Depth-Induced Breaking Characteristics for Hurricane-Generated Waves

Fumin Xu   Will Perrie

Bedford Institute of Oceanography, DFO, Canada

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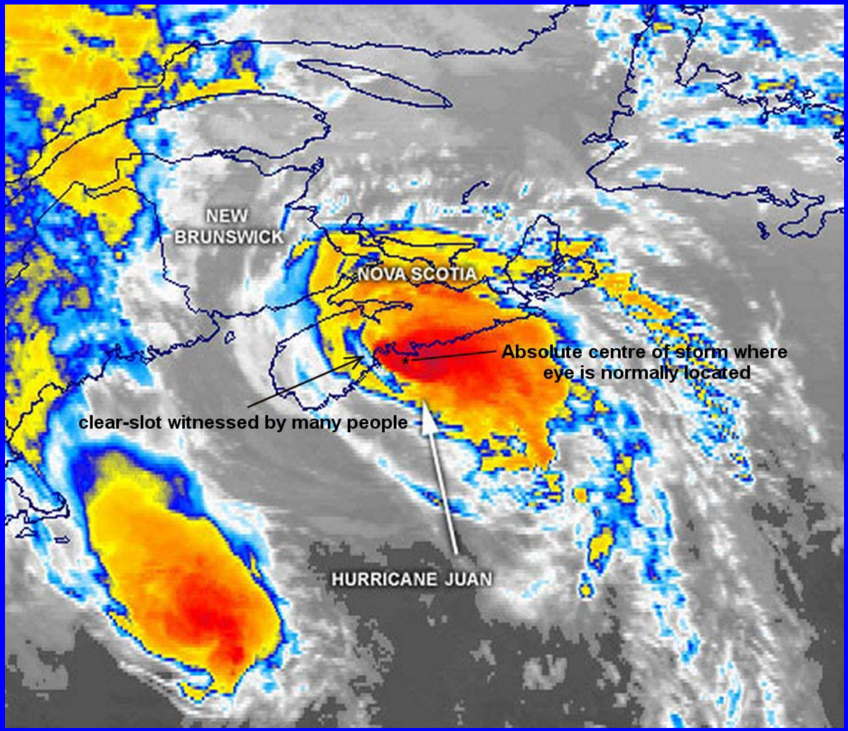
Outline

1. conclusions
2. Introduction of hurricane Juan
3. Observation buoys in Lunenburg Bay
4. A new depth-induced breaking model--based on Weibull distribution
   - 3.1 Breaking Criteria
   - 3.2 Weibull Distribution
   - 3.3 Breaking Wave Height formula based on Weibull distribution
5. Case Study-hurricane Juan generated waves in Lunenburg Bay
   - 4.1 field wind observations
   - 4.2 Analysis of observed Juan’s waves at DWR location
6. Simulation of Juan’s Waves in the bay----with the new depth-induced breaking method

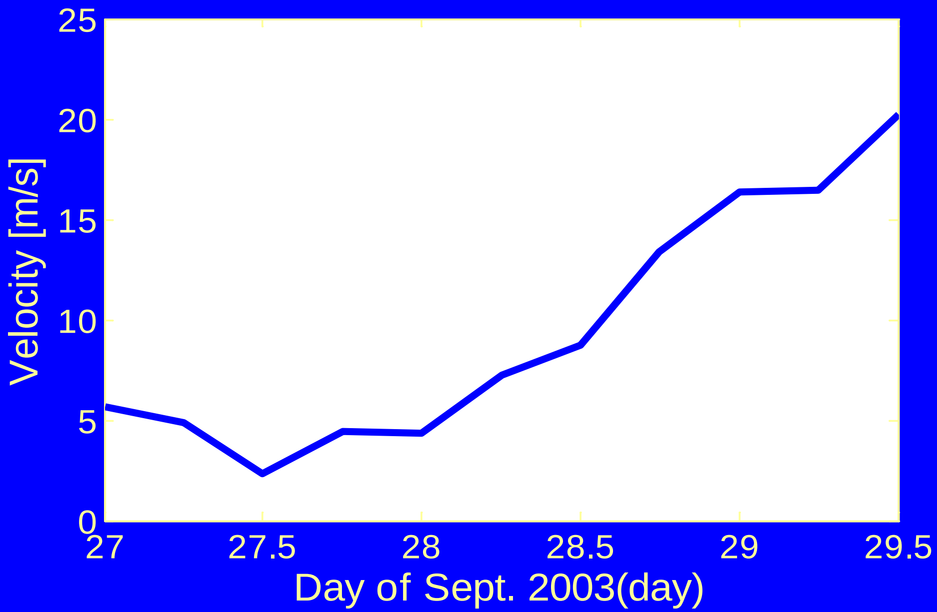
1. Conclusions

- **Swell dominates the waves** measured during hurricane Juan, which are narrow in spectral directional and frequency ranges.
- **Commonly accepted Rayleigh distribution is not** accurate description of hurricane-generated shallow water wave distributions.
- **Weibull distribution is more suitable** to describe hurricane-generated waves, and can describe narrow spectral directional, frequency ranges.
- **New depth-induced breaking model is developed**, based on Weibull distribution.
- **The new depth-induced breaking model is applied** to simulate waves generated during Juan in Lunenburg Bay; better results are obtained.

2. Introduction of Hurricane Juan



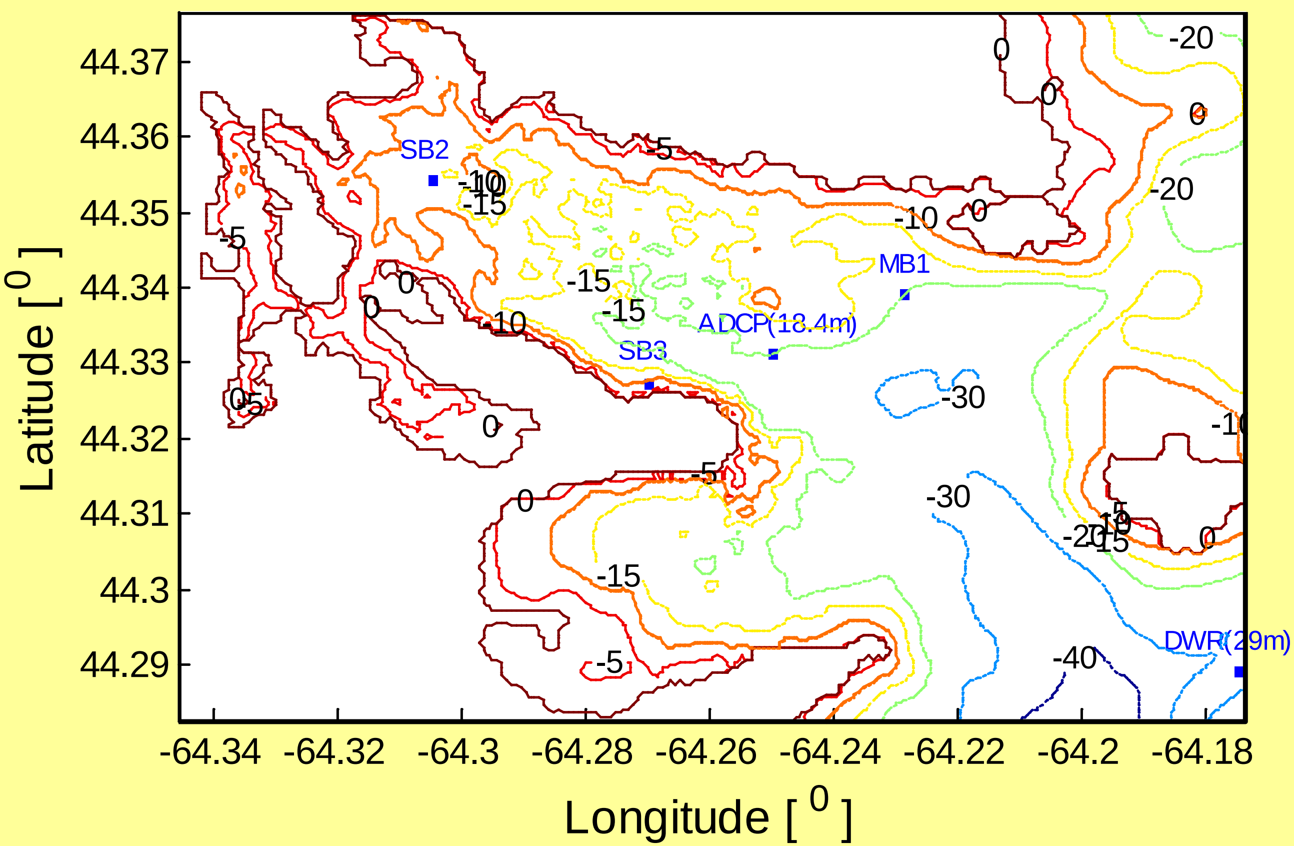
Satellite image at landfall



Juan's translation speed

- Hurricane Juan was one of the most damaging storms in the modern history of Nova Scotia.
- Juan was a rapidly propagating accelerating hurricane.

3.Observation buoys In Lunenburg Bay



DWR(64.18°W, 44.24°N) in 29m depth water

ADCP(64.25°W, 44.33°N) in 18.4m depth water

Wind observation stations: MB1, SB2 and SB3

Note: DWR was about 25 km to the left of Juan’s track

4. A New Depth-induced Breaking Model --based on Weibull distribution

4.1 Breaking Criteria

$$H_{\max} = 0.142\, L \tanh(kd)$$

$$H_b / d = 0.78$$

$$H_{\max} = \gamma d \qquad \gamma \in (0.6, 1.59)$$

4.2 Weibull Distribution

$$f\left(\frac{H}{\overline{H}}\right) = ab\left(\frac{H}{\overline{H}}\right)^{b-1} \exp\left[-a\left(\frac{H}{\overline{H}}\right)^{b}\right]$$

In Which $$a = \left[\frac{1}{\Gamma\left(1+\frac{1}{b}\right)}\right]^{b}$$

- When $a = 0.5$ and $b = 2$, Weibull distribution is Rayleigh distribution.

- With the increase of parameter b, the distribution will become narrower and sharper.

4.3 Breaking Wave Height Formula Based on Weibull Distribution

Definition and assumption

Define: H_m maximum possible wave heights for each water depth

Assume: the heights of all the waves which are breaking at these points are equal to the respective H_m

$f(H)$: wave height distribution

Q_b : breaking wave probability

A characteristic wave height H_b : $$H_b = \left[\int_0^\infty H^b df(H) \right]^{1/b}$$

Then: $Q_b = P_r(H = H_m)$ We can deduce: $\ln Q_b = -a\left(\frac{H_m}{\overline{H}}\right)^b$

$$\overline{H}^{\,b} = -a\frac{H_m^{\;b}}{\ln Q_b} \quad \text{---(1)}$$

$$\left(\frac{H_b}{\overline{H}}\right) = \left\{\int_0^{\infty}\left(\frac{H}{\overline{H}}\right)^b ab\left(\frac{H}{\overline{H}}\right)^{b-1}\exp\left[-a\left(\frac{H}{\overline{H}}\right)^b\right]d\left(\frac{H}{\overline{H}}\right)\right\}^{1/b}$$

Thus: $$\left(\frac{H_b}{\overline{H}}\right)^b = \left\{\int_0^{H_m}\left(\frac{H}{\overline{H}}\right)^b ab\left(\frac{H}{\overline{H}}\right)^{b-1}\exp\left[-a\left(\frac{H}{\overline{H}}\right)^b\right]d\left(\frac{H}{\overline{H}}\right)\right\} \quad \text{----------(2)}$$

From (1) and (2), we can get: $$\left(\frac{H_b}{H_m}\right)^b = -\frac{1-Q_b+Q_b\ln Q_b}{\ln Q_b}$$

We know: $$H_{rms} = \left[\int_0^\infty H^2 df(H) \right]^{1/2}$$

Suppose that: $$H_b \approx H_{rms}$$

Then we have: $$\left(\frac{H_{rms}}{H_m} \right)^b = -\frac{1 - Q_b + Q_b \ln Q_b}{\ln Q_b}$$

This transcendental equation **means that--** Q_b can be solved as a function of H_{rms}/H_m

SWAN model's breaking method (which based on Rayleigh distribution of Hs): $$\left(\frac{H_{rms}}{H_m} \right)^2 = -\frac{1 - Q_b}{\ln Q_b}$$

H_m Values under different b and Q_b

(1) with nonlinear term $Q_b \ln Q_b$ (2) without nonlinear term $Q_b \ln Q_b$

Q		0.02	0.05	0.1	0.2	0.3	0.5	0.7	0.8
b=6	(1)	12.53	12.84	13.02	13.07	12.95	12.44	11.54	10.83
	(2)	12.70	13.21	13.68	14.24	14.62	15.15	15.55	15.71
b=2	(1)	7.68	8.27	8.63	8.72	8.49	7.52	6.01	4.96
	(2)	8.00	9.01	10.00	11.28	12.20	13.59	14.67	15.14

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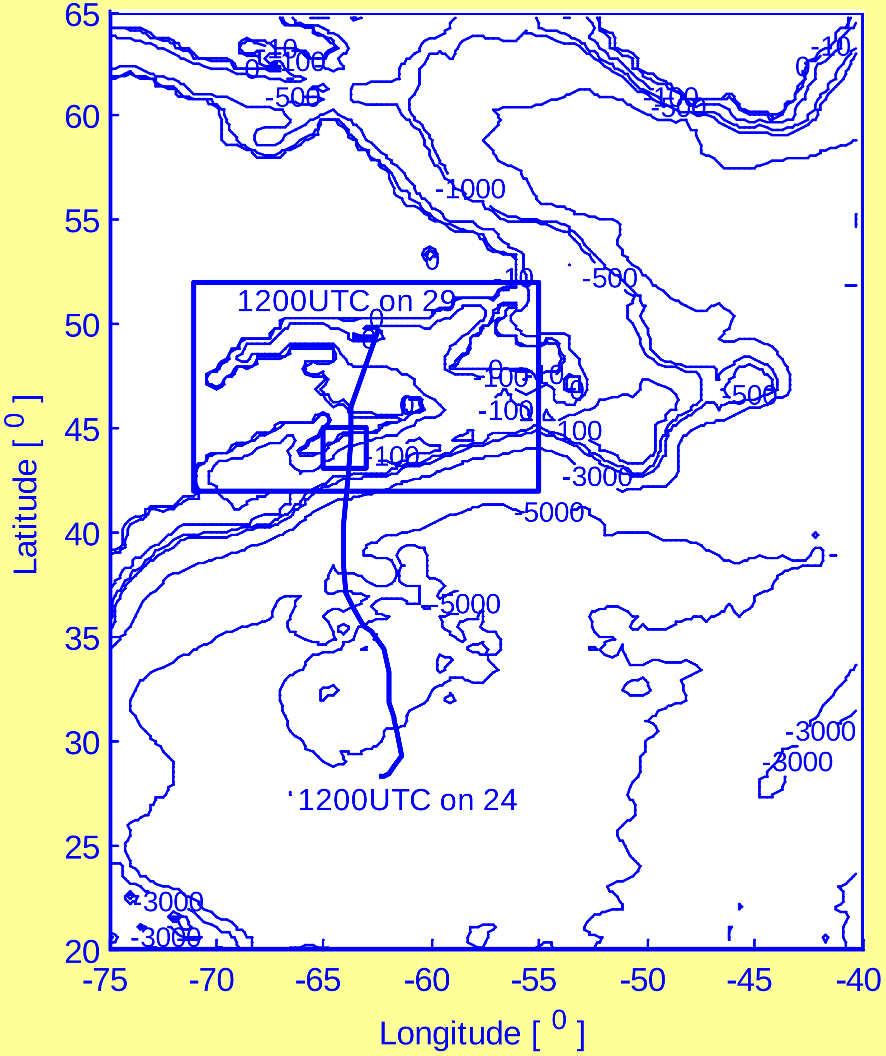
5. Case Study -- Hurricane Juan Generated waves in Lunenburg Bay



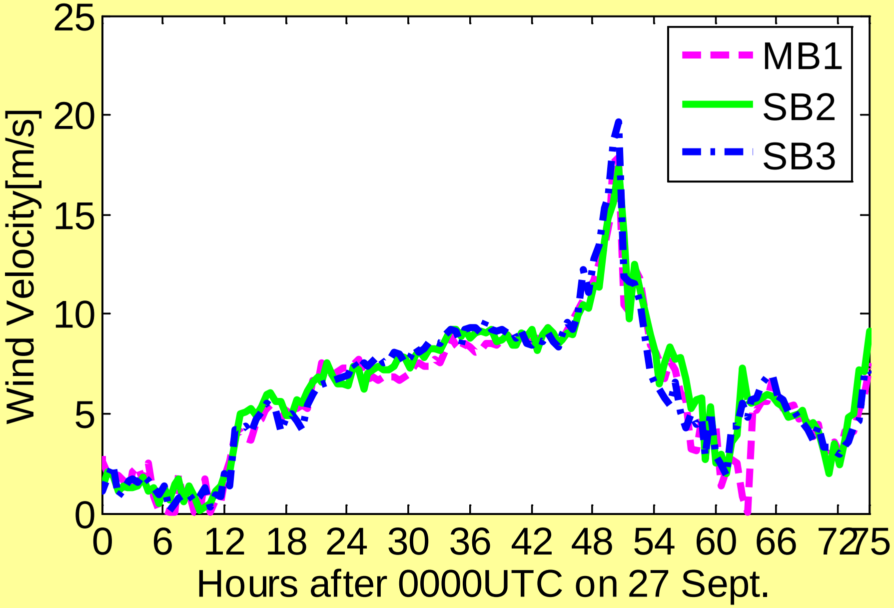
Juan's track, numerical simulation domains and topography

(1) Juan reached hurricane strength at 1200 UTC 26 Sept. near Bermuda;

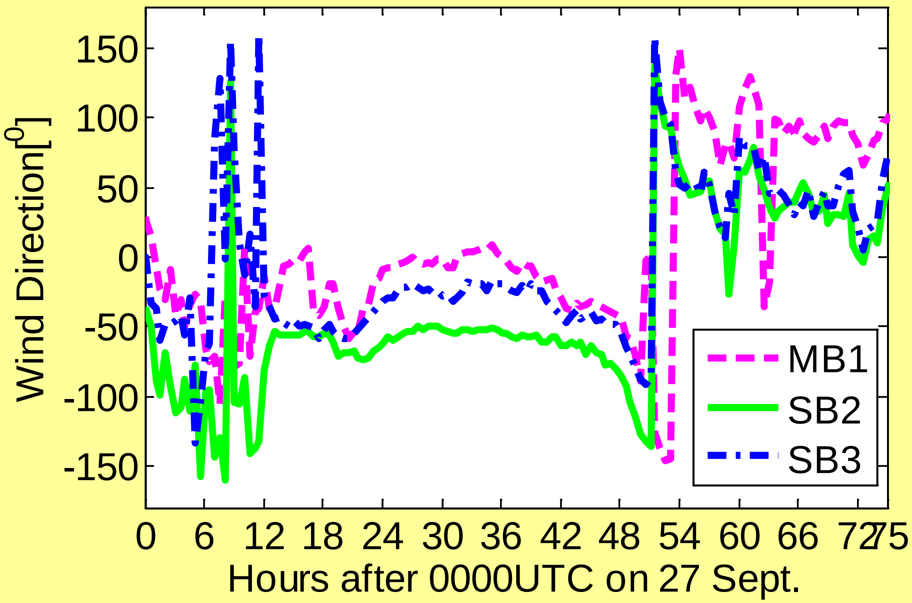
(2) maximum winds were 90 knots at 1800 UTC 27 Sept. at north of the Gulf Stream.

(3) Juan made landfall near Halifax (0300 UTC 29 Sept.), with winds of 85 knots.

5.1 Field Wind Observations



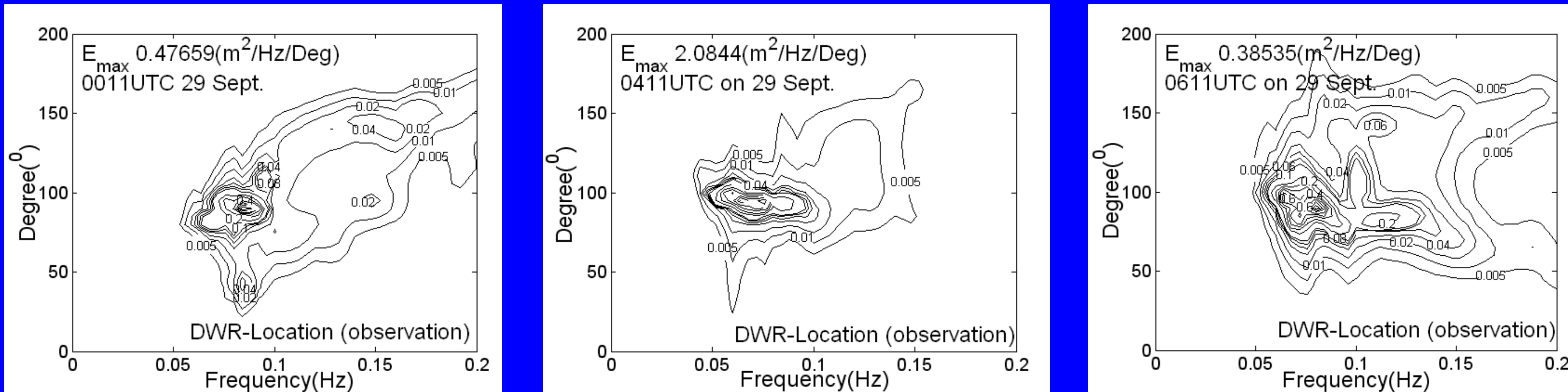
Wind velocity



Wind direction

Observed winds at the SB2, SB3 and MB1 stations

5.2 Observed Juan's Waves at DWR Location



observed 2D wave spectra at DWR during peak waves

We can see:

- Juan generated waves are extremely narrow in directional and frequency ranges.
- Swell dominates.
- The waves there can't be seen as random and ergodic → wave height distribution of Juan's waves are different from the commonly accepted Rayleigh distribution.

Thus: we use the new breaking model, which based on Weibull distribution, and can describe wider range of distributions.

6. Simulation of waves in the Bay

(1) <u>A nested four-domain simulation</u>

- coarse-resolution domain (75°W - 40°W, 20°N - 65°N)
- nested intermediate-res. domain (71°W - 55°W, 42°N- 52°N)
- 3rd nested fine-res. domain (65°W - 63°W, 43°N- 45°N)
- final very fine–res. nested Mahone Bay domain
  - (64.44°W ~ 64.05°W, 44.24°N ~ 44.60°N)

<u>The model nesting relationship:</u> WW3 $\Rightarrow$ WW3 $\Rightarrow$ SWAN $\Rightarrow$ SWAN

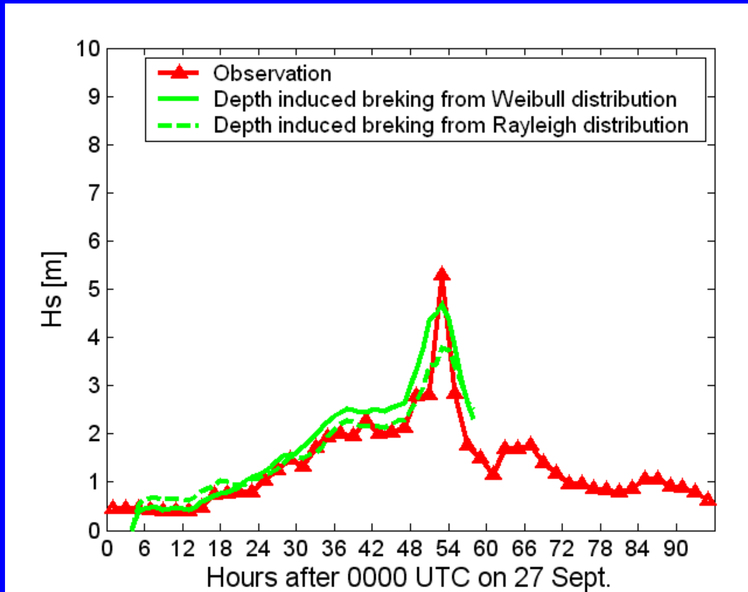
→ to get 2D wave spectra at open boundary points of Lunenburg Bay

<u>(2) Observed ½ - hourly 2D spectra at DWR</u>, combined with nesting

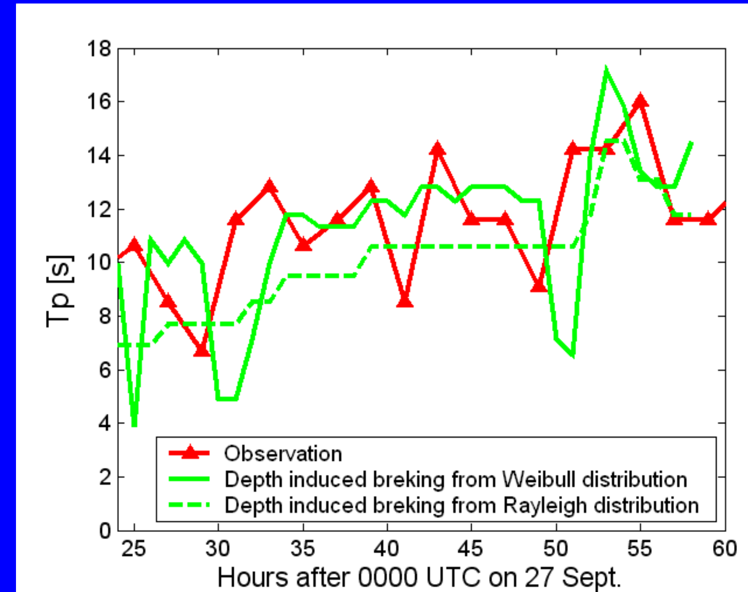
→ to get the 2D input spectra along open boundaries of Lunenburg Bay

→ ≈ maybe a quasi-observed input 2D boundary condition!

<u>(3) Replace the depth-induced breaking model</u> in SWAN with the breaking model based on Weibull distribution.



Significant wave height



Peak period

→ comparisons between simulations and observation at ADCP, using breaking models: (NEW) based on Weibull, and (OLD) based on Rayleigh

- **Results from the new depth-induced breaking** simulation are notably improved, especially peak period values

→ suggests the dominant swell is therefore simulated reasonably.

- Simulated waves are lower than observations during peak waves, and a little higher than observations before and after peak

→ **Reason**: Weibull parameter set as fixed value 6

Concluding remarks

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Thank you!