Two-Scale Approximation for Real Spectra

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OUTLINE / CONCLUSIONS

- TSA formulation – review
- Comparisons:
  - DIA, TSA, FBI (Full Boltzman Integral)
- JONSWAP cases
- Field data (i) Currituck Sound (ii) FRF waverider
- Error estimates
- Conclusions

→ TSA is quantitatively similar to FBI, whereas DIA has only some qualitative similar behavior and many serious errors

TSA Formulation

We need a new approximation that;

- **conserves action, energy, momentum**
- **number of degrees of freedom as spectrum**
- **not limited to $k_p h \geq 1$**
- **much more efficient than FBI**

$$\frac{\partial n(\underline{k}_1)}{\partial t} = \iint T(\underline{k}_1, \underline{k}_3)\, d\underline{k}_3$$

$$T(\underline{k}_1, \underline{k}_3) = 2 \oint [n_1 n_3 (n_4 - n_2) + n_2 n_4 (n_3 - n_1)] C(\underline{k}_1, \underline{k}_2, \underline{k}_3, \underline{k}_4)\, \theta\left(|\underline{k}_1 - \underline{k}_4| - |\underline{k}_1 - \underline{k}_3|\right) \left| \frac{\partial W}{\partial n} \right|^{-1} ds$$

Basis for Two-Scale Approximation

$$n = \hat{n} + n'$$

Broad scale characterization ↑ ↑ **local scale perturbation**

$$S_{nl}(f,\theta) = B + L + X$$

B = broad-scale interactions
L = local-scale interactions
X = cross-scale interactions

$$\begin{aligned}
N^3 = \; & \hat{n}_1\hat{n}_3(\hat{n}_4 - \hat{n}_2) + \hat{n}_2\hat{n}_4(\hat{n}_3 - \hat{n}_1) + \\
& n_1'n_3'(n_4' - n_2') + n_2'n_4'(n_3' - n_1') + \\
& \hat{n}_1\hat{n}_3(n_4' - n_2') + \hat{n}_2\hat{n}_4(n_3' - n_1') + \\
& n_1'n_3'(\hat{n}_4 - \hat{n}_2) + n_2'n_4'(\hat{n}_3 - \hat{n}_1) + \\
& \hat{n}_1 n_3'(\hat{n}_4 - \hat{n}_2) + \hat{n}_2 n_4'(\hat{n}_3 - \hat{n}_1) + \\
& n_1'\hat{n}_3(\hat{n}_4 - \hat{n}_2) + n_2'\hat{n}_4(\hat{n}_3 - \hat{n}_1) + \\
& \hat{n}_1 n_3'(n_4' - n_2') + \hat{n}_2 n_4'(n_3' - n_1') + \\
& n_1'\hat{n}_3(n_4' - n_2') + n_2'\hat{n}_4(n_3' - n_1')
\end{aligned}$$

$$\Rightarrow \quad \frac{\partial n_1}{\partial t} = B + \iint N_*^3 C \left| \frac{\partial W}{\partial n} \right|^{-1} ds\, k_3 d\theta_3 dk_3$$

N_*^3 terms neglect terms containing n_2' and n_4' - retain $\hat{n}_2$ and $\hat{n}_4$

$$N_*^3 = \hat{n}_2\hat{n}_4(n_3' - n_1') + n_1'n_3'(\hat{n}_4 - \hat{n}_2) + \hat{n}_1 n_3'(\hat{n}_4 - \hat{n}_2) + n_1'\hat{n}_3(\hat{n}_4 - \hat{n}_2)$$

$$\frac{\partial n_1}{\partial t} = \left(\frac{k}{k_0}\right)^{-19/2} \left\{ B\left(\frac{\varsigma}{\varsigma_0}\left(\frac{k}{k_0}\right)^p\right)^3 + \left\langle \begin{aligned} &\left(\frac{\varsigma}{\varsigma_0}\right)\left(\frac{k}{k_0}\right)^p \iint (\hat{n}_1 n'_3 + n'_1\hat{n}_3 + n'_1 n'_3)\Lambda_p(\hat{n}_2 - \hat{n}_4, \underline{k}_1, k_*, \theta_*, \mathrm{x}_1,..,\mathrm{x}_n)\, k_* d\theta_* dk_* \\ &+ \left(\left(\frac{\varsigma}{\varsigma_0}\right)\left(\frac{k}{k_0}\right)^p\right)^2 \iint (n'_1 - n'_3)\Lambda_d(\hat{n}_2\hat{n}_4, \underline{k}_1, k_*, \theta_*, \mathrm{x}_1,..,\mathrm{x}_n)\, k_* d\theta_* dk_* \end{aligned} \right\rangle \right\}$$

where

$$\Lambda_p(\hat{n}_2 - \hat{n}_4, \underline{k}_1, k_*, \theta_*, \mathrm{x}_1,...,\mathrm{x}_n) = \oint C \left|\frac{\partial W}{\partial n}\right|^{-1} (\hat{n}_4 - \hat{n}_2)\, ds$$

$$\Lambda_d(\hat{n}_2\hat{n}_4, \underline{k}_1, k_*, \theta_*, \mathrm{x}_1,...,\mathrm{x}_n) = \oint C \left|\frac{\partial W}{\partial n}\right|^{-1} \hat{n}_2\hat{n}_4\, ds$$

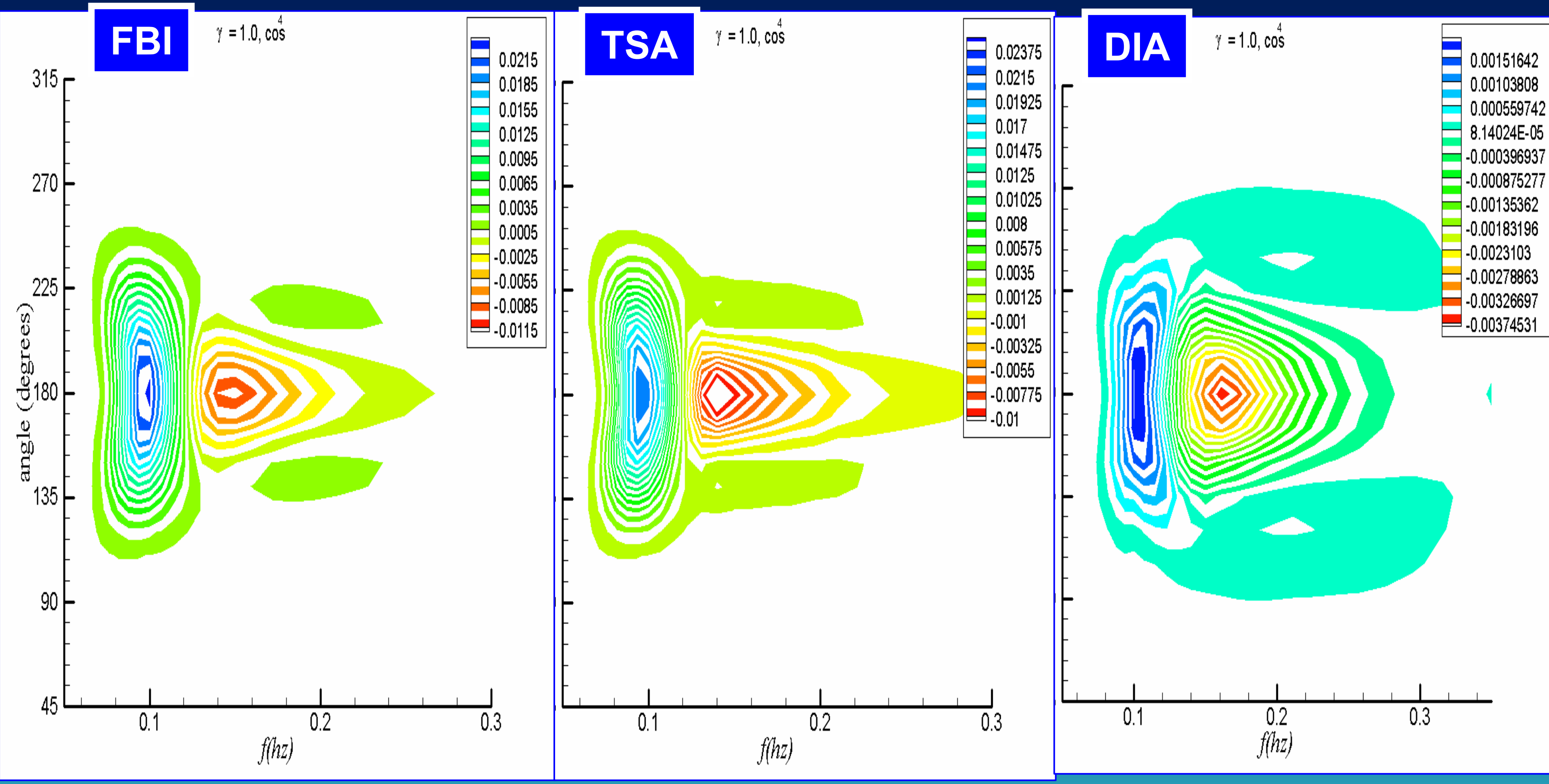
pre-calculated terms remove all calculations from innermost loop

(ς/ς_0)

is ratio of (actual / reference) linear scaling coefficients for broad-scale

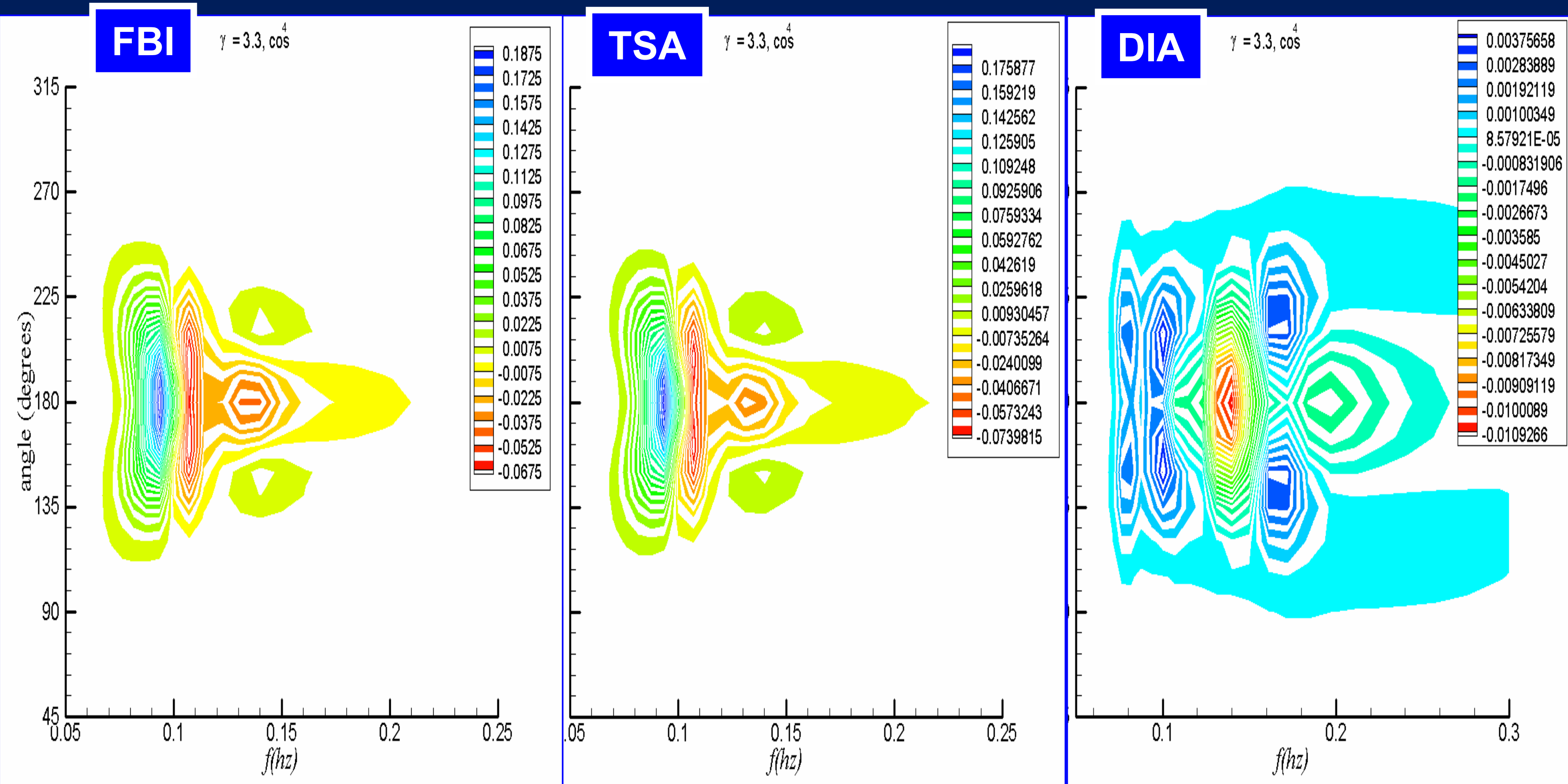
- Computer time ~ DIA

Case #1: JONSWAP cases



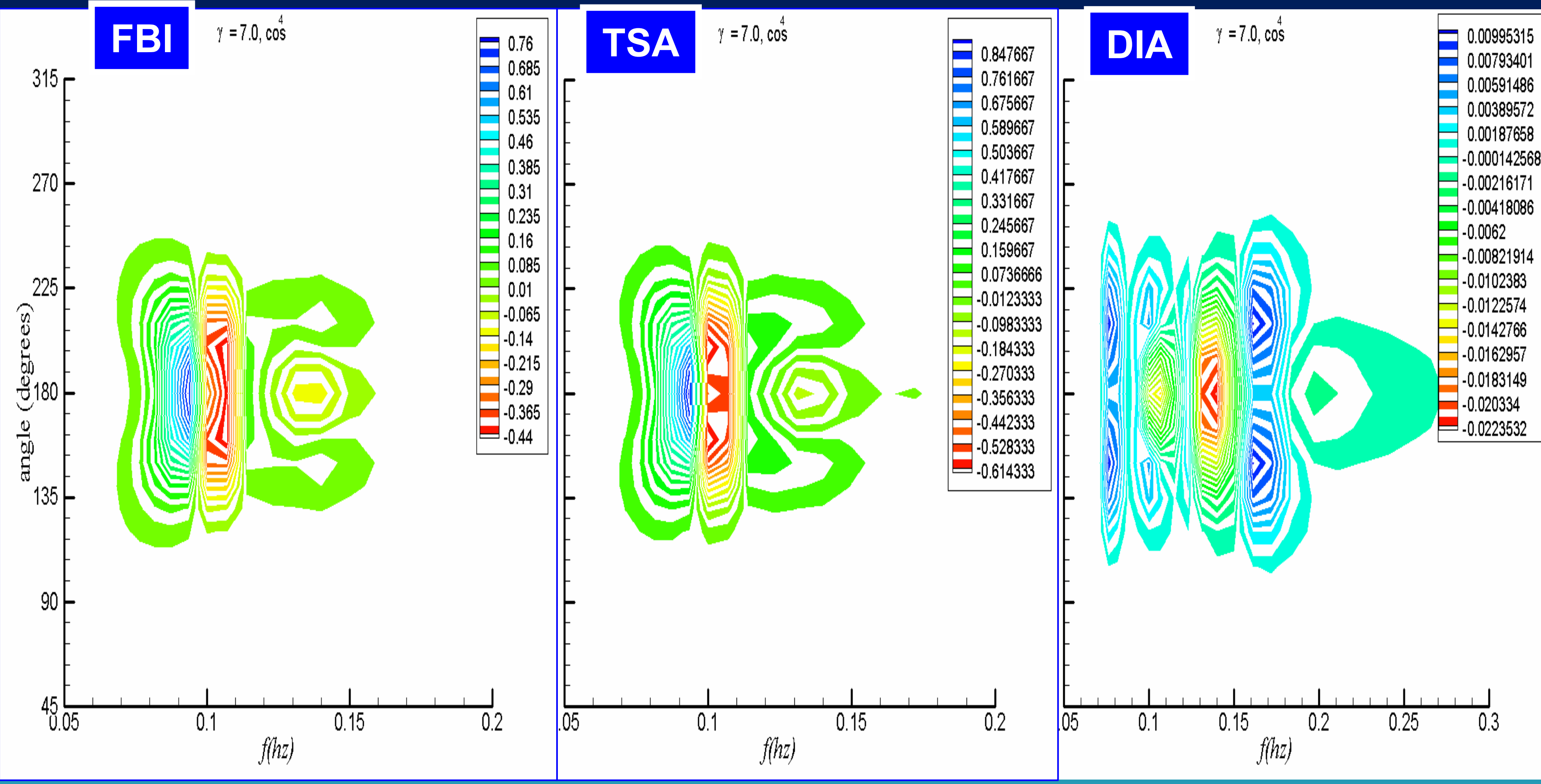
γ = 1.0

Case #1: JONSWAP cases



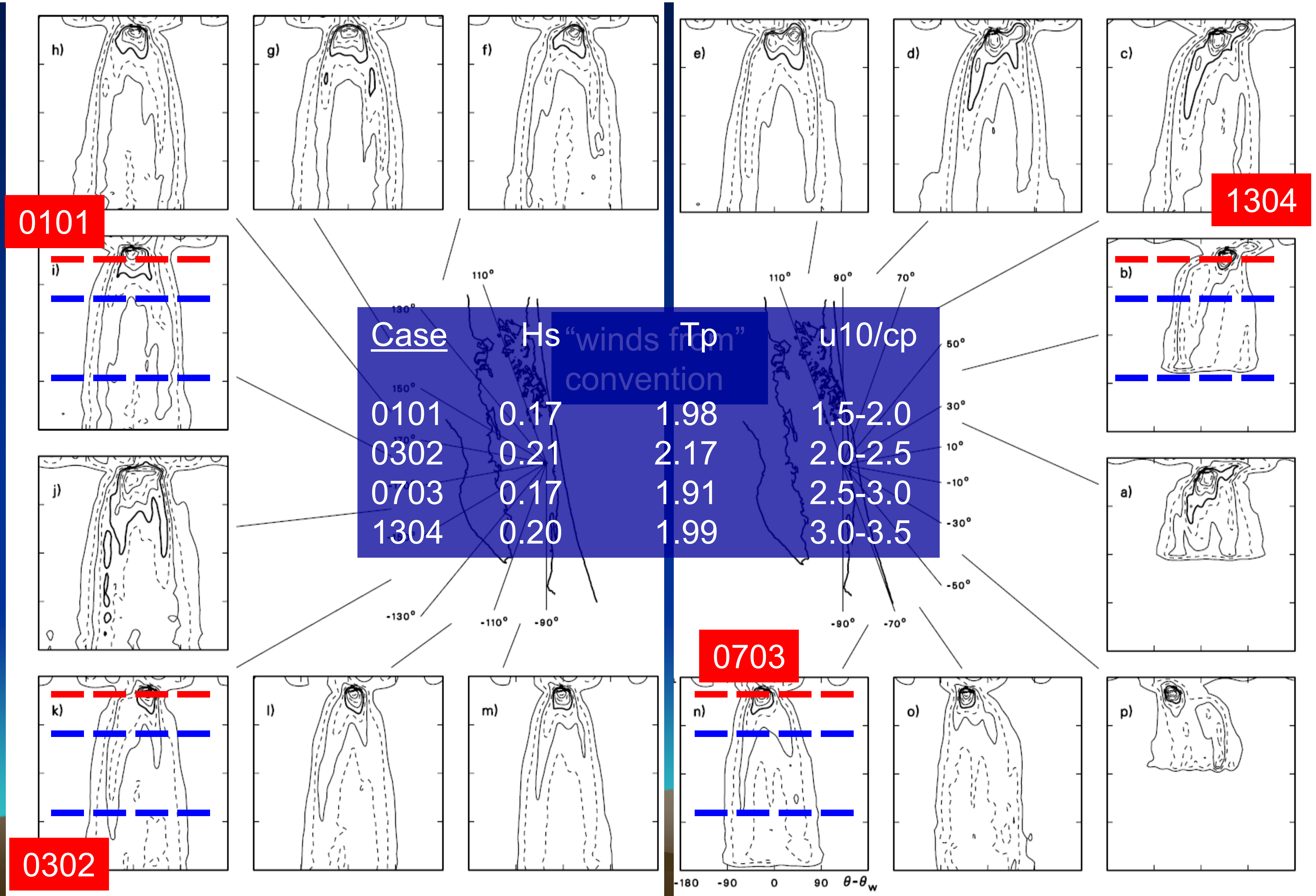
γ = 3.3

Case #1: JONSWAP cases

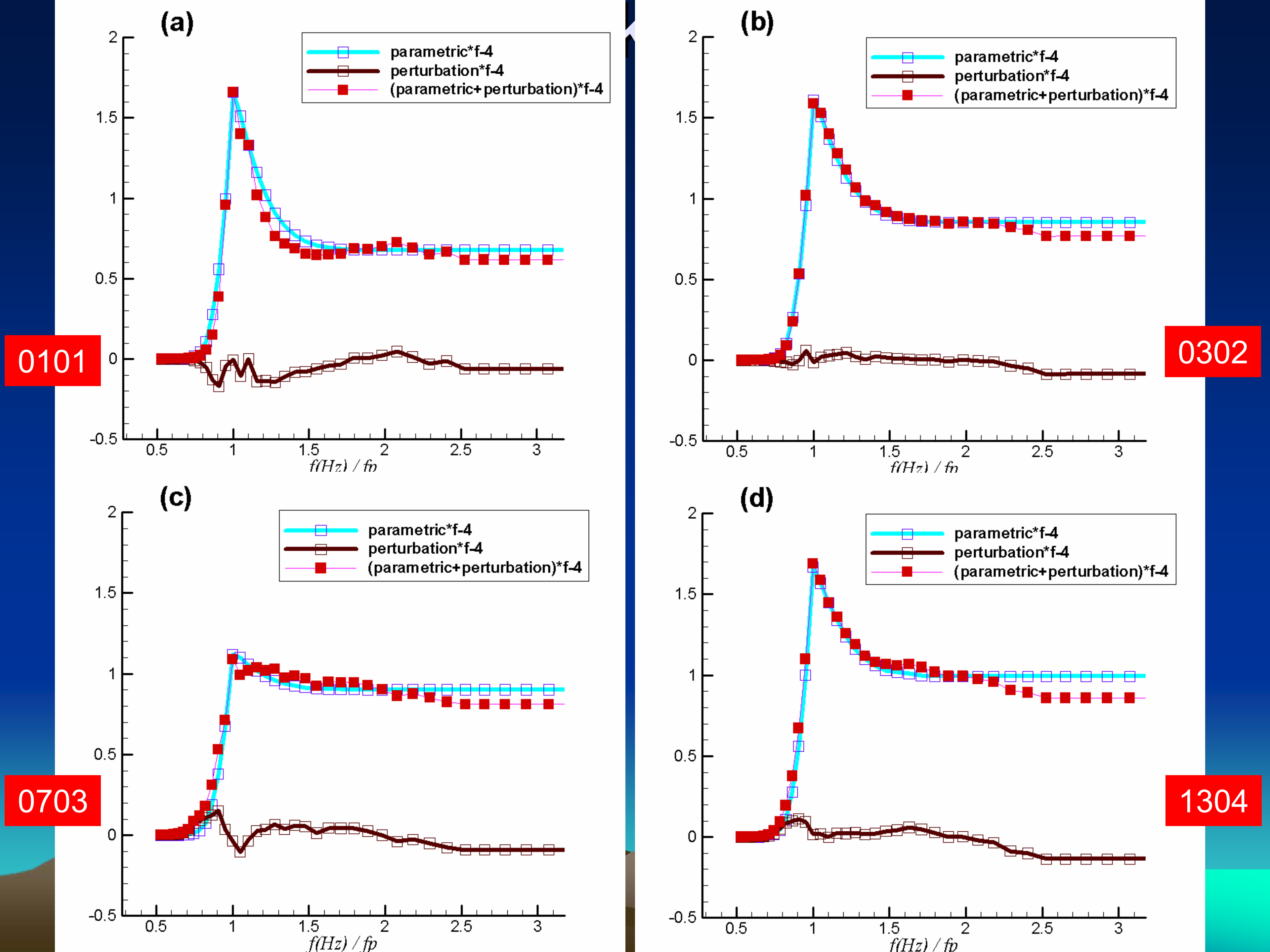


γ = 7.0

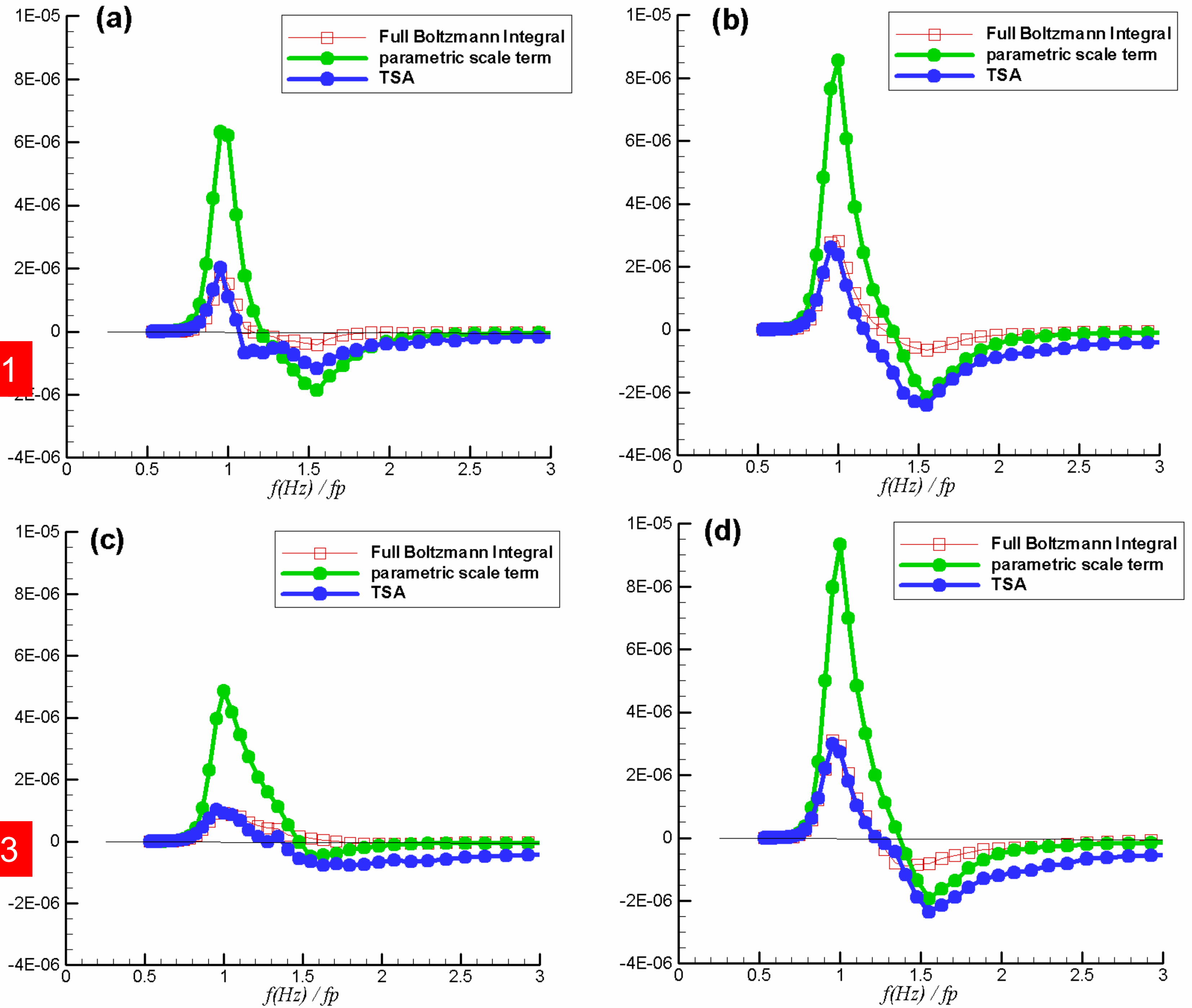
Case #2: Currituck data (Long & Resio 2007)



Case	Hs	Tp	u10/cp
0101	0.17	1.98	1.5-2.0
0302	0.21	2.17	2.0-2.5
0703	0.17	1.91	2.5-3.0
1304	0.20	1.99	3.0-3.5



(a)
parametric*f-4
perturbation*f-4
(parametric+perturbation)*f-4
f(Hz) / fp
0101
(b)
parametric*f-4
perturbation*f-4
(parametric+perturbation)*f-4
f(Hz) / fp
0302
(c)
parametric*f-4
perturbation*f-4
(parametric+perturbation)*f-4
f(Hz) / fp
0703
(d)
parametric*f-4
perturbation*f-4
(parametric+perturbation)*f-4
f(Hz) / fp
1304



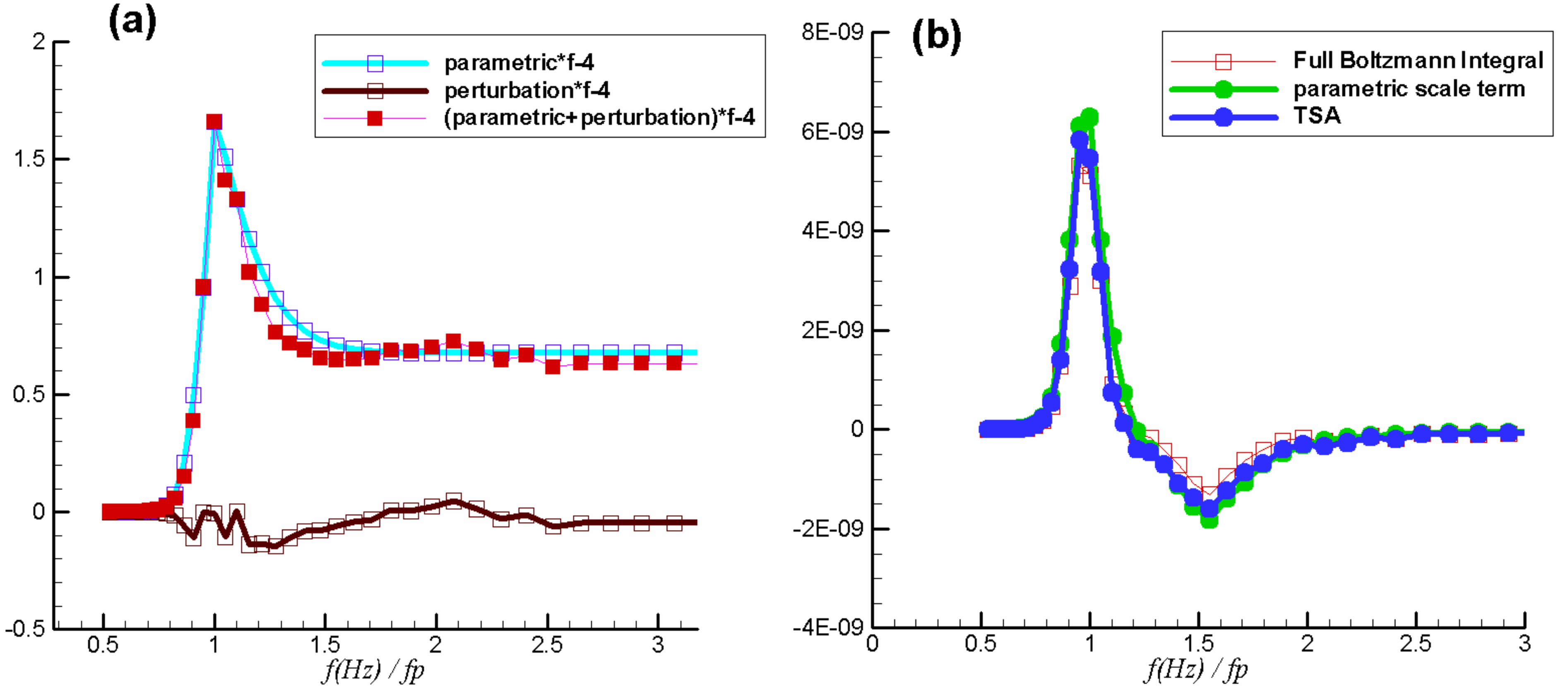
0101

0302

0703

1304

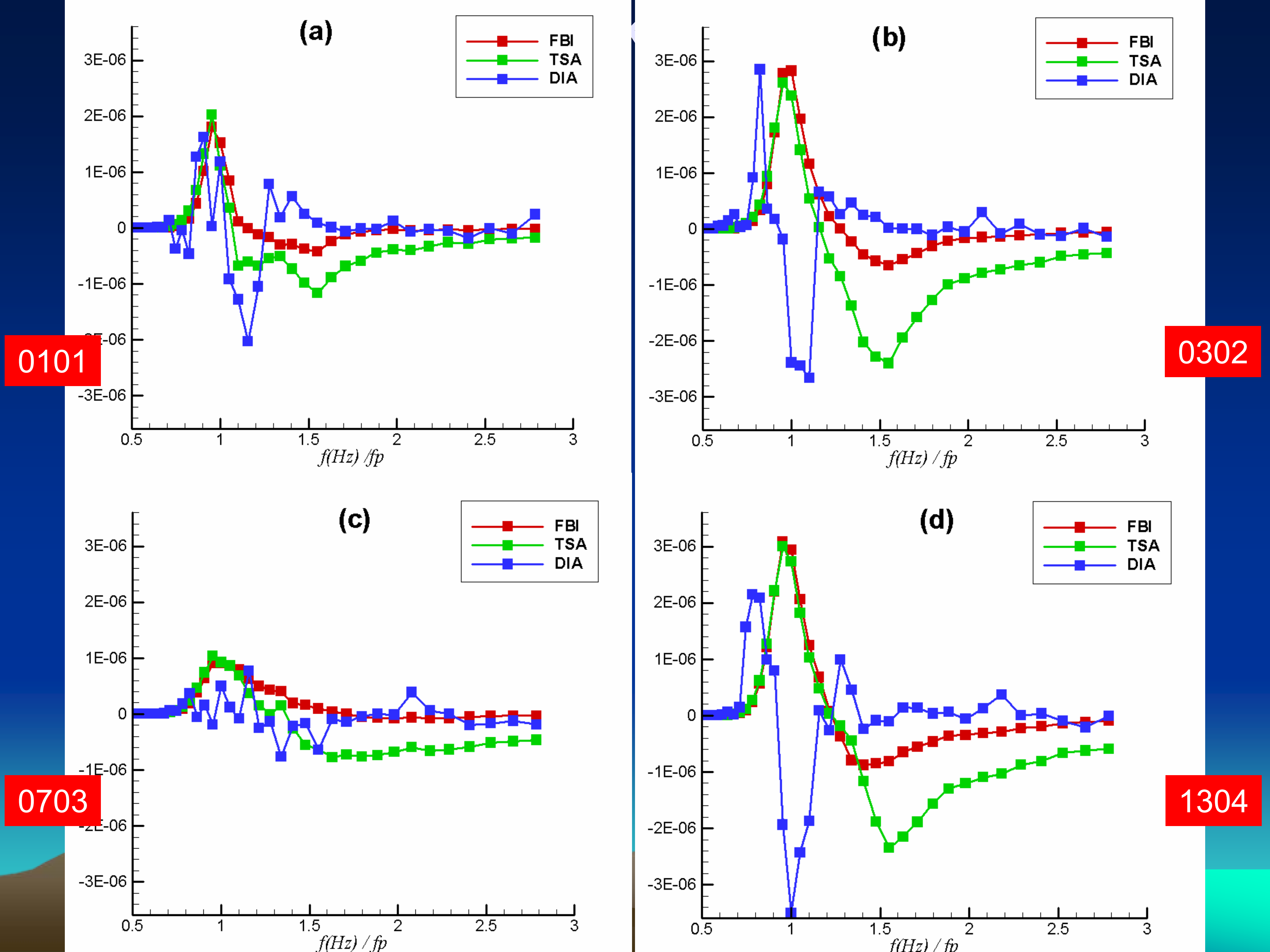
Currituck spectra



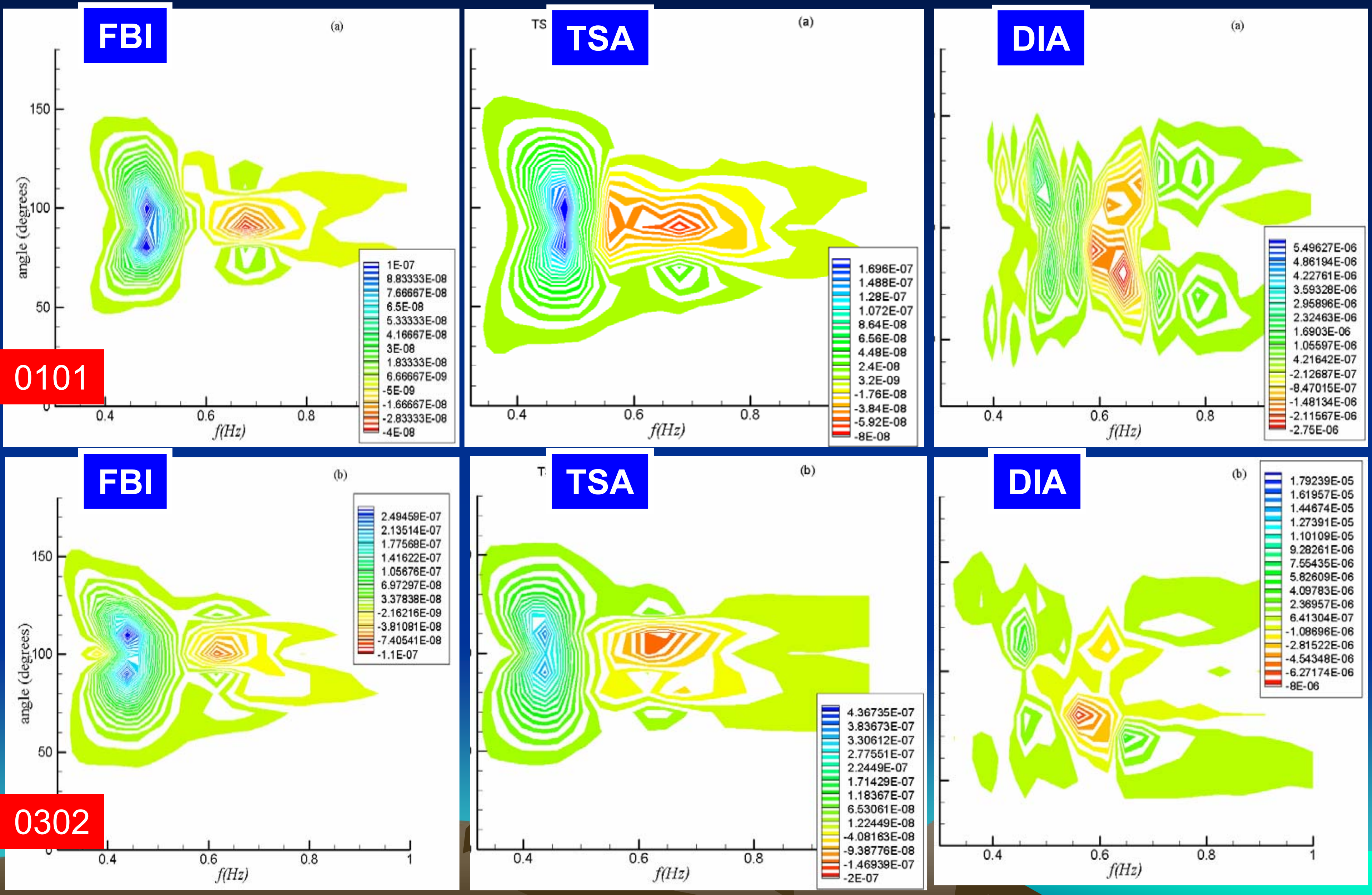
0101

-angular spreading of input $\cos^6$

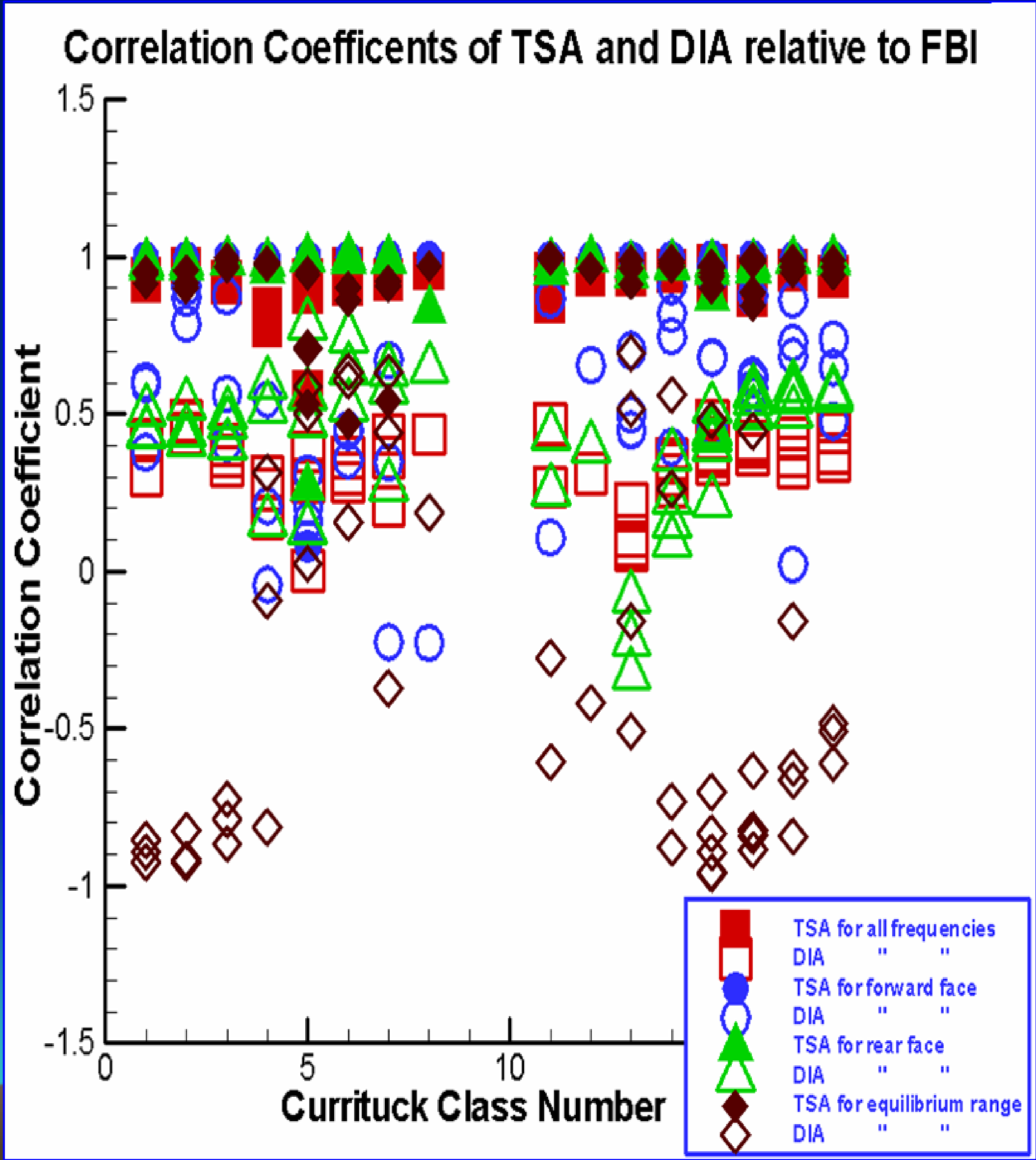
→ variations in directional spreading can affect nonlinear transfer rates

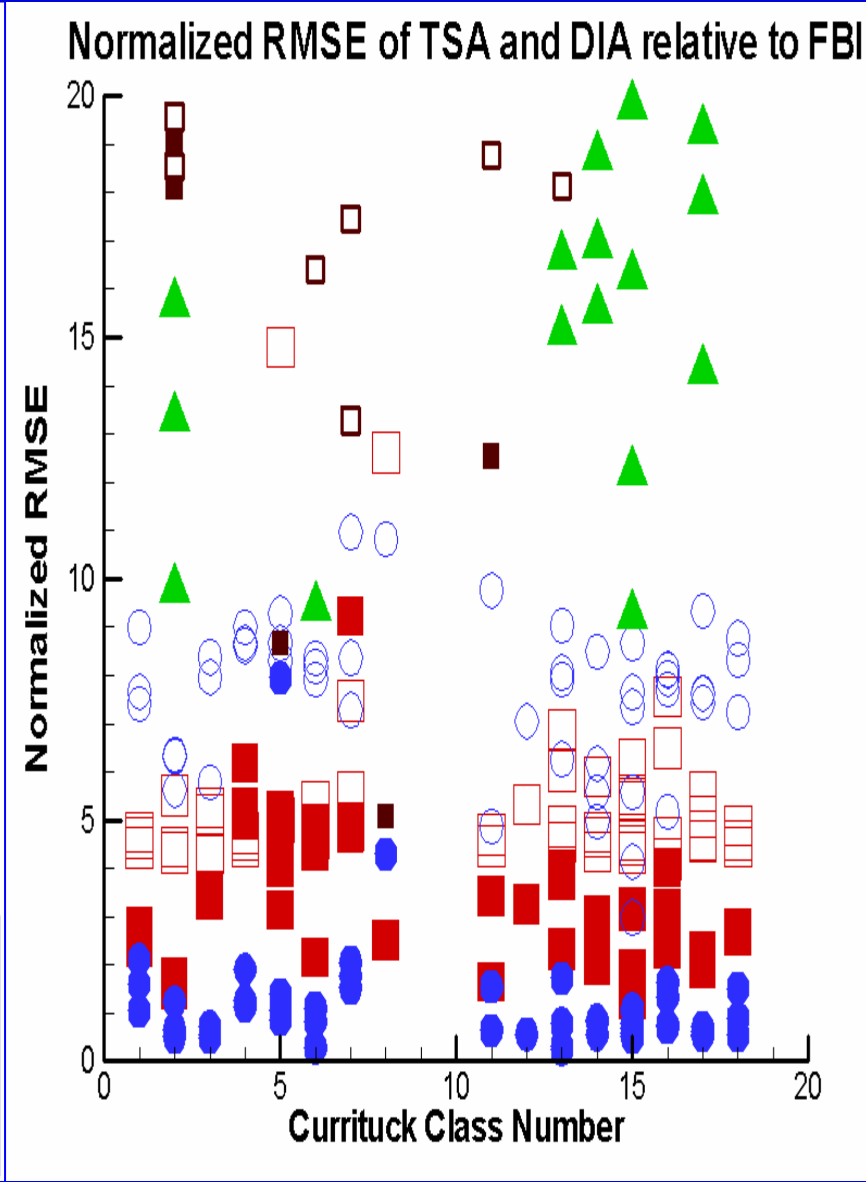

(a)
FBI
TSA
DIA
3E-06
2E-06
1E-06
0
-1E-06
-3E-06
0.5
1
1.5
2
2.5
3
f(Hz) /fp
0101
(b)
FBI
TSA
DIA
3E-06
2E-06
1E-06
0
-1E-06
-2E-06
-3E-06
0.5
1
1.5
2
2.5
3
f(Hz) / fp
0302
(c)
FBI
TSA
DIA
3E-06
2E-06
1E-06
0
-1E-06
-3E-06
0.5
1
1.5
2
2.5
3
f(Hz) / fp
0703
(d)
FBI
TSA
DIA
3E-06
2E-06
1E-06
0
-1E-06
-2E-06
-3E-06
0.5
1
1.5
2
2.5
3
f(Hz) / fp
1304

2-d Currituck spectra

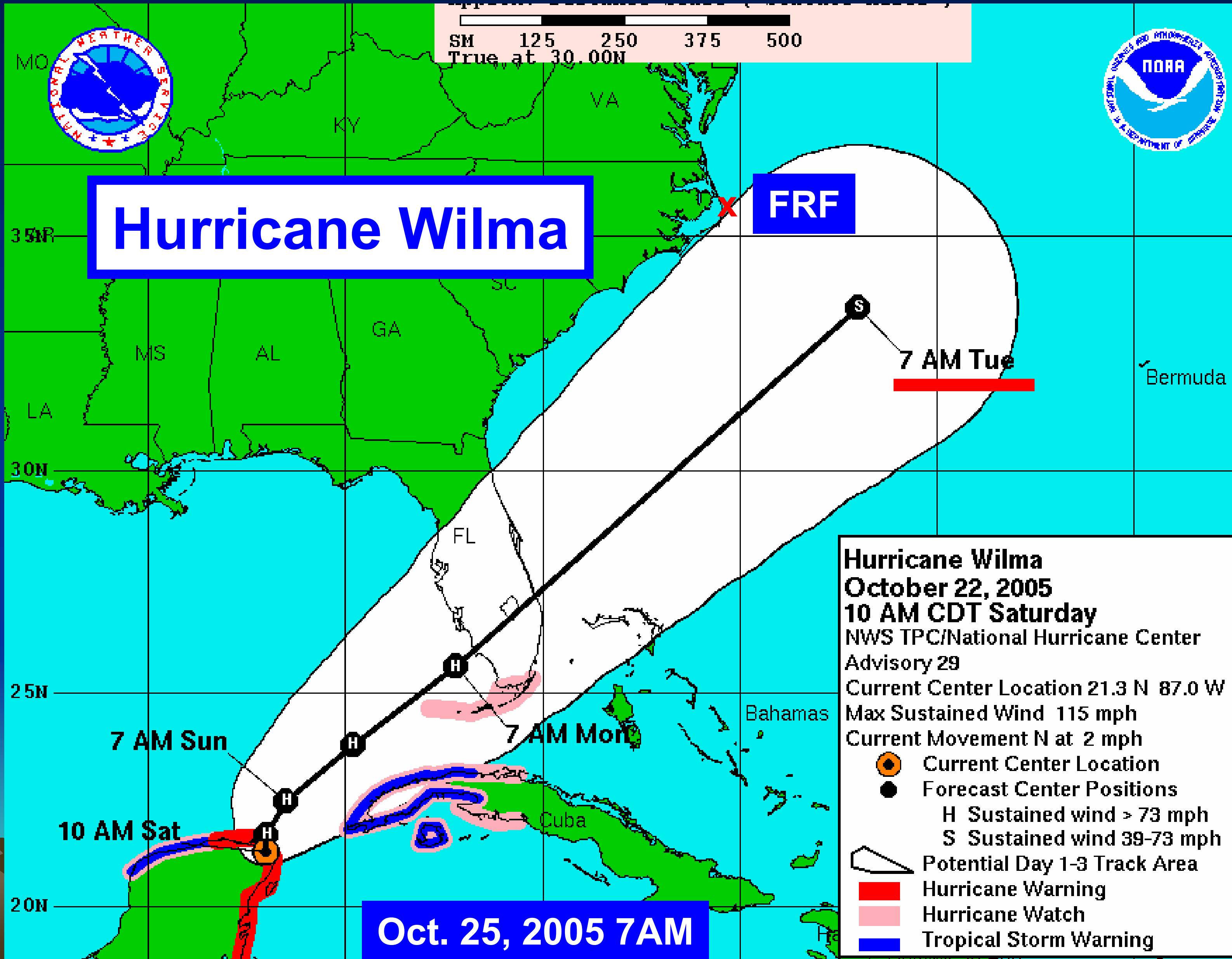


Error estimates: Currituck spectra



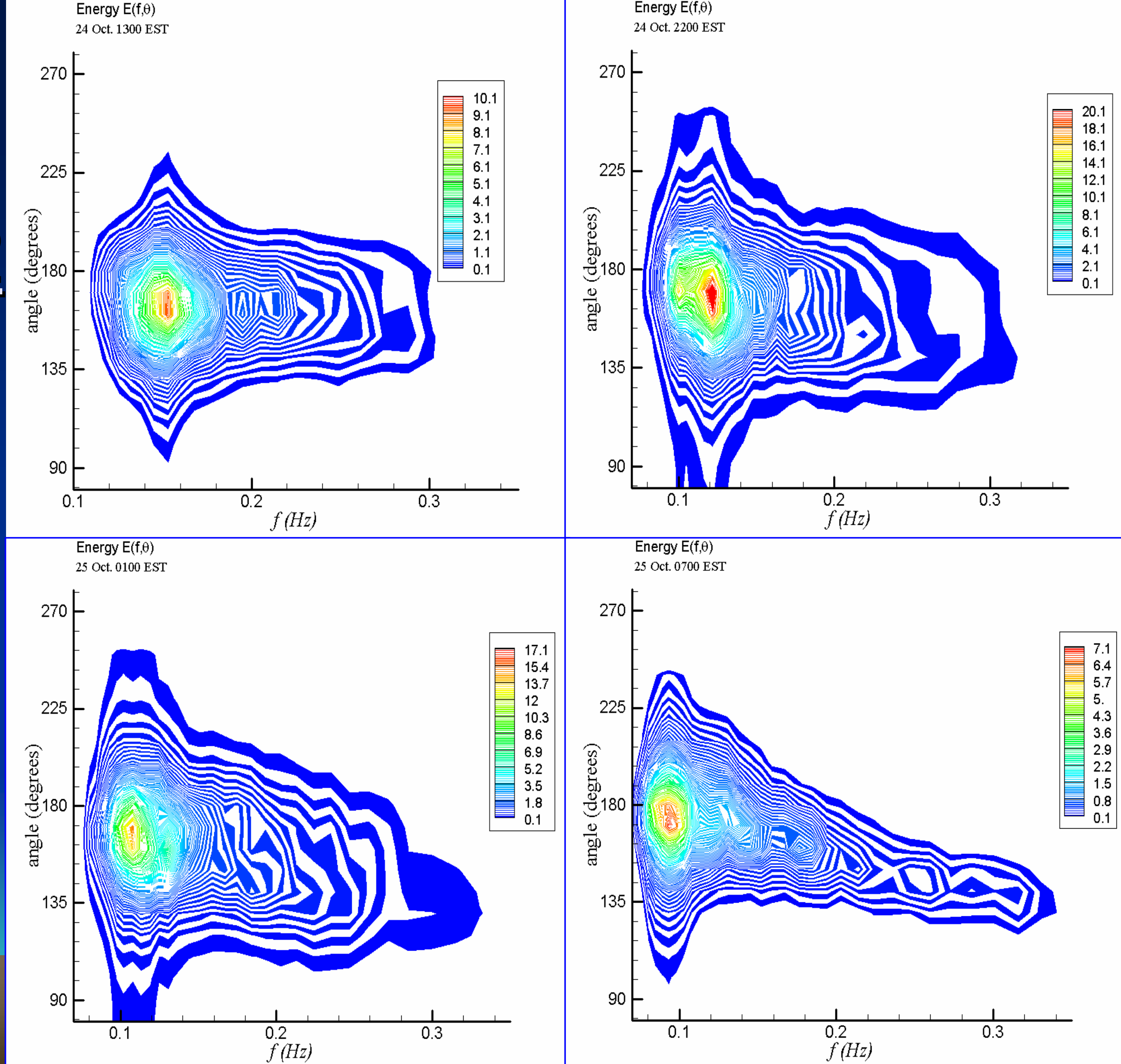


Case #3: FRF waverider spectra

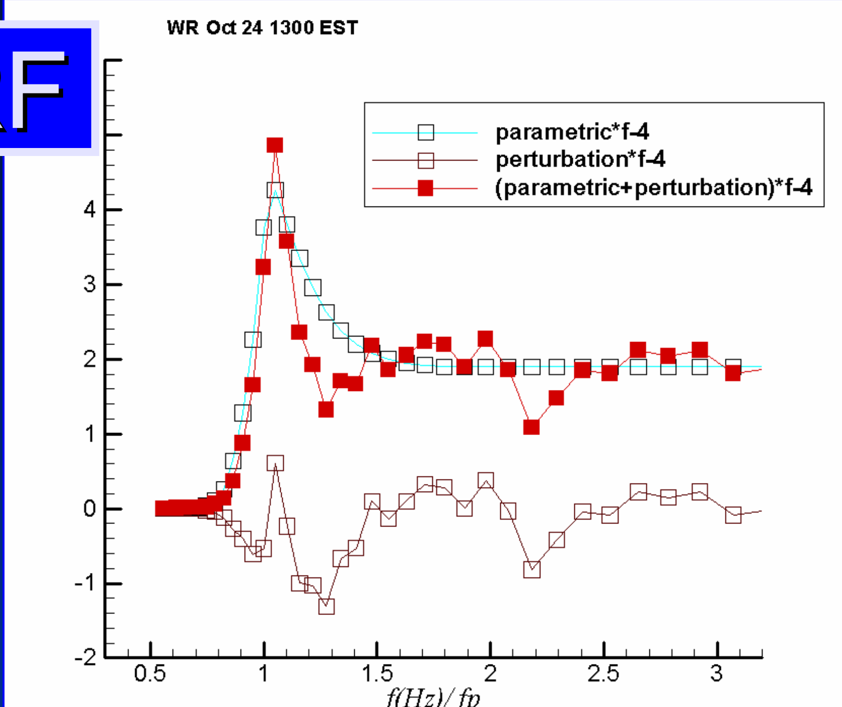


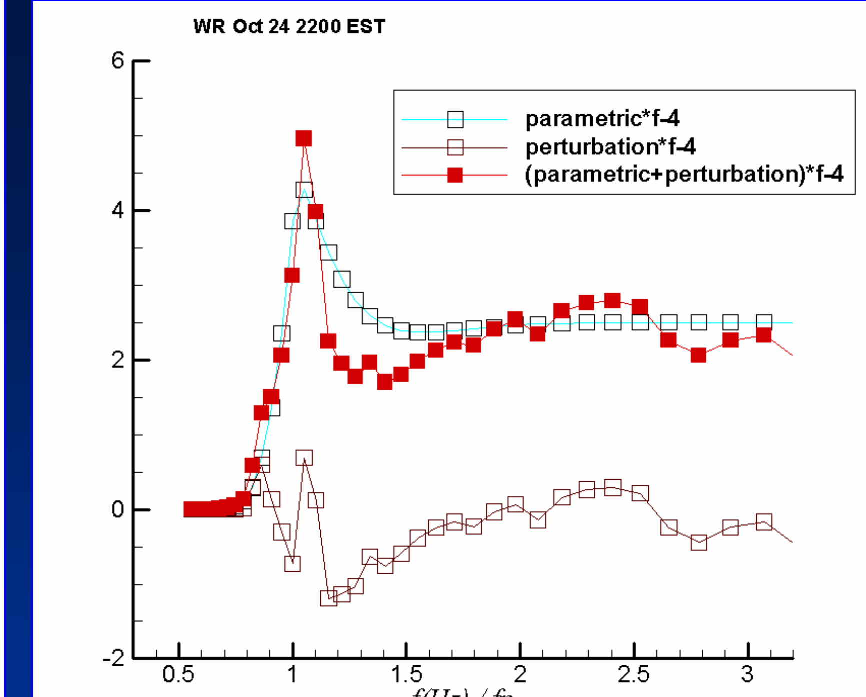
Case #2

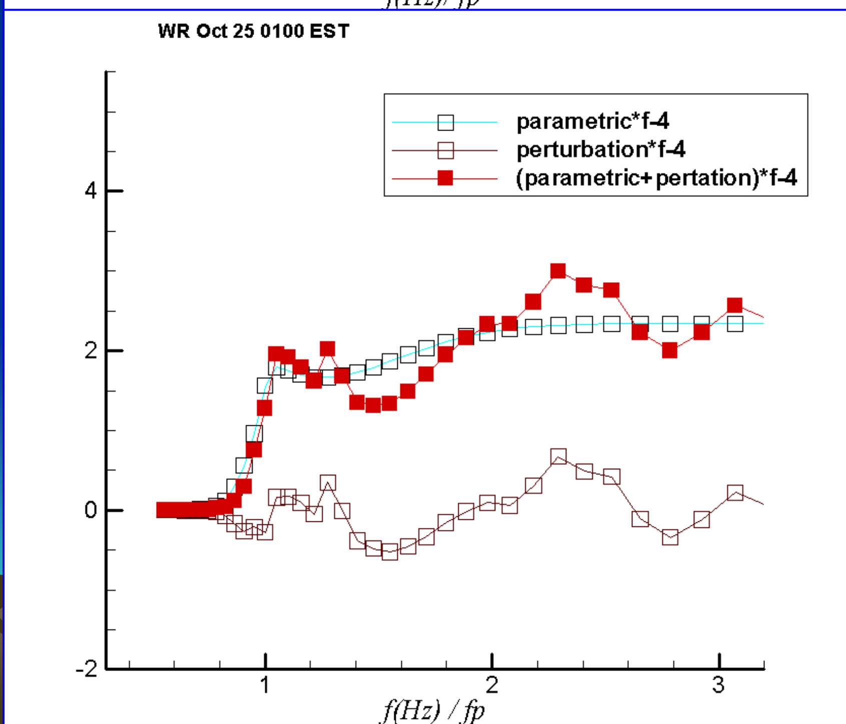
FRF spectra

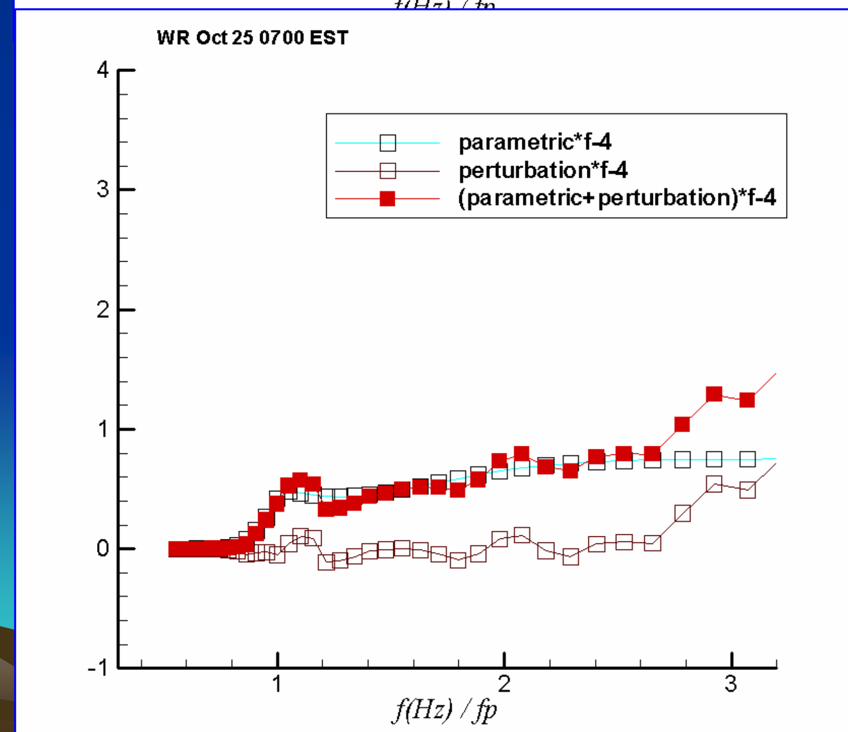


FRF

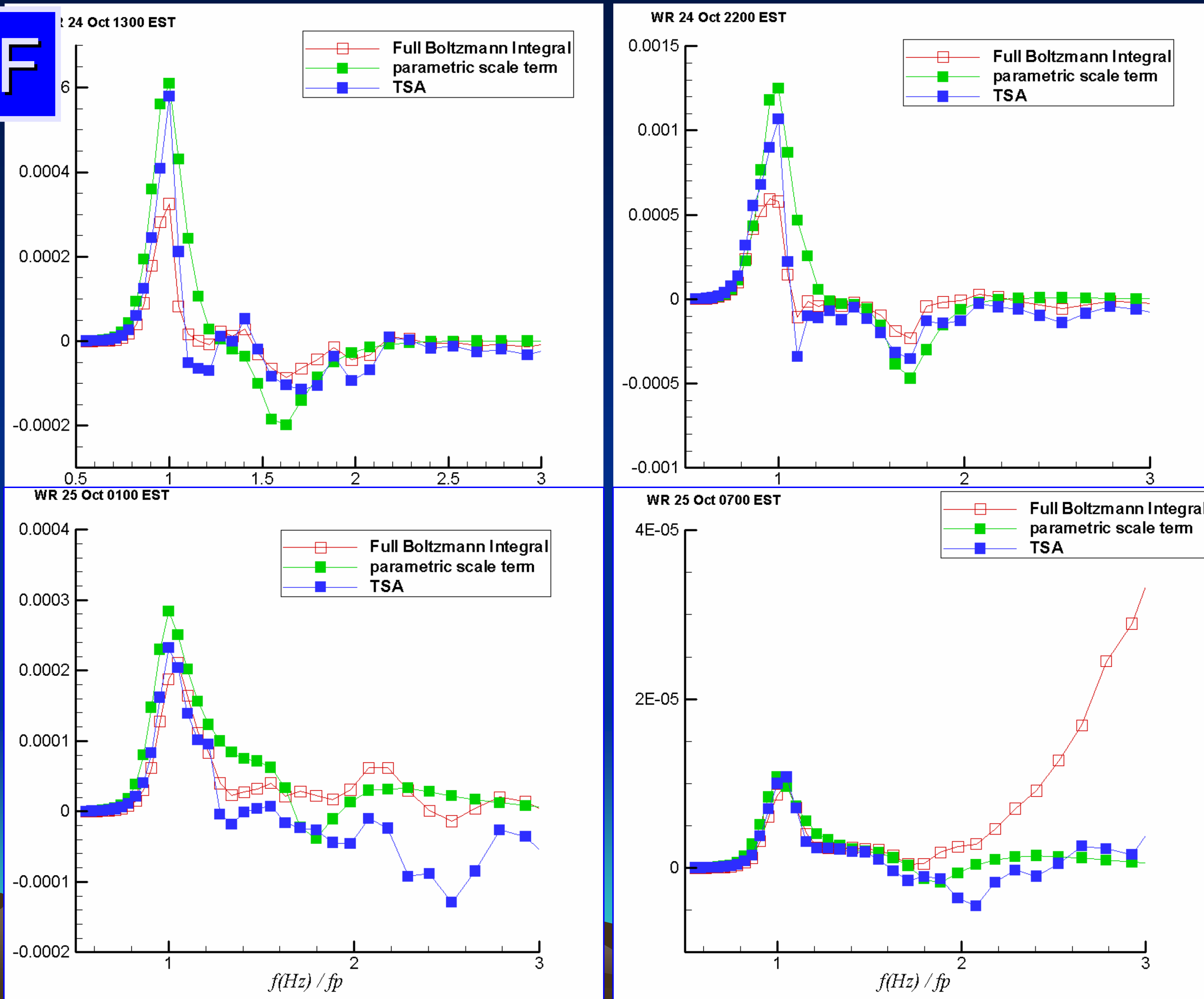

WR Oct 24 1300 EST
parametric*f-4
perturbation*f-4
(parametric+perturbation)*f-4
f(Hz)/ fp


WR Oct 24 2200 EST
parametric*f-4
perturbation*f-4
(parametric+perturbation)*f-4


WR Oct 25 0100 EST
parametric*f-4
perturbation*f-4
(parametric+pertation)*f-4
f(Hz) / fp

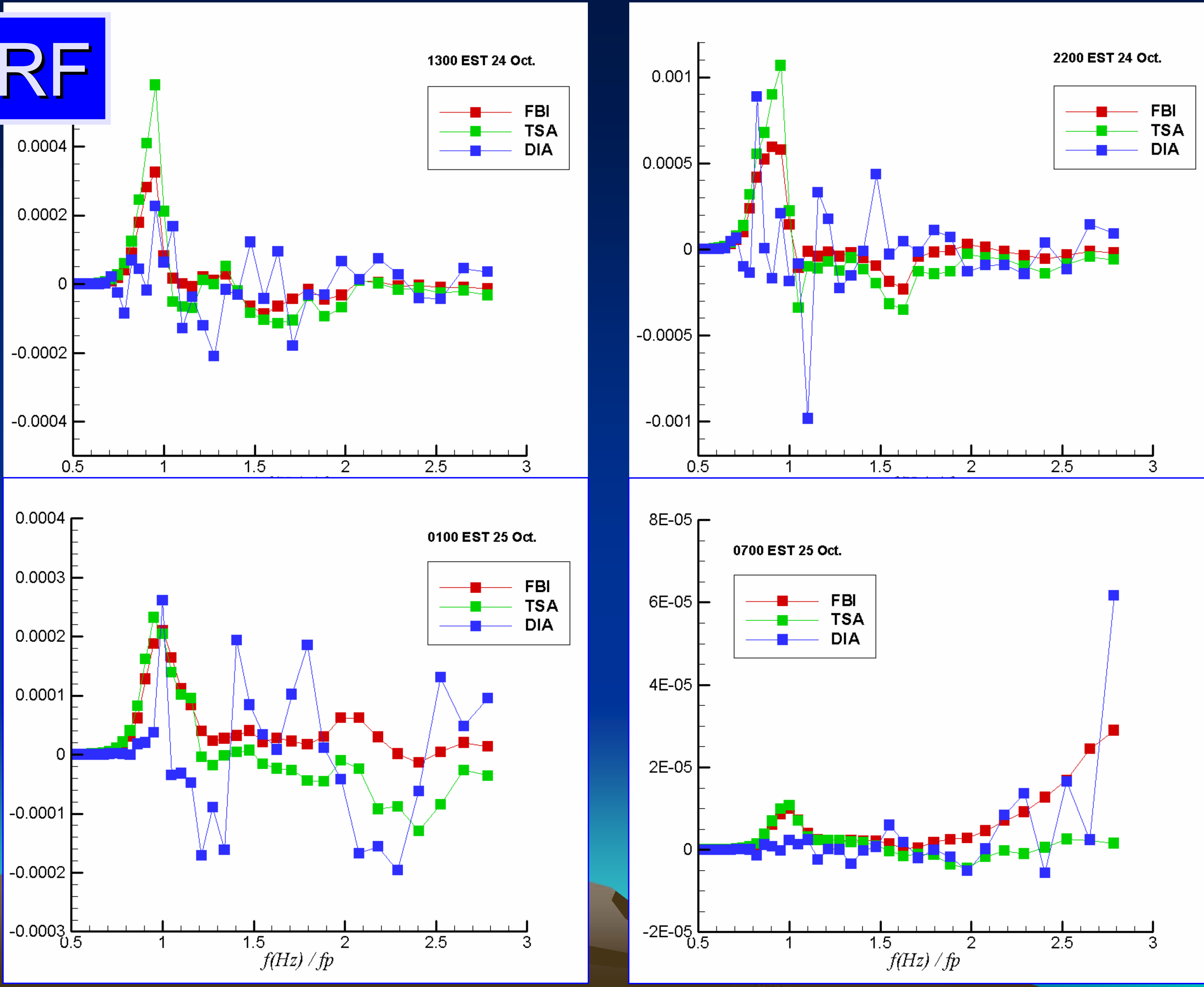

WR Oct 25 0700 EST
parametric*f-4
perturbation*f-4
(parametric+perturbation)*f-4
f(Hz) / fp

FRF

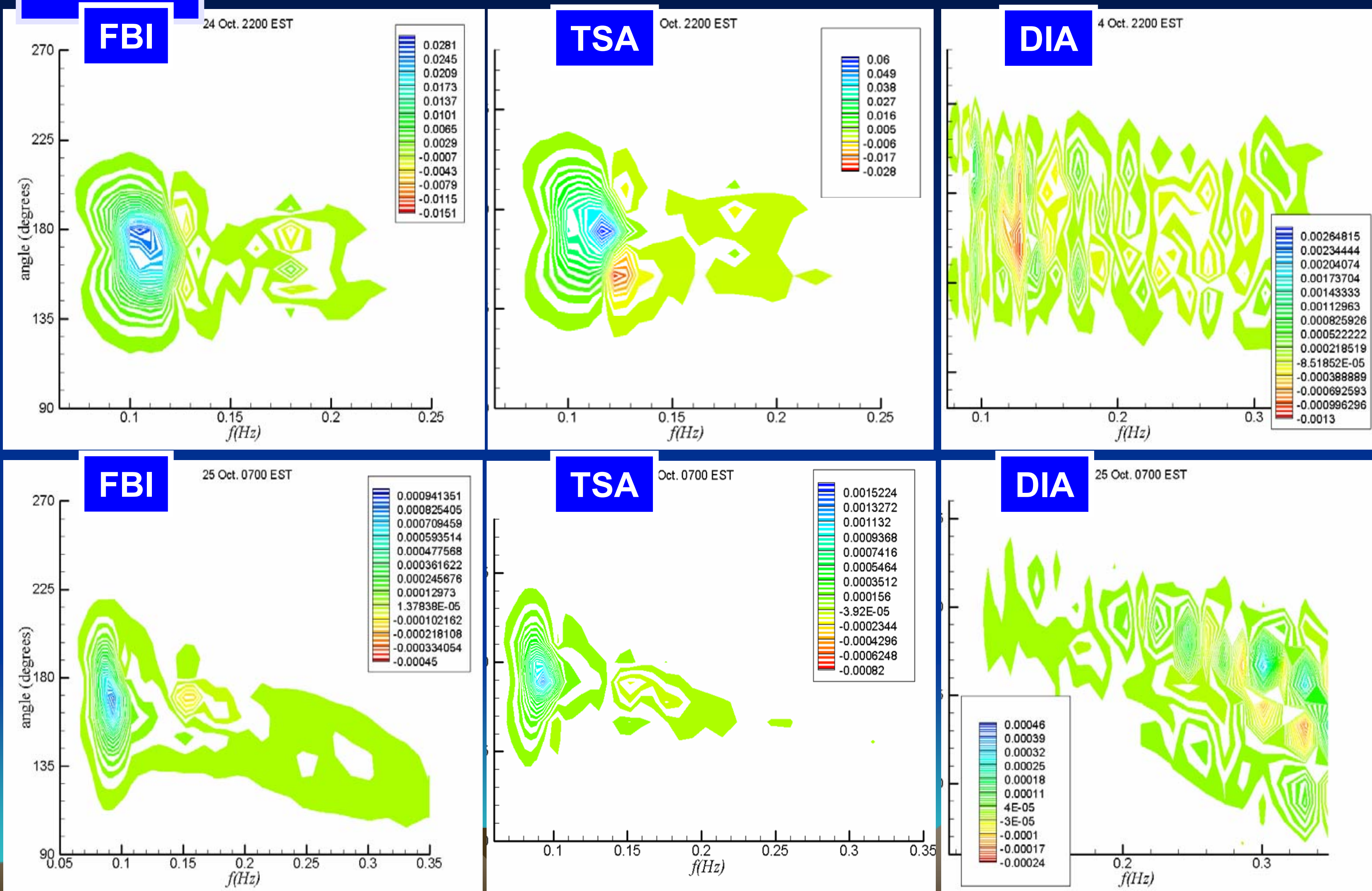


24 Oct 1300 EST
Full Boltzmann Integral
parametric scale term
TSA
6
0.0004
0.0002
0
-0.0002
0.5
1
1.5
2
2.5
3
WR 24 Oct 2200 EST
Full Boltzmann Integral
parametric scale term
TSA
0.0015
0.001
0.0005
0
-0.0005
-0.001
1
2
3
WR 25 Oct 0100 EST
Full Boltzmann Integral
parametric scale term
TSA
0.0004
0.0003
0.0002
0.0001
0
-0.0001
-0.0002
1
2
3
f(Hz) / fp
WR 25 Oct 0700 EST
Full Boltzmann Integral
parametric scale term
TSA
4E-05
2E-05
0
1
2
3
f(Hz) / fp

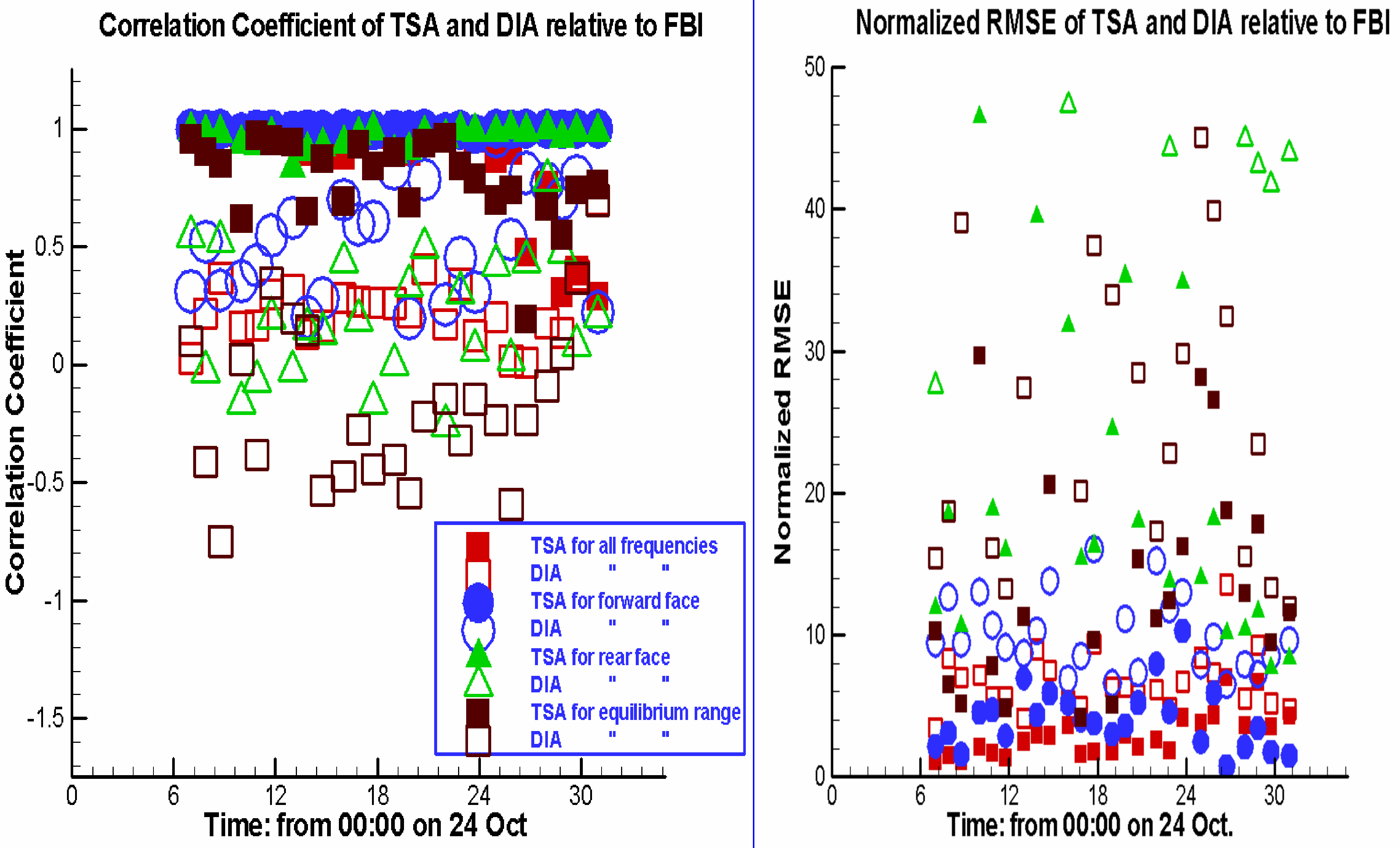
FRF


1300 EST 24 Oct.
2200 EST 24 Oct.
0100 EST 25 Oct.
0700 EST 25 Oct.
FBI
TSA
DIA
f(Hz) / fp

FRF


FBI
24 Oct. 2200 EST
0.0281
0.0245
0.0209
0.0173
0.0137
0.0101
0.0065
0.0029
-0.0007
-0.0043
-0.0079
-0.0115
-0.0151
angle (degrees)
270
225
180
135
90
0.1
0.15
0.2
0.25
f(Hz)
TSA
Oct. 2200 EST
0.06
0.049
0.038
0.027
0.016
0.005
-0.006
-0.017
-0.028
0.1
0.15
0.2
0.25
f(Hz)
DIA
4 Oct. 2200 EST
0.00264815
0.00234444
0.00204074
0.00173704
0.00143333
0.00112963
0.000825926
0.000522222
0.000218519
-8.51852E-05
-0.000388889
-0.000692593
-0.000996296
-0.0013
0.1
0.2
0.3
f(Hz)
FBI
25 Oct. 0700 EST
0.000941351
0.000825405
0.000709459
0.000593514
0.000477568
0.000361622
0.000245676
0.00012973
1.37838E-05
-0.000102162
-0.000218108
-0.000334054
-0.00045
angle (degrees)
270
225
180
135
90
0.05
0.1
0.15
0.2
0.25
0.3
0.35
f(Hz)
TSA
Oct. 0700 EST
0.0015224
0.0013272
0.001132
0.0009368
0.0007416
0.0005464
0.0003512
0.000156
-3.92E-05
-0.0002344
-0.0004296
-0.0006248
-0.00082
0.1
0.15
0.2
0.25
0.3
0.35
f(Hz)
DIA
25 Oct. 0700 EST
0.00046
0.00039
0.00032
0.00025
0.00018
0.00011
4E-05
-3E-05
-0.0001
-0.00017
-0.00024
0.2
0.3
f(Hz)

Error estimates: FRF



CONCLUSIONS

- DIA has difficulty in reproducing Snl
- Although DIA is calibrated to be similar to FBI in the low-frequency spectral region, the calibration is locally valid (~ γ=3.3)
- TSA appears more accurate than DIA for Currituck and waverider spectra from hurricane Wilma
- The present TSA can be extended to improve the B-scale treatment which improves TSA accuracy