{7}, \quad p = 10, \quad s = \frac{4}{3}$$

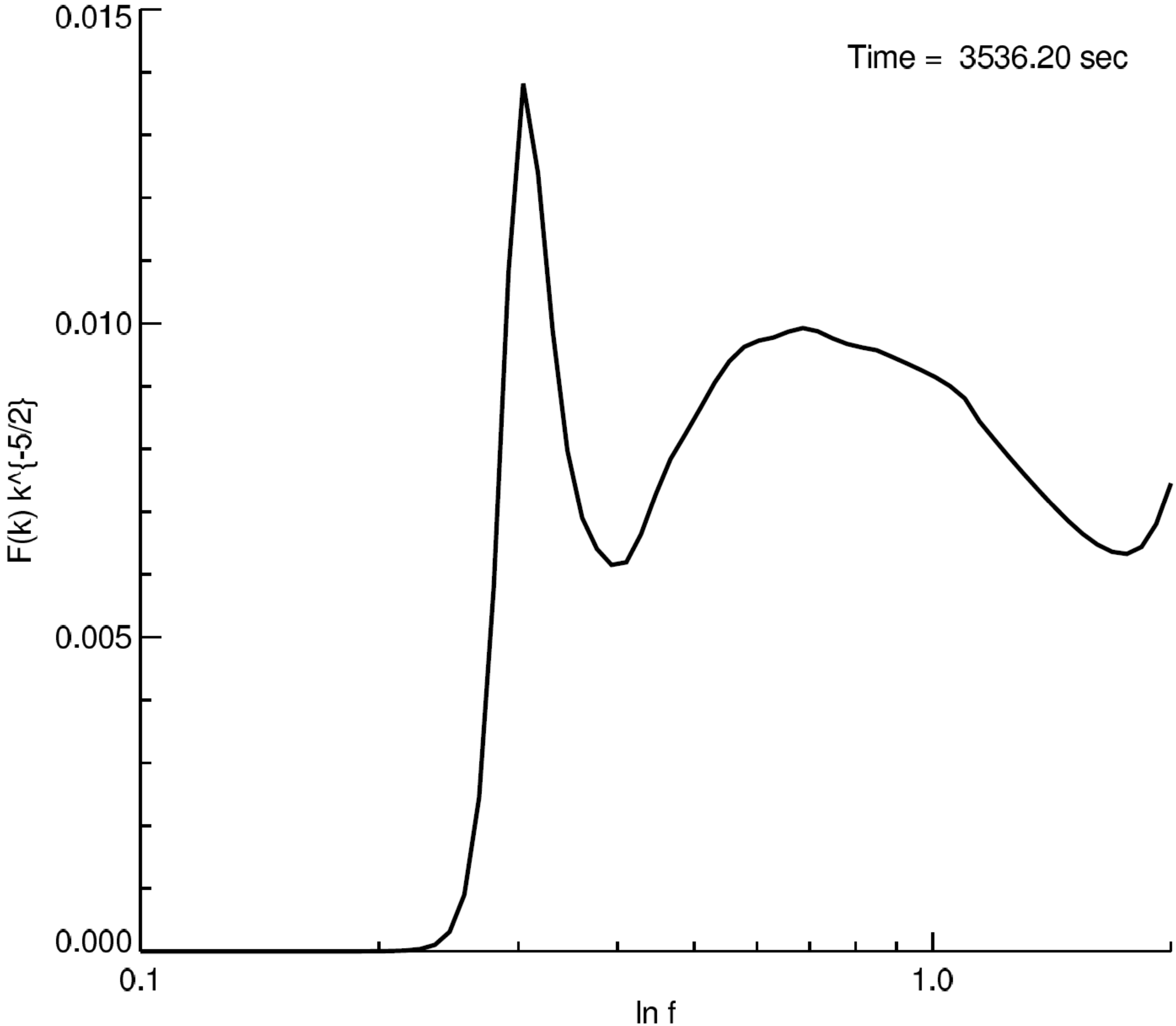
Wind input term

$$S_{wind}(\omega,\varphi) = 0.2\,\frac{\rho_{atm}}{\rho_w}\;\omega\left(\frac{\omega}{\omega_0}\right)^{4/3}\cos\varphi\;\;\varepsilon$$

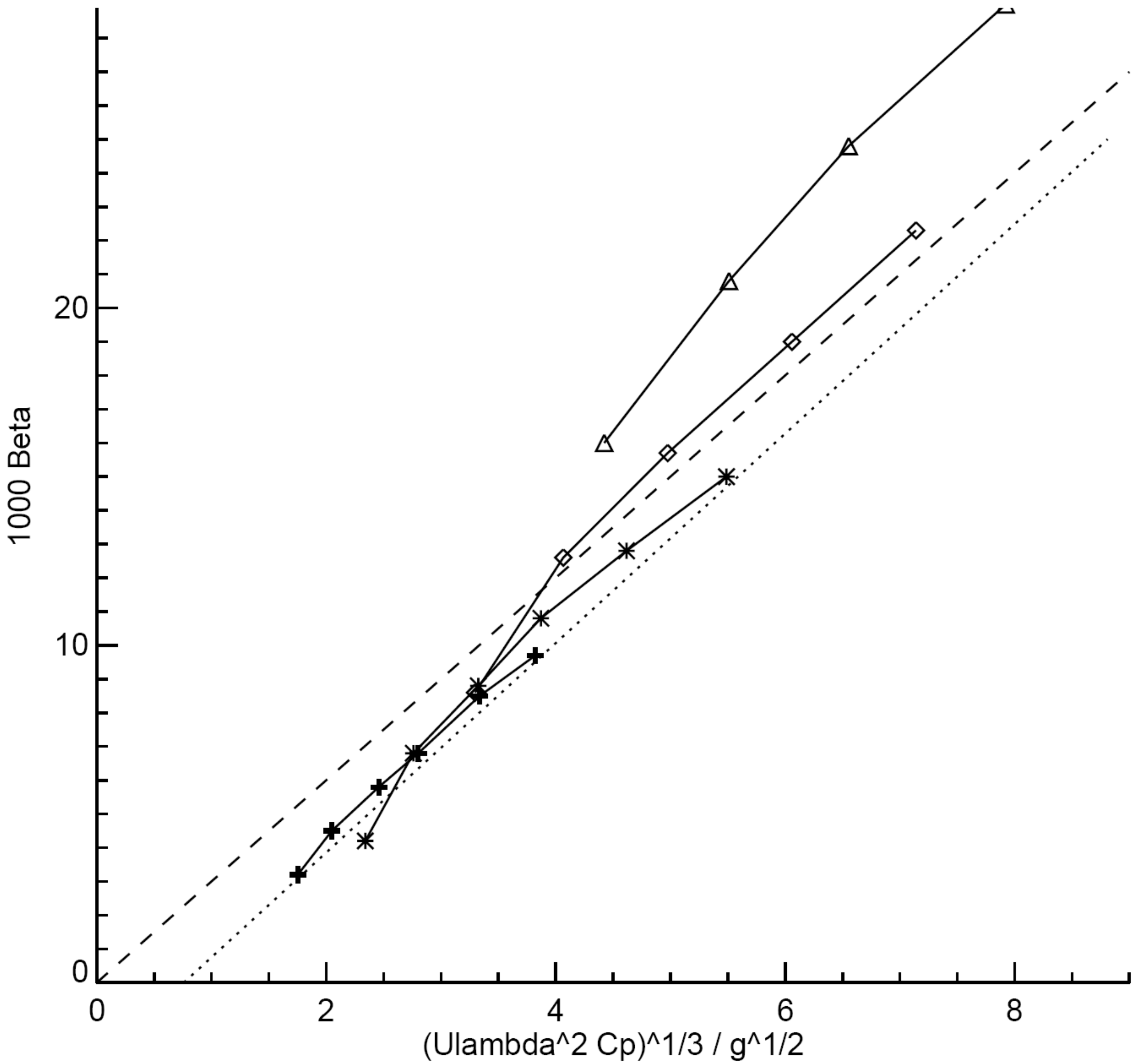
$$\omega_0 = \frac{g}{U}$$

Same as Resio, Perrie 1989


Time = 3536.20 sec
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
ln e
0.1
1.0
ln f



Compensated spectrum $F(k)k^{5/2}$ *as a function of* $\ln f$*, wind speed* $u = 10.0$ *m/sec*



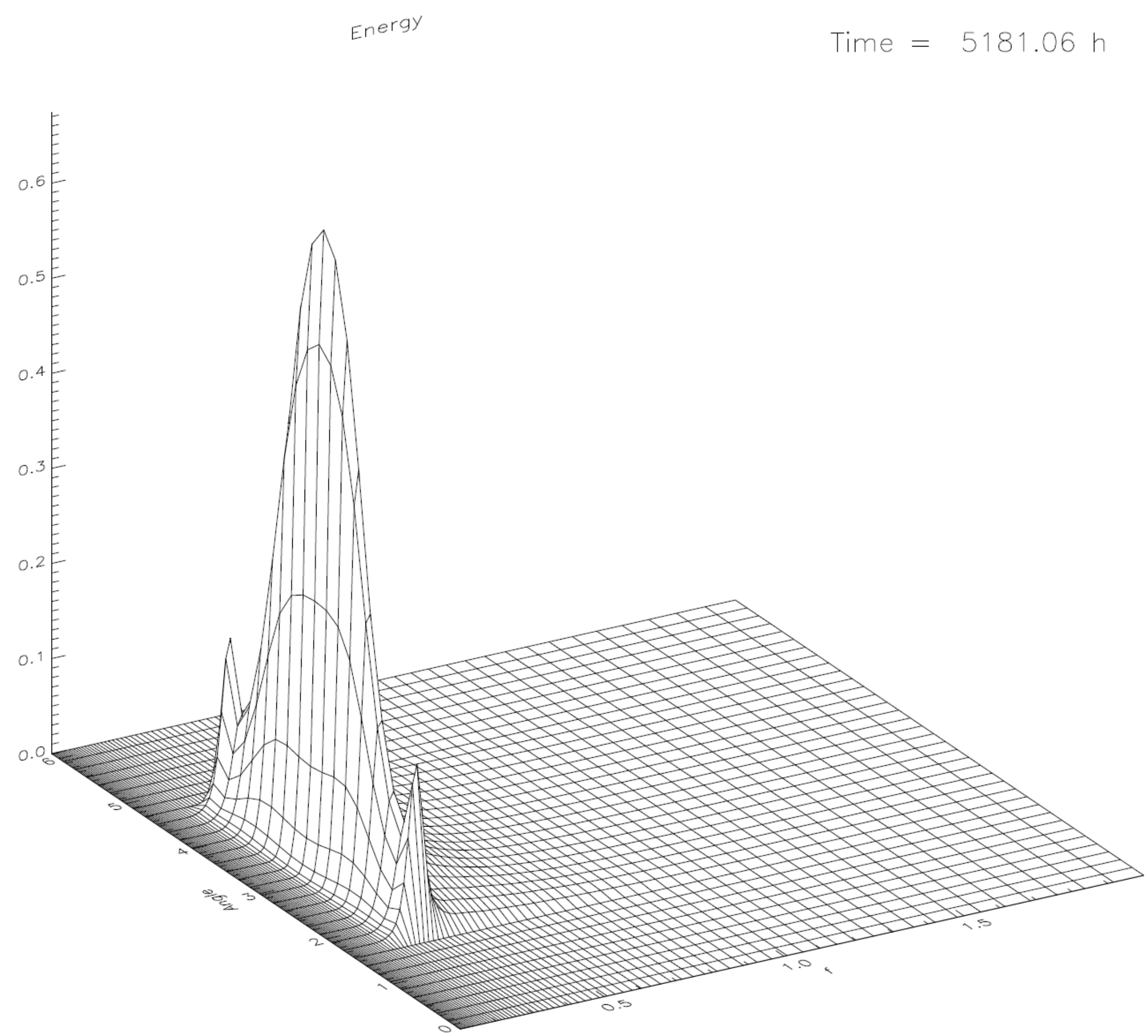
Experimental, theoretical and numerical evidence on the single graph for 1000β as a function of $(u_\lambda^2 c_p)^{1/3}/g^{1/2}$.

Fetch Limited Case

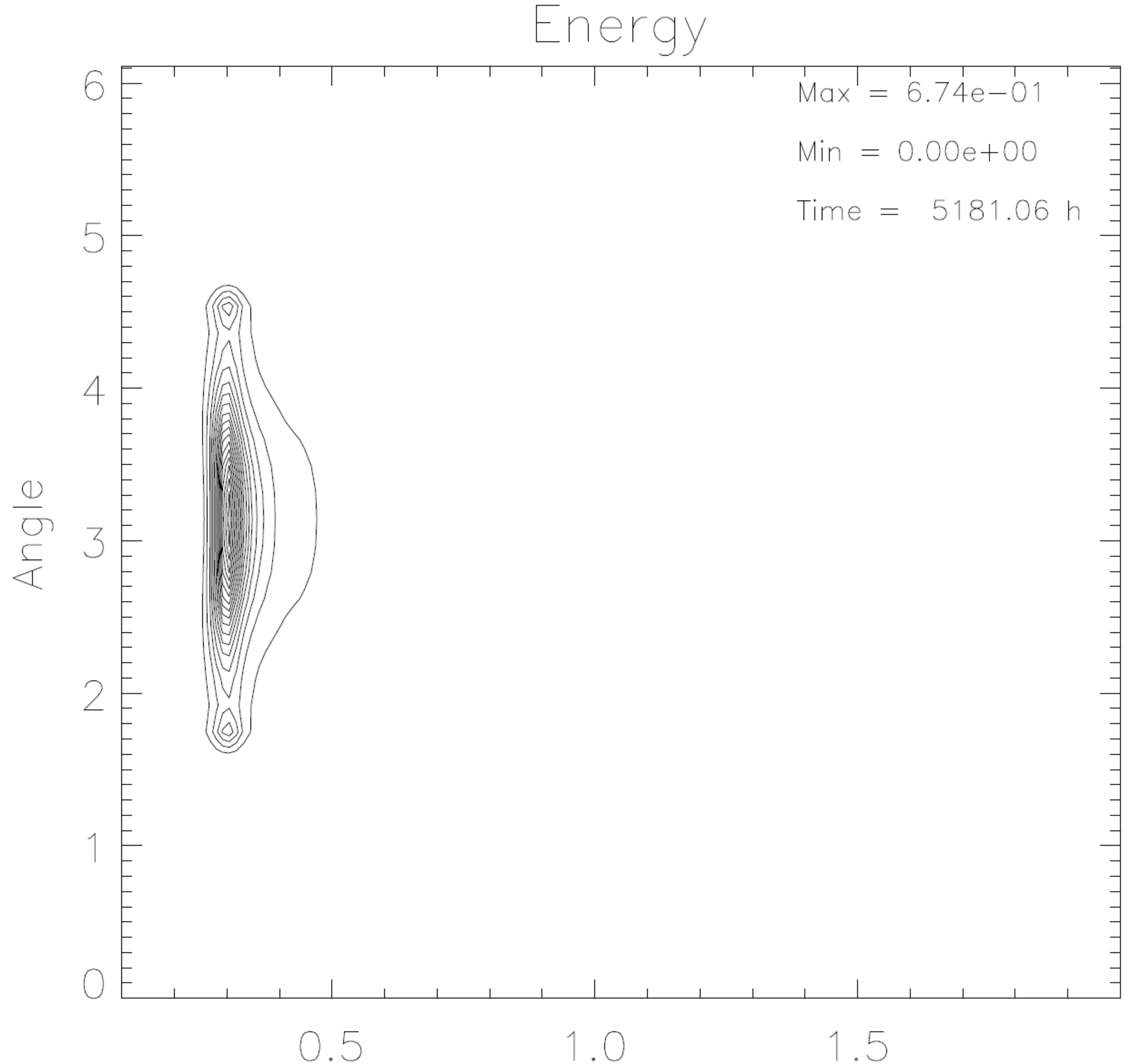
$$\frac{g\cos\varphi}{2\omega}\frac{\partial\varepsilon}{\partial x} = S_{nl} + S_{wind} + S_{diss}$$

$$\epsilon = x^{p+q} F(\omega x^q)$$

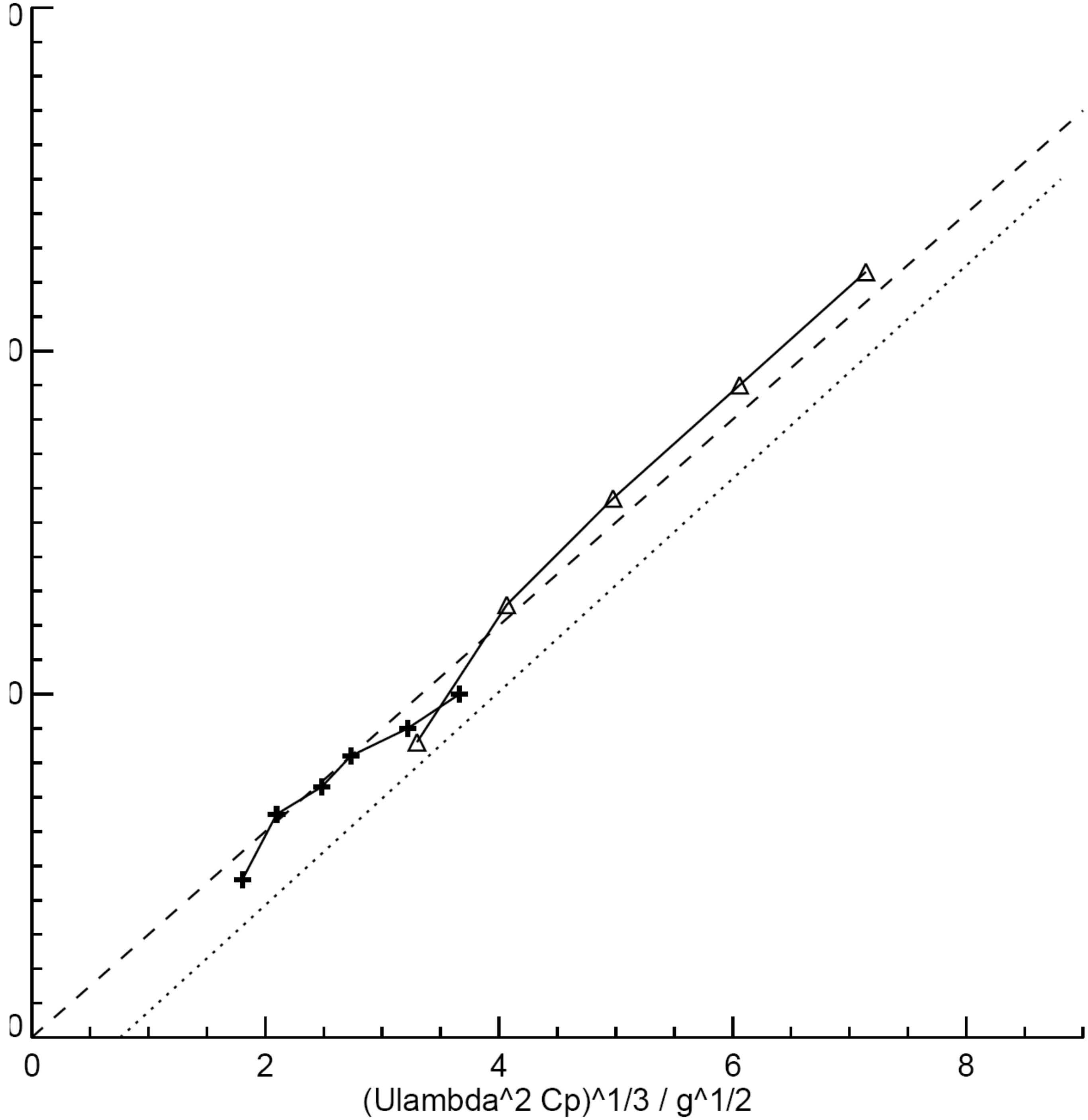
$$q = \frac{3}{10}, \quad p = 1, \quad s = \frac{4}{3}$$



Energy 3D spectrum distribution as a function of frequency ω and angle ϕ.



Energy spectrum line levels as a function of frequency ω and angle ϕ



0
0
0
0
0
2
4
6
8
(Ulambda^2 Cp)^1/3 / g^1/2

CONCLUSION

1. Resio, Perrie 1989 prediction is analytical solution of Hasselmann equation.
2. Universal for time and limited fetch domains.
3. Validated through numerical comparison with experimental data.
4. On-going validation through non-stationary numerical solution of Hasselmann equation

$$\frac{\partial \epsilon}{\partial t} = S_{nl}(\epsilon) + S_{wind} + S_{diss}$$

$$S_{nl} = \omega \left(\frac{\omega^5 \epsilon}{g^2} \right) \epsilon$$

$$\epsilon = t^{p+q} F(\omega t^q)$$

$$p = \frac{9q-1}{2}, \quad q = \frac{1}{s+1}$$

$$\epsilon = \frac{(U^2 C_p)^{1/3} g}{\omega^4}$$

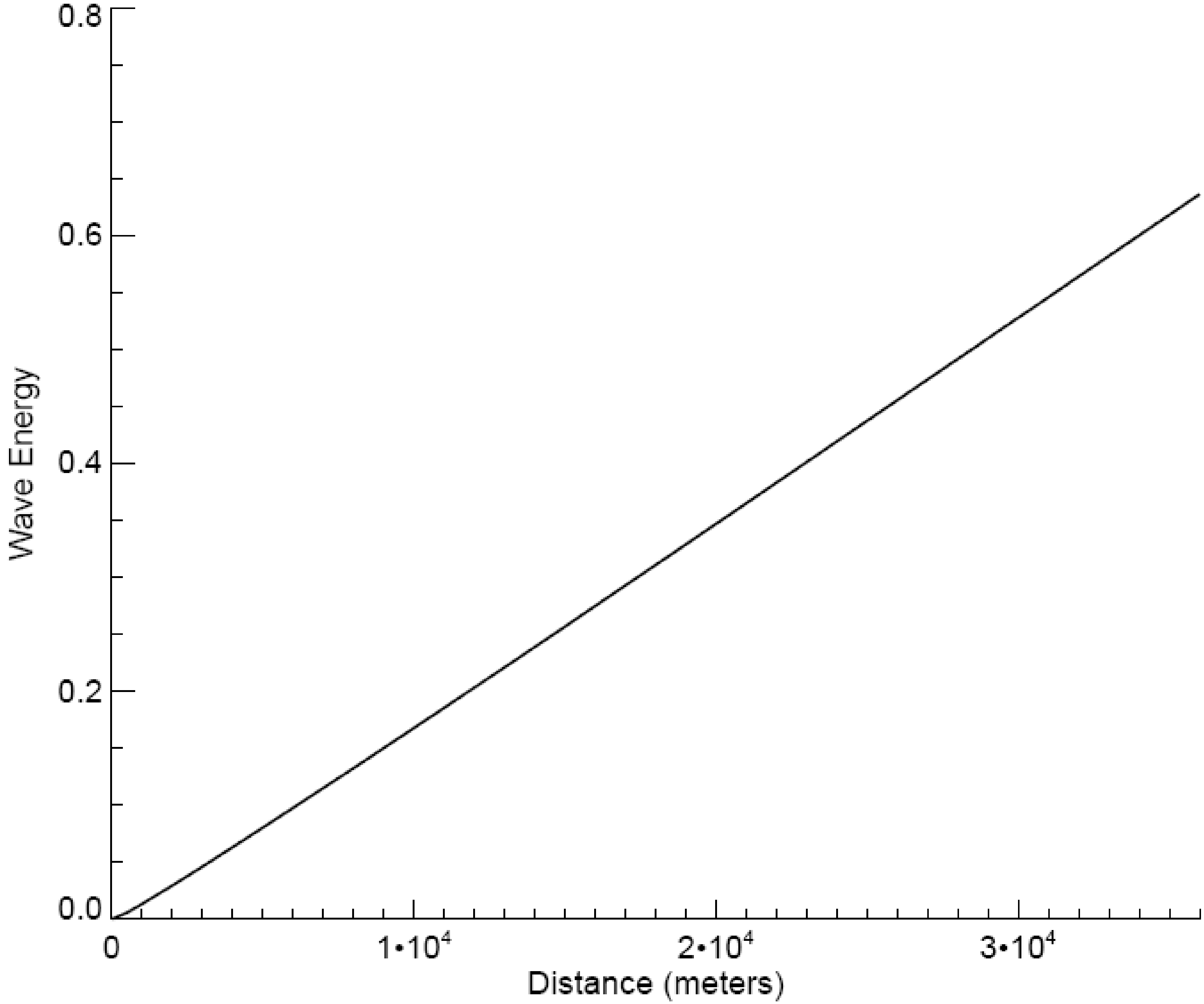
$$q = \frac{3}{7}, \quad p = 10, \quad s = \frac{4}{3}$$

$$\frac{1}{2} \frac{g}{\cos(\phi)} \frac{\partial \epsilon}{\partial x} = S_{nl}(\epsilon) + S_{wind} + S_{diss}$$

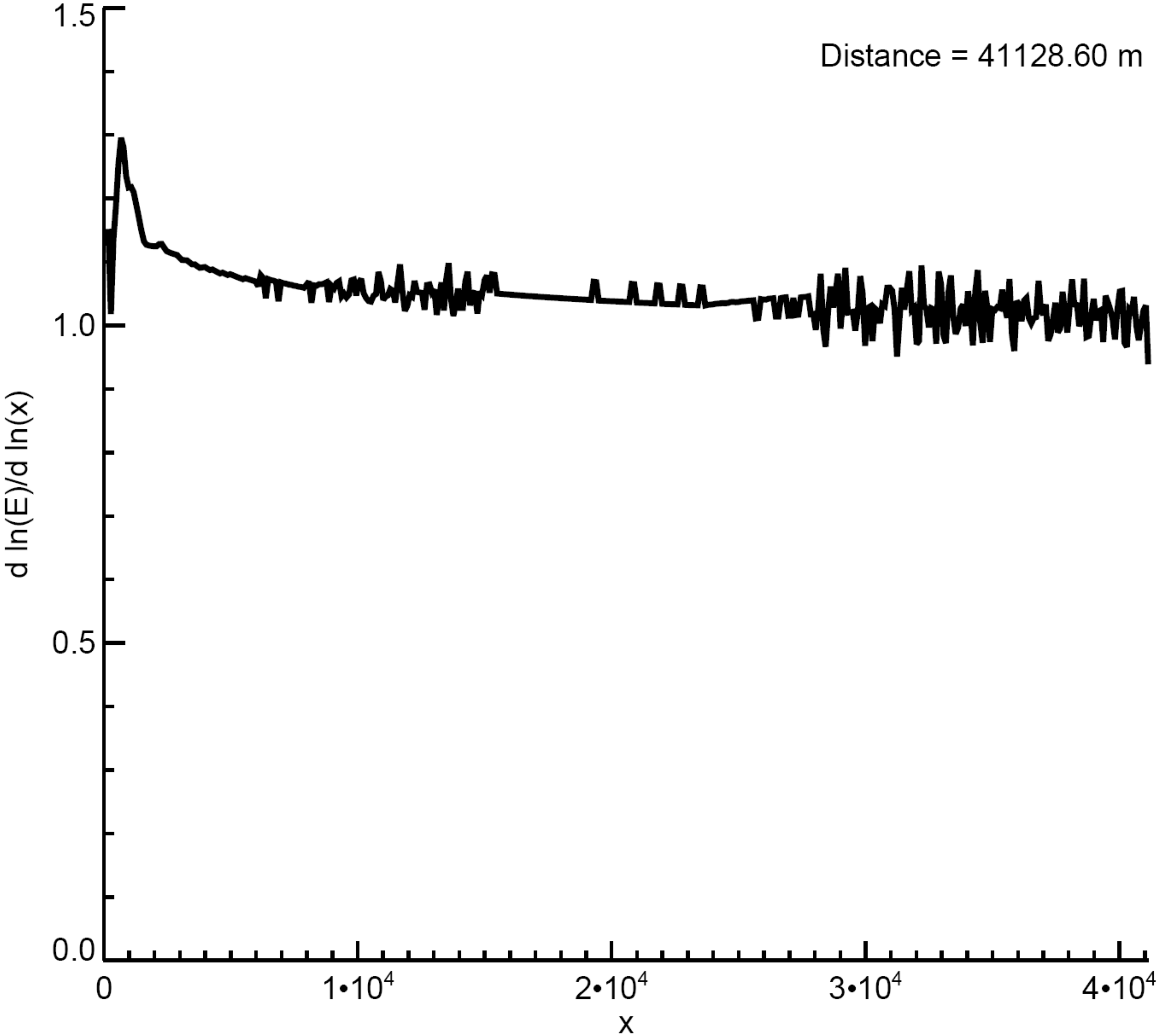
$$\epsilon = x^{p+q} F(\omega x^q)$$

$$p = \frac{10q-1}{2}, \quad q = \frac{1}{s+2}$$

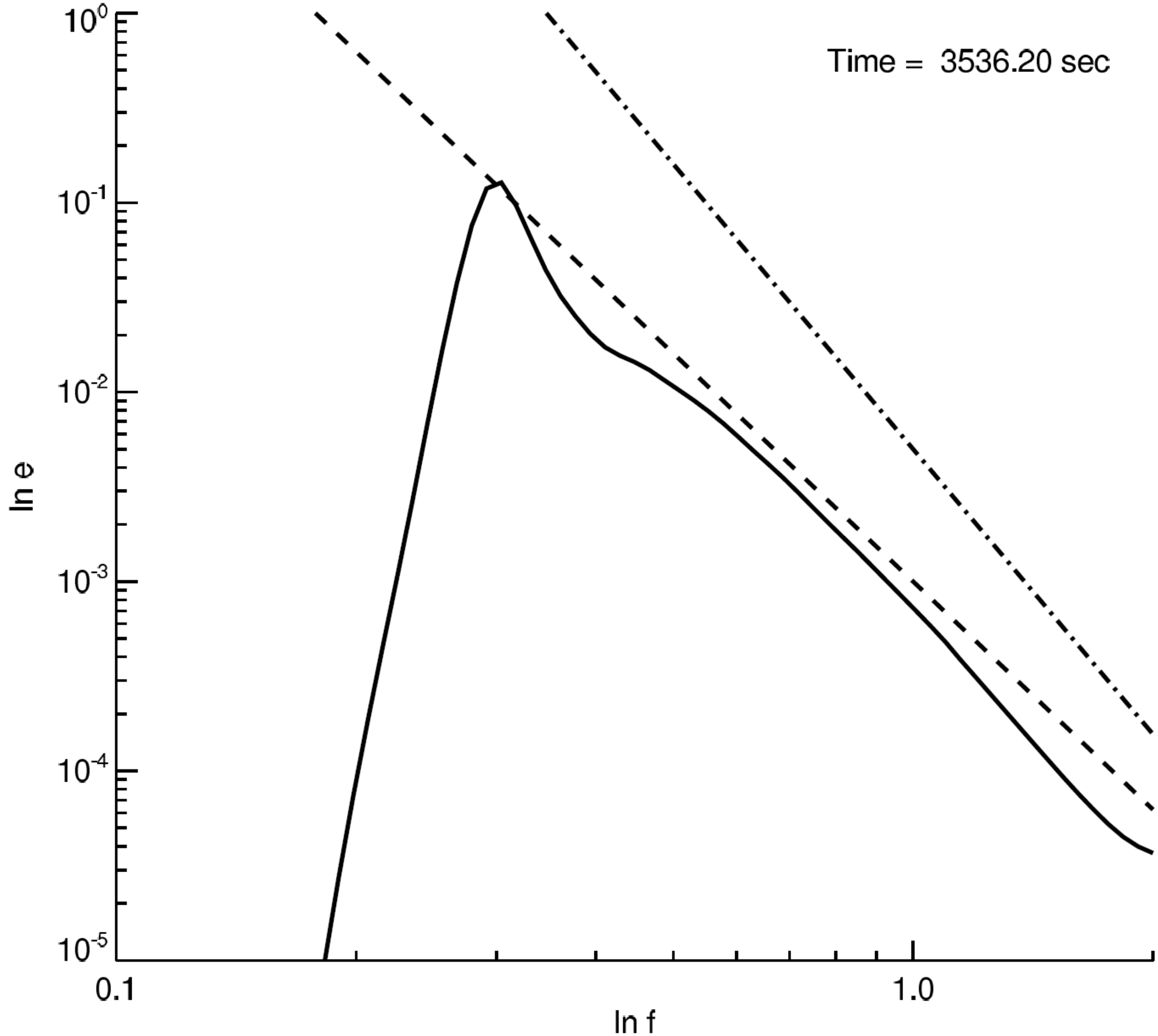
$$q = \frac{3}{10}, \quad p = 1, \quad s = \frac{4}{3}$$



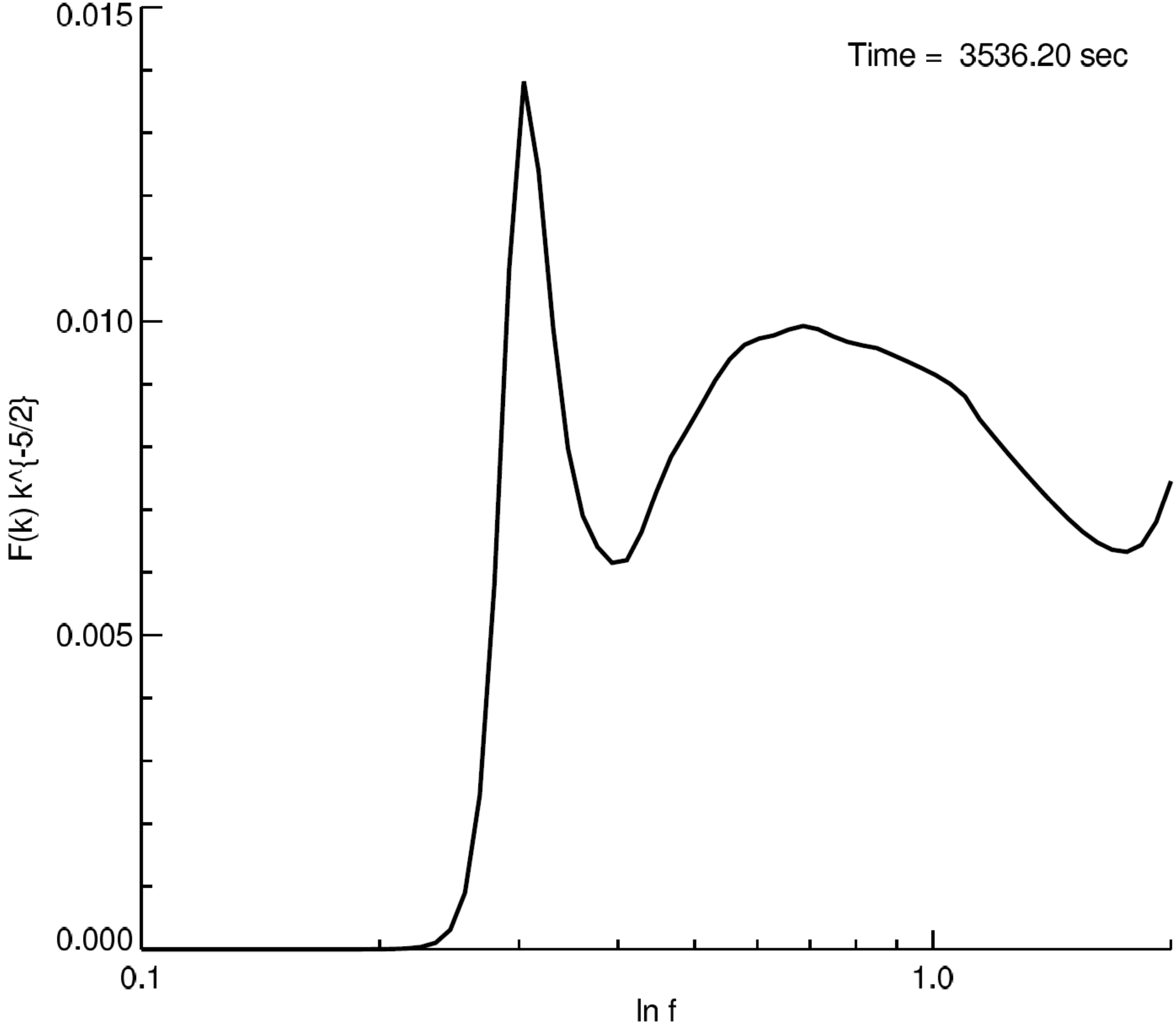
Energy as a function of time for limited fetch growth simulation



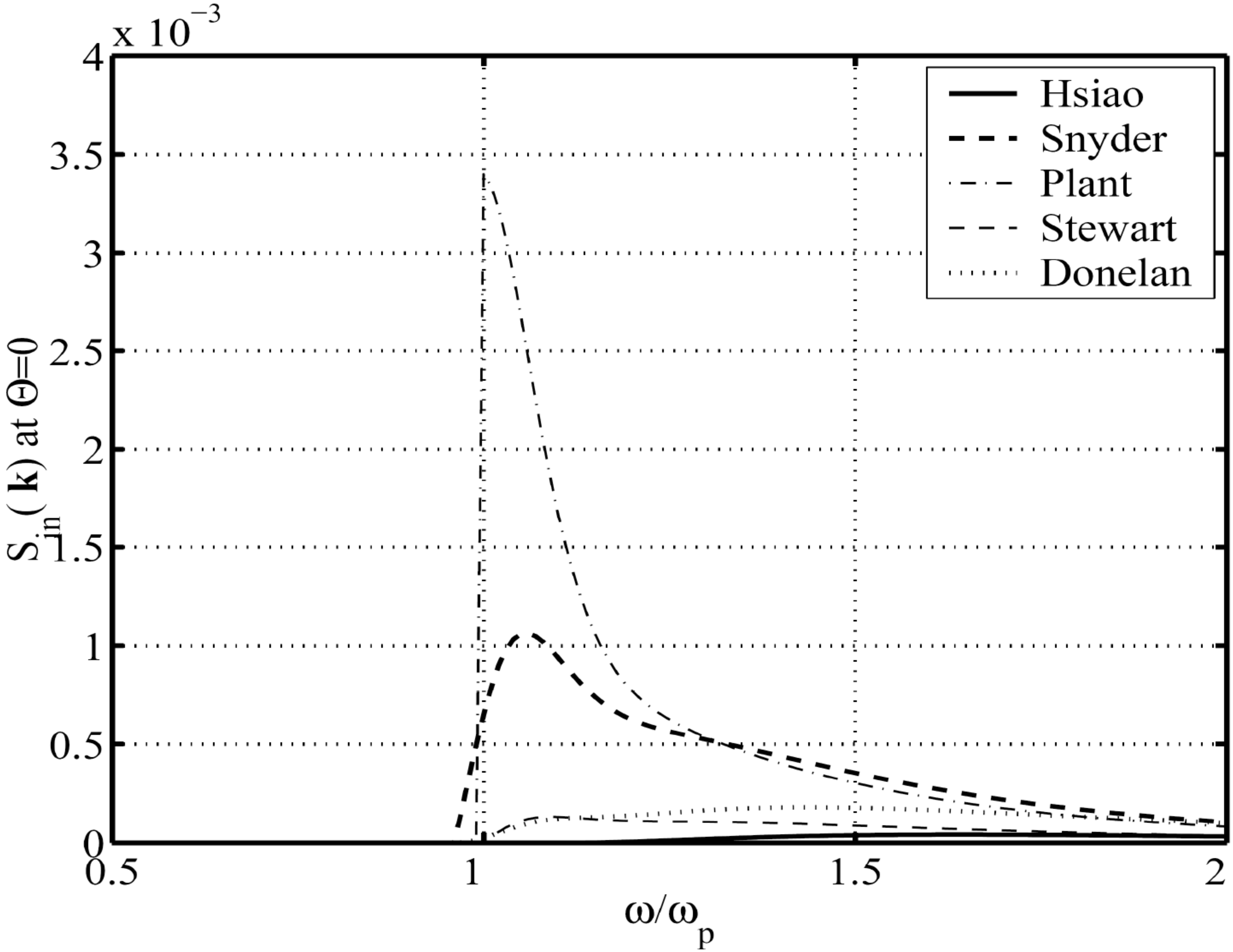
Index of energy growth $\frac{d\ln(\epsilon)}{d\ln x}$ as a function of fetch distance x.



Time = 3536.20 sec
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
ln e
0.1
1.0
ln f



Compensated spectrum $F(k)k^{5/2}$ *as a function of* $\ln f$, *wind speed* $u = 10.0$ *m/sec*



Badulin et al., NPG 2005

<u>**Objective:**</u>

1. Long-term: *get physically justified wind and dissipation input terms for operational models*
2. Short-term: *establish physically justified “reference point” for wind input term in “primitive situation”*

Two approaches:

1. **Physics of individual events**
   - ***individual wave-breaking events parameterization***
   - ***statistics of this events***
   - ***proper averaging to find wind input term***

2. **Statistical approach**
   - ***existing experimental data***
   - ***theoretical analysis***
   - ***numerical verification***

Nonlinear effects are the must!

No shortcuts like

$$\frac{\partial n_{\vec{k}}}{\partial t} = \gamma_{\vec{k}} n_{\vec{k}}$$

Non-locality is a trap!

Local modification of the spectrum immediately modifies the spectrum everywhere.

Consequence:

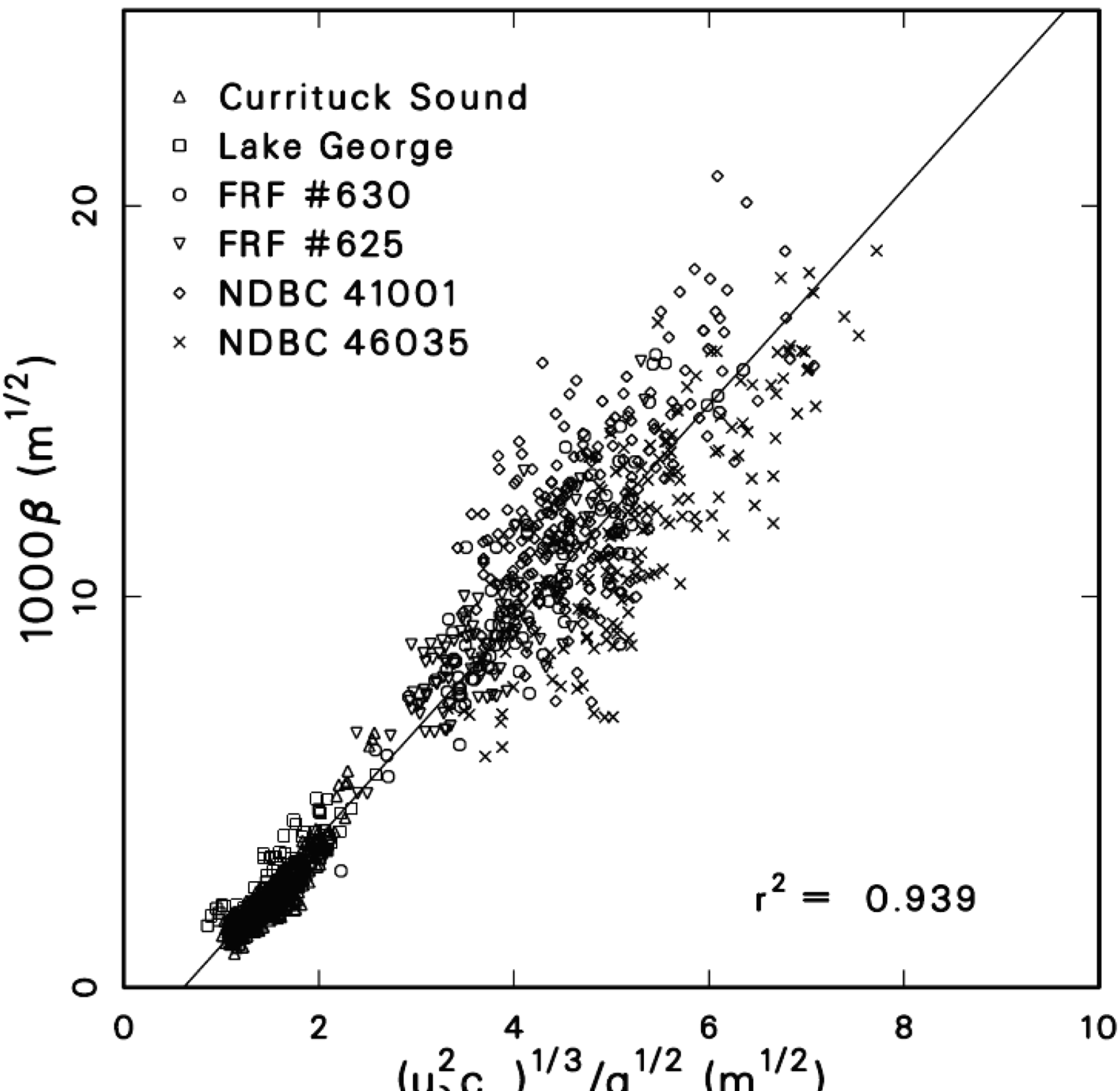
Local input terms modification of the spectrum is interpreted as a global one.

$$\varepsilon \sim \omega^{-4} \qquad F(k) = \beta k^{-5/2}$$

$$\beta = k^{5/2} F(k) \qquad \beta \sim \mathrm{V}$$

$$(u_\lambda^2 c_p)^{1/3} g^{-1/2}$$

Resio at al. 1987, 2004 and 2007


Currituck Sound
Lake George
FRF #630
FRF #625
NDBC 41001
NDBC 46035
$r^2 = 0.939$
1000β ($m^{1/2}$)
$(u_\lambda^2 c_p)^{1/3}/g^{1/2}$ ($m^{1/2}$)
0
10
20
0
2
4
6
8
10

$$\frac{\partial n_{\vec{k}}}{\partial t} = S_{nl} + S_{wind} + S_{diss}$$

$$S_{wind}(\omega, \phi) = \gamma(\omega, \phi)\ \ n_{\vec{k}}$$

Dissipation is defined by Phillips spectrum

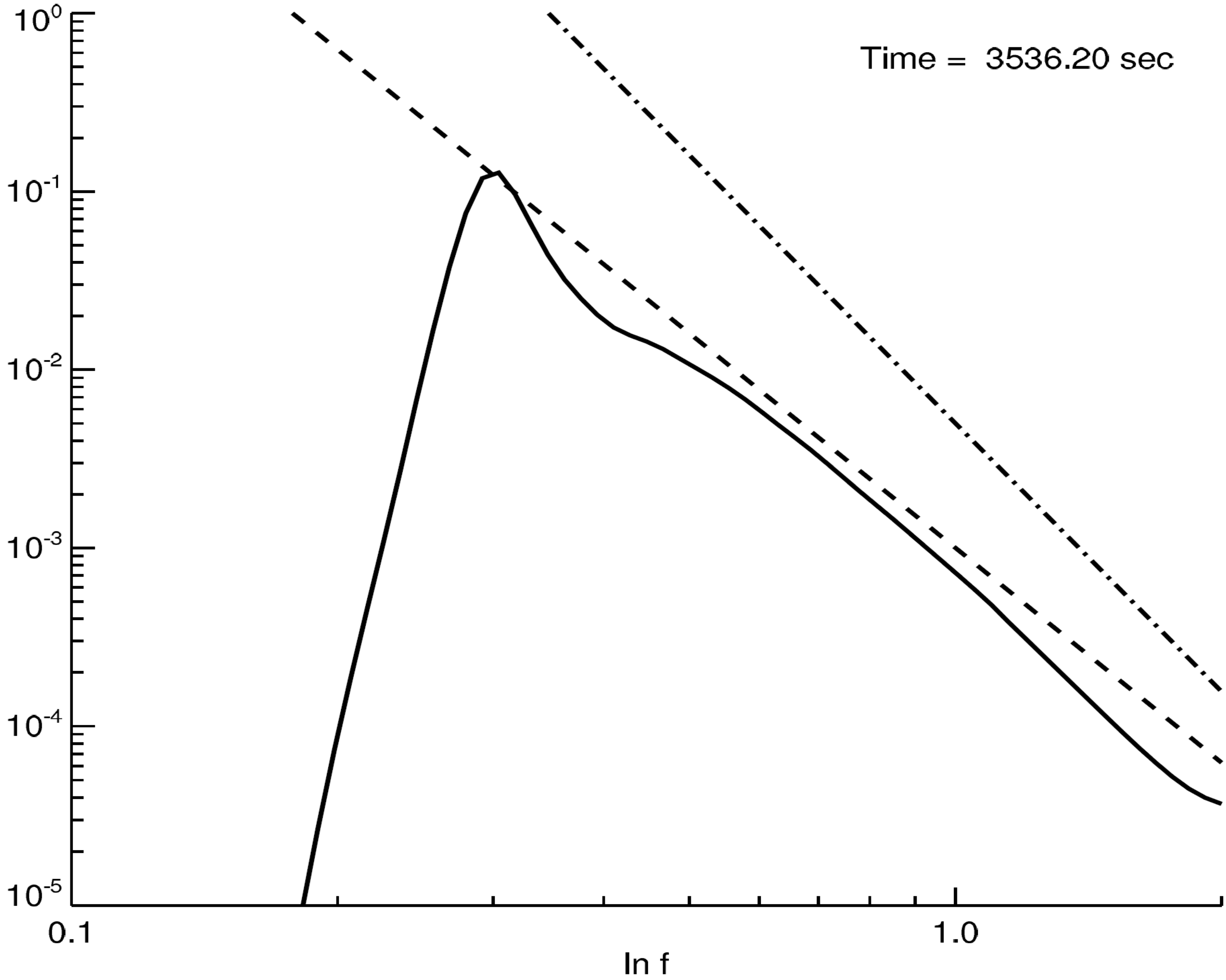
$$S_{diss} \sim \omega^{-5}$$

$$S_{wind}(\omega, \phi) = \gamma(\omega, \phi) \;\; n_{\vec{k}}$$

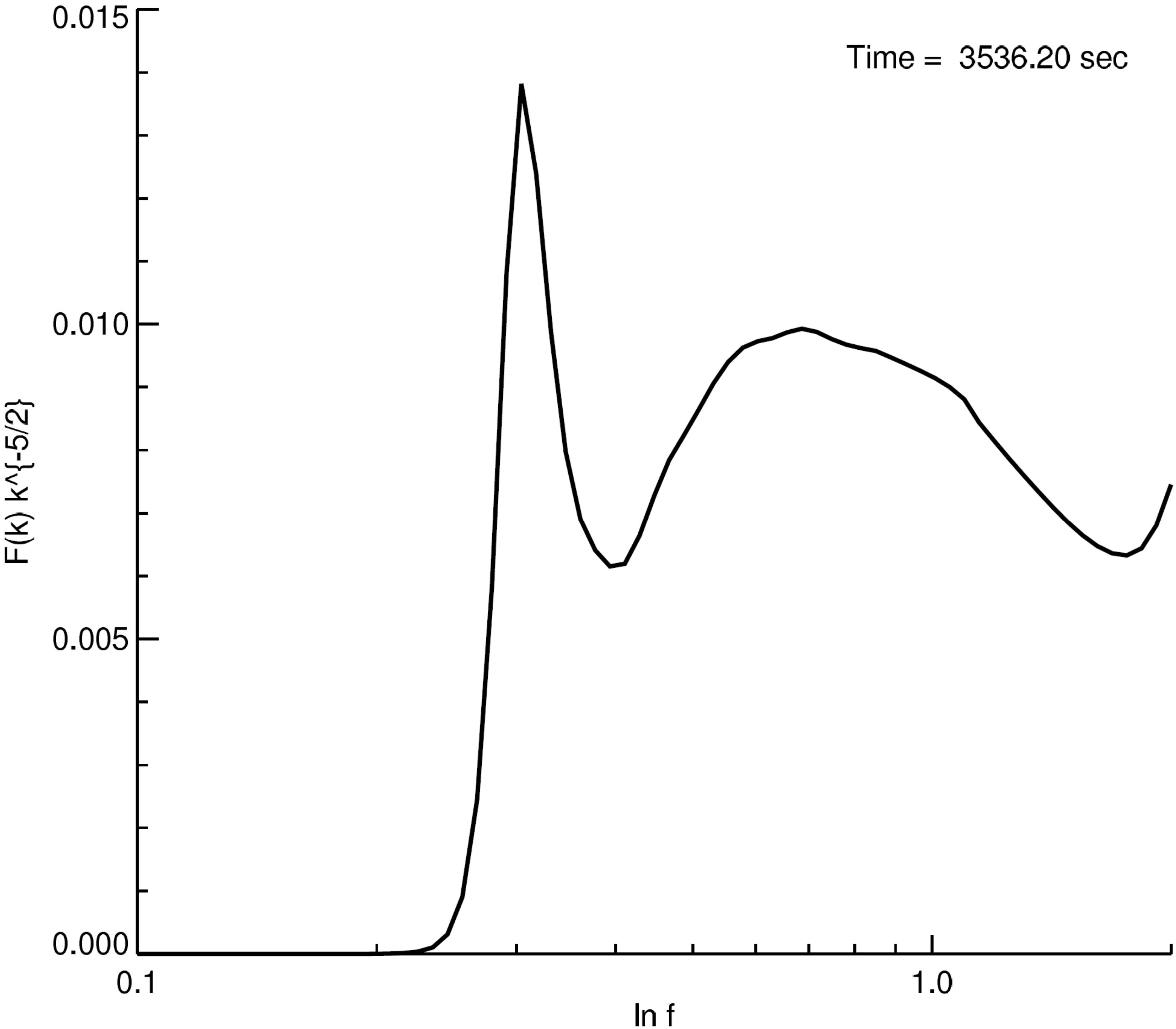
$$\gamma(\omega, \phi) \quad = \quad -\alpha \frac{\rho_{air}}{\rho_{water}} \omega \left(\omega/\omega_0 - 1\right)^{\frac{4}{3}} \cos\phi$$

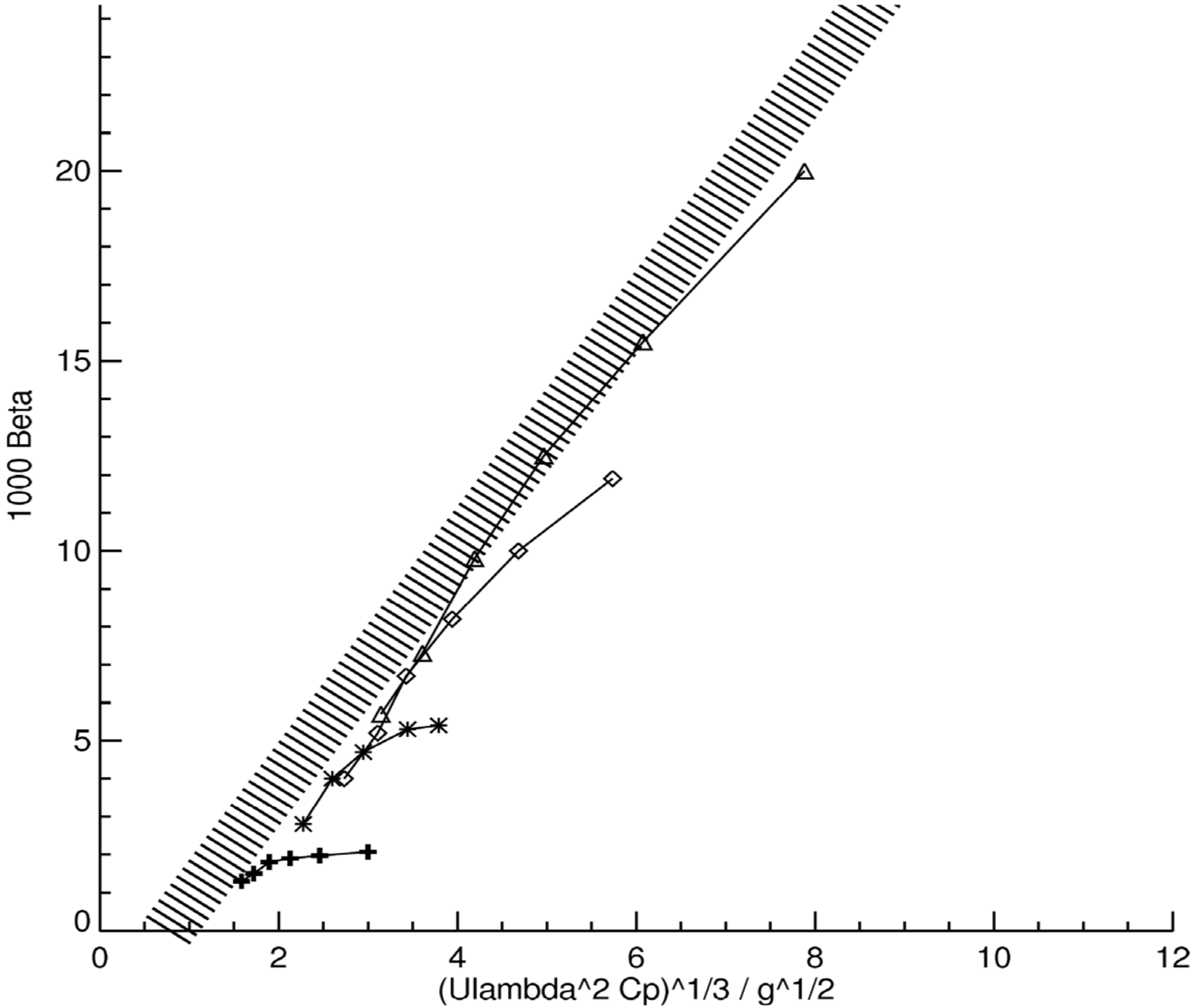
$$\alpha \quad = \quad 0.2 \qquad\qquad \rho_{air}/\rho_{water} \quad = \quad 1.3 \cdot 10^{-3}$$

$$\omega_0 \quad = \quad g/u$$



Time = 3536.20 sec
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.1
1.0
ln f


Time = 3536.20 sec
0.015
0.010
0.005
0.000
F(k) k^{-5/2}
0.1
1.0
ln f



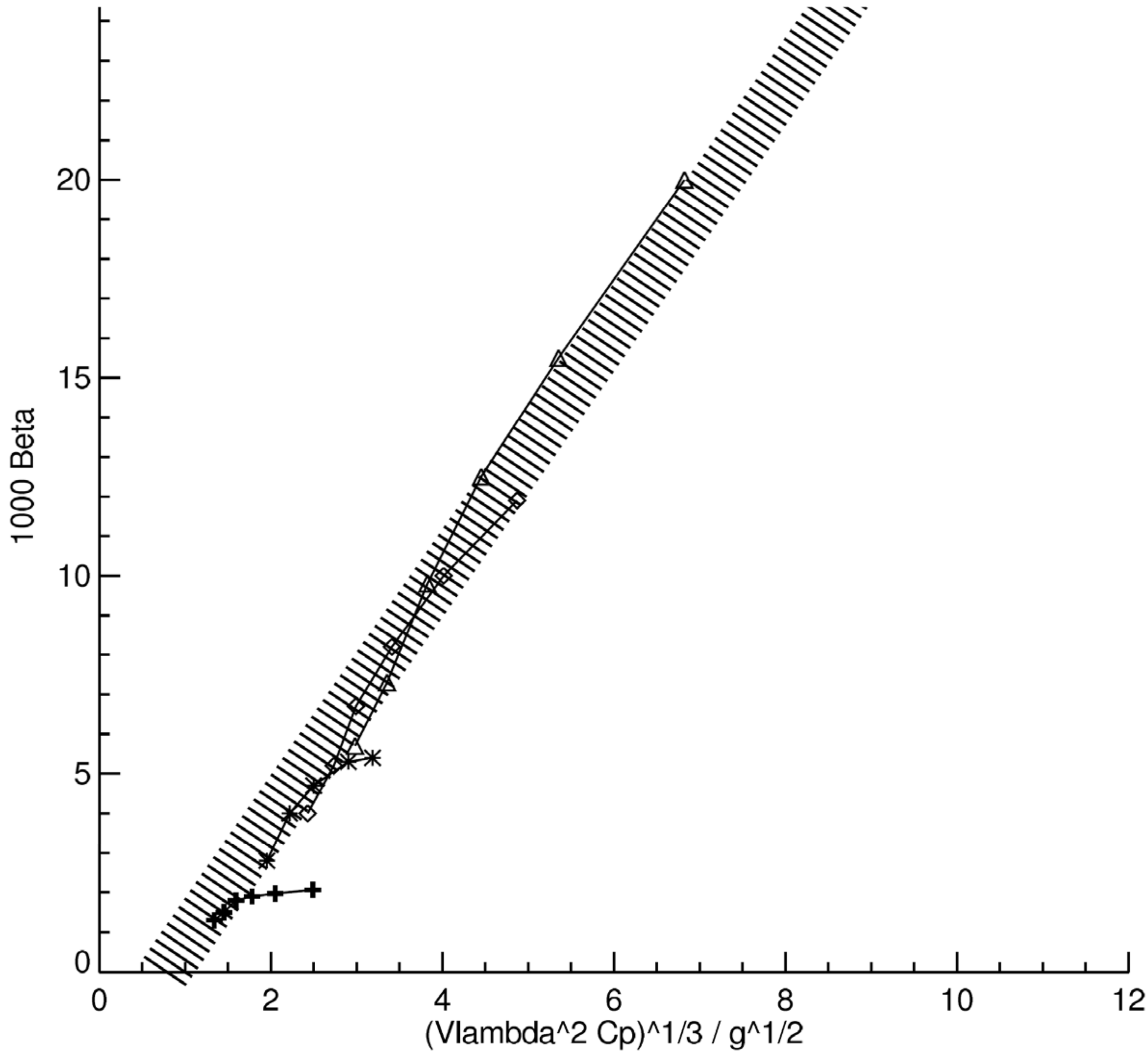
Crosses – $u = 2.5$, *stars* – $u = 5.0$, *rectangles* – $u = 10.0$, *triangles* – $u = 20.0$

Let's rescale velocity: $v = \epsilon u$

$$S_{wind}(\omega,\phi) \quad = \quad -\epsilon^{4/3}\alpha\omega\left(\frac{\omega}{g}v - \frac{1}{\epsilon}\right)^{4/3}\cos\phi \;\; n(\omega,\phi)$$

$$v = \epsilon u \quad \epsilon = 0.7$$

v	***u***
2.5	**3.6**
5	**7.14**
10	**14.3**
20	**28.6**



20
15
10
5
0
1000 Beta
0
2
4
6
8
10
12
(Vlambda^2 Cp)^1/3 / g^1/2

Theoretical consideration of Hasselmann equation:

$$\frac{\partial n_{\vec{k}}}{\partial t} = S_{nl} + \gamma(\omega, \phi)\epsilon$$

$$\gamma(\omega, \phi) = \alpha\omega^{1+s} f(\phi)$$

Self-similar solution:

$$\epsilon = t^{p+q} F(\omega t^q)$$

$$q = \frac{1}{s+1} \qquad p = \frac{9q-1}{2}$$

$$\xi = \frac{p-3q}{q} = \frac{3q-1}{2q}$$

$$\epsilon(\omega,\phi) = \frac{\mu g u^{1-\xi} C_p^{\xi}}{\omega^4}$$

For $\xi = 1/3$ we get exactly

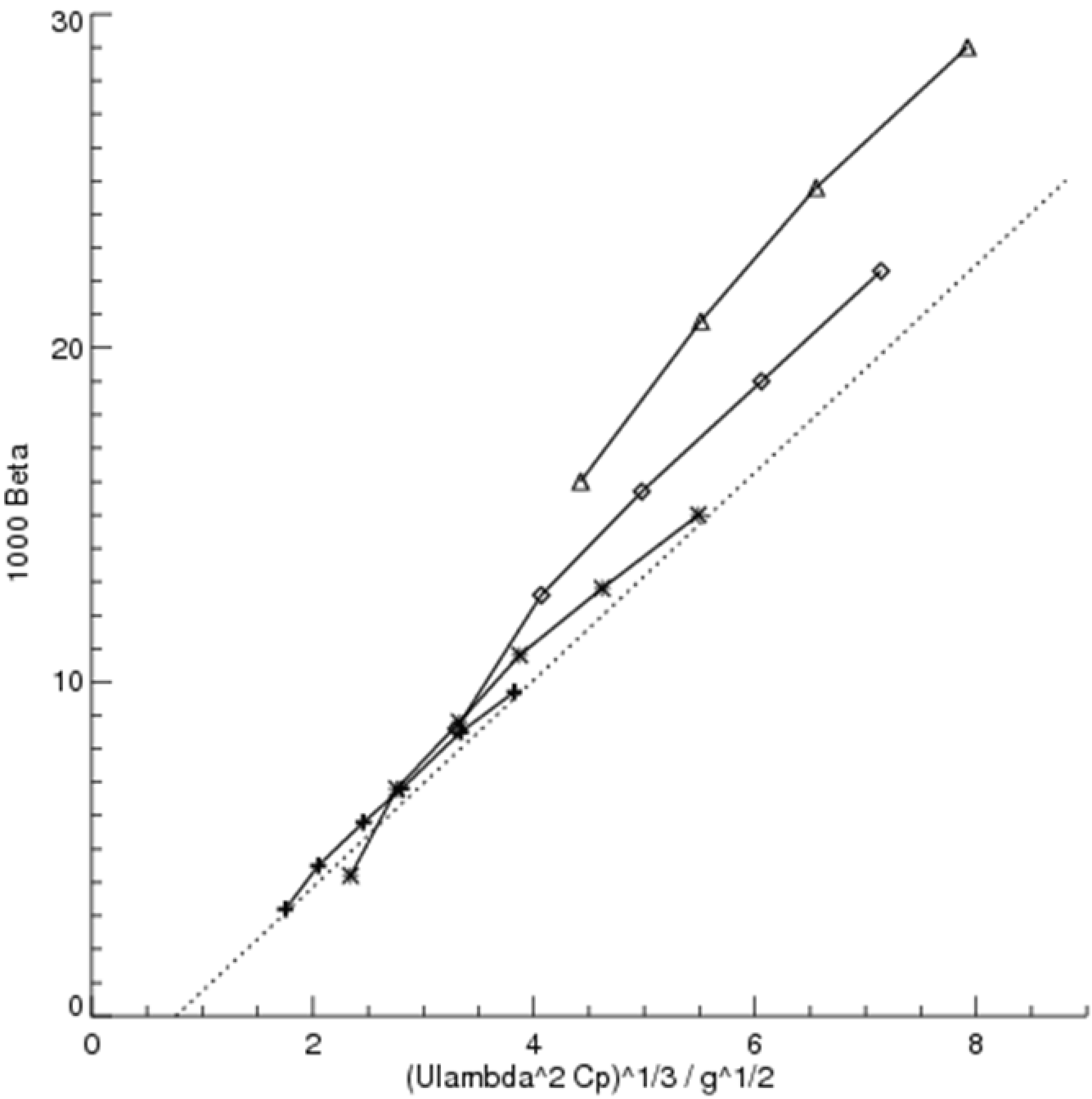
Resio et al. 2004 *dependence*

$$s = 4/3$$

or

New wind input term: $\gamma \simeq \omega^{7/3}$

Don Resio (1989) had similar parameters from fluxes consideration.



30
20
10
0
1000 Beta
0
2
4
6
8
(Ulambda^2 Cp)^1/3 / g^1/2

Conclusion:

1. **Old wind input terms need to be re-scaled**
2. **Nonlinearity effects are <u>CRUSIAL</u> for the right wind input term**
3. **New wind input term satisfies:**
   - *Experimental regression line*
   - *Theoretical solution*
   - *Numerical simulation*

$$\alpha_4 = 0.00553, u_0 = 1.93\, m/sec$$

$$u_\lambda = \frac{u_\star}{\kappa} \ln \frac{z}{z_0}$$

$$\kappa = 0.41$$

$$z = \lambda \cdot 2\pi/k_p$$

$$z_0 = \alpha_C u_\star^2/g$$

$$\alpha_C = 0.015$$

$$\varepsilon \sim \omega^{-4} \qquad F(k) = \beta k^{-5/2}$$

$$\beta = k^{5/2} F(k) \qquad \beta \sim V$$

$$(u_\lambda^2 c_p)^{1/3} g^{-1/2}$$

$$\beta = \frac{1}{2}\alpha_4 \left[(u_\lambda^2 c_p)^{1/3} - u_0 \right] g^{-1/2}$$

Resio at al. 1987, 2004 and 2007

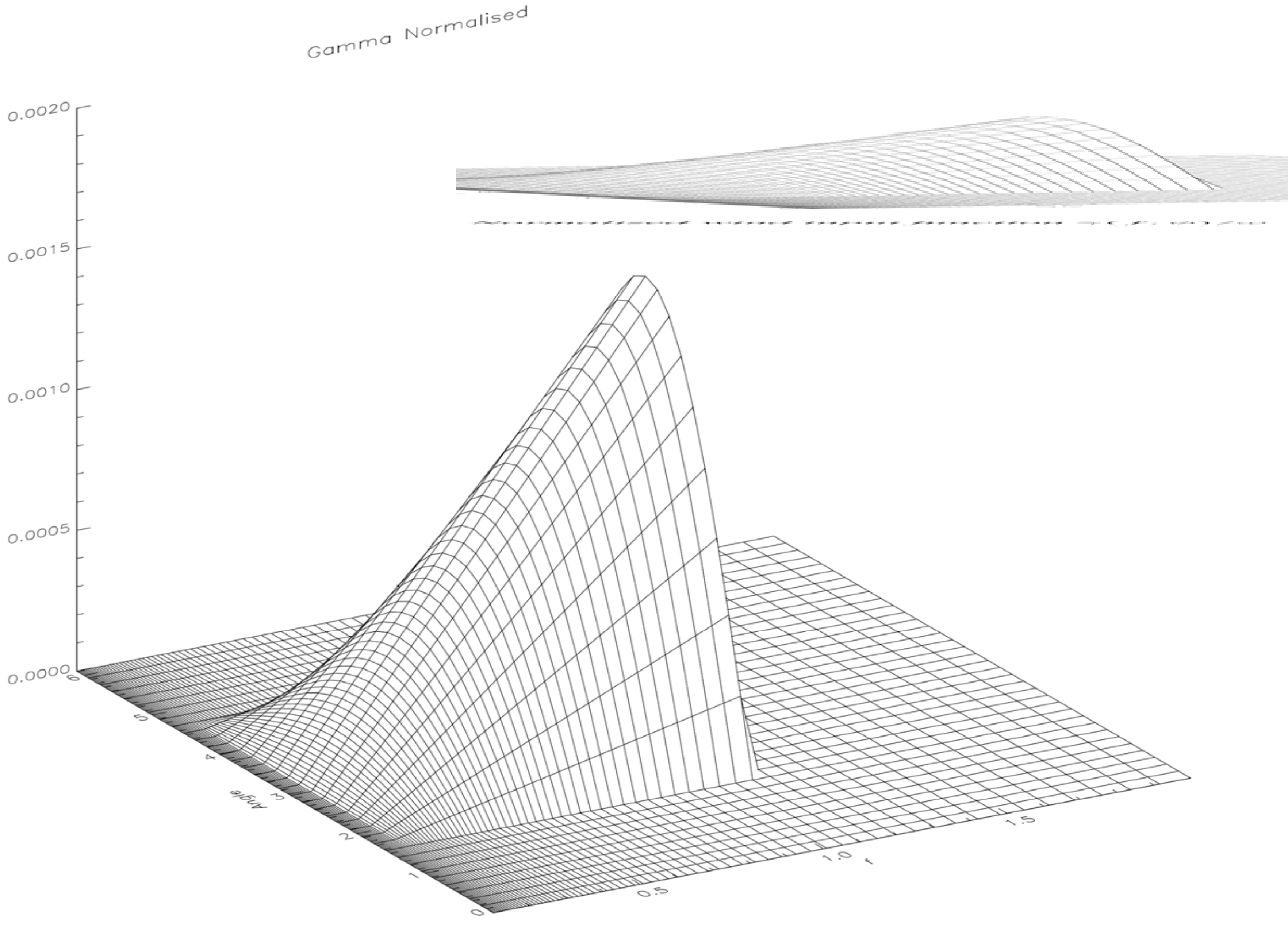
$$\frac{\partial n_{\vec{k}}}{\partial t} = S_{nl} + S_{wind} + S_{diss}$$

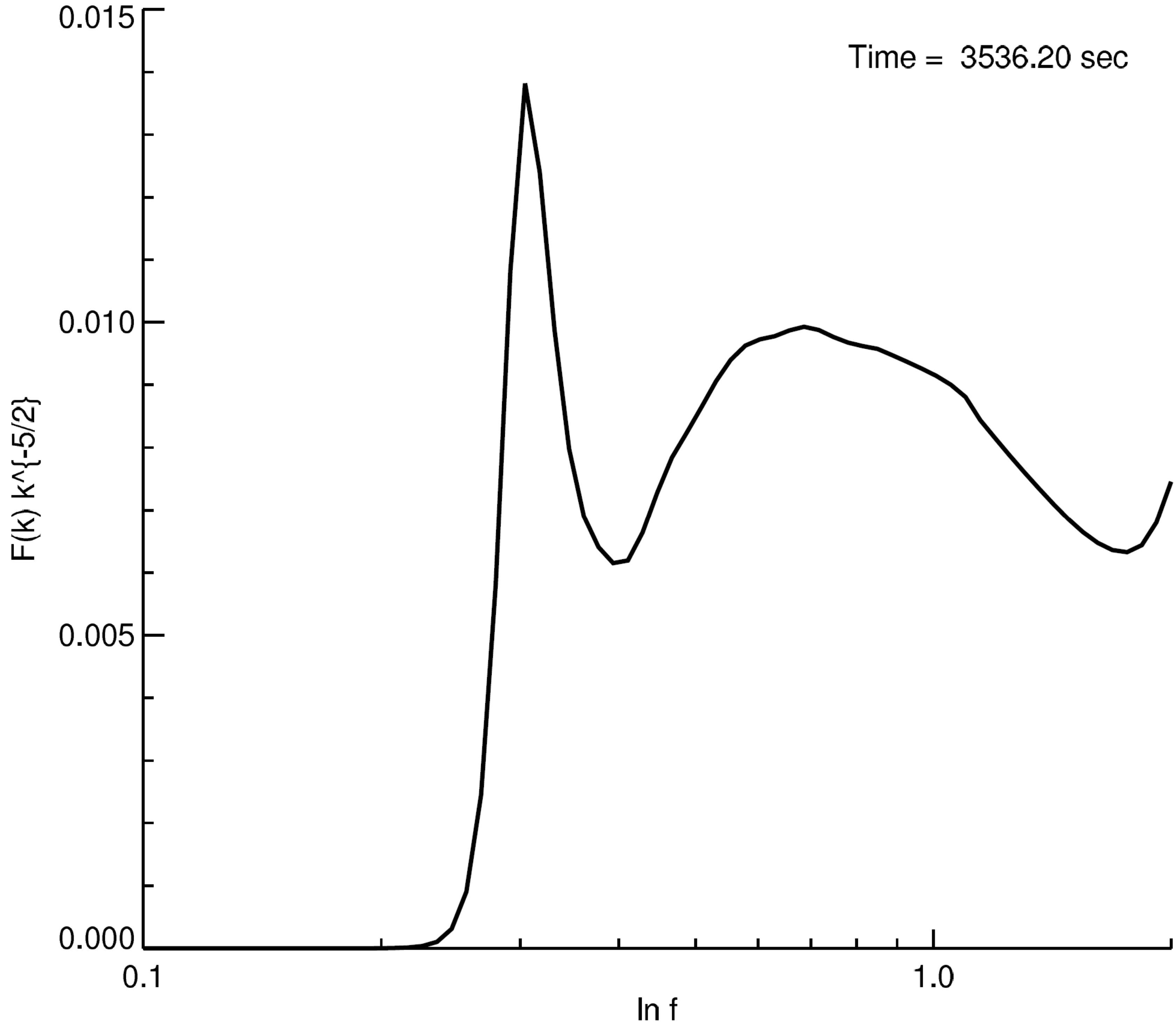
$$S_{wind}(\omega, \phi) = \gamma(\omega, \phi) \;\; n_{\vec{k}}$$

$$\gamma(\omega, \phi) \quad = \quad -\alpha \frac{\rho_{air}}{\rho_{water}} \omega \left(\omega/\omega_0 - 1\right)^{\frac{4}{3}} \cos\phi$$

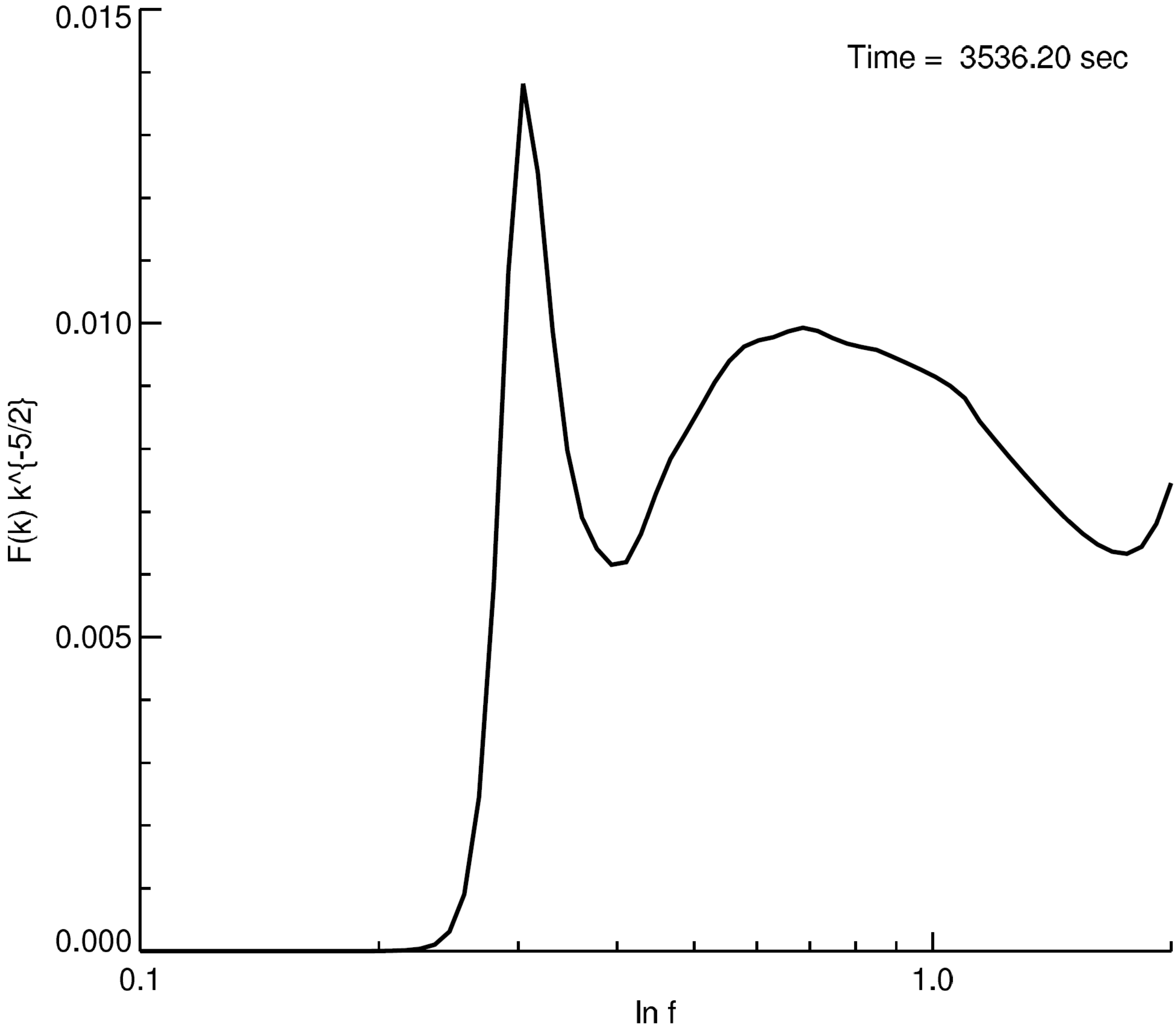
$$\alpha \quad = \quad 0.2 \qquad\qquad \rho_{air}/\rho_{water} \quad = \quad 1.3 \cdot 10^{-3}$$

$$\omega_0 \quad = \quad g/u$$

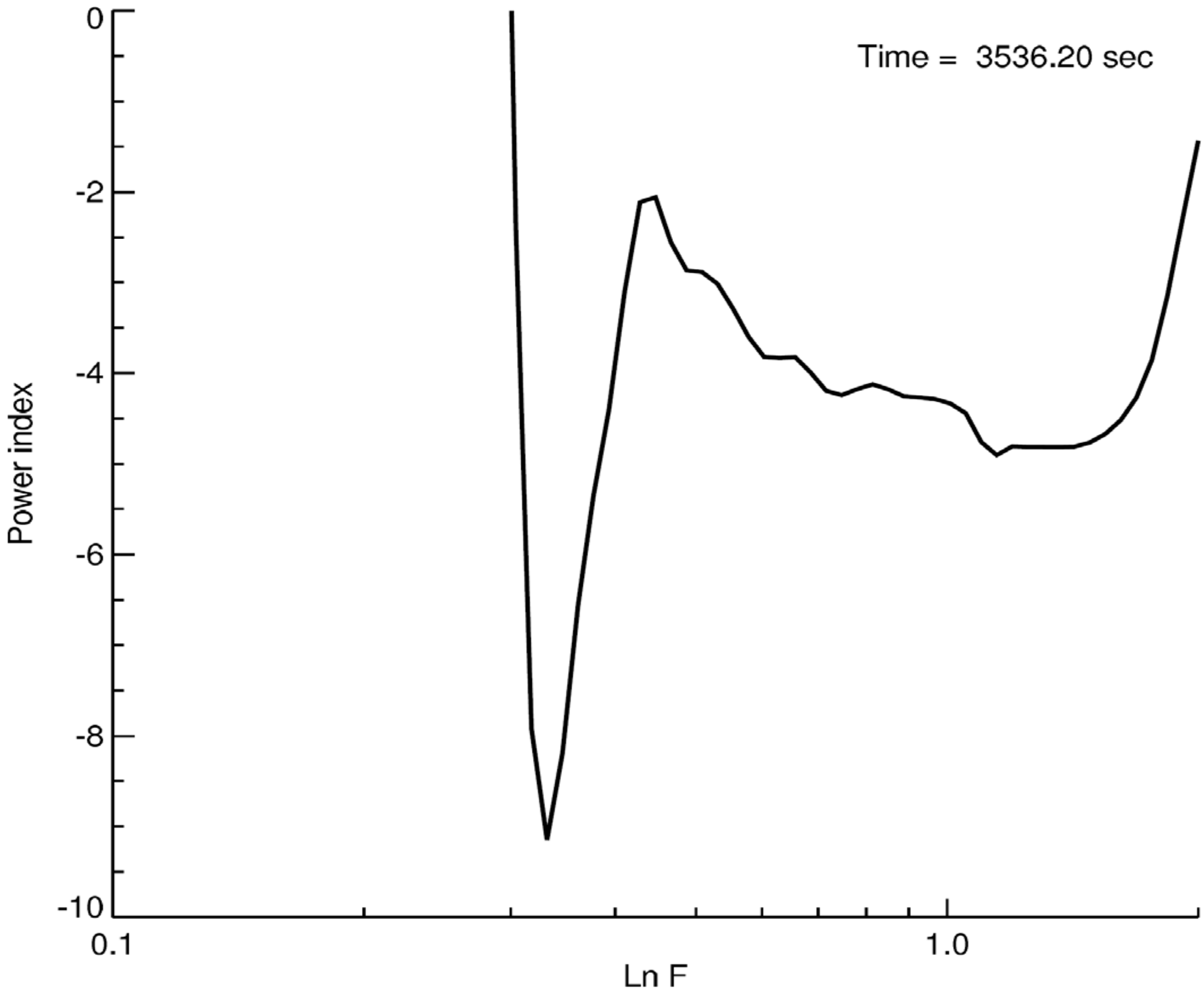

Gamma Normalised
0.0020
0.0015
0.0010
0.0005
0.0000
Angle
0
1
2
4
0.5
1.0
1.5
r



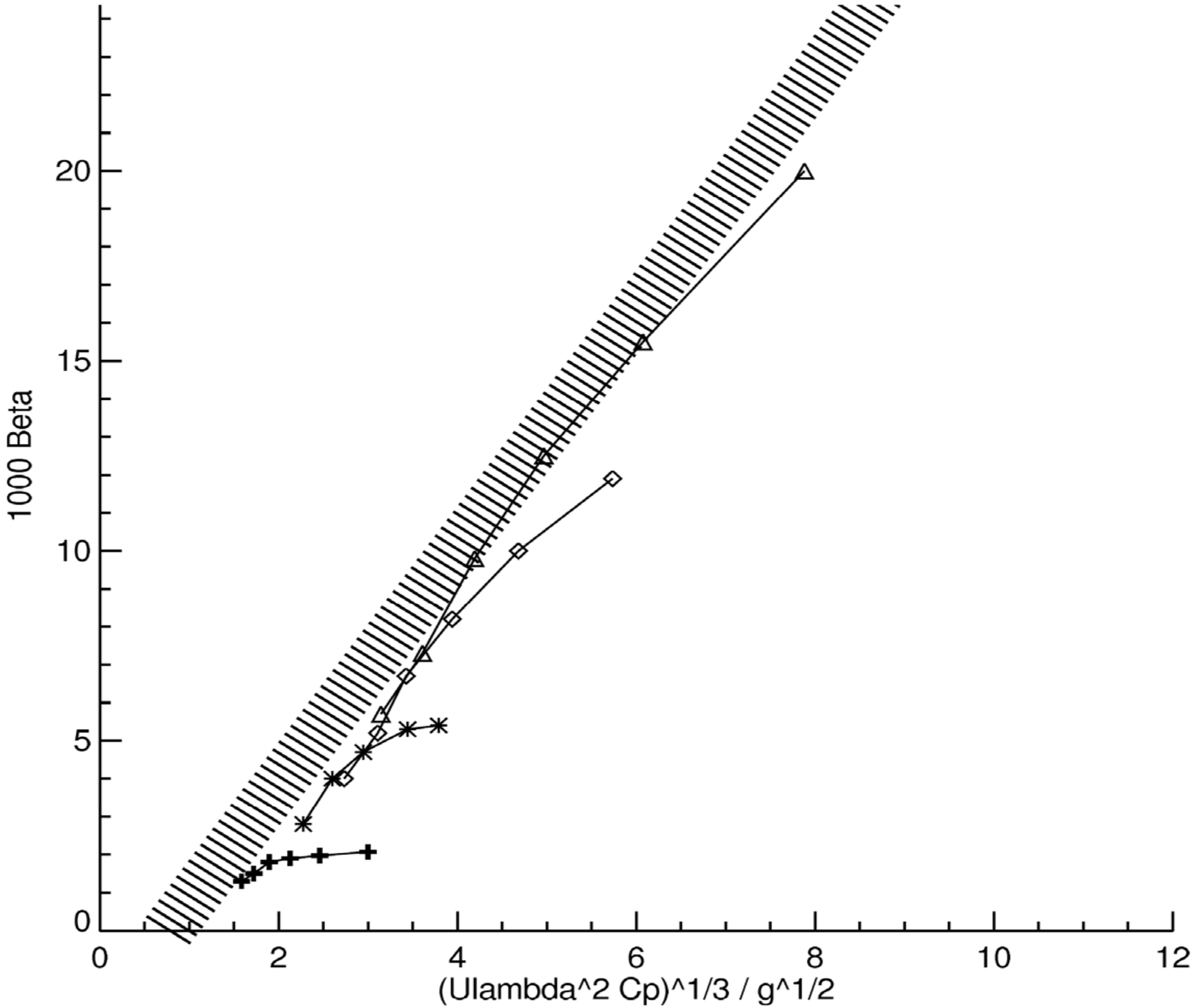
Time = 3536.20 sec
0.015
0.010
0.005
0.000
F(k) k^{-5/2}
0.1
1.0
ln f



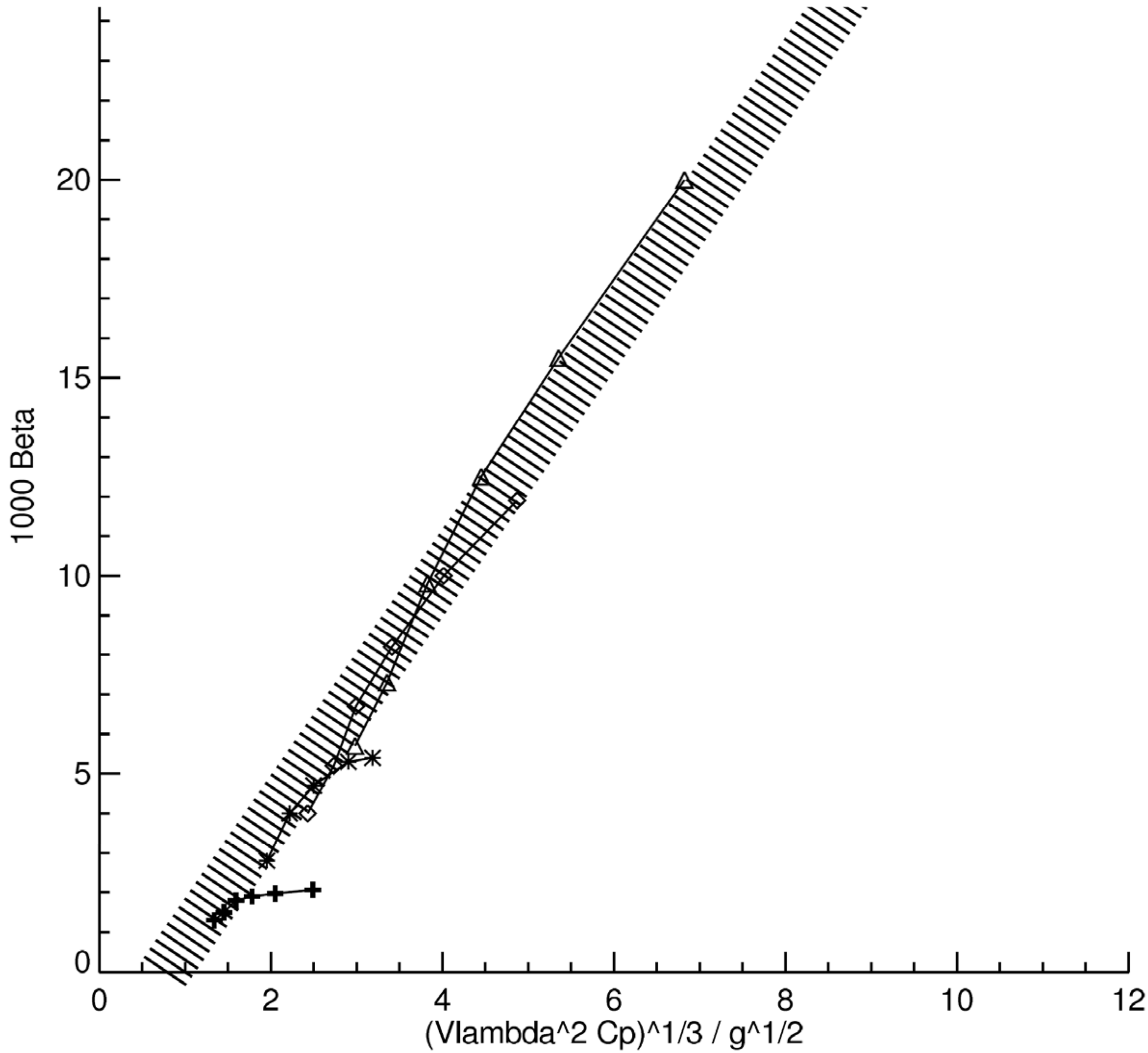
Time = 3536.20 sec
0.015
0.010
0.005
0.000
F(k) k^{-5/2}
0.1
1.0
ln f



Power index of energy spectrum $\frac{d \ln \epsilon(f)}{d \ln f}$, *wind speed* $u = 10.0$ *m/sec*



Crosses – $u = 2.5$, *stars* – $u = 5.0$, *rectangles* – $u = 10.0$, *triangles* – $u = 20.0$



1000 Beta
20
15
10
5
0
0
2
4
6
8
10
12
(Vlambda^2 Cp)^1/3 / g^1/2

Let's rescale velocity: $v = \epsilon u$

$$S_{wind}(\omega, \phi) \quad = \quad -\epsilon^{4/3} \alpha \omega \left(\frac{\omega}{g} v - \frac{1}{\epsilon} \right)^{4/3} \cos\phi \;\; n(\omega, \phi)$$

$$v = \epsilon u \quad \epsilon = 0.7$$

v	*u*
2.5	3.6
5	7.14
10	14.3
20	28.6

Self-similar interpretation of the regression line

$$\frac{\partial n_{\vec{k}}}{\partial t} = S_{nl} + \gamma(\omega, \phi)\epsilon$$

$$\gamma(\omega, \phi) = \alpha\omega^{1+s} f(\phi)$$

Self-similar solution:

$$\epsilon = t^{p+q} F(\omega t^q)$$

$$q = \frac{1}{s+1} \qquad\qquad p = \frac{9q-1}{2}$$

Zakharov-Filonenko asymptotics:

$$F(\xi) \simeq \xi^{-4} \;\; at \;\; \xi >> 1$$

Then for large frequencies

$$\epsilon \simeq \frac{t^{p-3q}}{\omega^4}$$

From another side

$$\epsilon(\omega, \phi) = \frac{\mu g u^{1-\xi} C_p^{\xi}}{\omega^4}$$

$$\omega_p \simeq t^{-q} \qquad t \simeq C_p^{1/q} \qquad C_p \simeq t^q$$