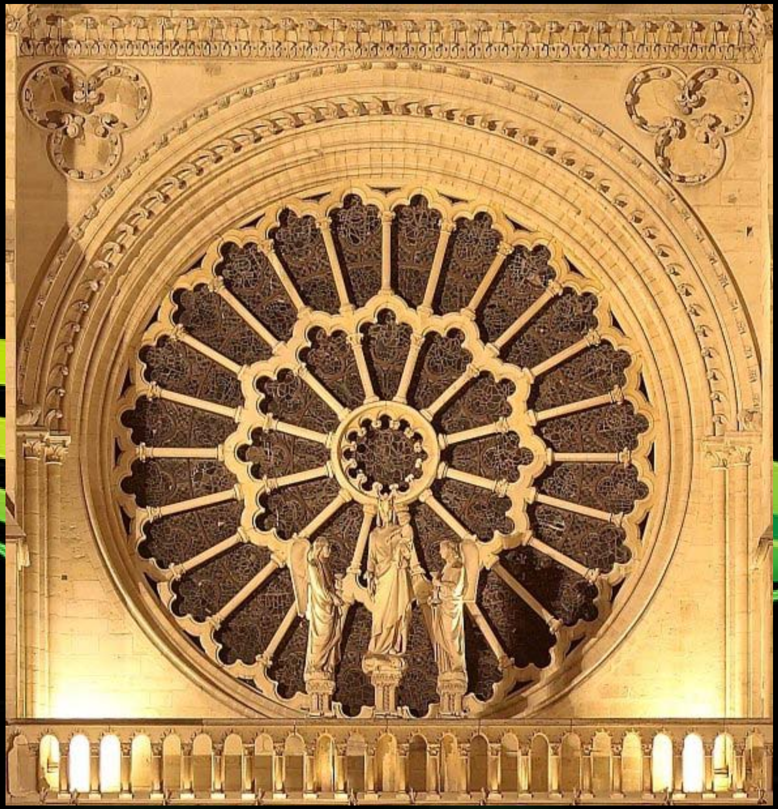
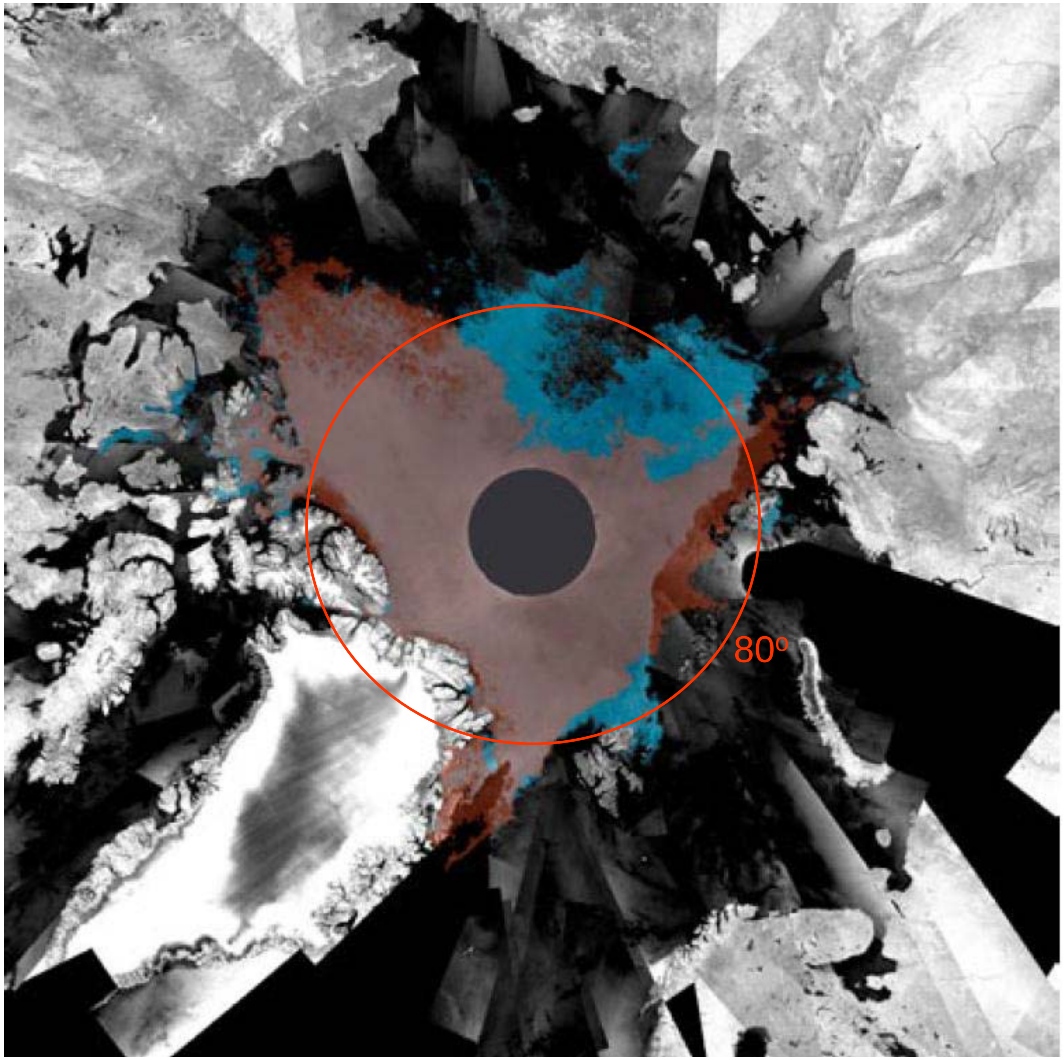


Spherical Multiple-Cell Grid to cover the Arctic

Jian-Guo Li 21 Oct 2009

Arctic ice by ESA Envisat ASAR in September 2007 and 2008



Arctic ice is retreating.

Global wave model needs extension to cover the Arctic.

Light brown: ice appear in both Septembers.

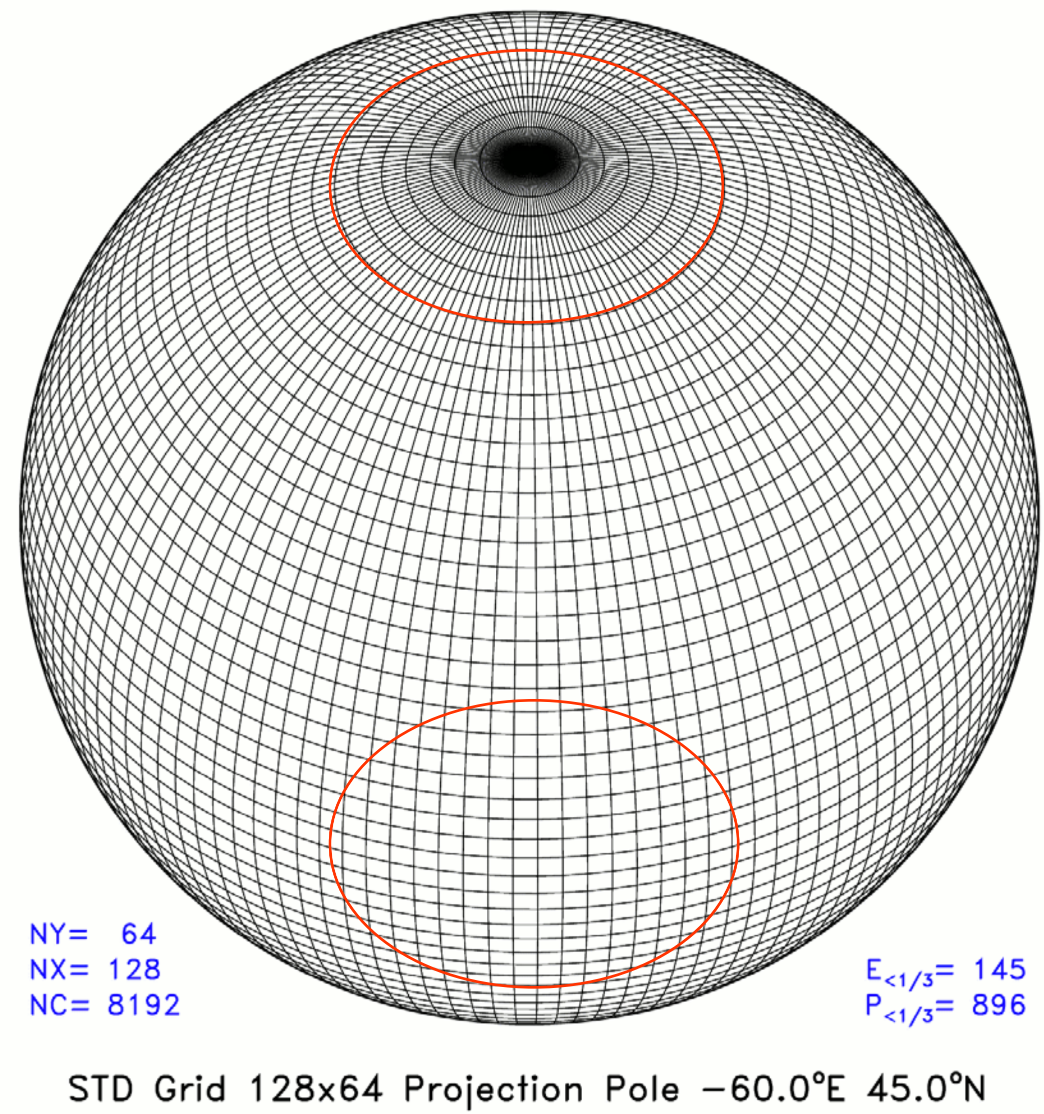
Dark brown: ice free in Sept 2008 but covered in Sept 2007.

Blue: ice free in Sept 2007 but covered in 2008.

Highest ice-free latitude 86º, present model 82º.

Worst scenario: Arctic ice free in summer 2030 (?)

Polar problem with standard grid



Transportation equation in spherical system:

$$\frac{\partial \psi}{\partial t} + \frac{\partial (u\psi)}{\partial x} + \frac{\partial (\upsilon\psi\cos\varphi)}{\cos\varphi\,\partial y} = 0$$

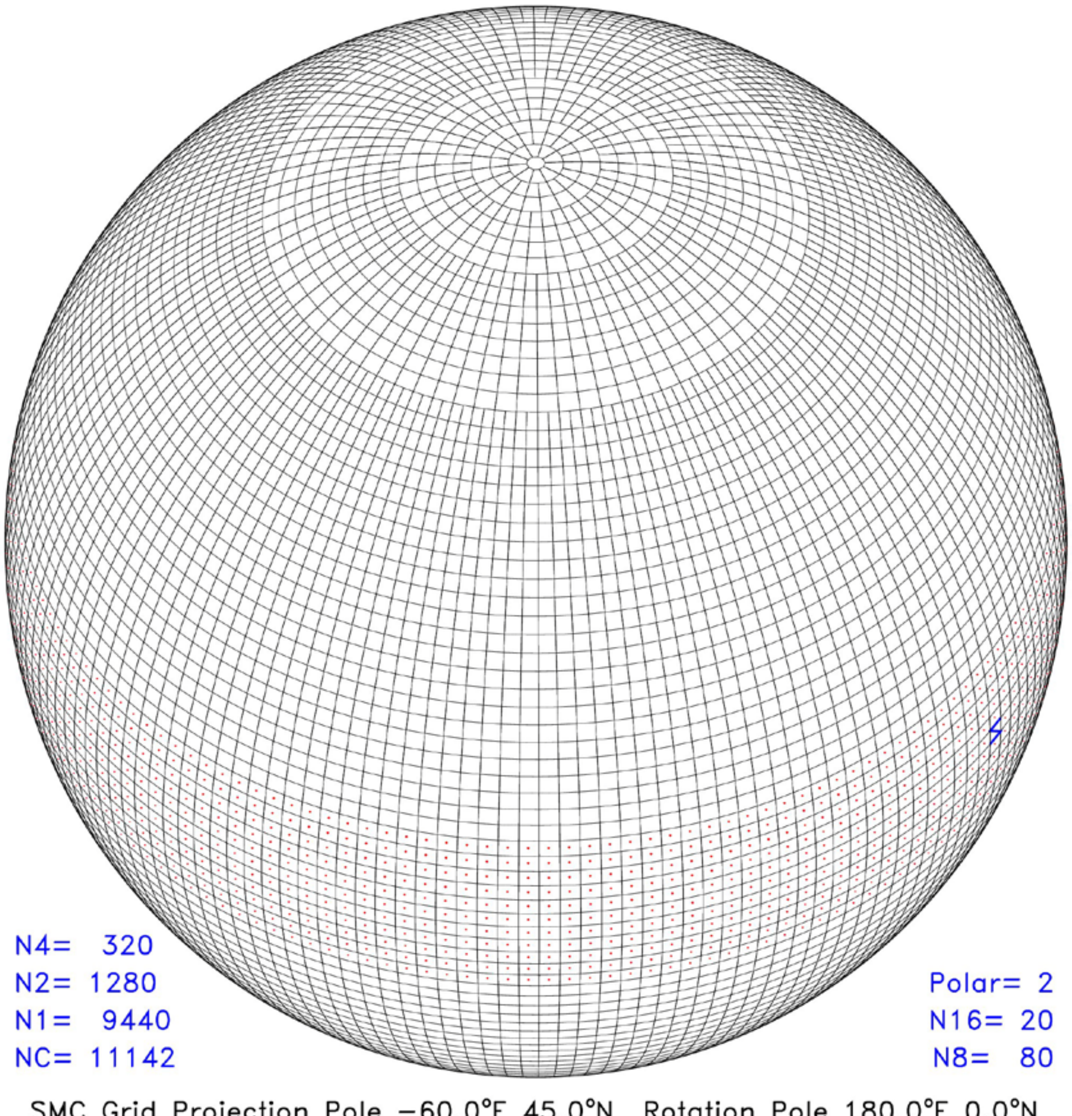
The Pole is a singular point
Flow has to go around it, not crossing it.

Severe CFL restriction on Eulerian advection time step near the Pole.

Disparity in space resolution

$E_{<1/3}$=145, $P_{<1/3}$=896.

SMC 2^o grid with polar cells



Polar singularity removed by using round polar cells.

Integration equ. replaces differential one for polar cells.

Balanced space resolution

$E_{<1/3}=243$, $P_{<1/3}=371$.

$\Delta\lambda=2.25^o$ $\Delta\varphi=2$

Regular 160x90=14,400

SMC 11,142 Reduced 22%

Summary and conclusions

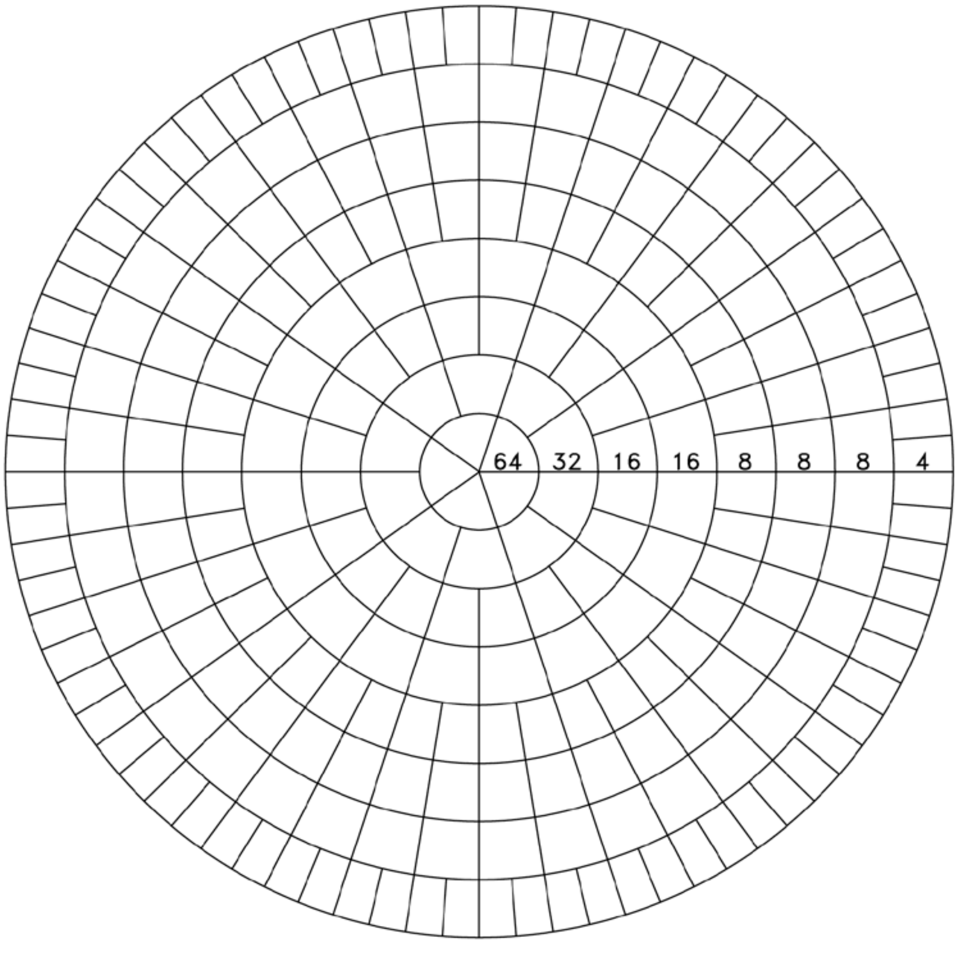
- Spherical Multiple-Cell (SMC) grid is developed to solve the polar transportation problem in global lat-lon grid for application in ocean surface wave model.
- A round polar cell is introduced with integral conservation equation to remove the Pole singularity and enable cross-pole transportation.
- Solid-body rotation tests are used to demonstrate the performance of SMC grid with Upstream Non-Oscillatory (UNO) advection schemes.
- Application of SMC grid in global ocean surface shows its unstructured flexibility and transportation efficiency, saving 45% in memory and computation.
- Numerical results confirm that SMC grid with UNO schemes are suitable for global, including cross-pole, transportation.

Contents

This presentation covers the following areas

- The polar transportation problem
- Spherical Multiple-Cell (SMC) grid
- Solid-body rotation test
- SMC grid for global ocean surface
- Wave spectral transport in the Arctic
- Summary and conclusions

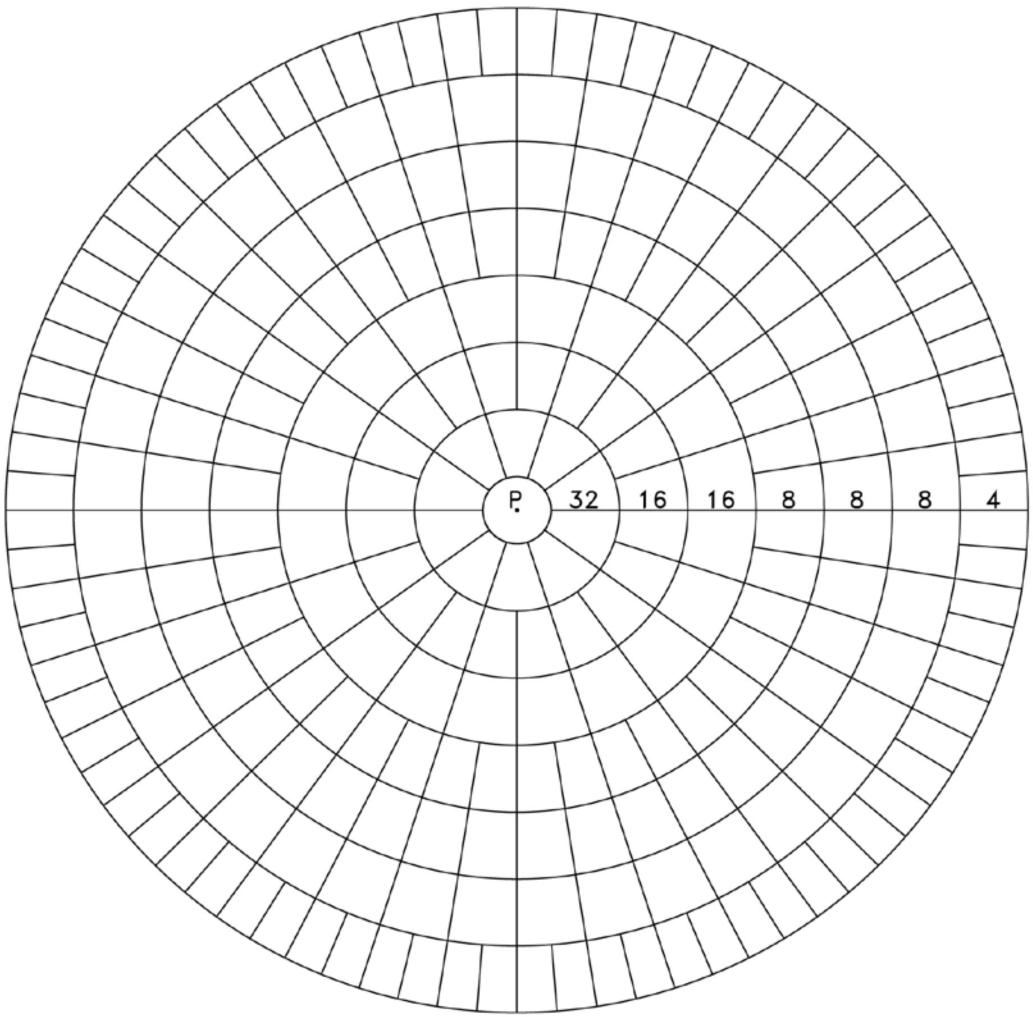
Polar problem with reduced grid



Reduced grid (Rasch 1994) relaxes the CFL restriction at high latitudes. But the polar singularity remains unsolved.

The Pole is still a singular point and flow has to go around it, not crossing it.

SMC grid with a round polar cell



Conservation equation for the polar cell:

$$\frac{\partial}{\partial t}\iint_A \psi \, dA = -\oint_{C_A} \psi \mathbf{v} \cdot d\mathbf{s}$$

Discrete form:

$$\psi_P^{n+1} - \psi_P^{n} = \pm \frac{\Delta t}{A_P} \sum_{i=1}^{m} \psi_i^{*} \upsilon_i \Delta s_i$$

Singularity removed;

Instance cross-pole transportation enabled

Upstream Non-Oscillatory (UNO) advection schemes

$$\psi_j^{n+1} = \psi_j^n + \left(u_{j-1/2}\psi_{j-1/2}^{MF} - u_{j+1/2}\psi_{j+1/2}^{MF}\right)\Delta t/\Delta x_j$$

$$\psi_{j+1/2}^{MF} = \psi_C^n + \left(x_{MF} - x_C\right)G_C$$

$$x_{MF} - x_C = 0.5 sign\left(u_{j+1/2}\right)\left(\Delta x_C - \left|u_{j+1/2}\right|\Delta t\right)$$

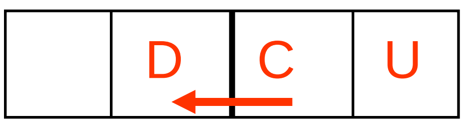
Details see:

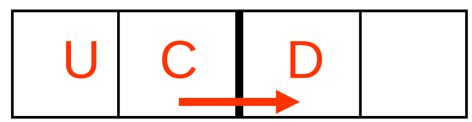
Li, J.G. (2008)
Mon. Wea. Rev.,
136, 4709-4729.

UNO2

You Know Too

$$G_C = Sign\left(G_{DC}\right)\min\left(\left|G_{DC}\right|,\left|G_{CU}\right|\right) \qquad G_{AB} \equiv \left(\psi_A - \psi_B\right)/\left(x_A - x_B\right)$$





$$G_C = G_{DC} - \frac{4\left(x_D - x_{MF}\right)}{3}\left(\frac{G_{DC} - G_{CU}}{x_D - x_U}\right) \qquad for \left|G_{DC} - G_{CU}\right| \le 1.2\left|G_{DU}\right|;$$

UNO3

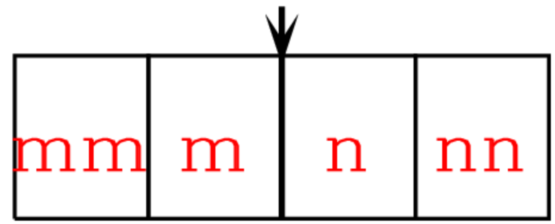
$$Else \quad G_C = 2Sign\left(G_{DC}\right)\min\left(\left|G_{DC}\right|,\left|G_{CU}\right|\right) \qquad for\ G_{DC}G_{CU} > 0$$

$$Otherwise \quad G_C = Sign\left(G_{DC}\right)\min\left(\left|G_{DC}\right|,\left|G_{CU}\right|\right)$$

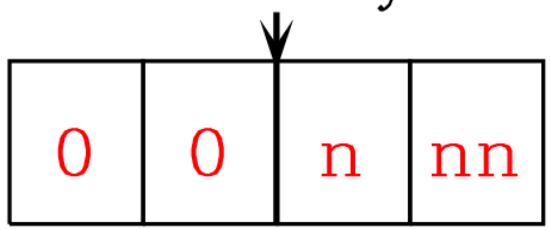
Unstructured SMC grid flux face pointer array

- Internal and boundary faces are treated alike in 1-D array. No boundary for global model.
- Single point island is extended by 0-cells, allowing single-island blocking.
- Two-D spherical surface advection is done by 3 loops: u- and v-face flux loops and cell update loop.

Internal face



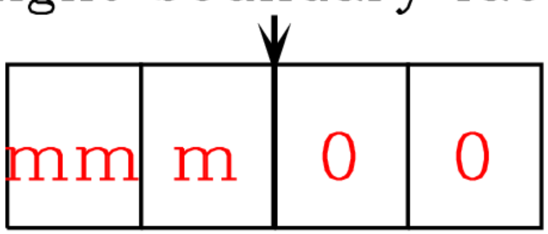
Left boundary face



Details see:

Li, J.G. (2003) *Bounary-Layer Meteorology,*. **107**, 289-322.

Right boundary face



Solid-body rotation velocity

Non-divergent 2-D flow on the spherical surface

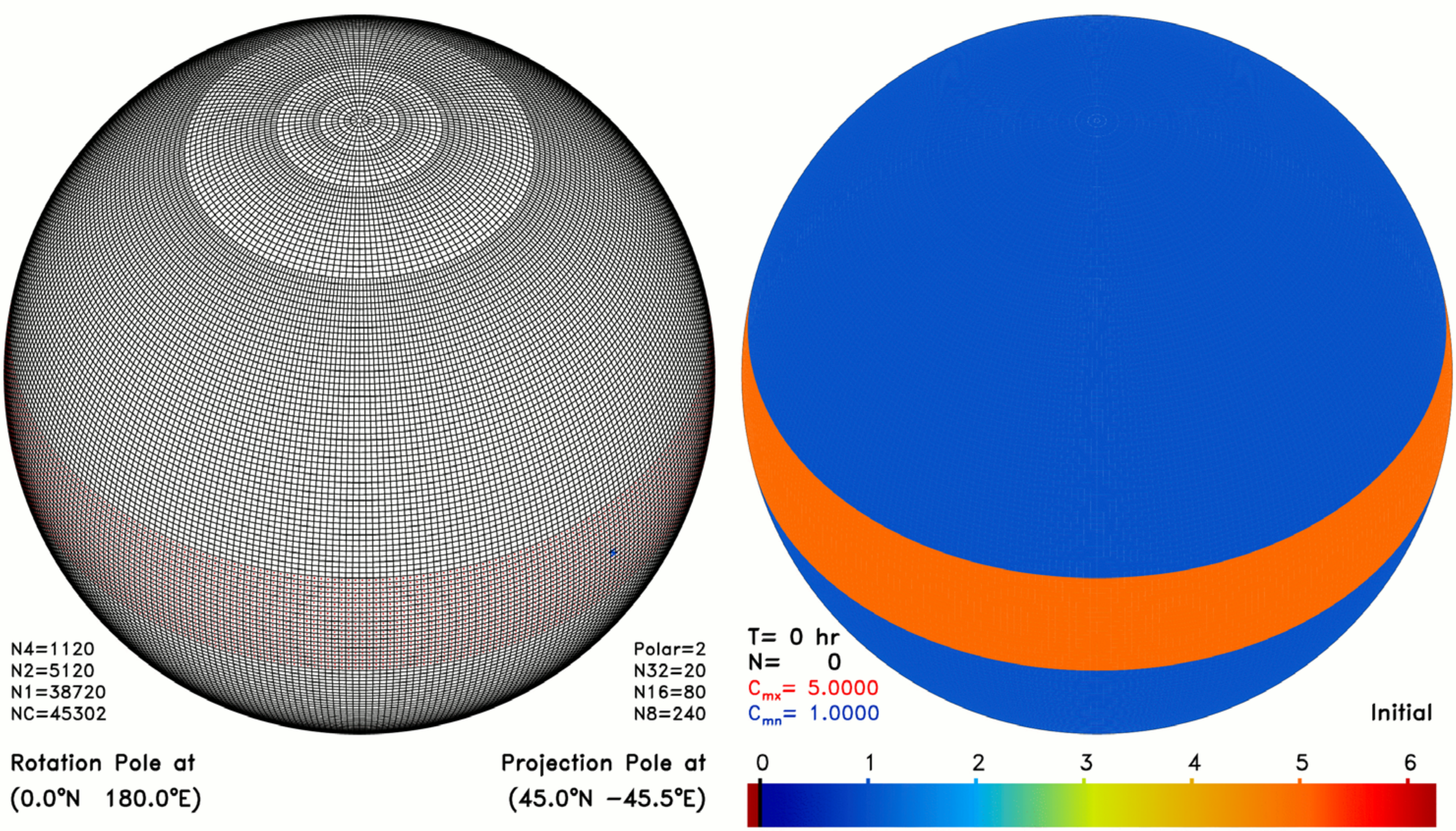
$$u = \omega r \left[\cos\alpha \cos\varphi - \sin\alpha \sin\varphi \cos\left(\lambda - \lambda_P\right) \right]$$

$$\upsilon = \omega r \sin\alpha \sin\left(\lambda - \lambda_P\right), \qquad \alpha \equiv \pi/2 - \varphi_P$$

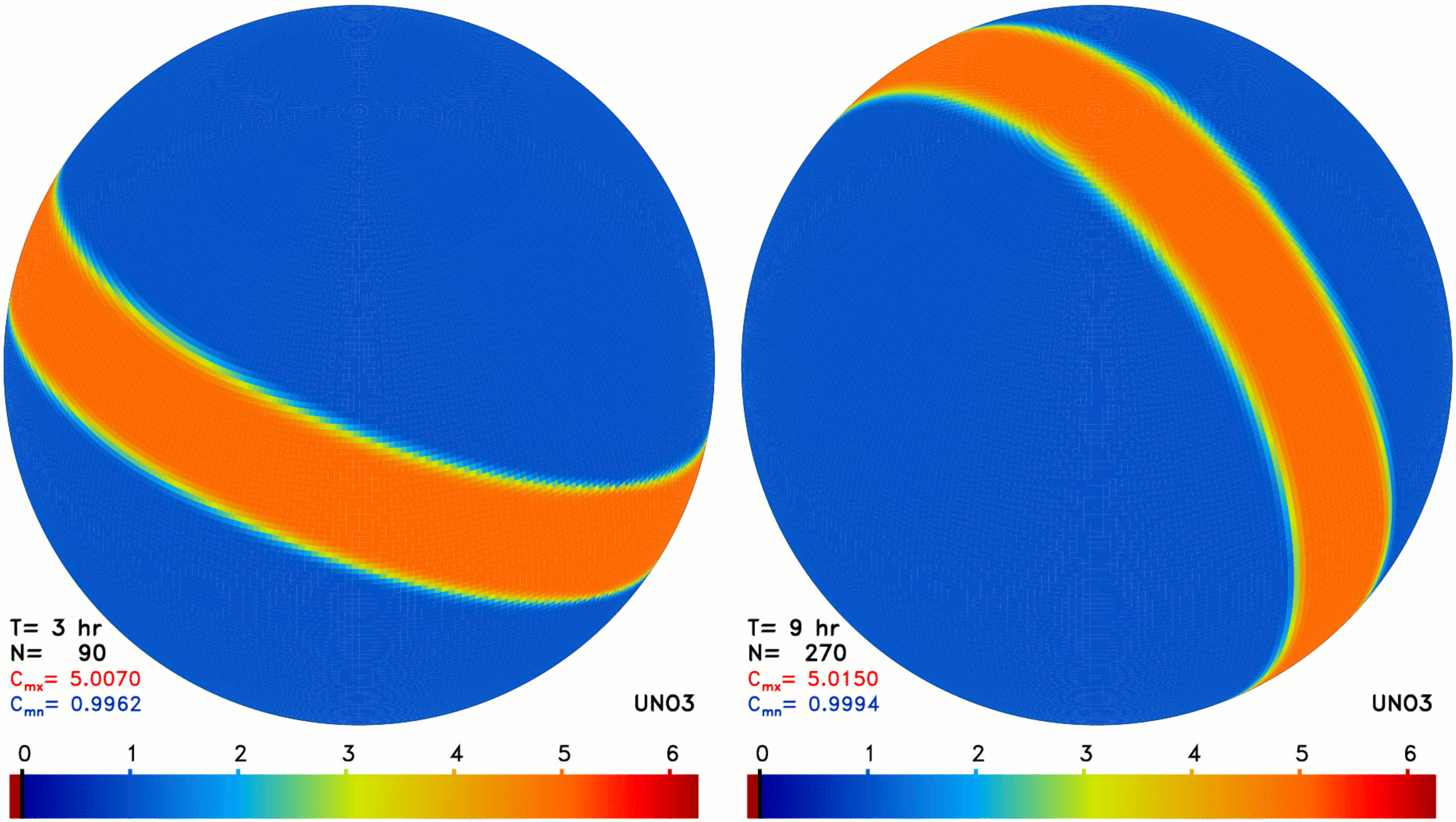
Where *r* is radius, λ longitude, φ latitude.

The rotation pole is at $(\lambda_P, \varphi_P) = (180^o, 0^o)$ and the constant angular speed $\omega = 10^o$ per hour.

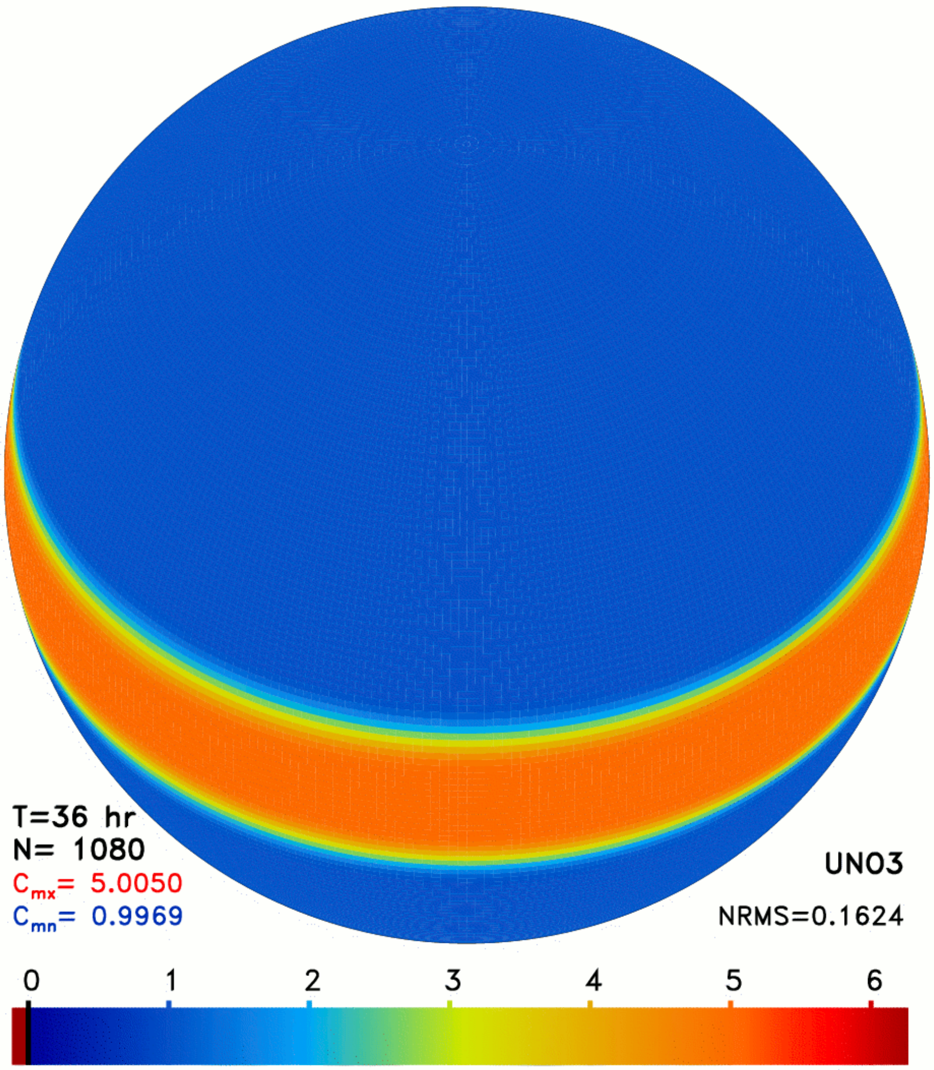
Spherical Step Function (SSF) solid-body rotation on SMC 1° grid UNO3 scheme

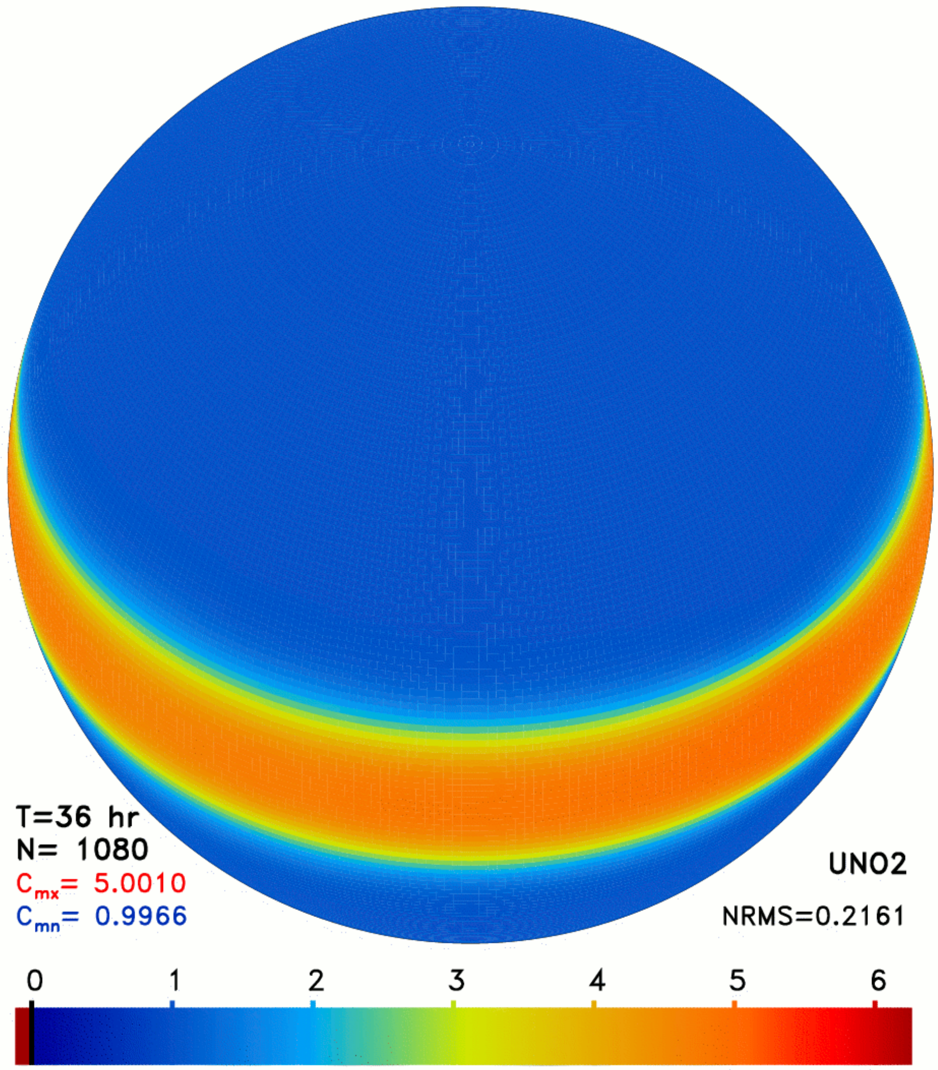


SSF s-b rotation on SMC 1º grid with UNO3 scheme



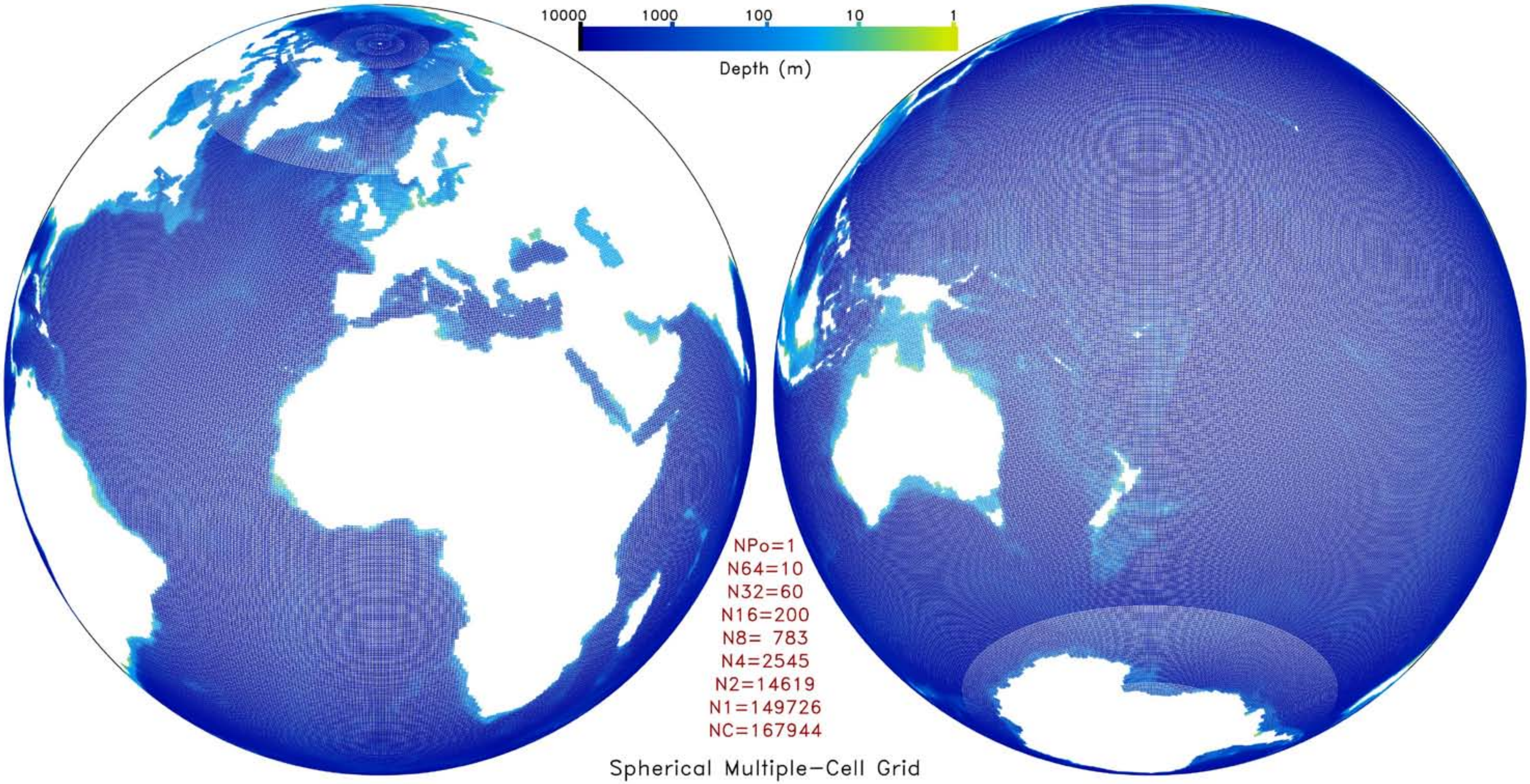
SSF s-b rotation on SMC 1º grid with UNO3 and UNO2 schemes





SMC 40 km grid for global model

DXmin = 31 km

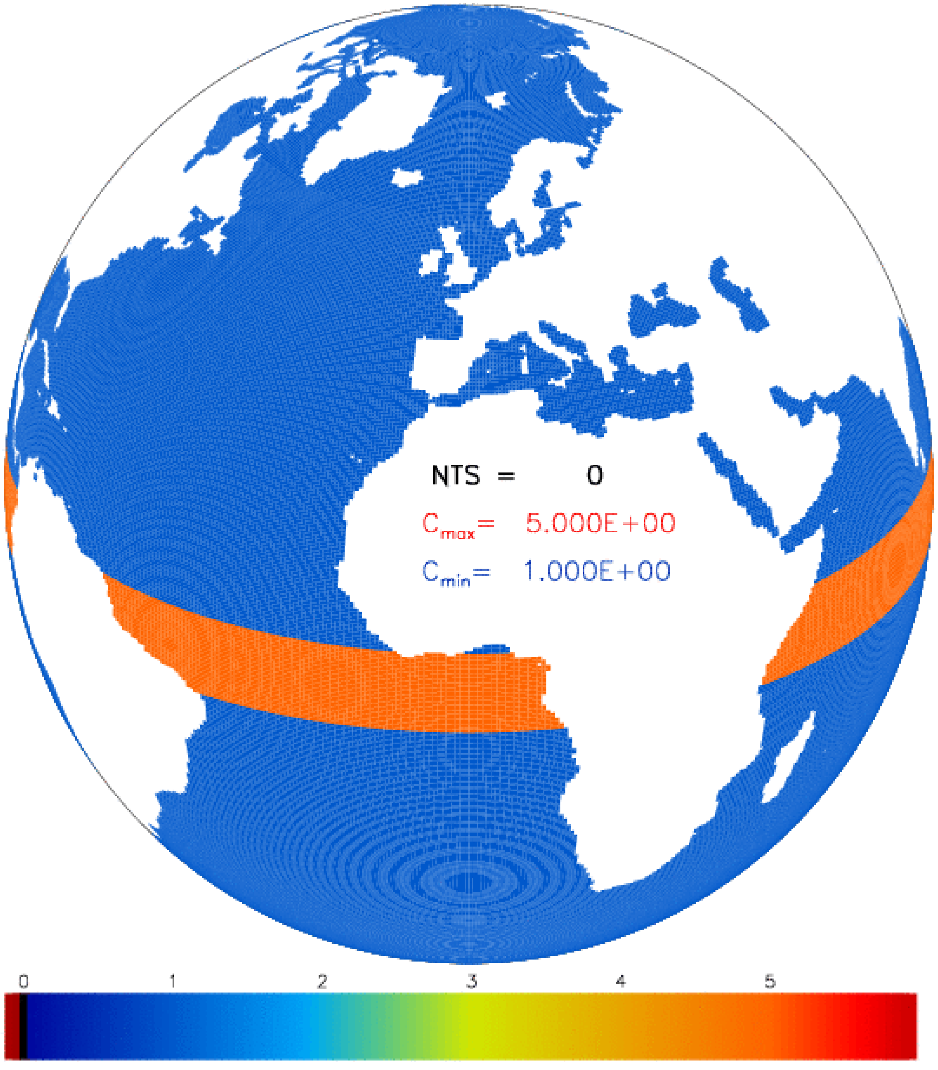


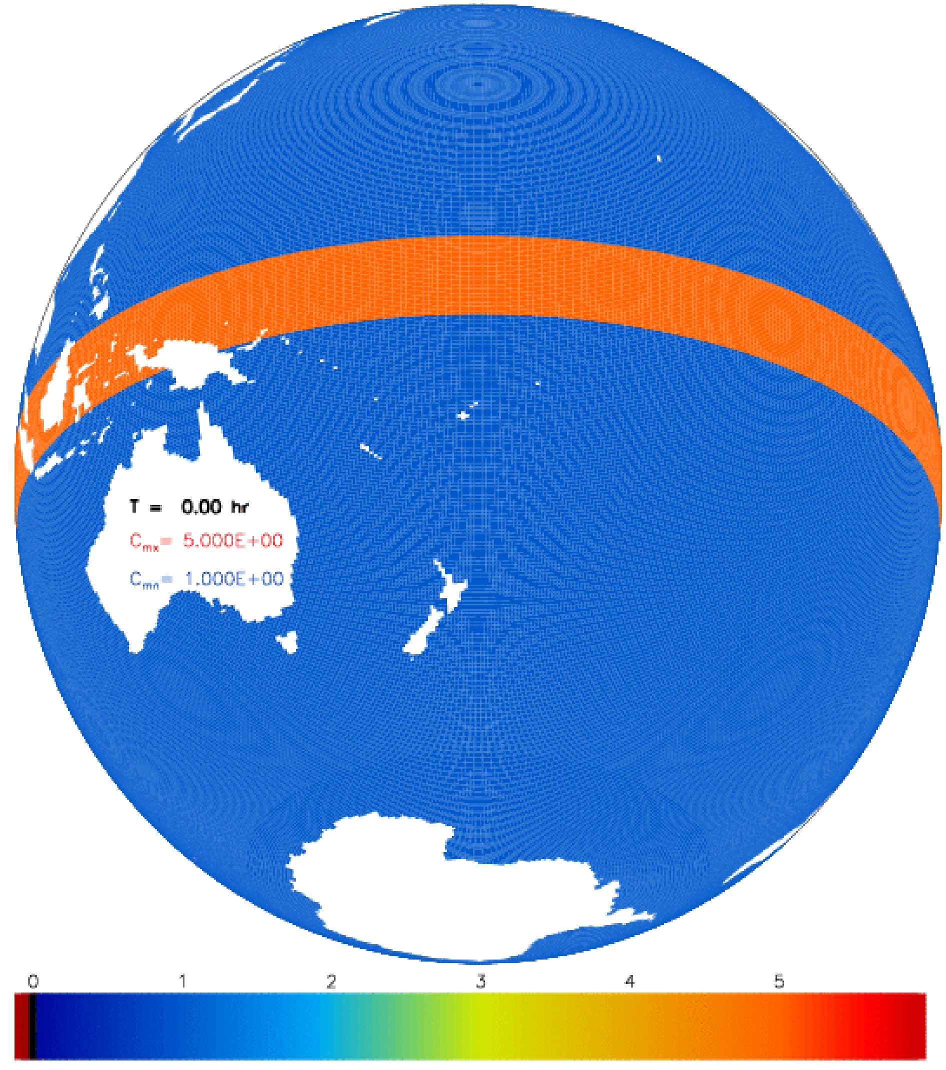
Regular lat-lon grid 480*640 ($\Delta\lambda$=0.5625° $\Delta\varphi$=0.375°)
SMC grid total cell number 167 944, reduction 45%

SMC grid single points islands

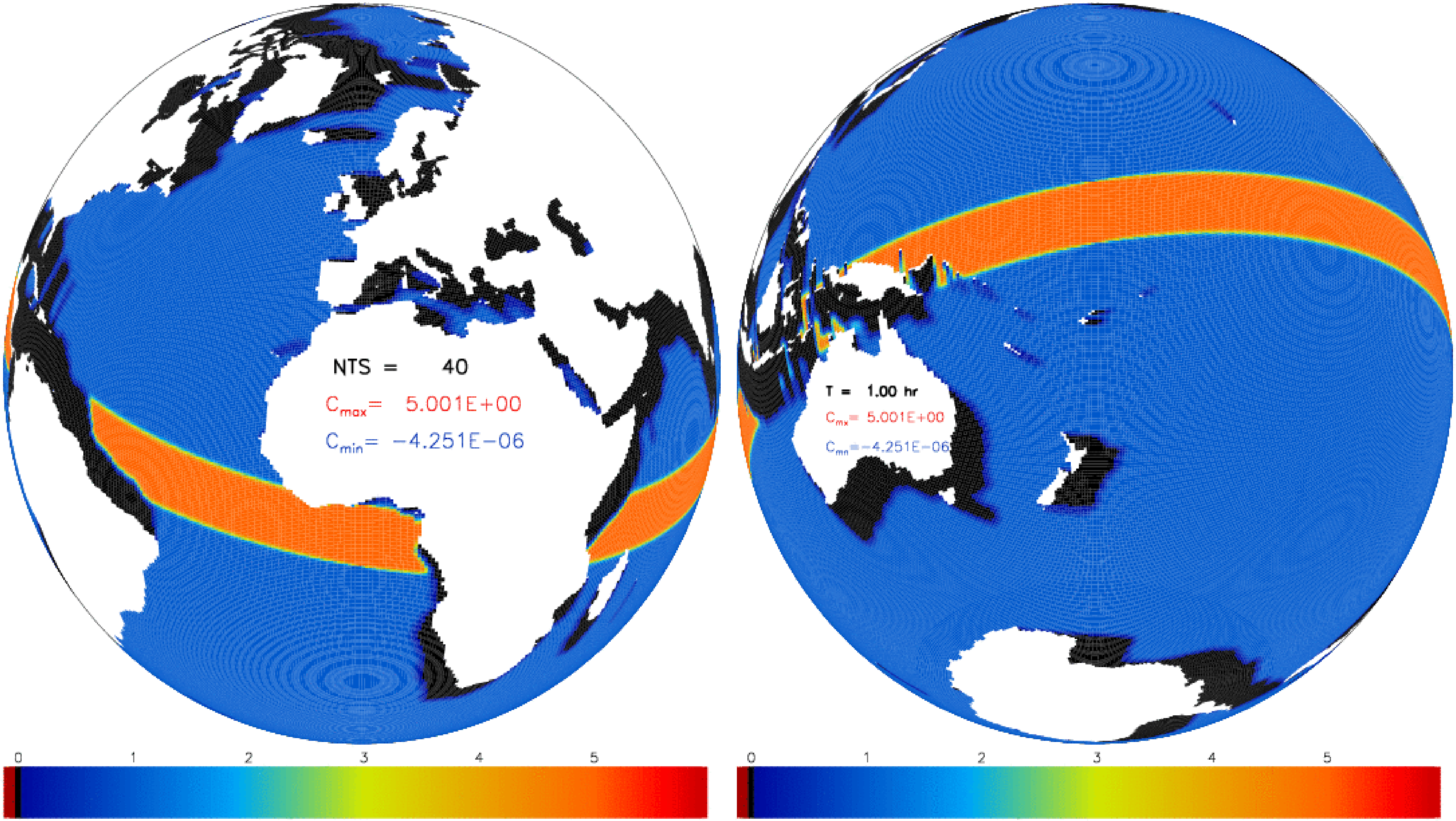
SSF rotation on SMC with UNO3

Met Office

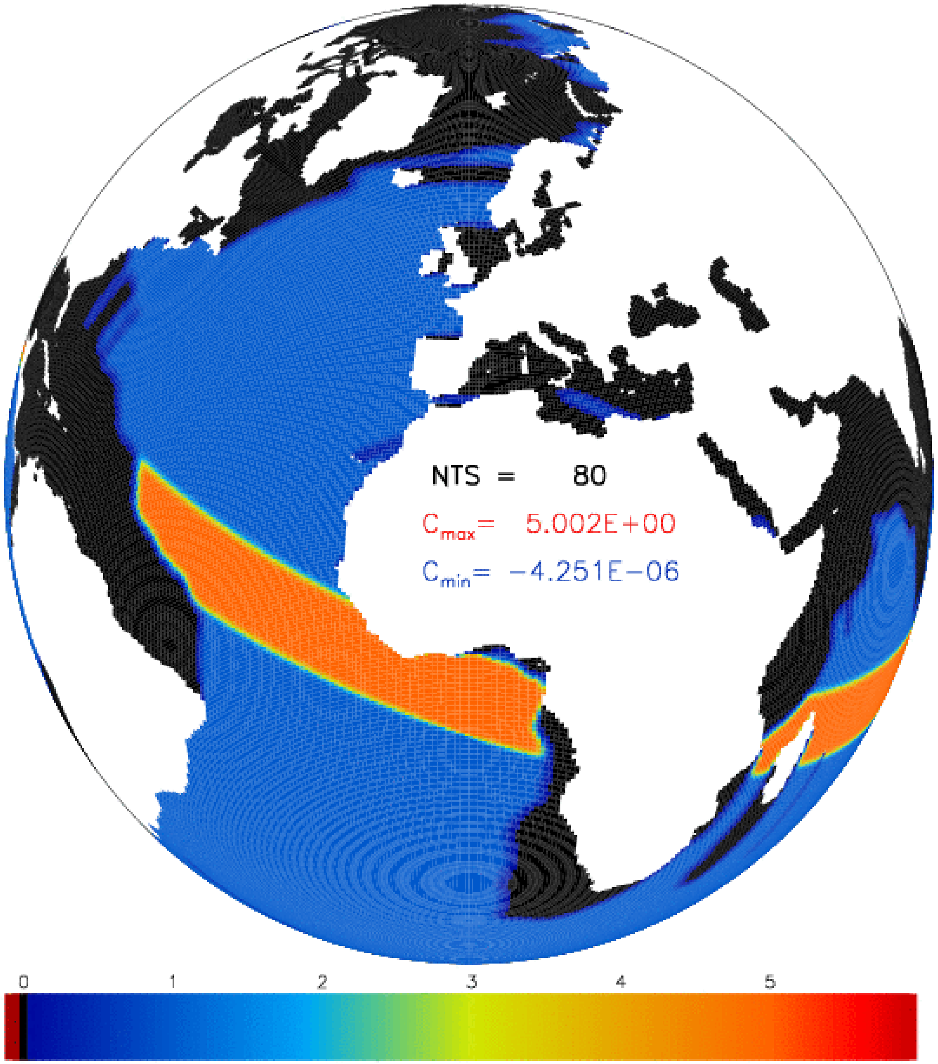


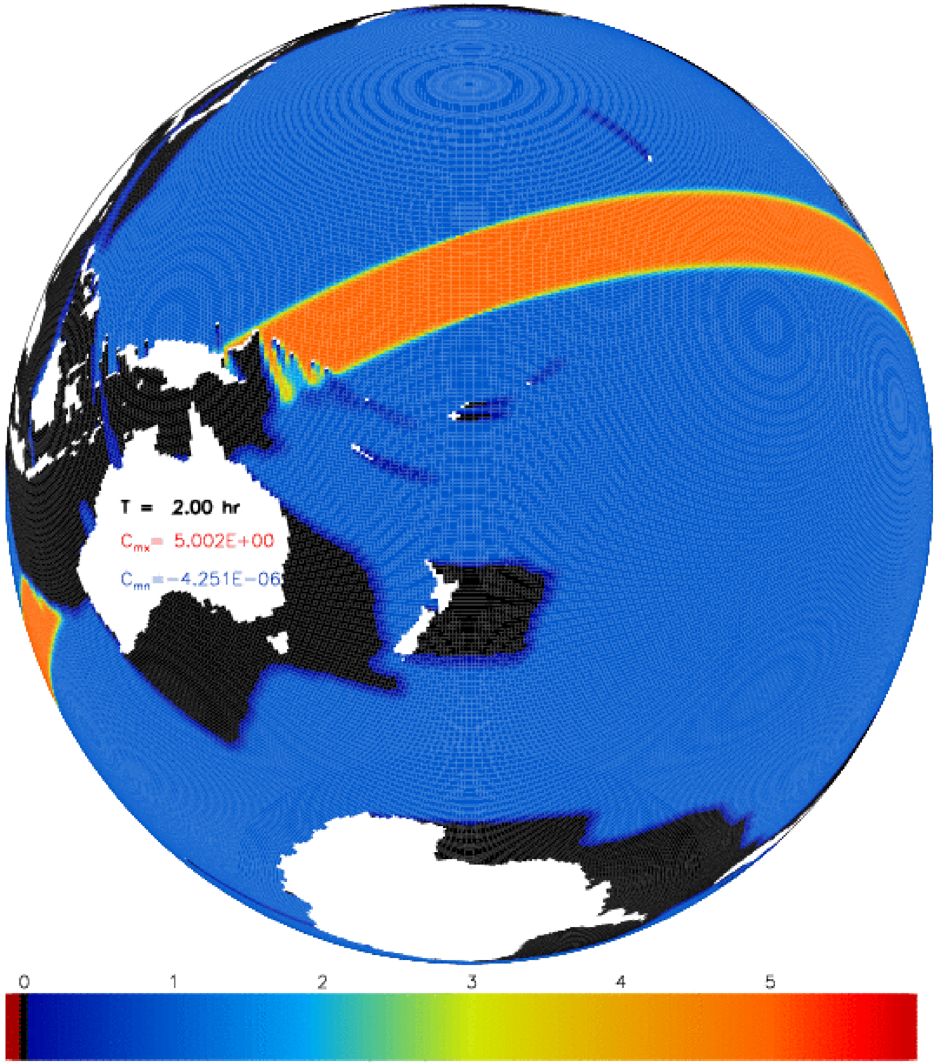


SSF rotation on SMC with UNO3

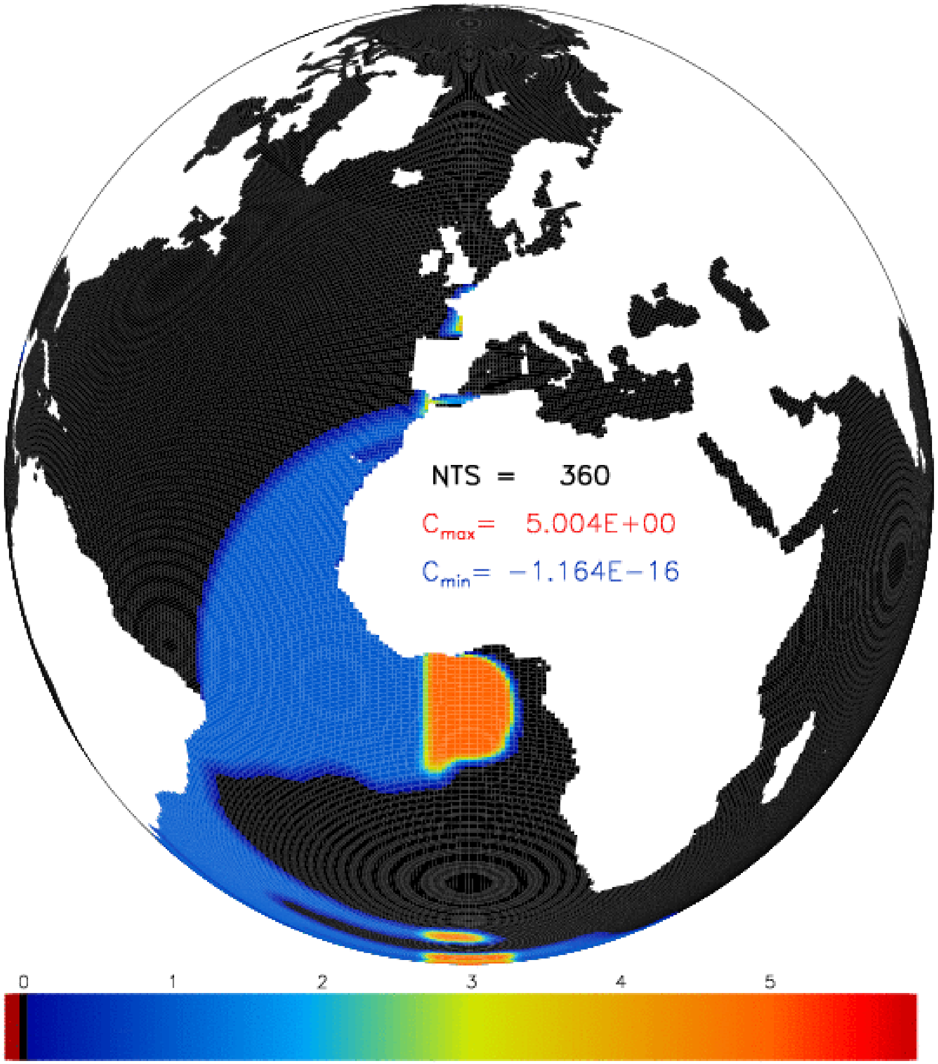


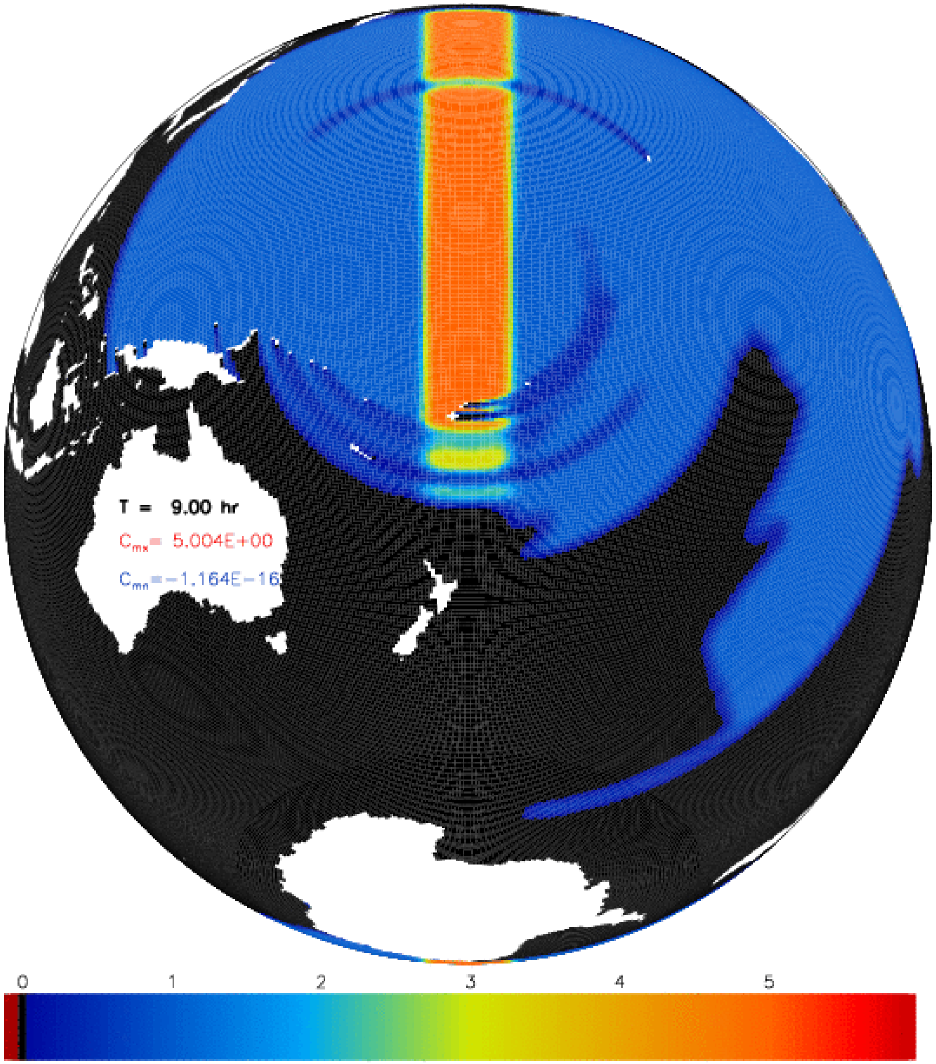
SSF rotation on SMC with UNO3



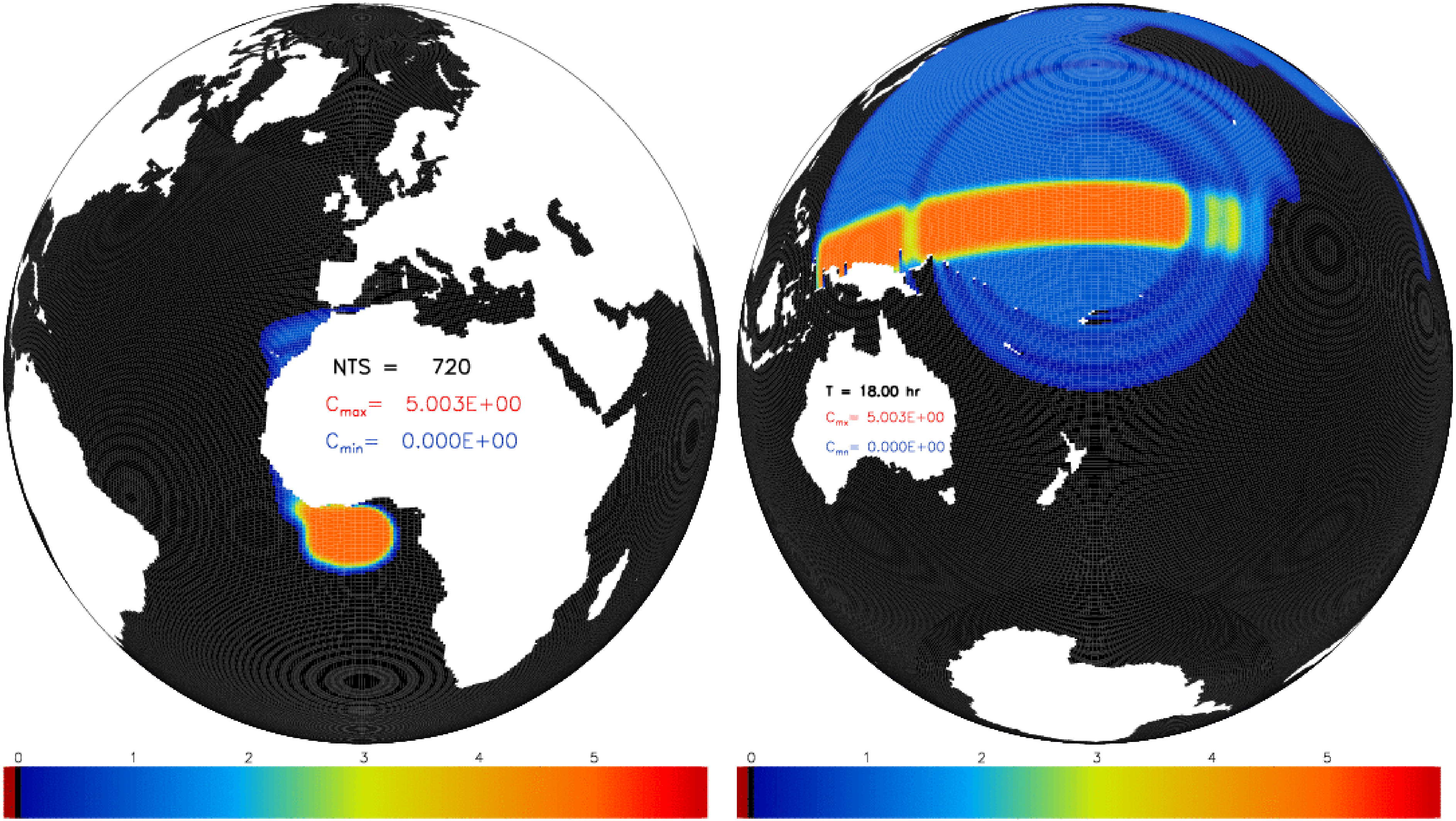


SSF rotation on SMC with UNO3

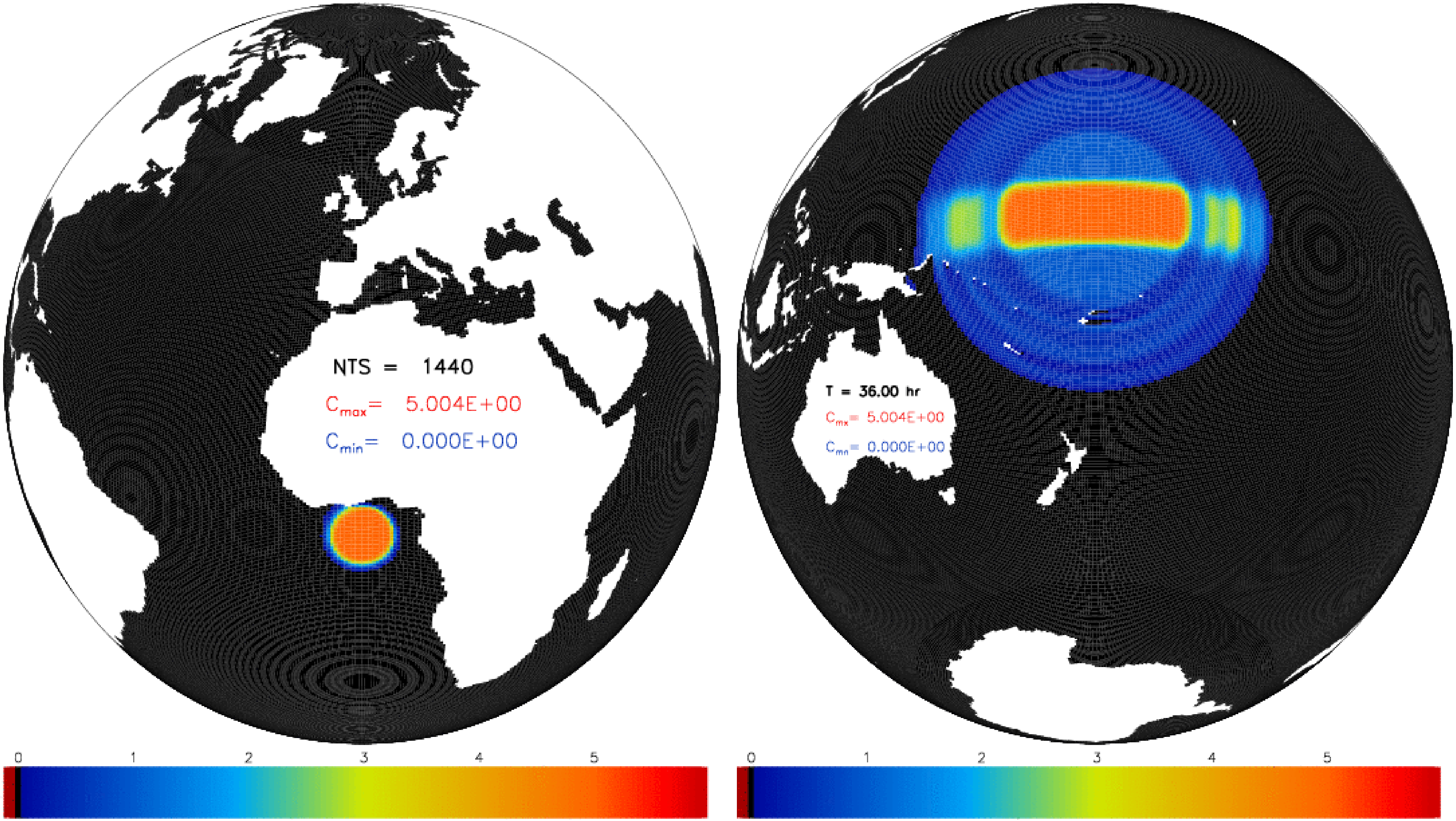




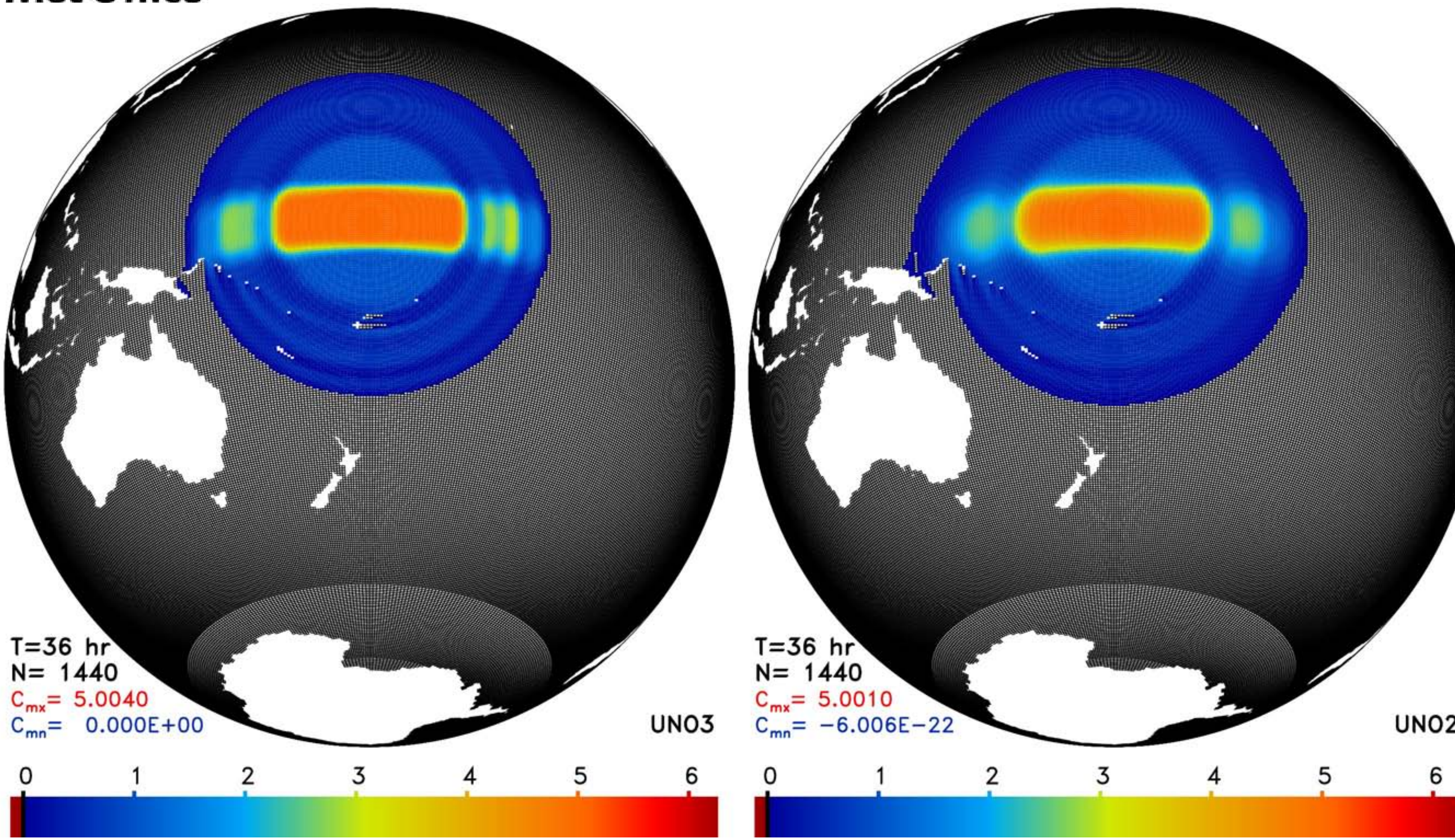
SSF rotation on SMC with UNO3



SSF rotation on SMC with UNO3



Comparison of UNO3 and UNO2



Wave spectral transport in Arctic

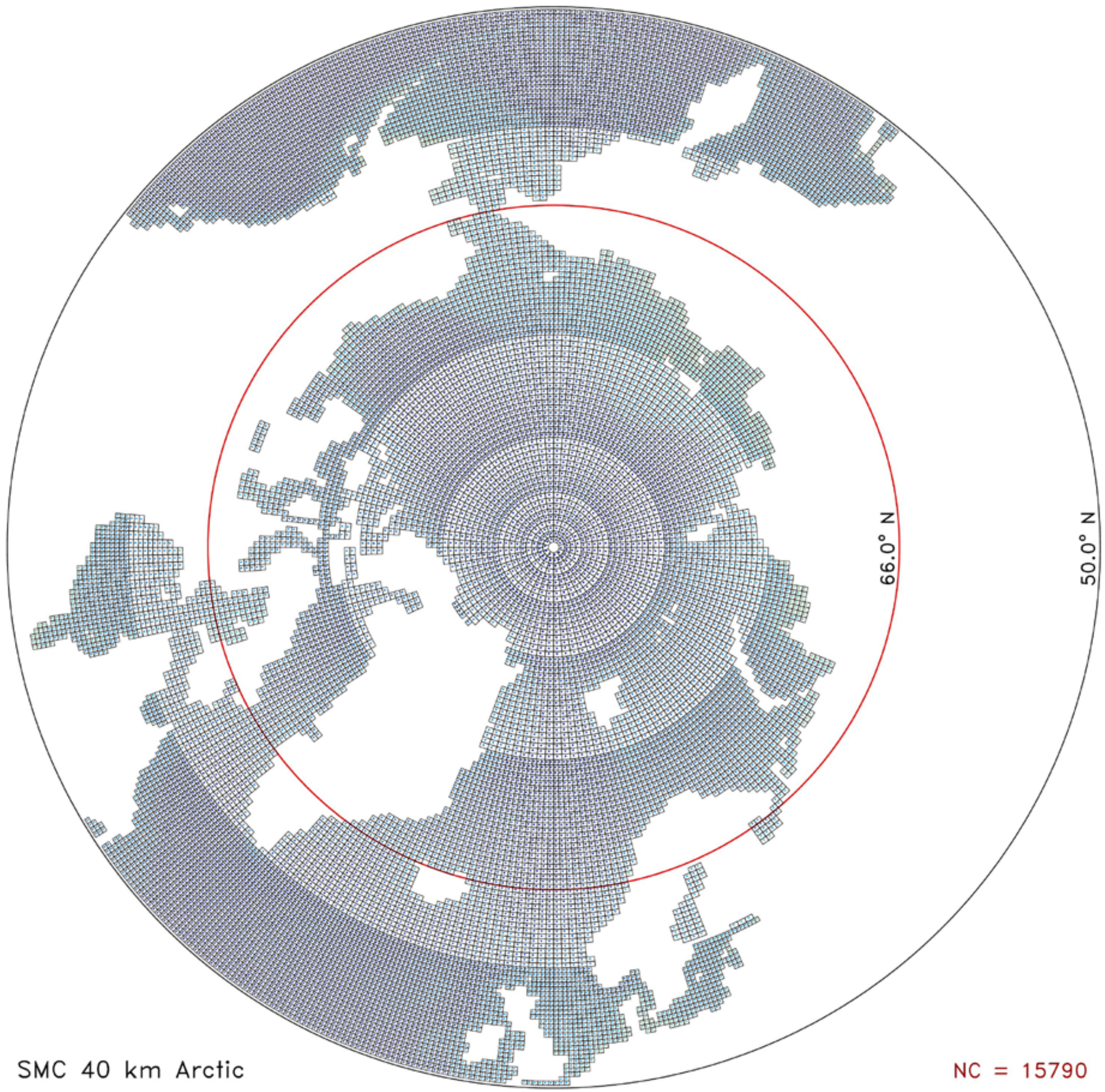
Arctic part > 66° N

Fixed reference dir.

Lower part 50-66°N

Std local east dir.

Two rows of extra cells as boundaries for each parts.

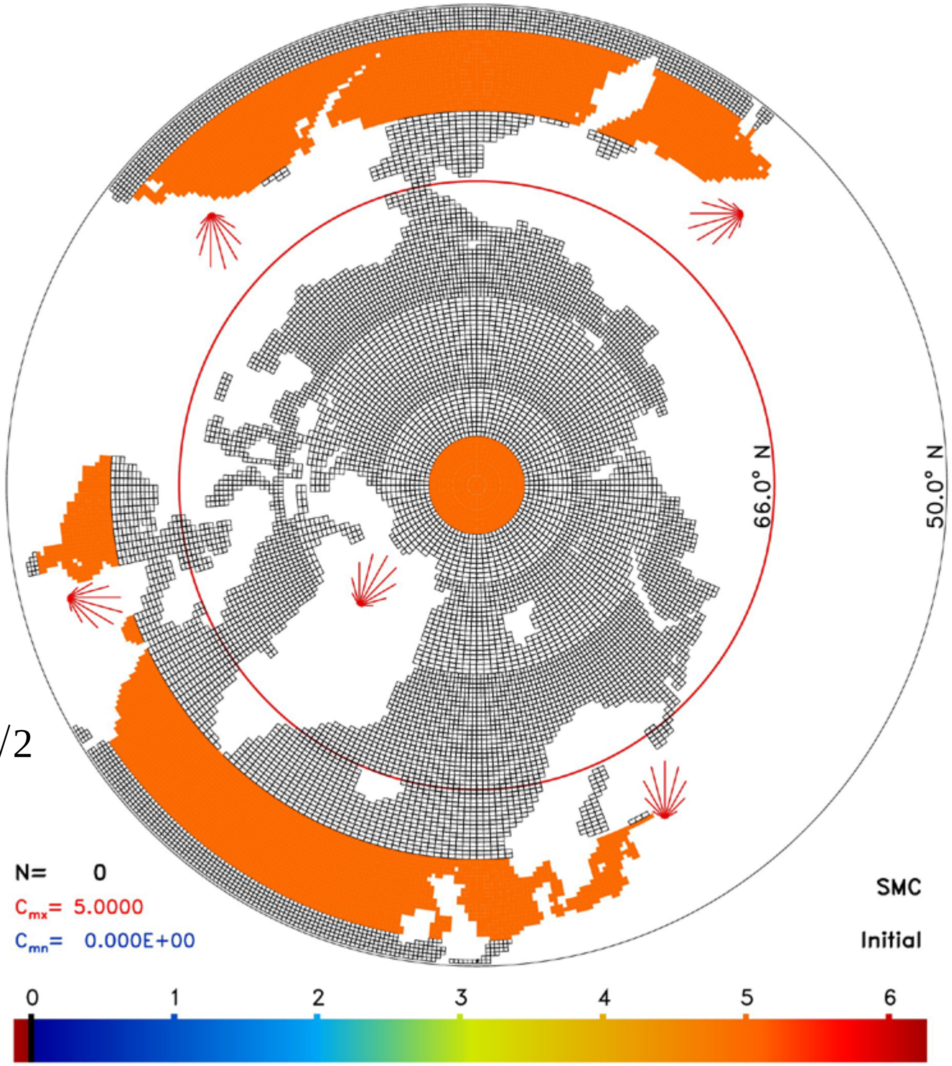


Wave spectral transport in Arctic

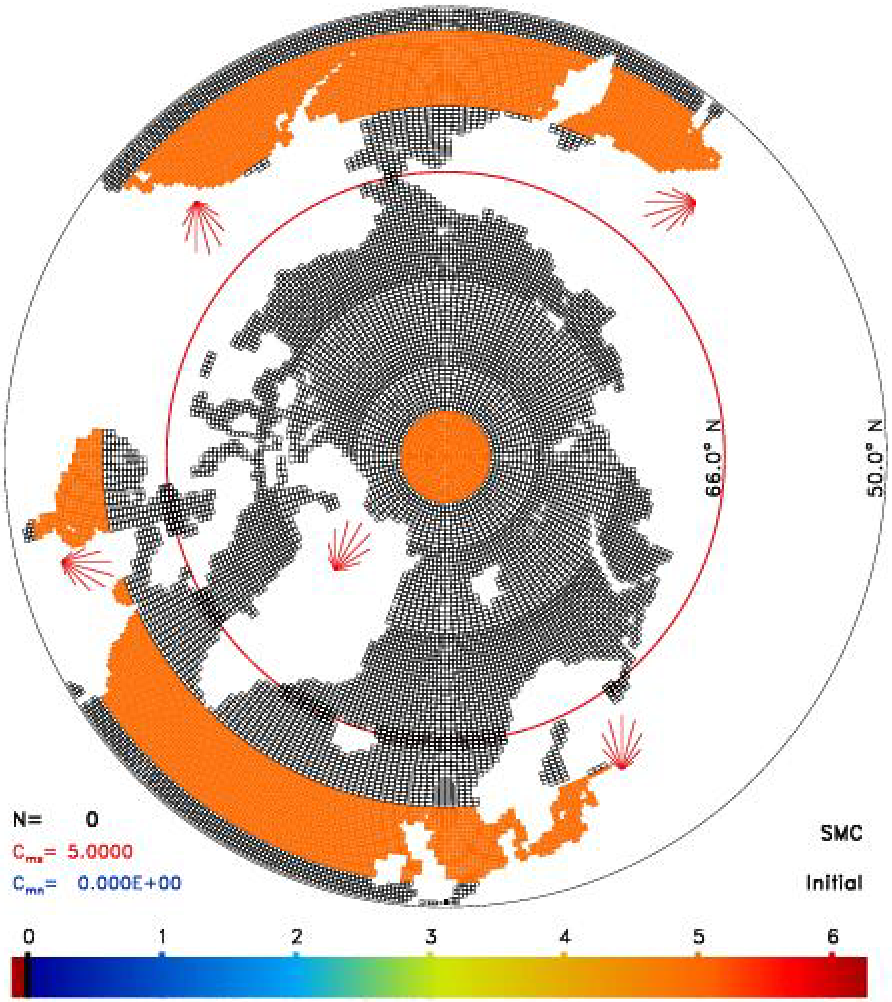
Initial wave spectral as typical wind-sea.

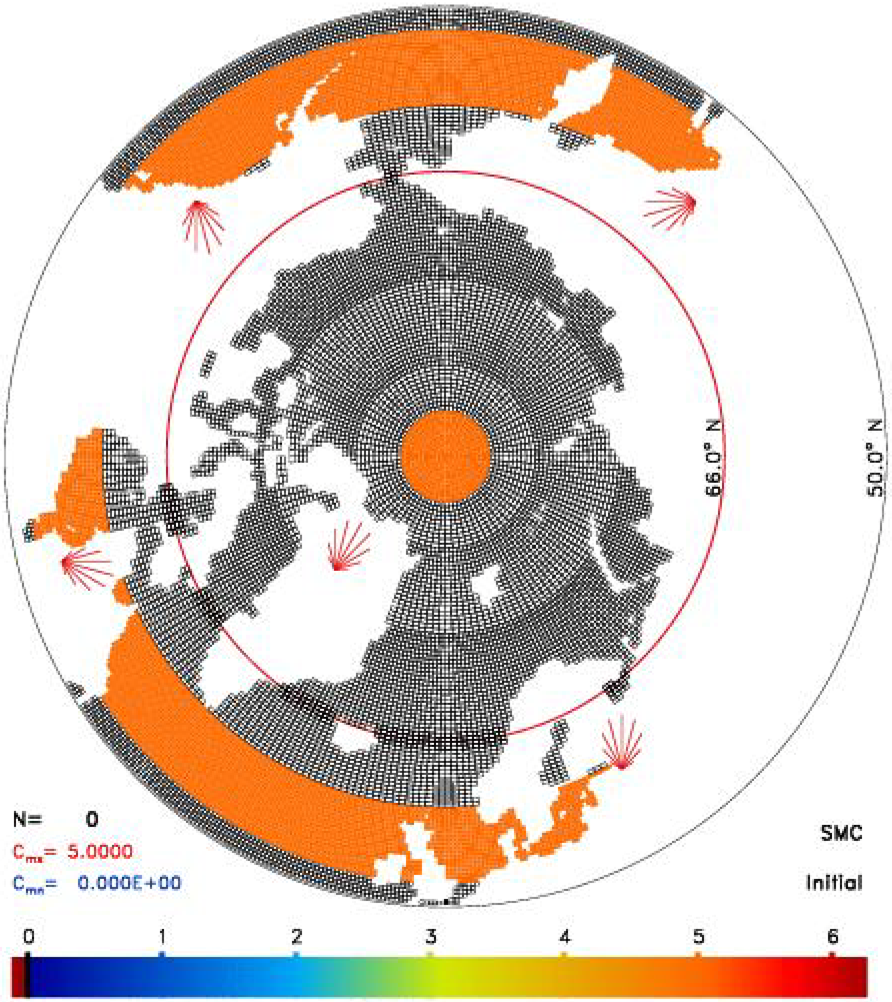
Peak direction at 60º from reference dir., local east in lower part and fixed dir in Arctic part

$$E(\theta) = \begin{cases} E_0 \cos^2(\theta - \pi/3), & for\ |\theta - \pi/3| \le \pi/2 \\ 0, & Otherwise \end{cases}$$



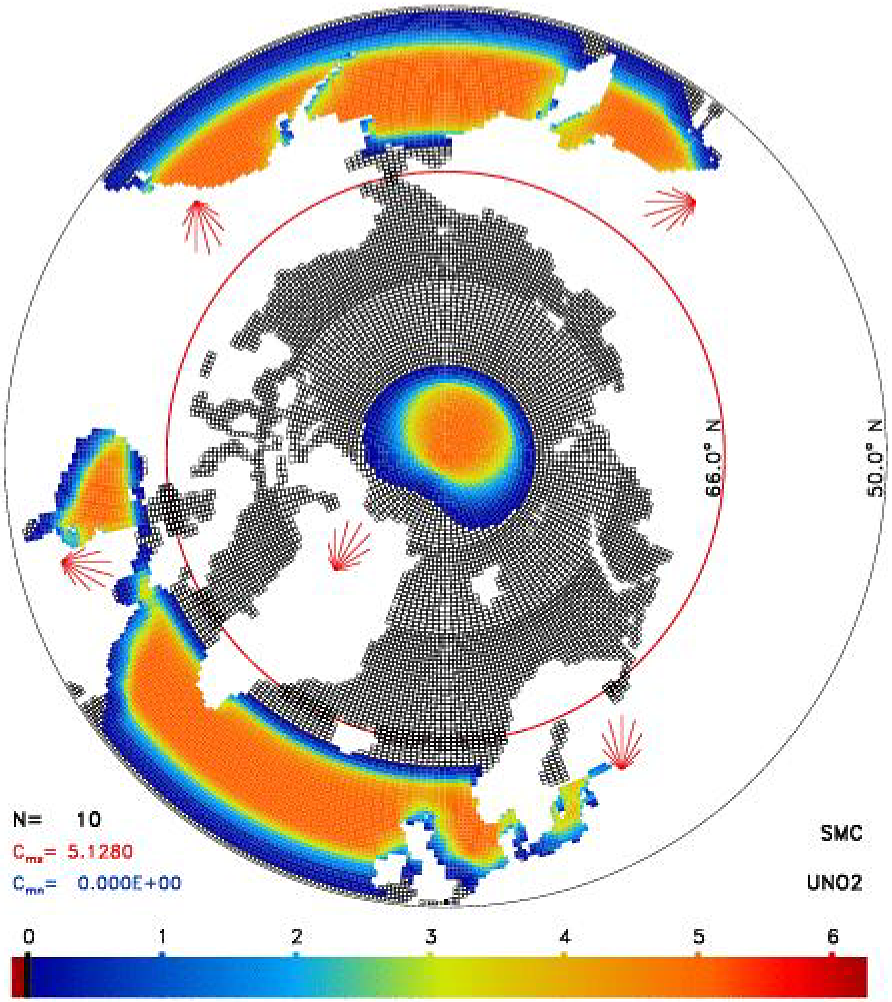
Wave spectral transport in Arctic

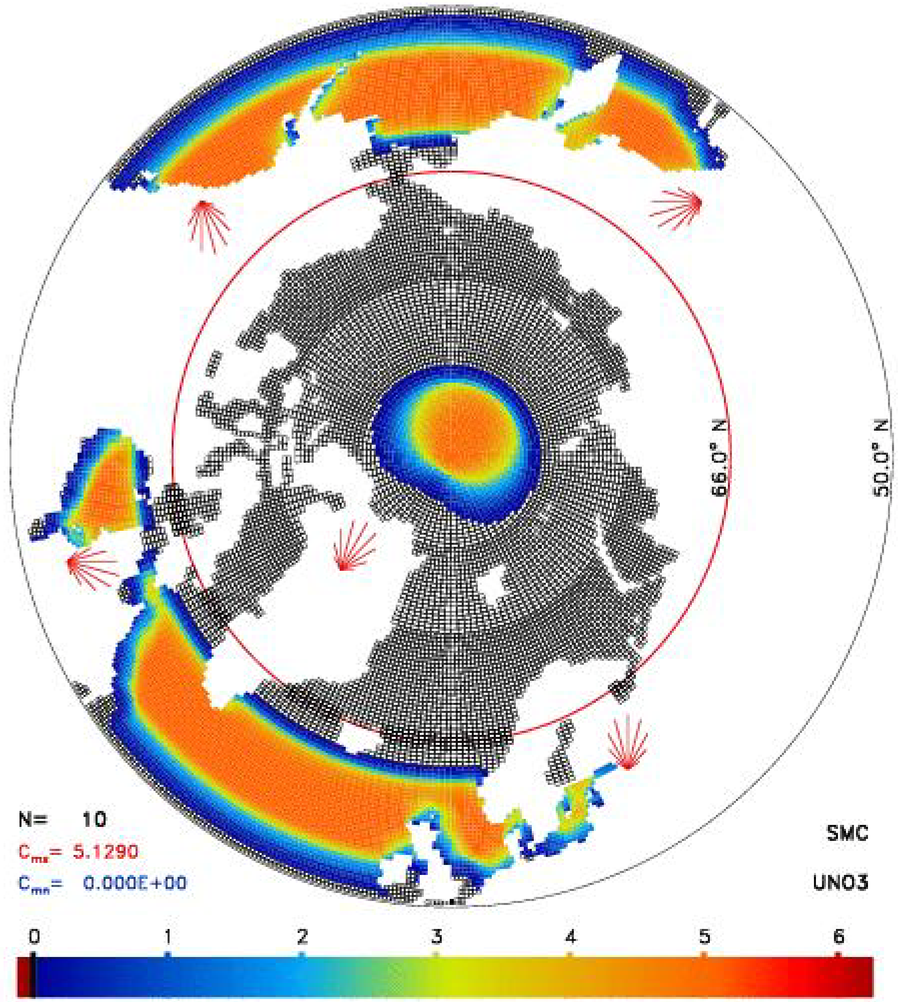




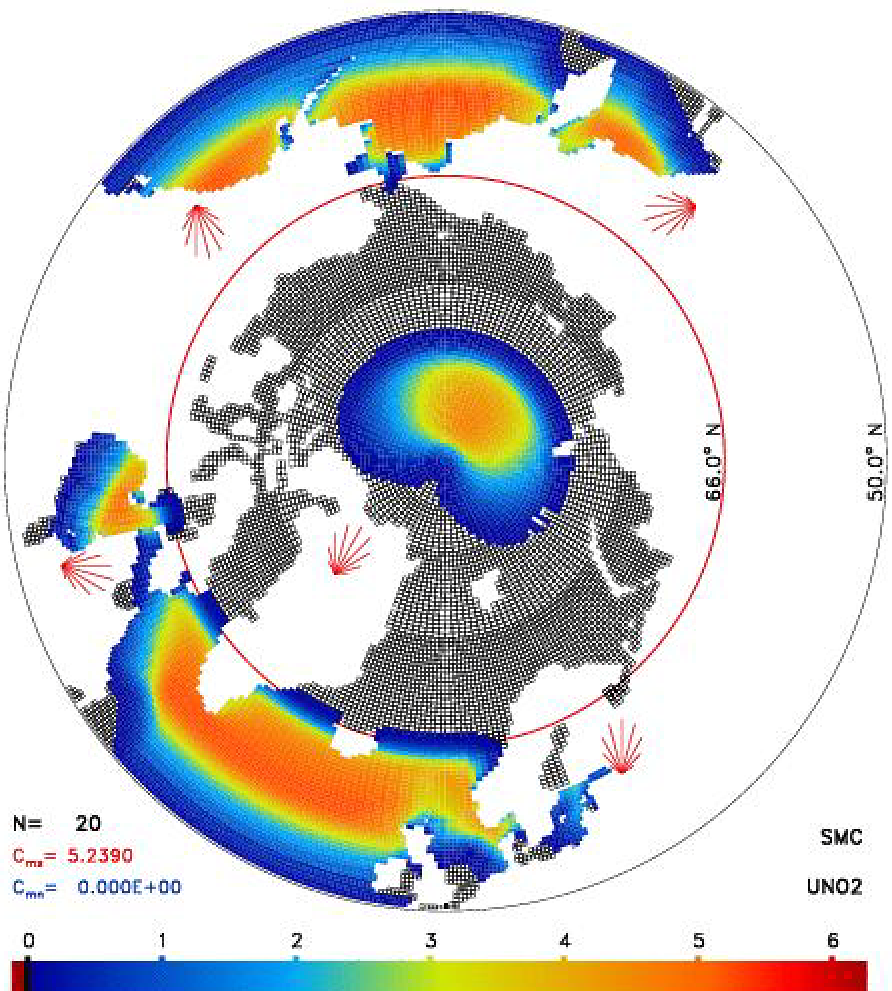
Met Office

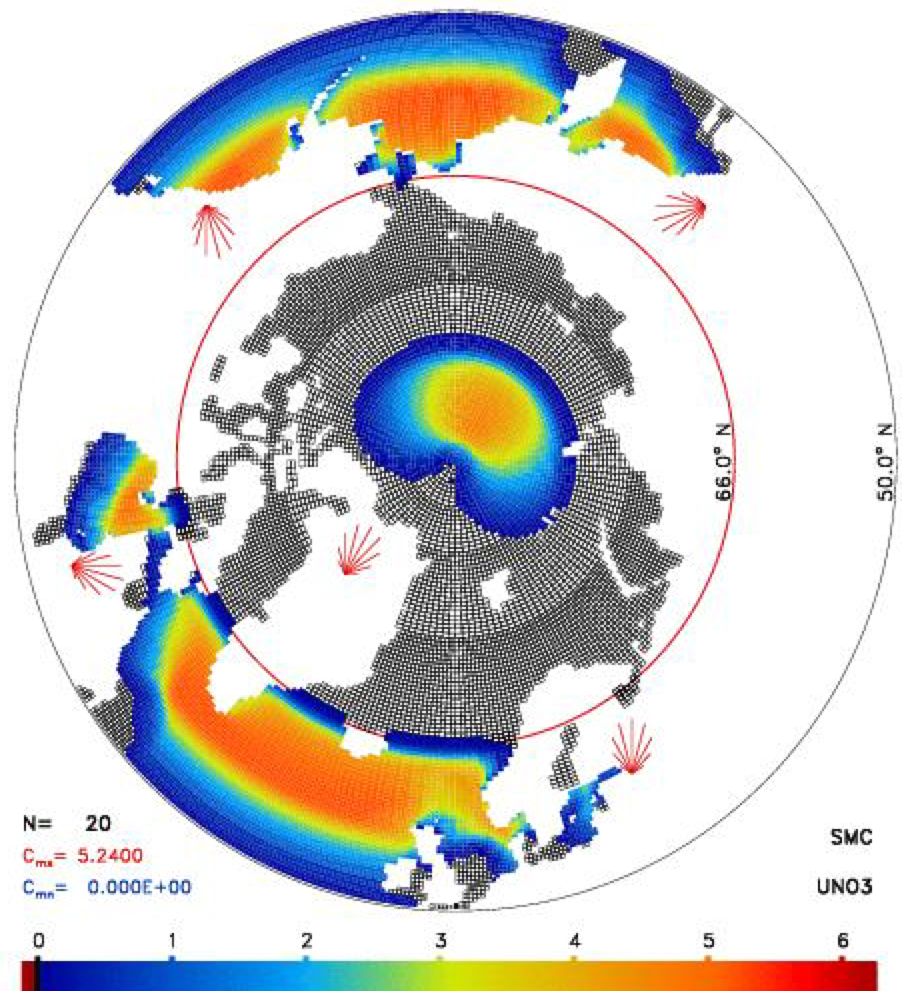
Wave spectral transport in Arctic





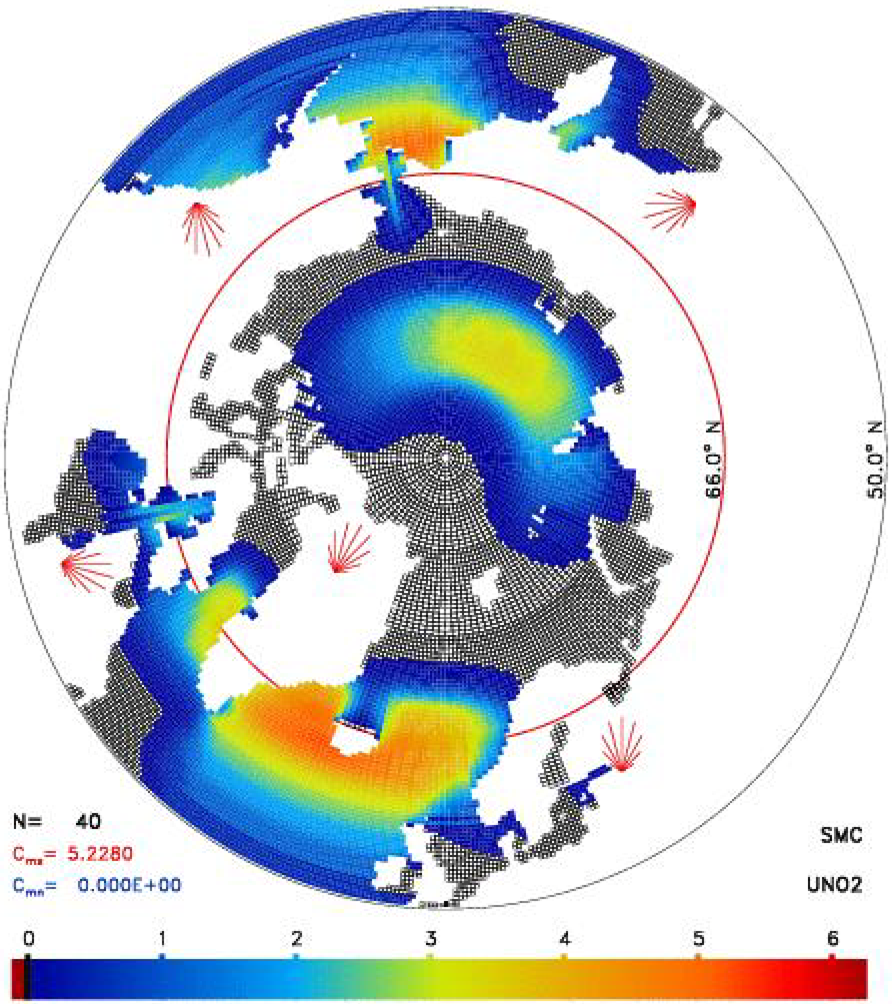
Wave spectral transport in Arctic

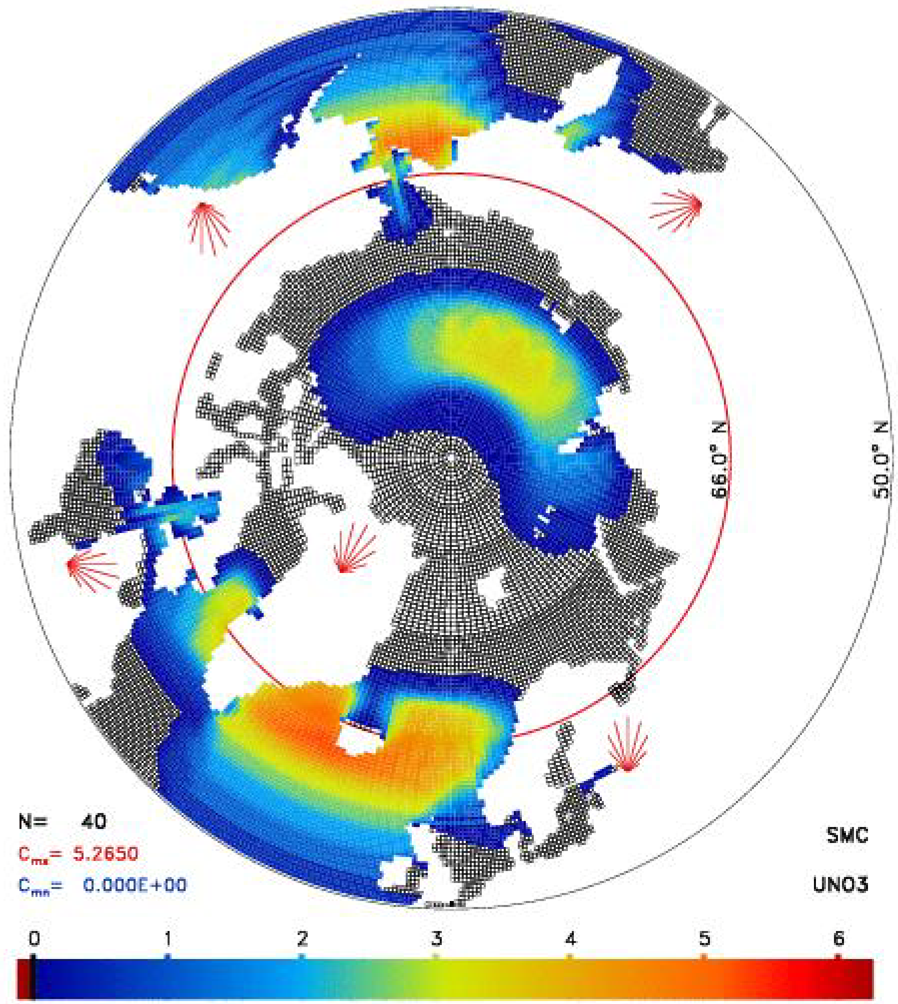




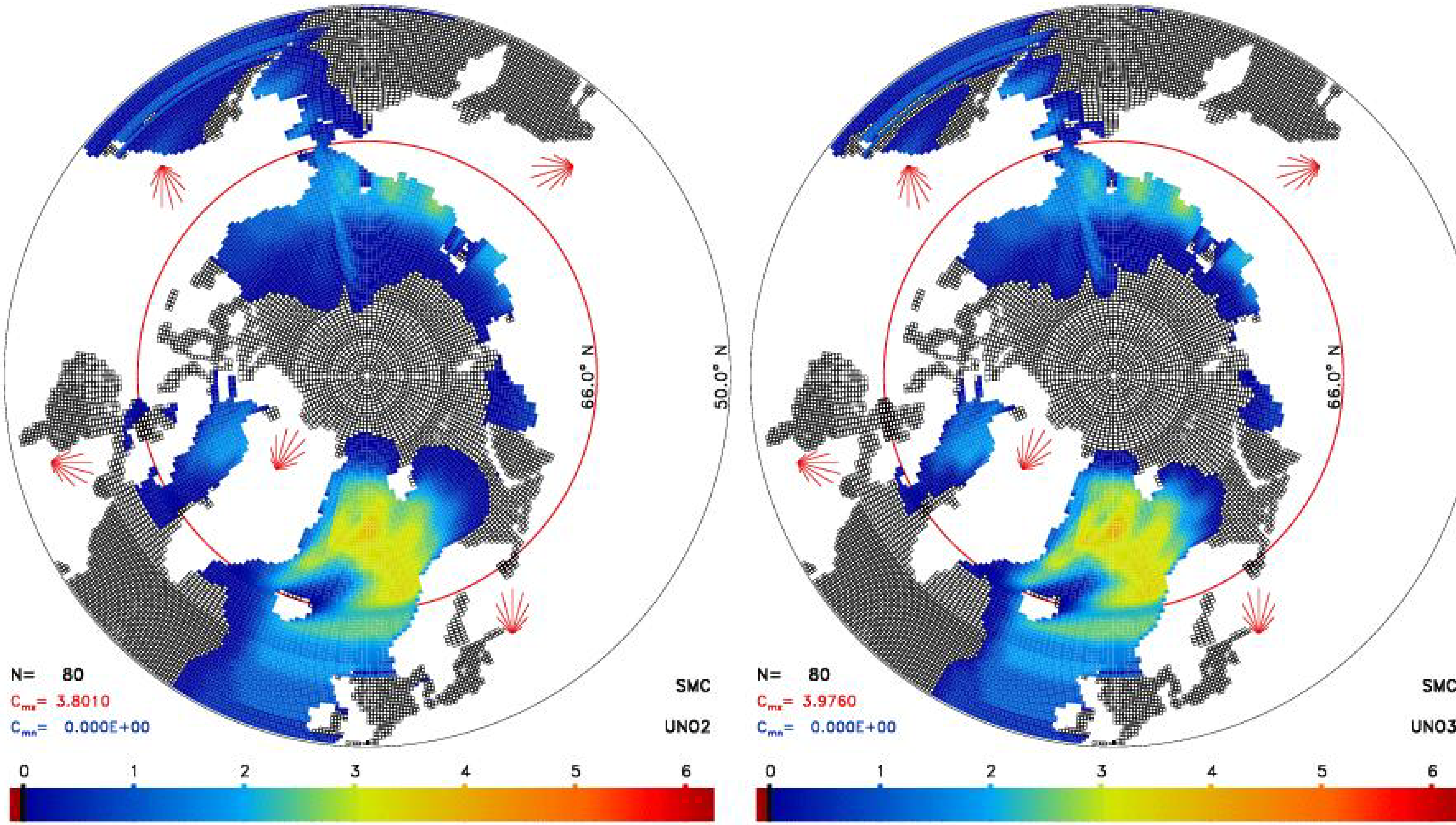
Met Office

Wave spectral transport in Arctic

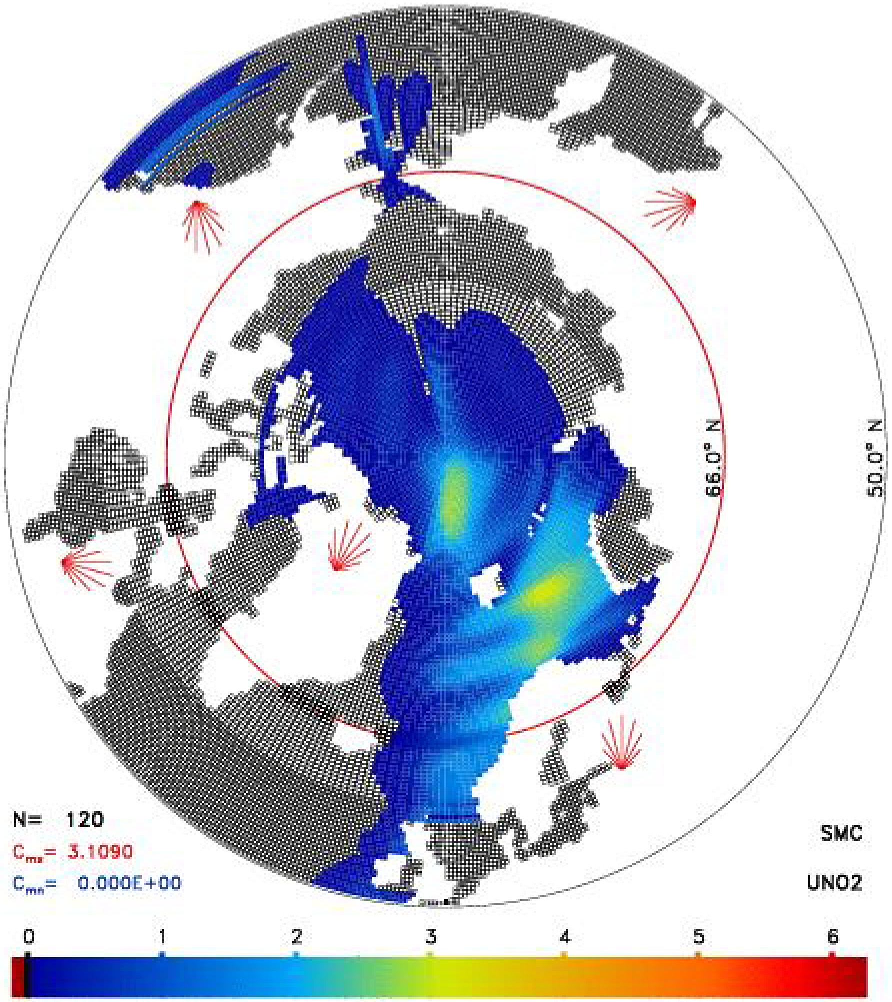


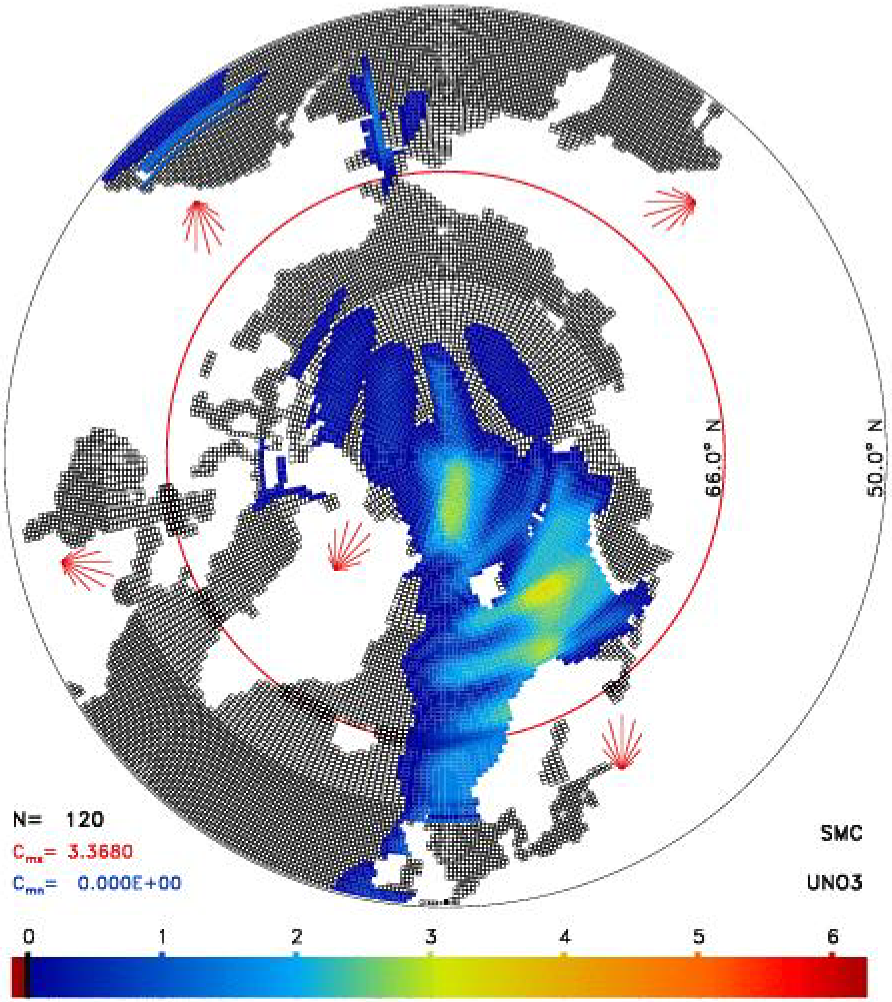


Wave spectral transport in Arctic

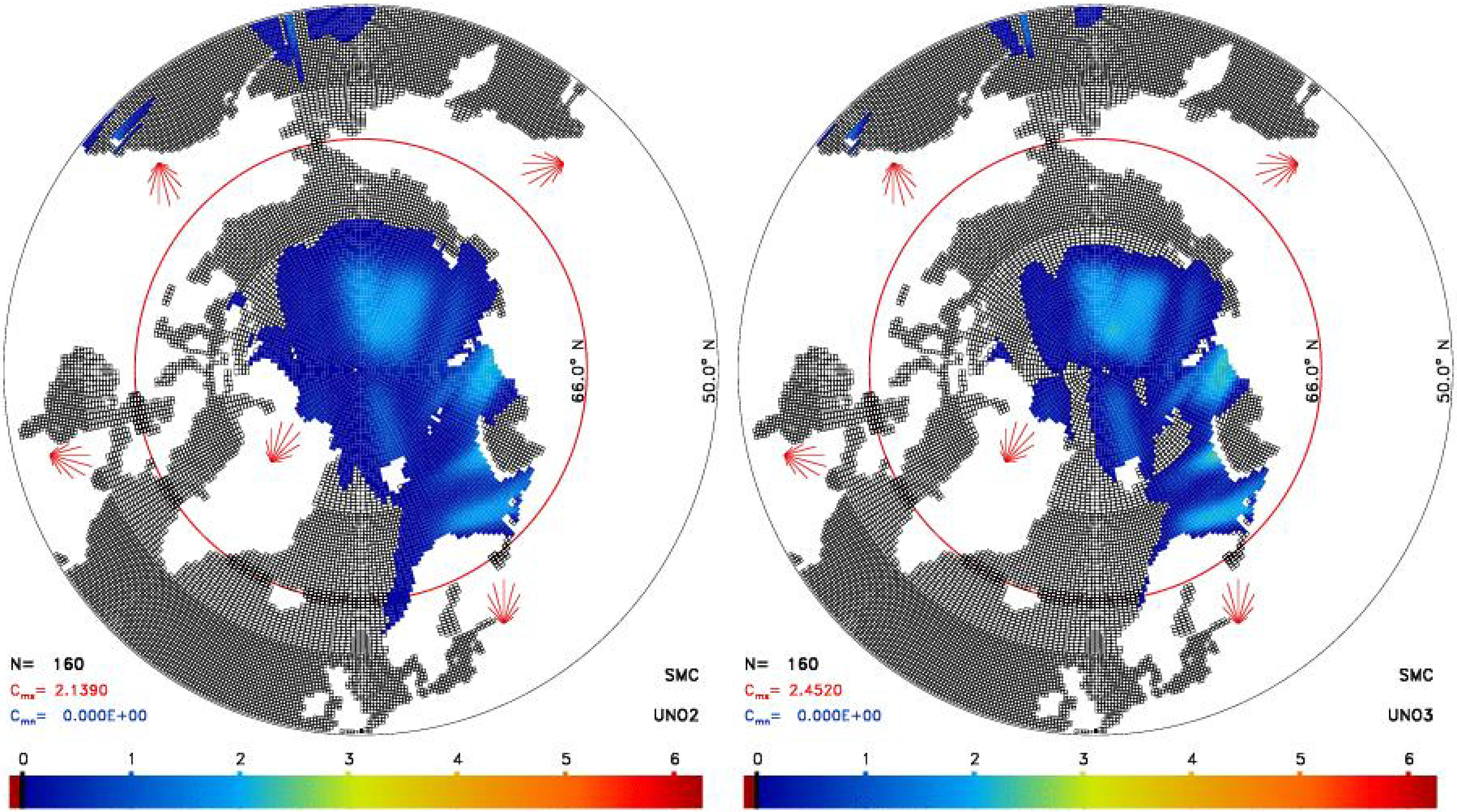


Wave spectral transport in Arctic

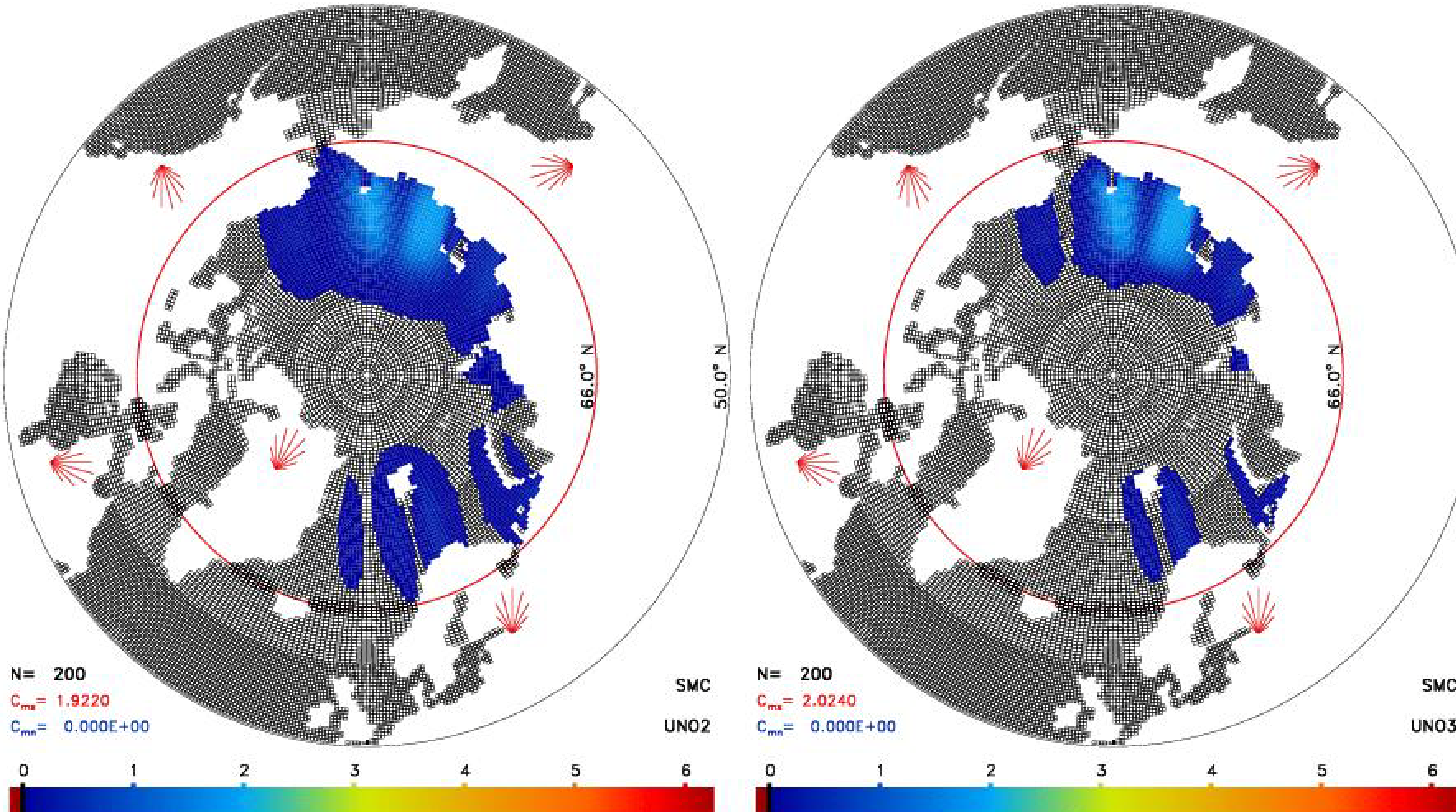




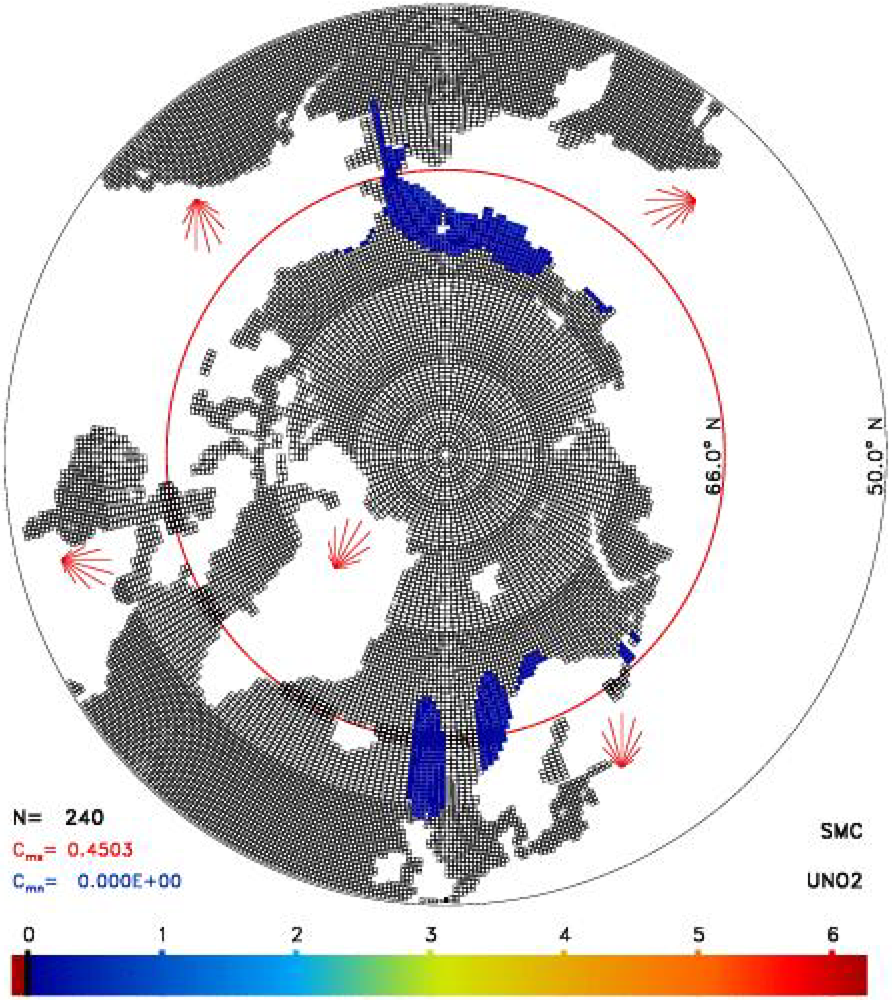
Wave spectral transport in Arctic

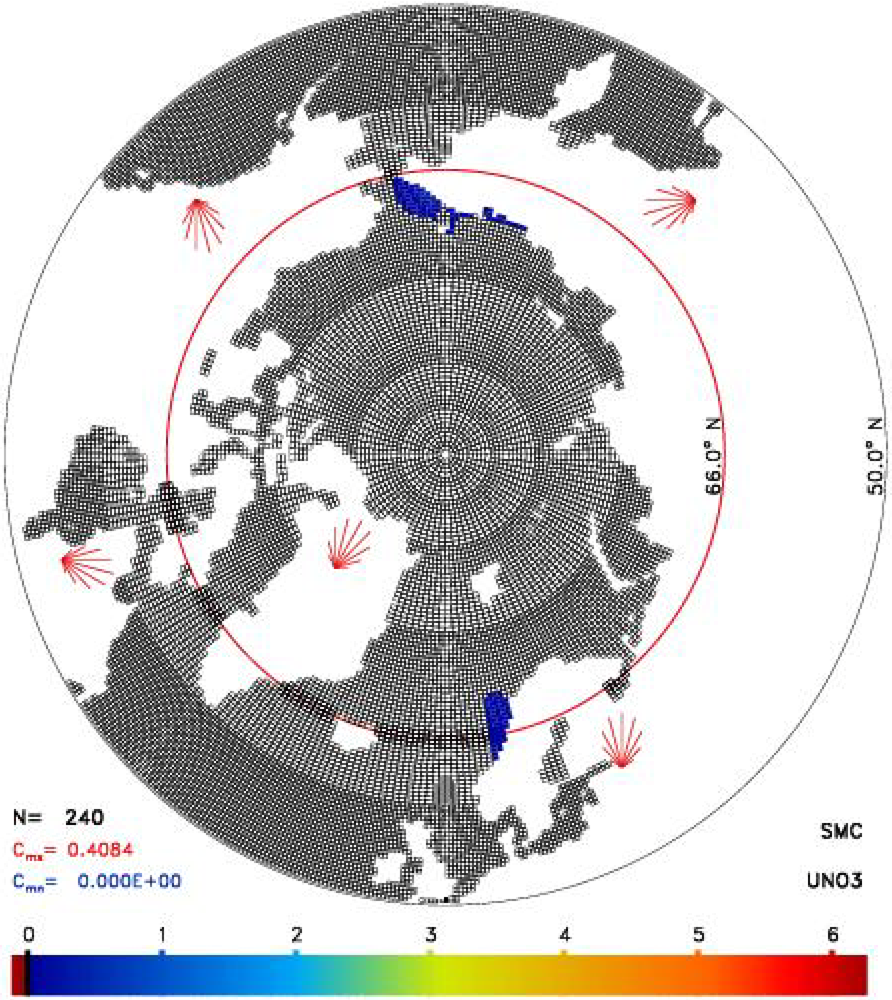


Wave spectral transport in Arctic



Wave spectral transport in Arctic



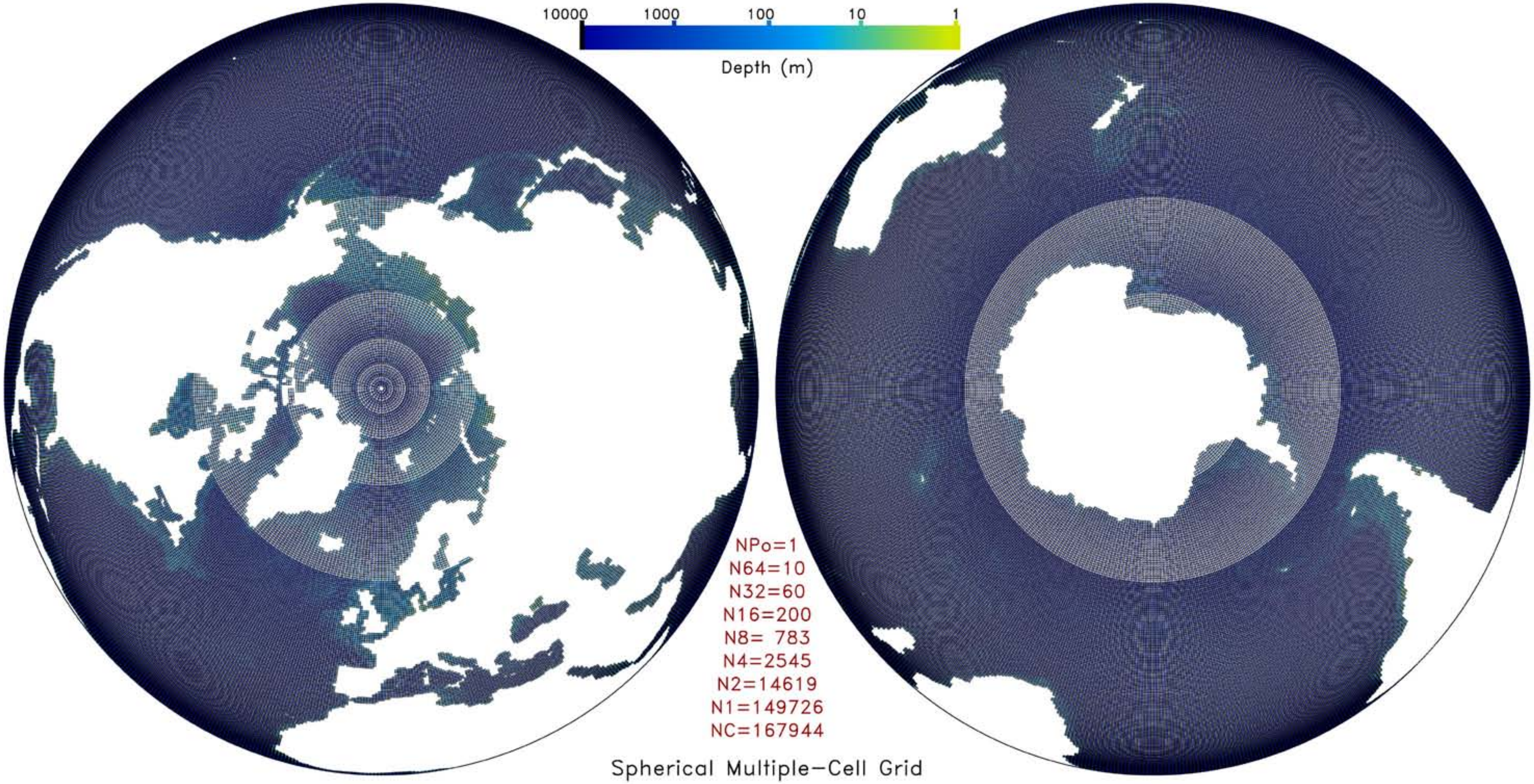


Summary and conclusions

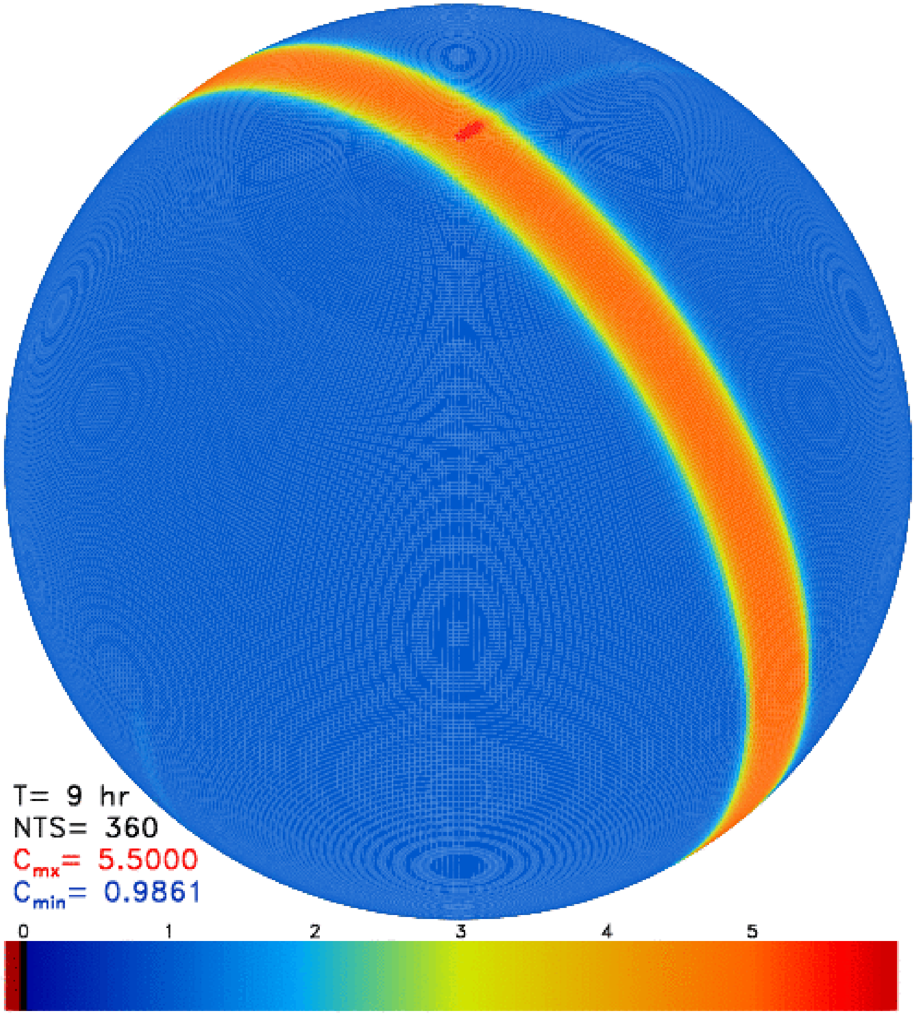
- Spherical Multiple-Cell (SMC) grid is developed to solve the polar transportation problem in global lat-lon grid for application in ocean surface wave model.
- A round polar cell is introduced with integral conservation equation to remove the Pole singularity and enable cross-pole transportation.
- Solid-body rotation tests (SSF and CB) and spherical deformation test are used to demonstrate the performance of SMC grid with Upstream Non-Oscillatory (UNO) advection schemes.
- Application of SMC grid in global ocean surface shows its unstructured flexibility and transportation efficiency, saving 45% in memory and computation.
- Numerical results confirm that SMC grid with UNO schemes are suitable for global, including cross-pole, transportation.

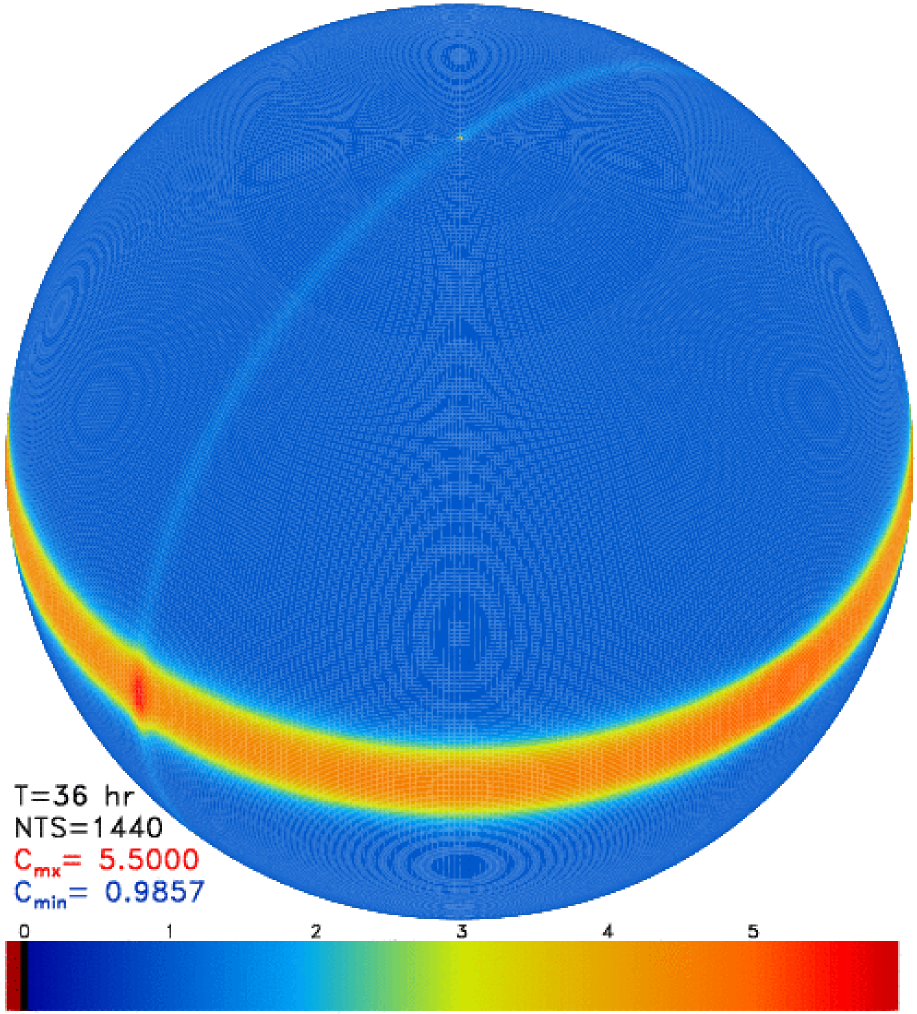
SMC Grid Polar Views --- 40 km

Regular lat-lon grid 480*640 = 307,200   SMC grid 167,944   reduction 45%

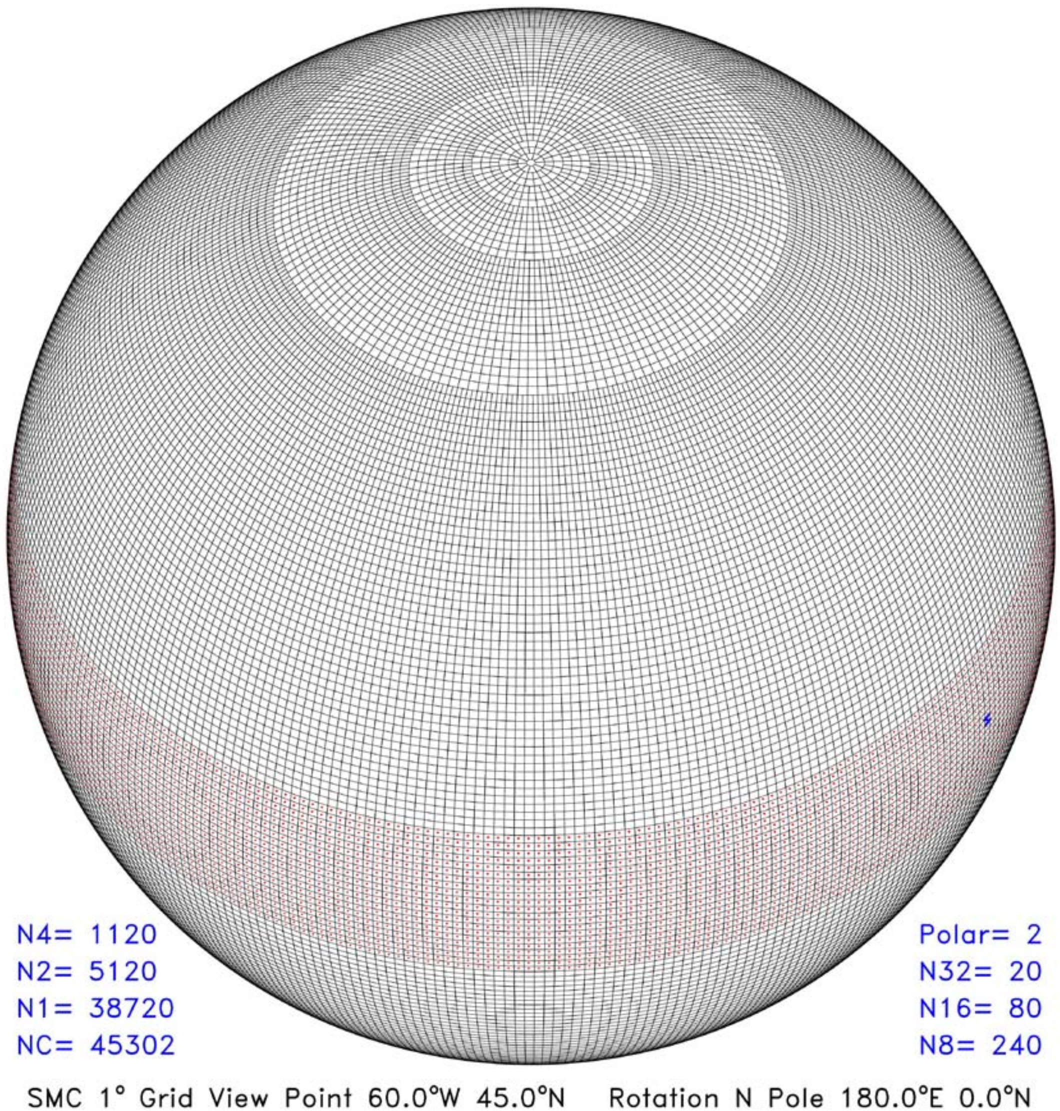


Pole blocking on reduced grid





SMC 1^o grid with polar cells



Polar singularity removed by using round polar cells.

Integration equ. replaces differential one for polar cells.

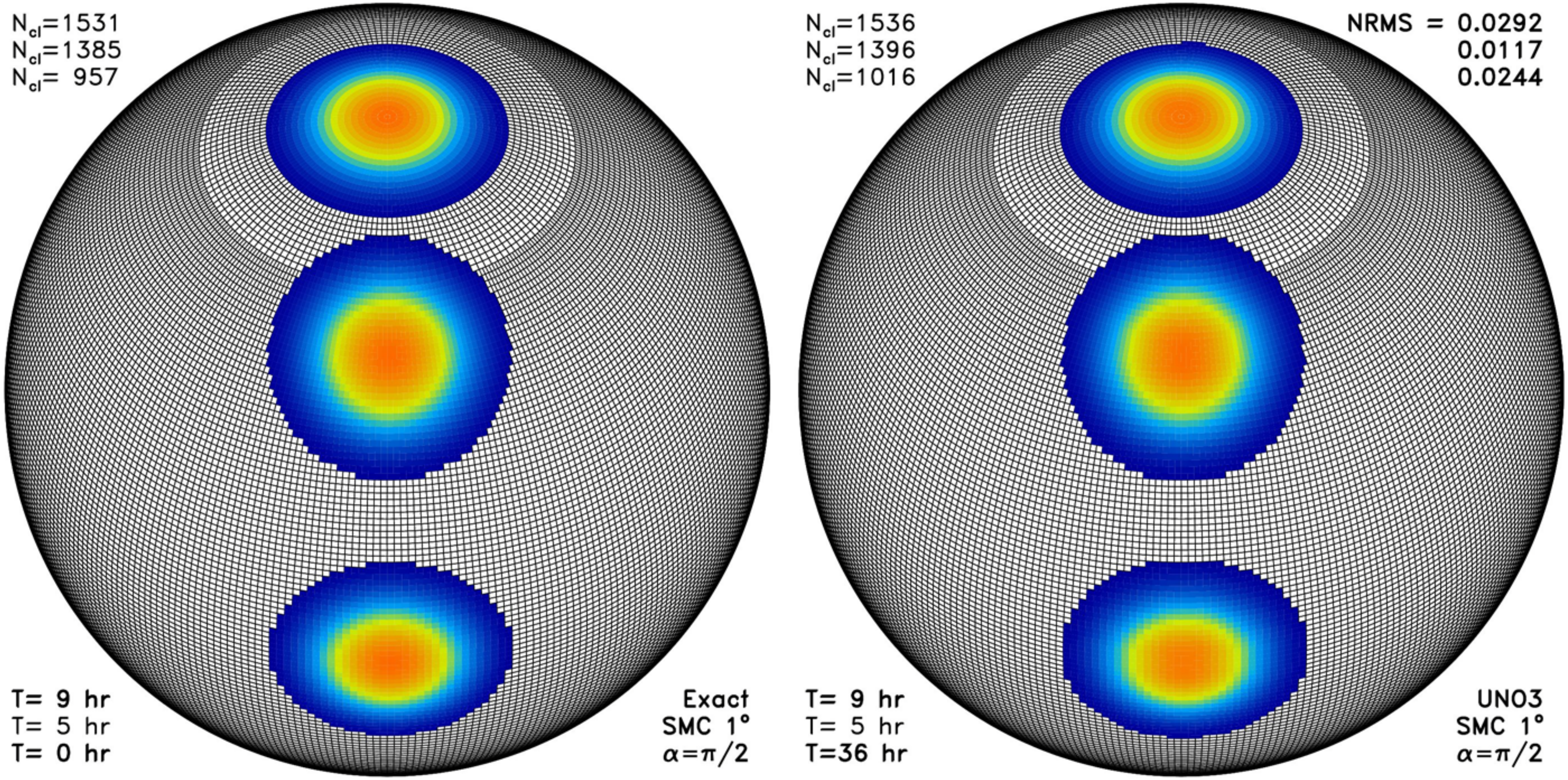
Max size-32 cells next to polar cells.

$\Delta\lambda$=1.125^o $\Delta\varphi$=1°

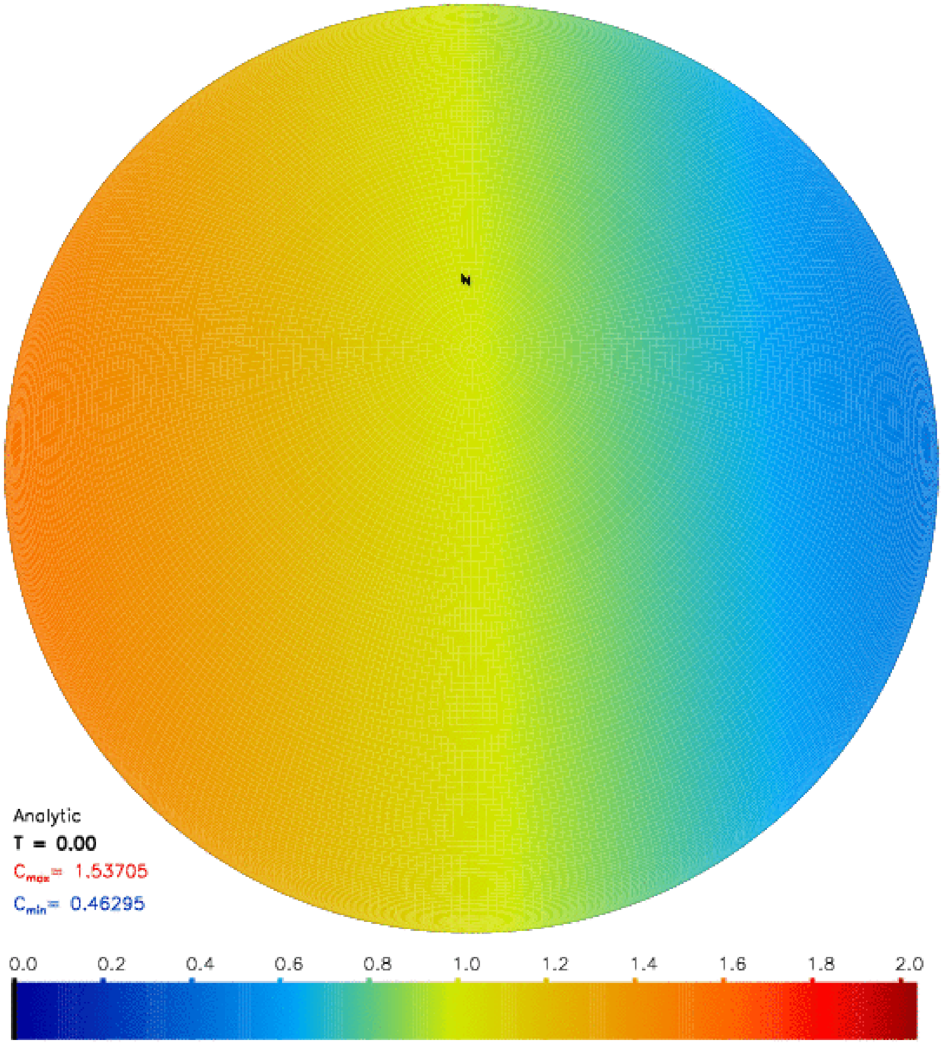
Regular 320x180=57,600

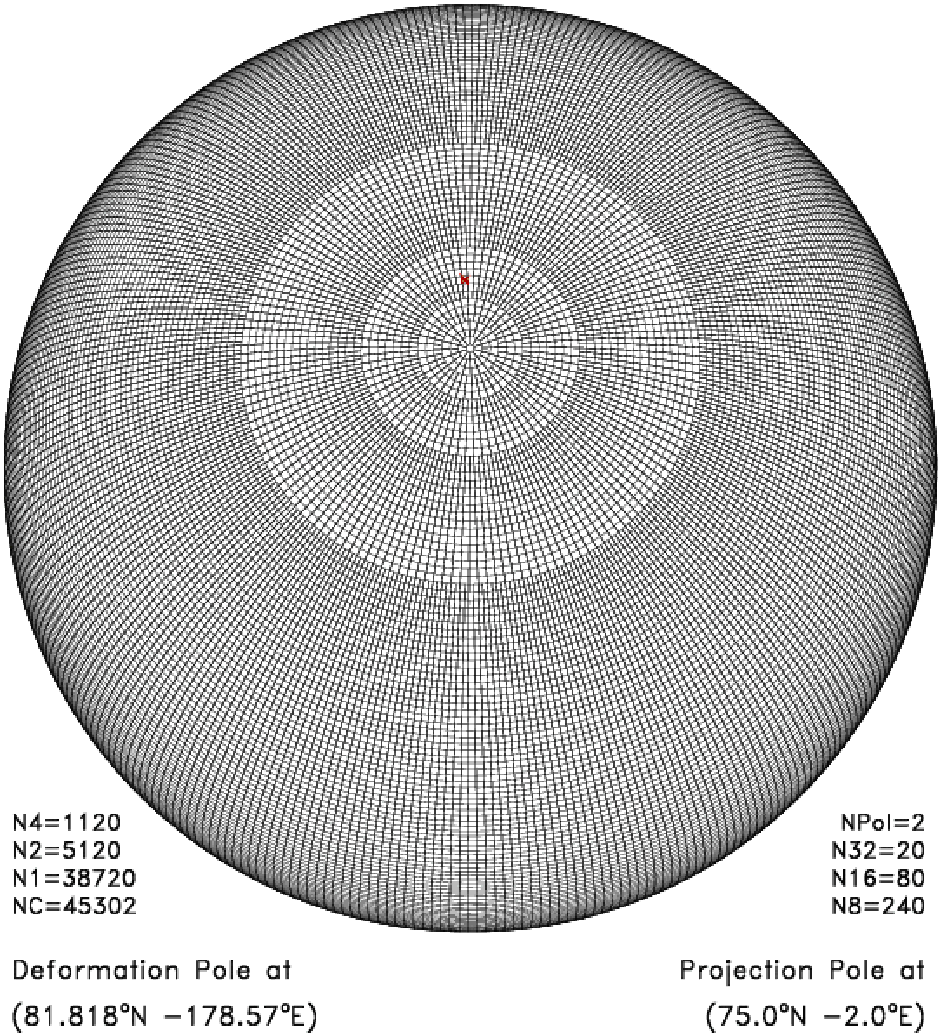
SMC 45,302 Reduced 21%

Cosine bell solid-body rotation

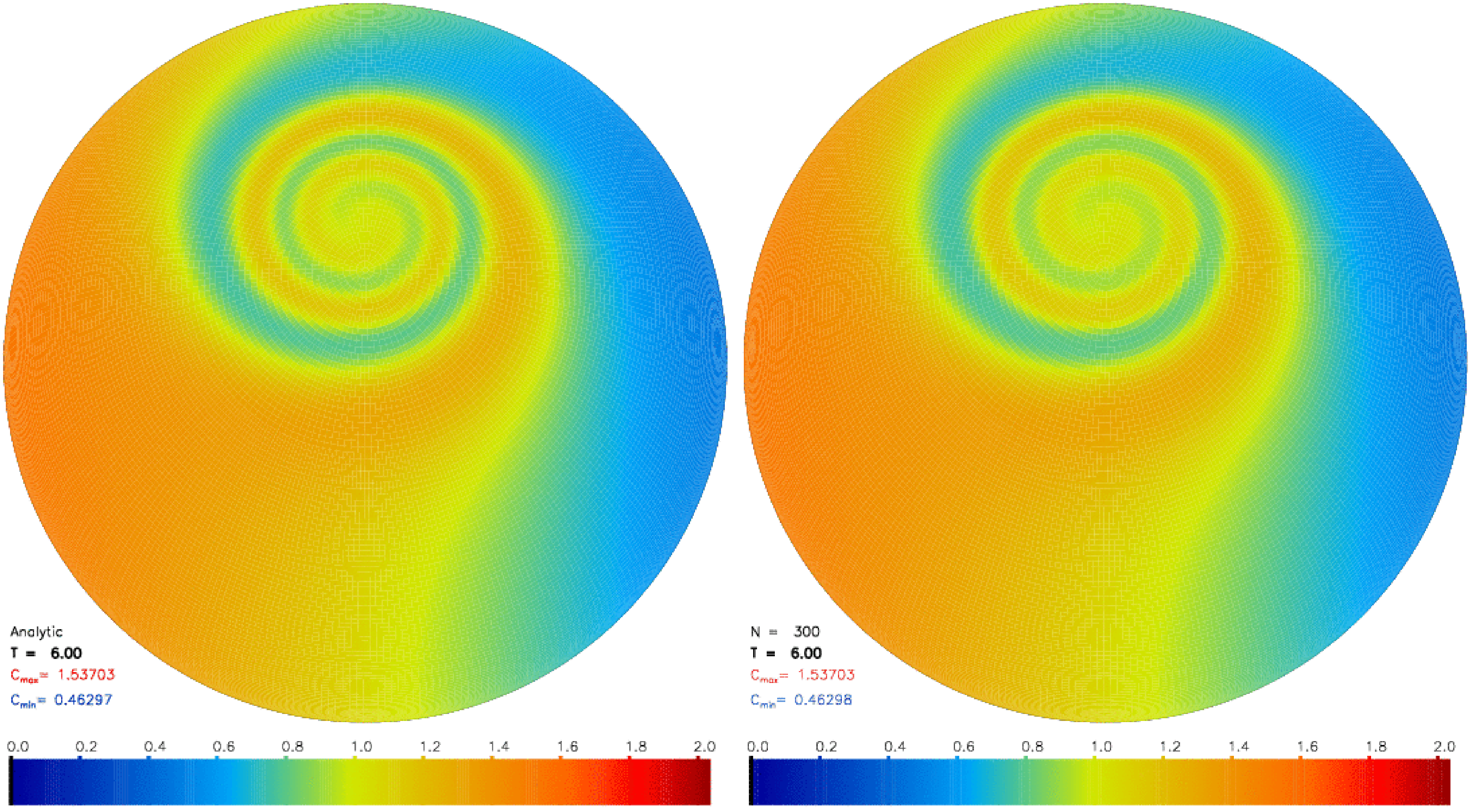


Spherical deformation flow on SMC 1º grid with UNO3 scheme

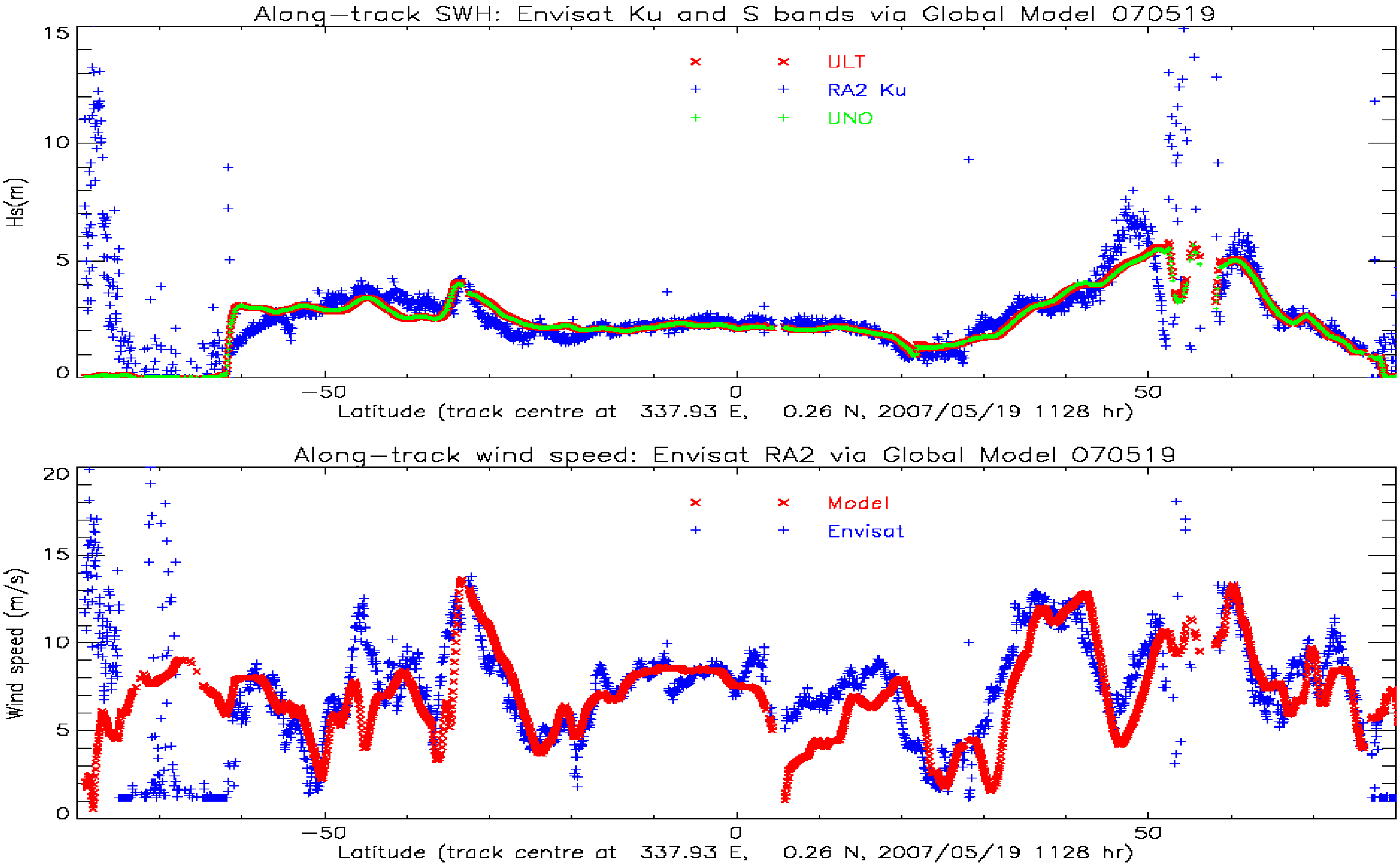




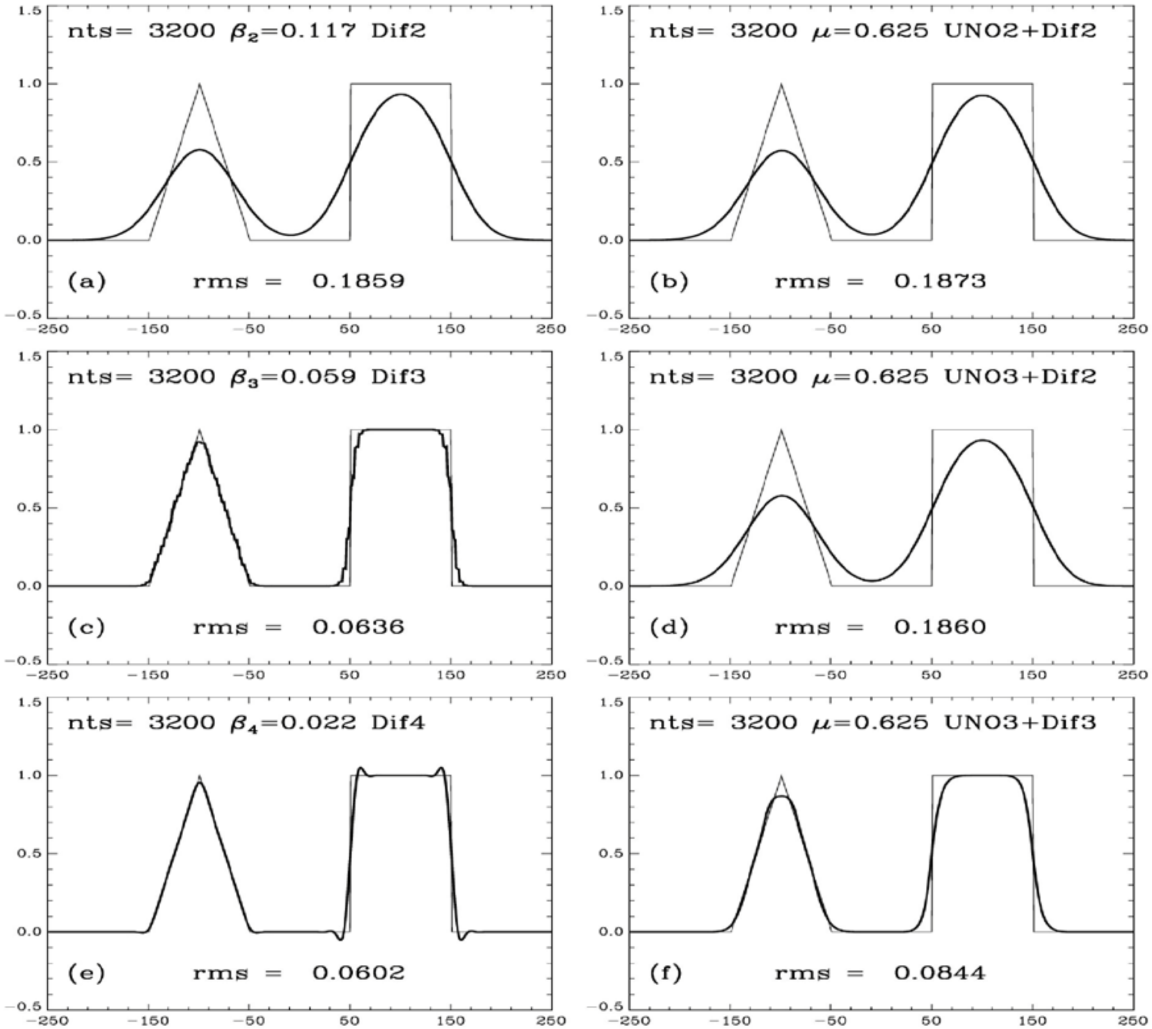
Spherical deformation flow on SMC grid with UNO3 scheme



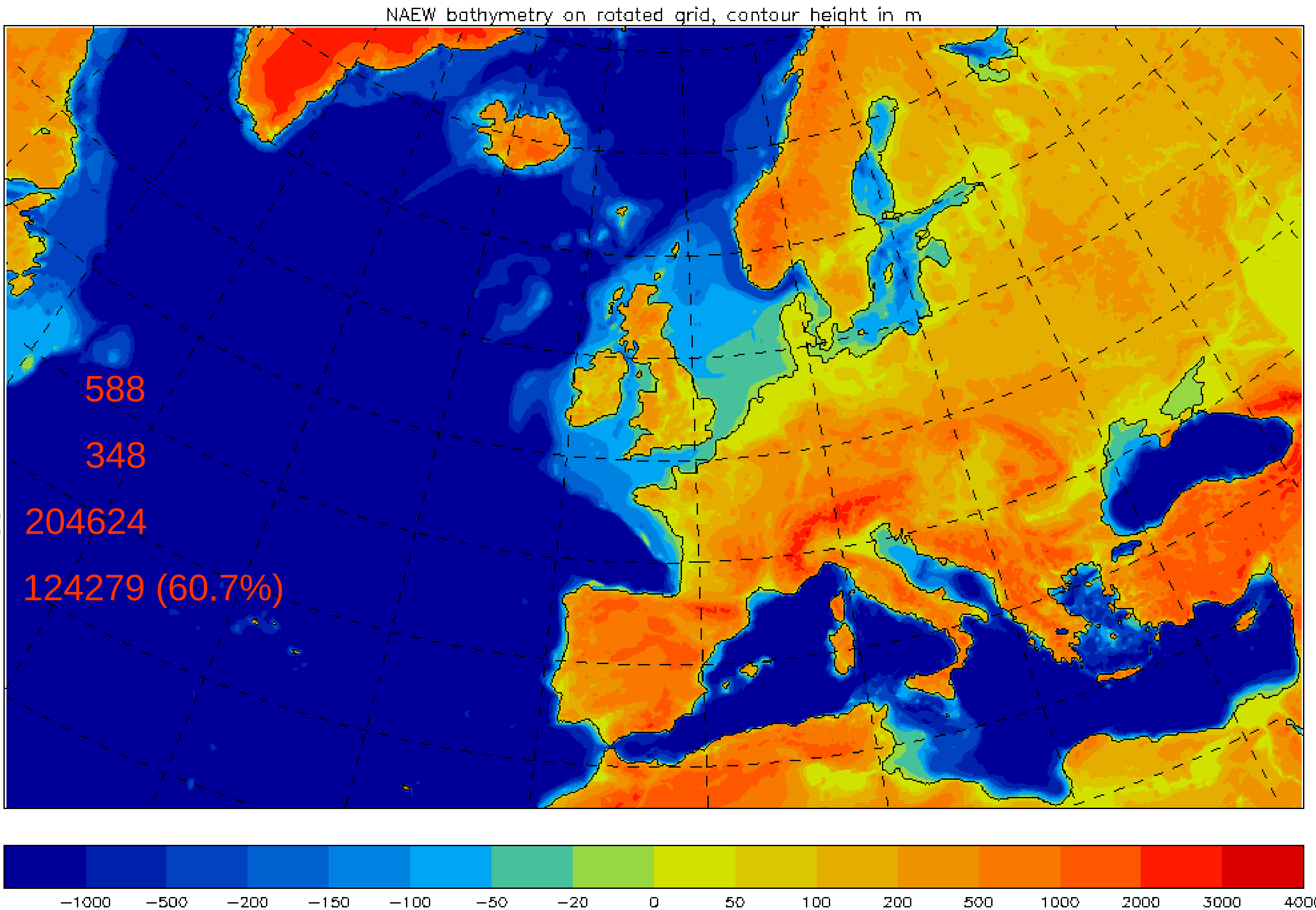
Comparison of ULTks and UNO2 in wave model



Implicit diffusion of UNO2/3



NAEWW3 Model domain



longitudes : 588

latitudes : 348

grid points : 204624

sea points : 124279 (60.7%)