Progress in the neural network parameterisation of the mapping of wave–spectra onto nonlinear interaction source terms

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1 Introduction

The further development of operational wave–models requires a replacement of the approximative methods for the calculation of the nonlinear wave-wave interaction by fast and accurate methods. Attempts have been made with extensions of the well known discrete interaction approximation, [Hasselmann et al., 1985], but they are so far not generally applicable since tuning for different classes of wave spectra is required. The same problem occurs for methods based on diffusion operators. For a comprehensive discussion see [Cavaleri et al., 2007].

This paper describes a neural net (NN) parametrisation of the mapping between wave–spectra and the corresponding nonlinear wave–wave interaction. The idea of applying neural networks in this context was first introduced by [Krasnopolsky et al., 2001], [Krasnopolsky et al., 2002]. There the wave spectra as well as the nonlinear interaction source terms are assumed to be separable functions of frequency and direction which are approximated by expansion series, respectively. The neural network is used to map the two sets of expansion coefficients. This assumption was dropped in a successive work by [Tolman et al., 2005] where the authors used two–dimensional Empirical Orthogonal Functions for the expansion of single peaked spectra.

In contrast to this approach, we demonstrated the feasibility of the direct mapping of wave spectra onto the nonlinear interaction source term (see [Wahle et al., 2009]). Additionally, no assumptions (such as separability or convergence of an expansion series) about the wave spectra and nonlinear interaction terms were made. Furthermore, our dataset utilises the complexity of wave spectra in an operational wave model:

The choice of the training data was essential for the successful construction of a NN, since a NN has good interpolation properties but produces unpredictable

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output when forced to extrapolate. We used simulated spectra from a hindcast in the North-Atlantic for the one month period of January 1995 with the wave model WAM cycle 4, [WAMDI group, 1988]. From this dataset of over five million spectra, a representative subset containing spectra originating from wind sea and swell and complex combinations of the two was selected. Figure 1 shows some examples of highly complex spectra included in our training dataset.

We expected that the training for wave spectra representing multi–modal wave systems would be more difficult than for single peaked spectra. Therefore the more complex cases should be well represented. To be able to enrich the number of multi–modal spectra in the training data set, we performed an automatic classification of the wave spectra using the cluster algorithm by [Schiller, 1980].

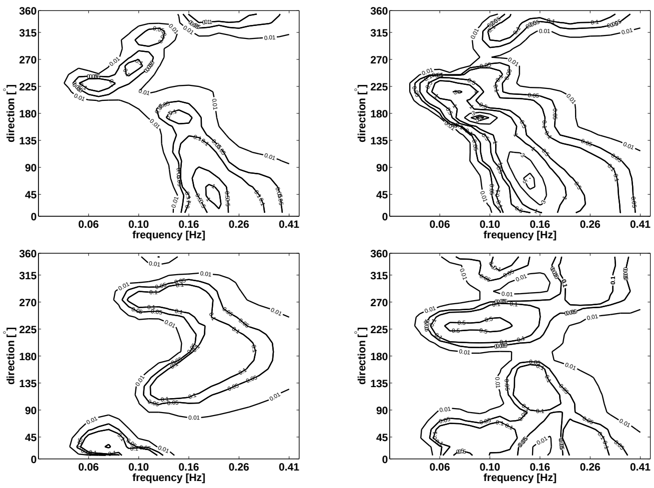

Figure 1: Examples of complex wave spectra included in the dataset used for the training of the neural networks.

The next step was to choose an appropriate structure for the NN. The spectra were calculated on a frequency–direction grid of size (25x24). Thus, both spectra and nonlinear interaction source terms were 600 dimensional. Still, the number of independent parameters (or intrinsic dimensionality) was assumed to be much smaller. In order to establish the necessary complexity of the NN it is important to have an estimate of the intrinsic dimensionality of the spectra, *i.e.* the minimum number of variables needed to represent the spectra. For this we constructed auto-associative neural nets (AANN) which map the spectra onto themselves while compressing them in between. The finding of the intrinsic dimensionality led to the decision about the actual NN structure for the mapping of the wave spectra onto the nonlinear interaction source terms.

In the following section 2 we will summarise the results of our feasibility study: After a short introduction, the functioning of the auto-associative neural net is illustrated in 2.1. The design and training of the actual neural network for the nonlinear interaction source term is described in section 2.2. We then summarise our results and discuss the relevant steps towards an operational usage of the procedure in an outlook given in section 3.

2 Neural networks

Wave spectra and the nonlinear interaction source term are related via a six–dimensional Boltzmann integral. A computational efficient method to parametrise this complex functional relation is the usage of NN's. This is possible since a NN with at least one hidden layer – a layer between the input and the output layer – is able to approximate any continuous function (Universal Approximation Theorem) as was shown by [Hornik, 1991].

So a NN — in this context — is a computational tool for function approximation. The NN maps the input vector (in our application the two–dimensional wave spectra arranged as vectors with 600 components) nonlinearly onto the output vector (the corresponding nonlinear interaction source terms).

The free parameters of the parametrisation are fixed during the so–called training phase of the NN. The training is time consuming. But it needs to be done only once, whereas the subsequent usage of a NN is very fast.

A more detailed introduction to NN's is beyond the scope of this paper. For a detailed description see *e.g.* [Bishop, 1995].

To train our NN's we used the program developed by [Schiller, 2000]

2.1 Auto–associative neural network of wave–spectra

In order to achieve the goal of directly mapping the wave spectra onto the corresponding nonlinear interaction source term, it is essential to find their intrinsic dimensionality (*the number of independent variables*) to be able to fix an appropriate NN structure.

Even if this step is not necessary, the training of auto-associative NN's is instructive and well suited to find the intrinsic dimensionality. An AANN is a particular NN which maps the input vector onto itself. At one stage of the mapping — in the so called bottleneck layer — the dimensionality is reduced.

We applied AANN's with different degrees of reduction of dimension to the 600 components of the wave spectra. We found that an AANN with 39 bottleneck neurons gave a good parametrisation for all different classes of wave spectra contained in our dataset.

Figure 2 shows three examples of the performance of this AANN. The left panel shows the original wave spectra. The middle panel shows the output of the AANN — the mapping of the original wave spectra onto themselves. The right panel shows the directionally integrated wave spectra. The examples exhibit an increasing complexity of the wave spectra. The AANN output strongly resembles the original wave spectra throughout the spectral space.

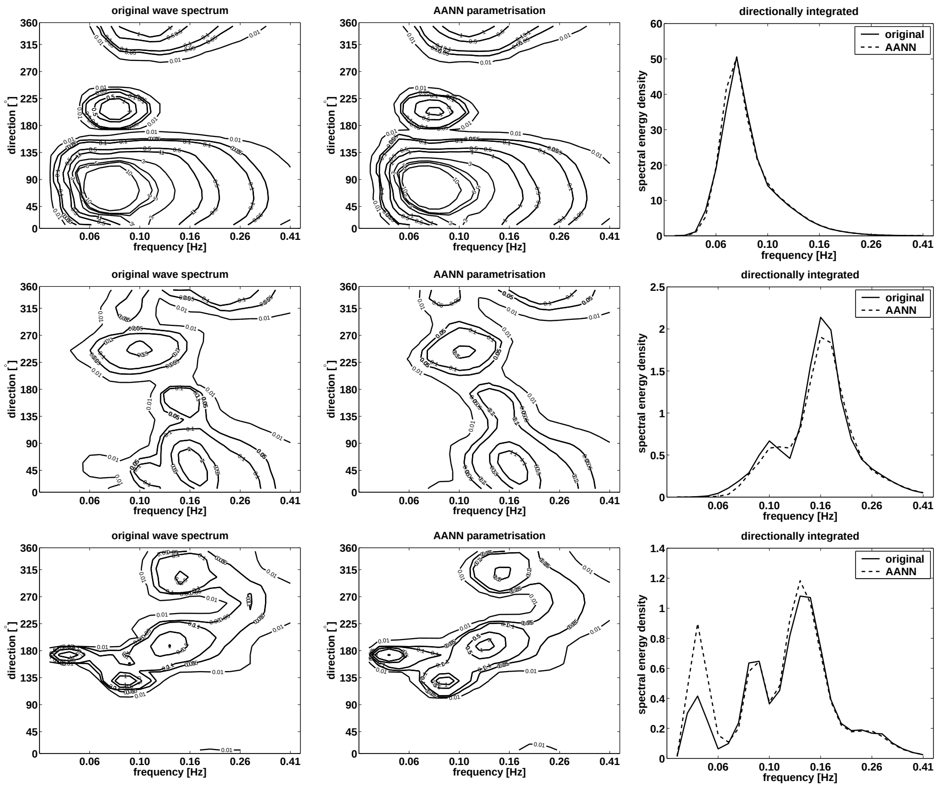

Figure 2: Examples of the performance of the AANN: left panel shows original wave spectra, panel in the middle shows corresponding AANN parametrisation, and right panel shows the directionally integrated spectra.

2.2 Nonlinear wave–wave interaction

We then trained the neural network for the direct derivation of the nonlinear interaction source (S_{nl}) terms from wave spectra.

To do so, the (exact) nonlinear interaction source terms corresponding to each of the spectra had to be calculated first. The method first suggested by [Webb, 1978] and known as the WRT method with further improvements by [Van Vledder, 2006] was applied for this purpose.

The in– and output layer of the NN consists of 600 neurons as the discretised wave spectra and the S_{nl}–terms are interpreted as 600–dimensional vectors. Firstly, the incoming wave spectrum is compressed to 39 dimensions, as suggested by the AANN (see previous section). Subsequently, the nonlinear interaction source term is derived.

Figure 3 shows results of the performance of the NN. The top row shows the original wave spectra which served as input for the NN. The next row shows the corresponding exact S_{nl}–terms calculated with the WRT–method and below it

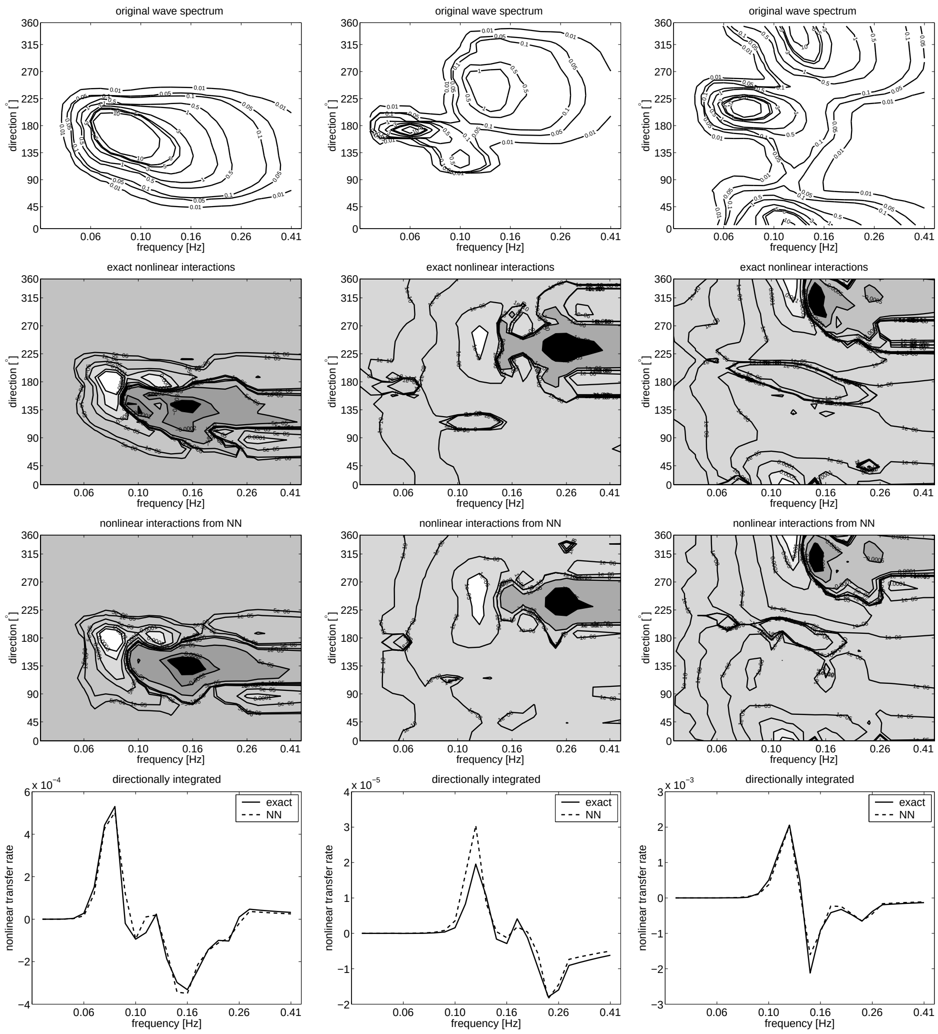

Figure 3: Examples of the performance of the NN for emulating the WRT method: upper row shows original wave spectra, next row the corresponding exact S_{nl}–term and third row shows its emulation by the NN. In the plots of the S_{nl}–term the black areas correspond to most negative values and the white areas to the biggest positive values. The gray areas in between cover ranges of four orders of magnitude for the S_{nl}–terms for either sign. The lower row shows the directionally integrated S_{nl}–terms.

the NN results (the WRT emulation by the NN) are shown. The bottom row shows the directionally integrated S_{nl}–terms. In all cases the NN emulation is very similar to the exact solution. The directionally integrated plots highlight the quality of the fit. The method is shown to be applicable, not only for single swell systems, but also for the combination of swell and wind sea.

3 Outlook and Conclusions

The feasibility of a neural network based method for direct mapping of discrete wave spectra onto the corresponding (exact) nonlinear wave–wave interaction source (S_{nl}) terms has been demonstrated.

Additionally, we incorporated the complexity of wave spectra in an operational wave model. Still, the NN is able to emulate the WRT method calculations for single and multi mode wave spectra much faster and more accurate then the approximations implemented in nowadays operational wave models.

However, the following improvements of the method are needed in order to incorporating it into an operationally wave model:

The training of the NN's should be repeated. We already started with extensive trials on different net architectures (*e.g.*, different mapping/ demapping part). In particular, we assume it promising to train a AANN for both, the wave spectra and the nonlinear interaction source terms and to map the both bottleneck layers onto each other. Thus, more complex net architectures can be tried out for the NN which maps the two bottleneck layers onto each other, since the number of in– and output variables is one order of magnitude smaller than in the present NN.

Finally, it has to be shown that this NN emulation gives robust and accurate results when implemented in a numerical wave model. As suggested by [Krasnopolsky et al., 2005], and [Krasnopolsky et al., 2007], parallel runs of the wave model with the original parametrisation of the exact nonlinear interaction and with its NN emulation should be performed to do so. Additionally, some critical test cases, such as the evolution of wave spectra and the suppression of instabilities, as shown by [Resio, 1991], and [Young, 1993] should be evaluated thoroughly, in order to test the quality of the NN emulation. When operationally running the wave model, a quality control block as suggested by [Krasnopolsky et al., 2008] could determine whether the NN emulation will be used or not.

References

[Bishop, 1995] Bishop C.M., (1995). Neural Networks for Pattern Recognition. Clarendon Press, Oxford.

[Cavaleri et al., 2007] Cavaleri L., Alves J.-H.G.M., Ardhuin F., Babanin A., Banner M., Belibassakis K., Benoit M., Donelan M., Groeneweg J., Herbers T.H.C., Hwang P., Janssen P.A.E.M., Janssen T., Lavrenov I.V., Magne R., Monbaliu J., Onorato M., Polnikov V., Resio D., Rogers W.E., Sheremet A., McKee Smith J., Tolman H.L., Van Vledder G., Wolf J., Young I., (2007).

Wave modelling The state of the art. Progress in Oceanography 75(4), 603–674.

[Hasselmann et al., 1985] Hasselmann S., Hasselmann K., Allender J.A., Barnett T.P., (1985). Computations and parametrizations of the non–linear energy transfer in a gravity wave spectrum. Part II: Parametrization of the non–linear transfer for application in wave models. Journal of Physical Oceanography 15, 1378–1391.

[Hornik, 1991] Hornik K., (1991). Approximation capabilities of multilayer feedforward networks. Neural Networks 4(2), 251–257.

[Krasnopolsky et al., 2001] Krasnopolsky V.M., Chalikov D.V., Tolman H.L., (2001). Using neural network for parametrization of nonlinear interactions in wind wave models. In: International Joint Conference on Neural Networks, 15–19 July, 2001, Washington DC, pp. 1421–1425, ISBN 0-7803-7044-9.

[Krasnopolsky et al., 2002] Krasnopolsky V.M., Chalikov D.V., Tolman H.L., (2002). A Neural Network Technique to Improve Computational Efficiency of Numerical Oceanic Models. Ocean Modelling 4, 363–383.

[Krasnopolsky et al., 2005] Krasnopolsky V.M., Fox-Rabinovitz M.S., Chalikov D.V., (2005). New Approach to Calculation of Atmospheric Model Physics: Accurate and Fast Neural Network Emulation of Long Wave Radiation in a Climate Model. Monthly Weather Review 133(5), 1370–1383.

[Krasnopolsky et al., 2007] Krasnopolsky V.M., (2007). Neural Network Emulations for Complex Multidimensional Geophysical Mappings: Applications of Neural Network Techniques to Atmospheric and Oceanic Satellite Retrievals and Numerical Modeling. Reviews of Geophysics 45, RG3009, doi:10.1029/2006RG000200.

[Krasnopolsky et al., 2008] Krasnopolsky V.M., Fox-Rabinovitz M.S., Tolman H.L., Belochitski A.A., (2008). Neural network approach for robust and fast calculation of physical processes in numerical environmental models: Compound parameterization with a quality control of larger errors. Neural Networks 21, 535–543.

[Resio, 1991] Resio D., Perrie W., (1991). A numerical study of nonlinear energy fluxes due to wave-wave interactions. Part I. Methodology and basic results. Journal of Fluid Mechanics 223, 603–629.

[Schiller, 1980] Schiller H., (1980). Cluster Analysis of Multiparticle Final States for Medium–Energy Reactions. Fizika elementarnyk chastits i atomnogo yadra 11, 182–235, translated in Soviet Journal of Particles and Nuclei 11(1), 71–90.

[Schiller, 2000] Schiller H., (2000). Feedforward–backpropagation neural net program ffbp1.0. GKSS–report 2000/37, ISSN 0344–9629.

[Tolman et al., 2005] Tolman H.L., Krasnopolsky V.M., Chalikov D.V., (2005). Neural network approximation for nonlinear interactions in wave spectra: direct mapping for wind seas in deep water. Ocean Modelling 8, 253–278.

[Van Vledder, 2006] Van Vledder G.Ph., (2006). The WRT method for the computation of non–linear four–wave interactions in discrete spectral wave models. Coastal Engineering 53, 223–242.

[Wahle et al., 2009] Wahle K., Günther H., Schiller, H., (2009). Neural network parameterisation of the mapping of wave spectra onto nonlinear four-wave interactions. Ocean Modelling 30, 48–55.

[WAMDI group, 1988] WAMDI group, (1988). The WAM model — a third generation ocean wave prediction model. Journal of Physical Oceanography 18, 1775–1809.

[Webb, 1978] Webb D.J., (1978). Non–linear transfers between sea waves. Deep Sea Research 25(3), 279–298.

[Young, 1993] Young I.R., Van Vledder G.Ph., (1993). A review of the central role of nonlinear interactions in wind–wave evolution. Philosophical Transactions of the Royal Society A 342, 505–524.