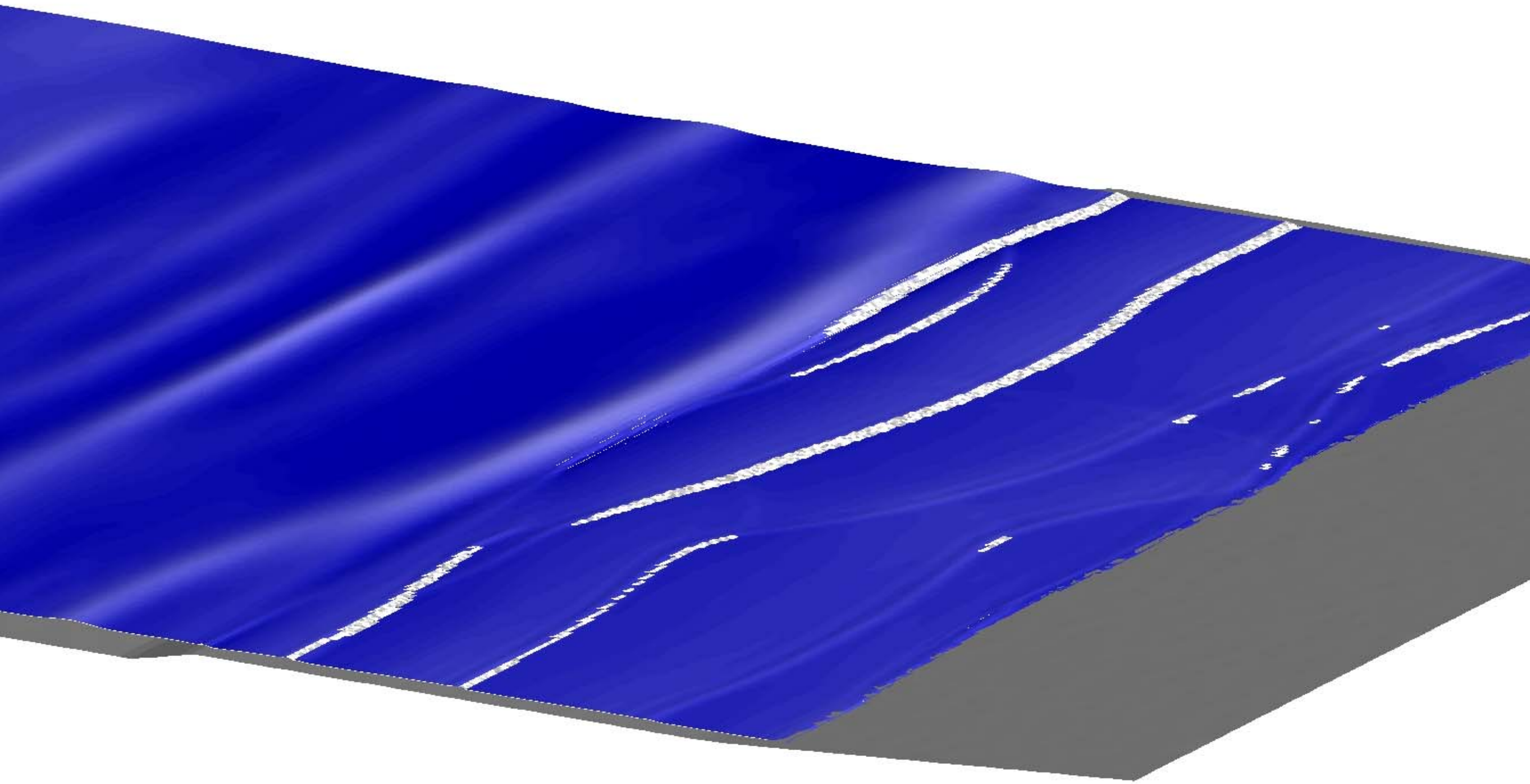
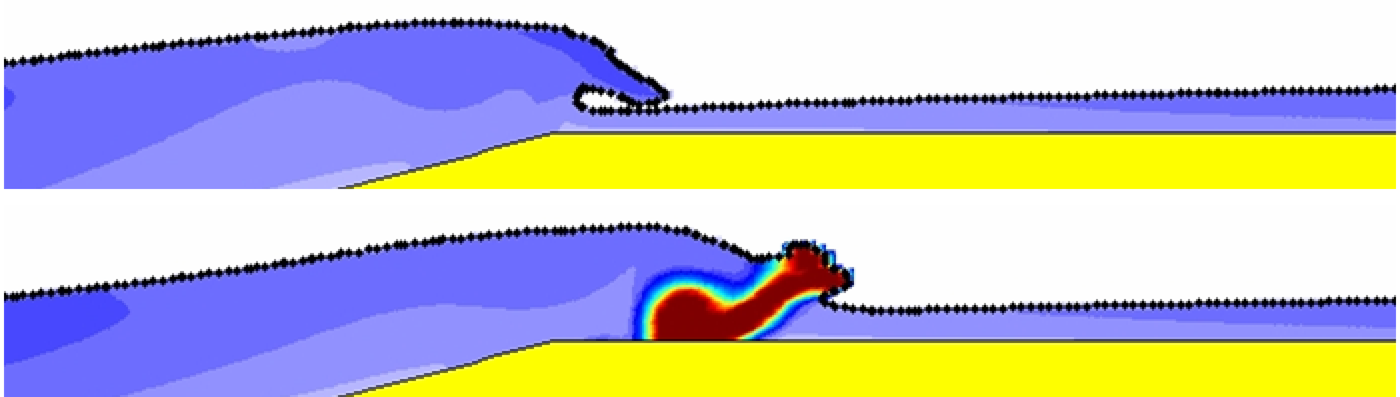
Phase-Resolving Simulation of Wave Evolution over a Shallow Shelf

Patrick Lynett, Texas A&M University



Motivation for Study

- Investigate the behavior of dissipation mechanisms in phase-resolving, time-domain simulation
  - Unidirectional and directional random waves over a reef
  - Breaking & bottom friction
  - Boussinesq & RANS (unidirectional) models



Methodology/Outline

- Introduce experimental data and numerical models (Boussinesq and 2DV RANS)
- Comparisons, focusing on:
  - Wave height
  - Mean water level
  - Spectral transformation
  - Peak period
  - Directionality
- Dependence of mean water level on friction factor

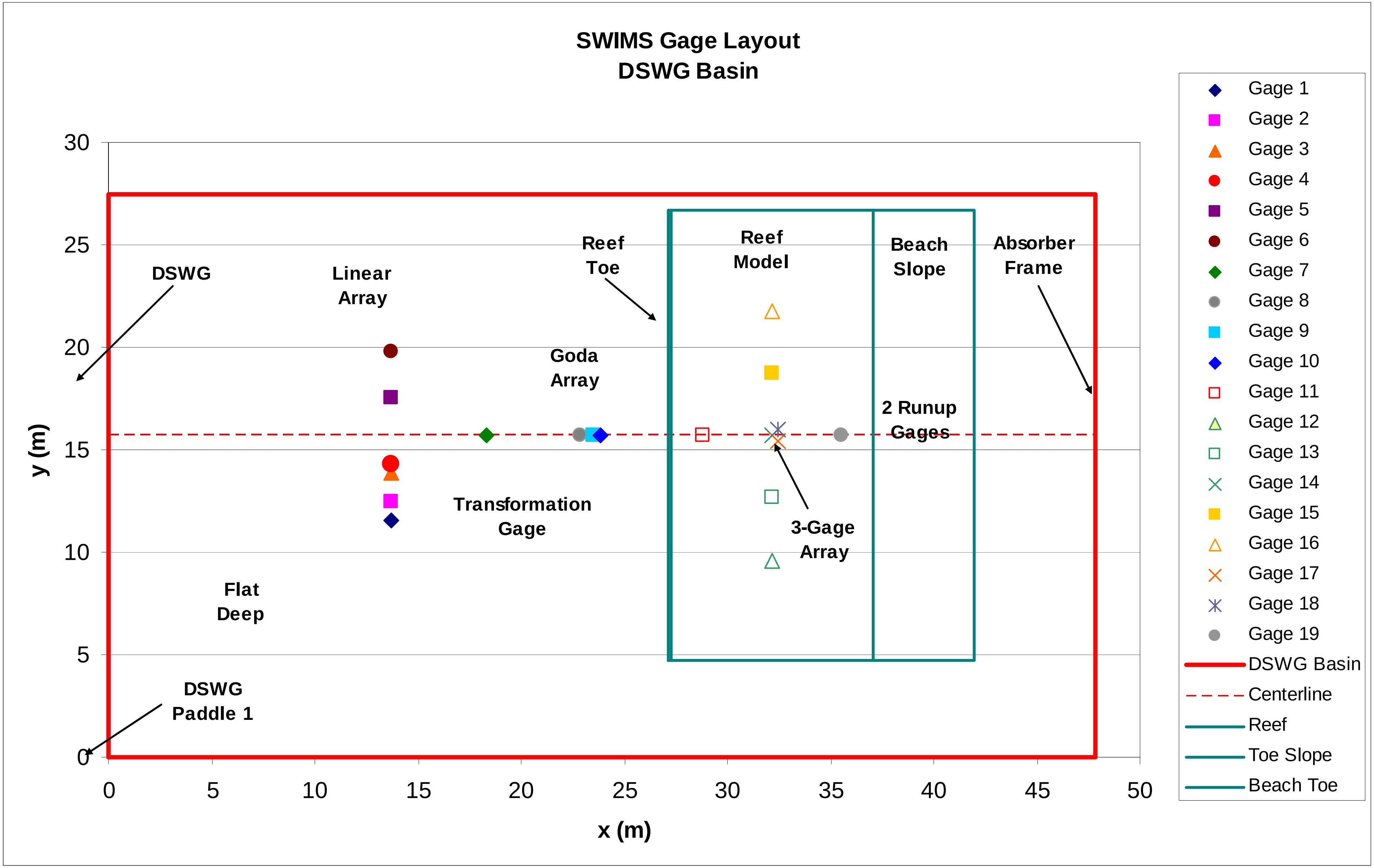
Summary of Conclusions

- If not interested in vertical and turbulent detail of the flow, Boussinesq provides equal accuracy to RANS
  - Wave height, water level, spectral transformation
- Shelf resonance is a dominant factor near the shoreline
  - Peak spectral period increases tenfold
- For the shallow reef, setup is proportional to bottom friction
  - Opposite pattern found with waves breaking up a constant slope
- Directionality plays a role in height and setup

Experimental Setup

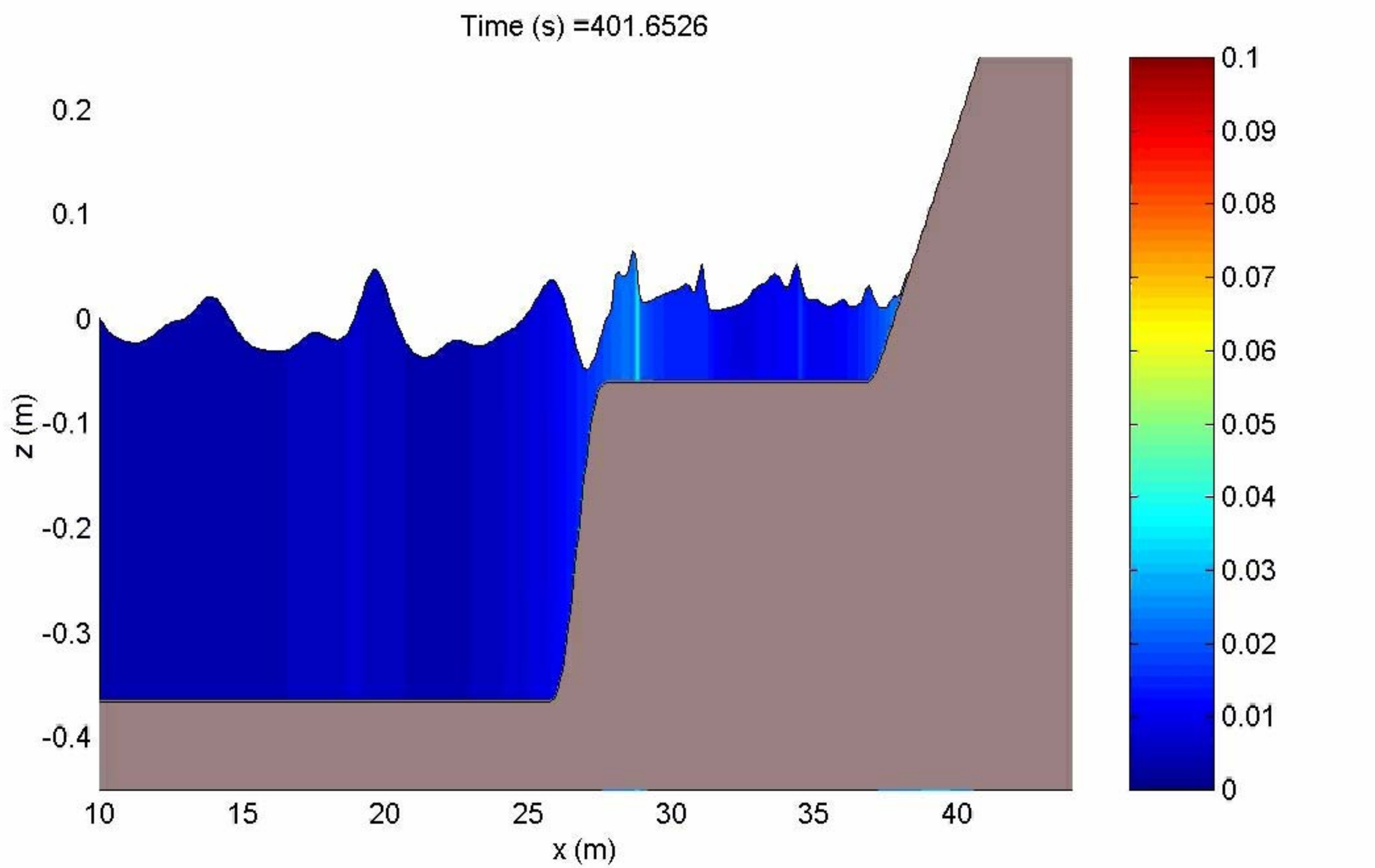
- Experiments performed by Don Ward *et al* at ERDC
  - Performed in 48m by 27m basin
  - Directional wavemaker
  - Model scale 1:25
  - Two reef configurations
    - 0.37 m “deep” water depth, 0.06 cm reef depth
    - 0.43 m “deep” water depth, 0.12 reef depth
  - Bottom is smooth everywhere
  - Waves
    - TMA spectrum unidirectional and 20 degree spread
    - Wave height 0.12 m
    - Peak period of 2s and 3s

Experimental Setup



Simulations & Comparisons – Unidirectional Waves

- Shelf depth=0.06 m, T_p=3s



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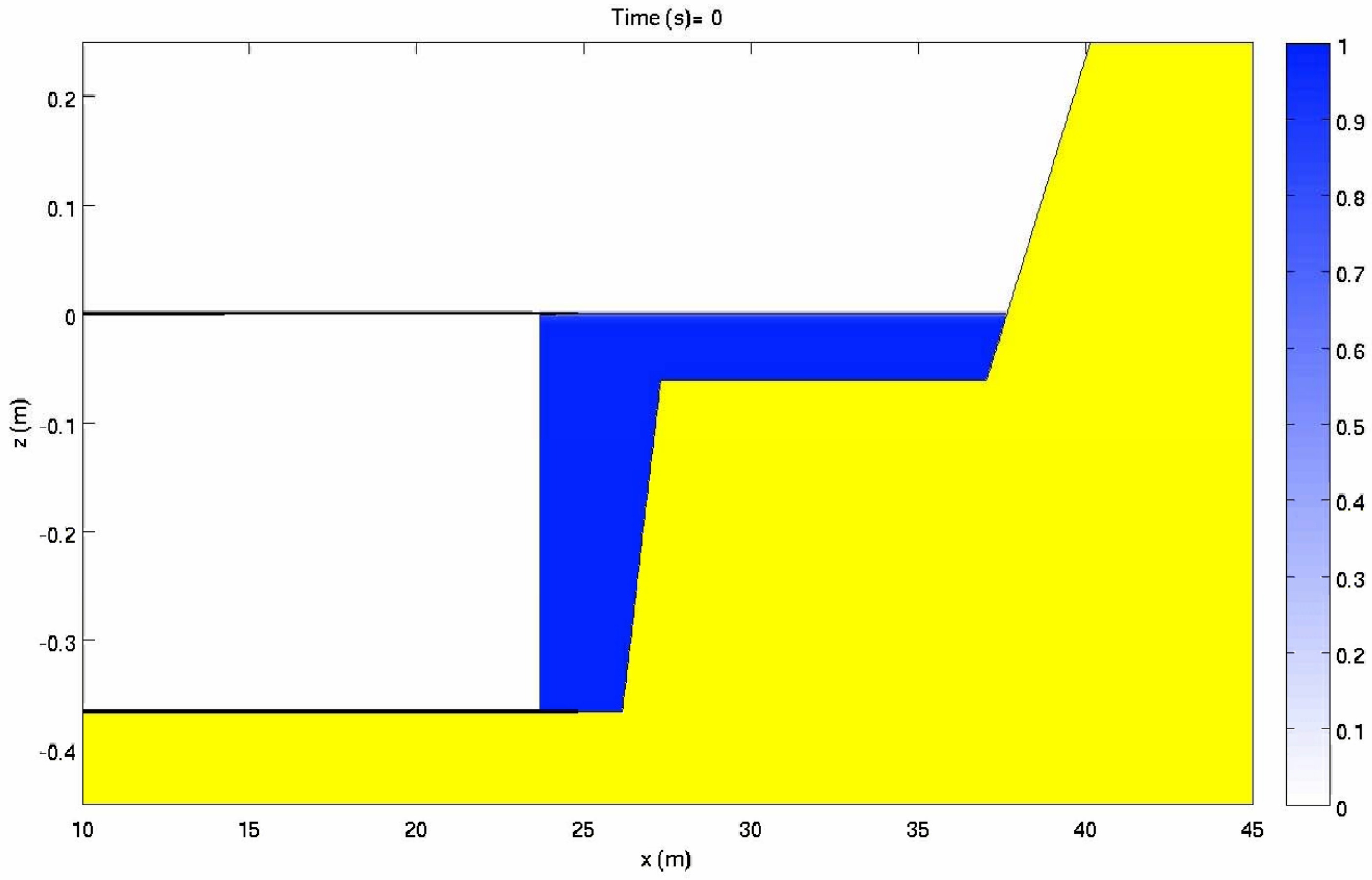

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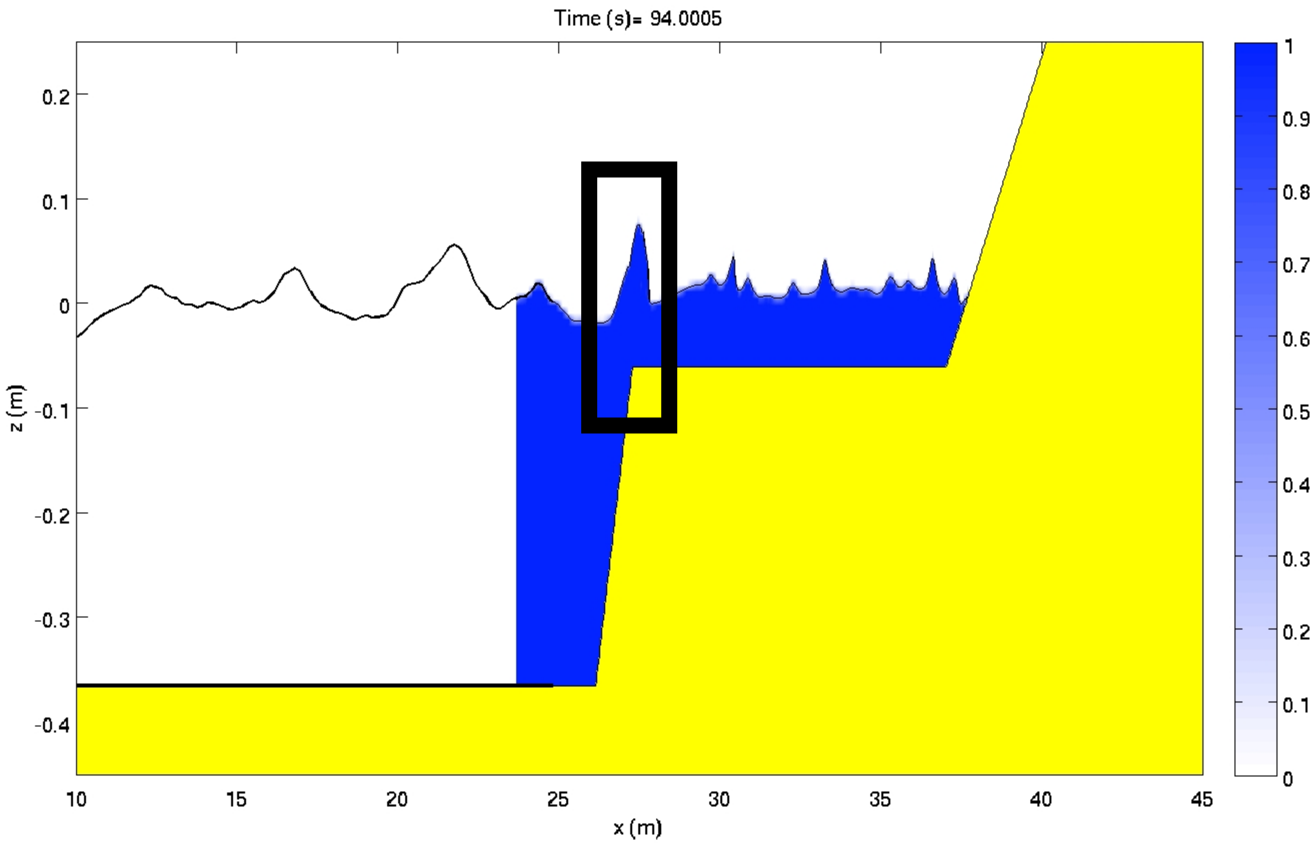

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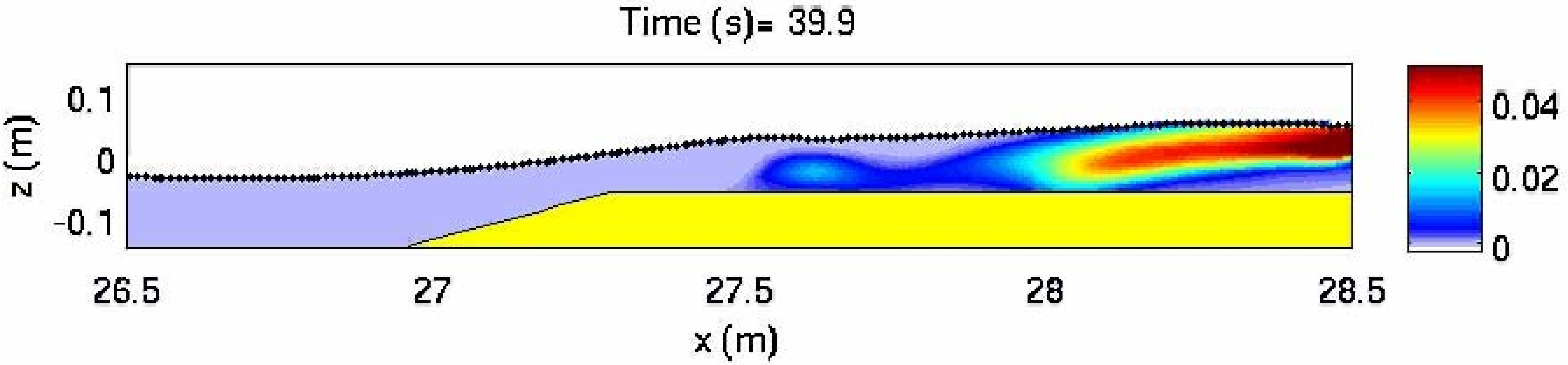
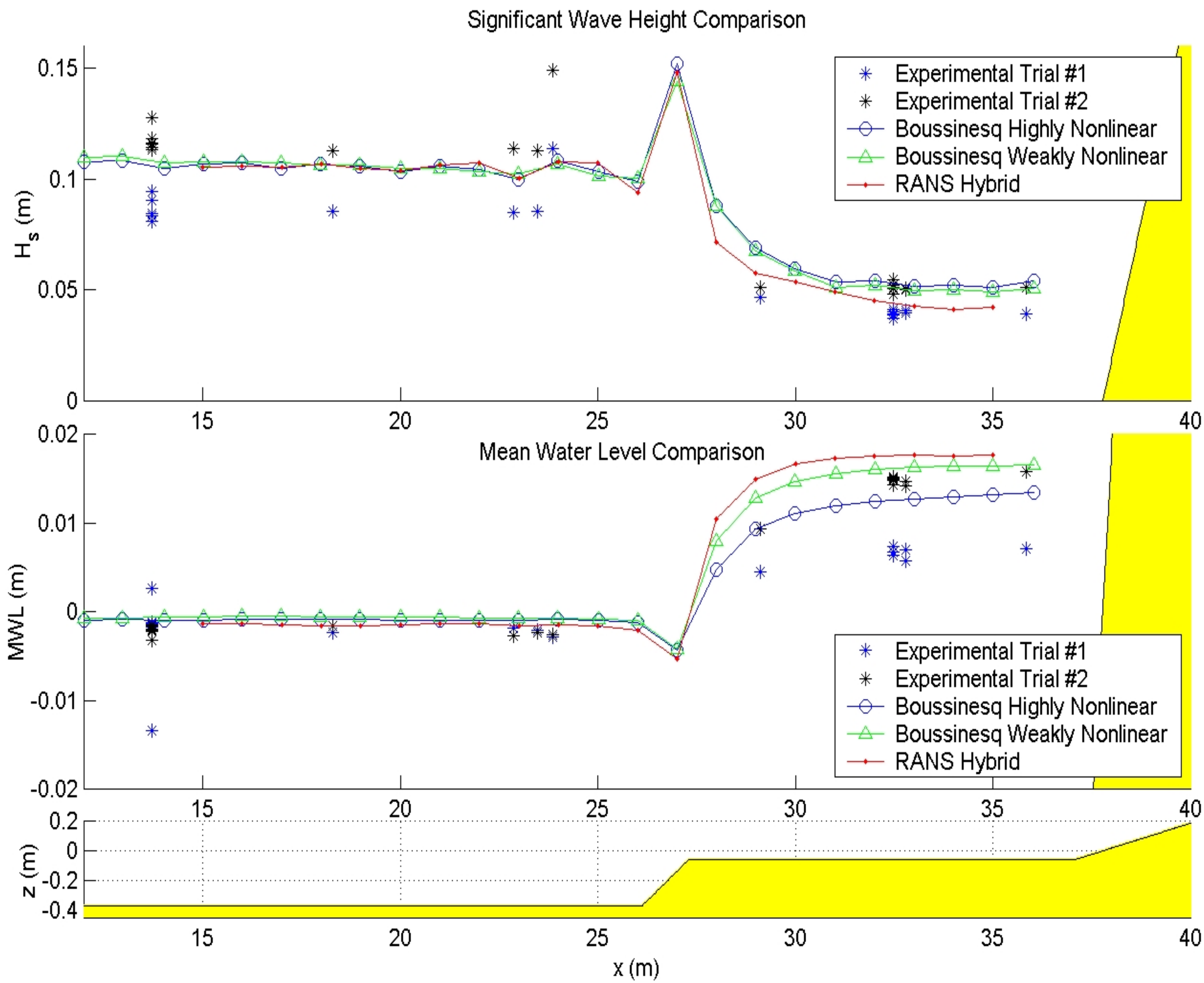
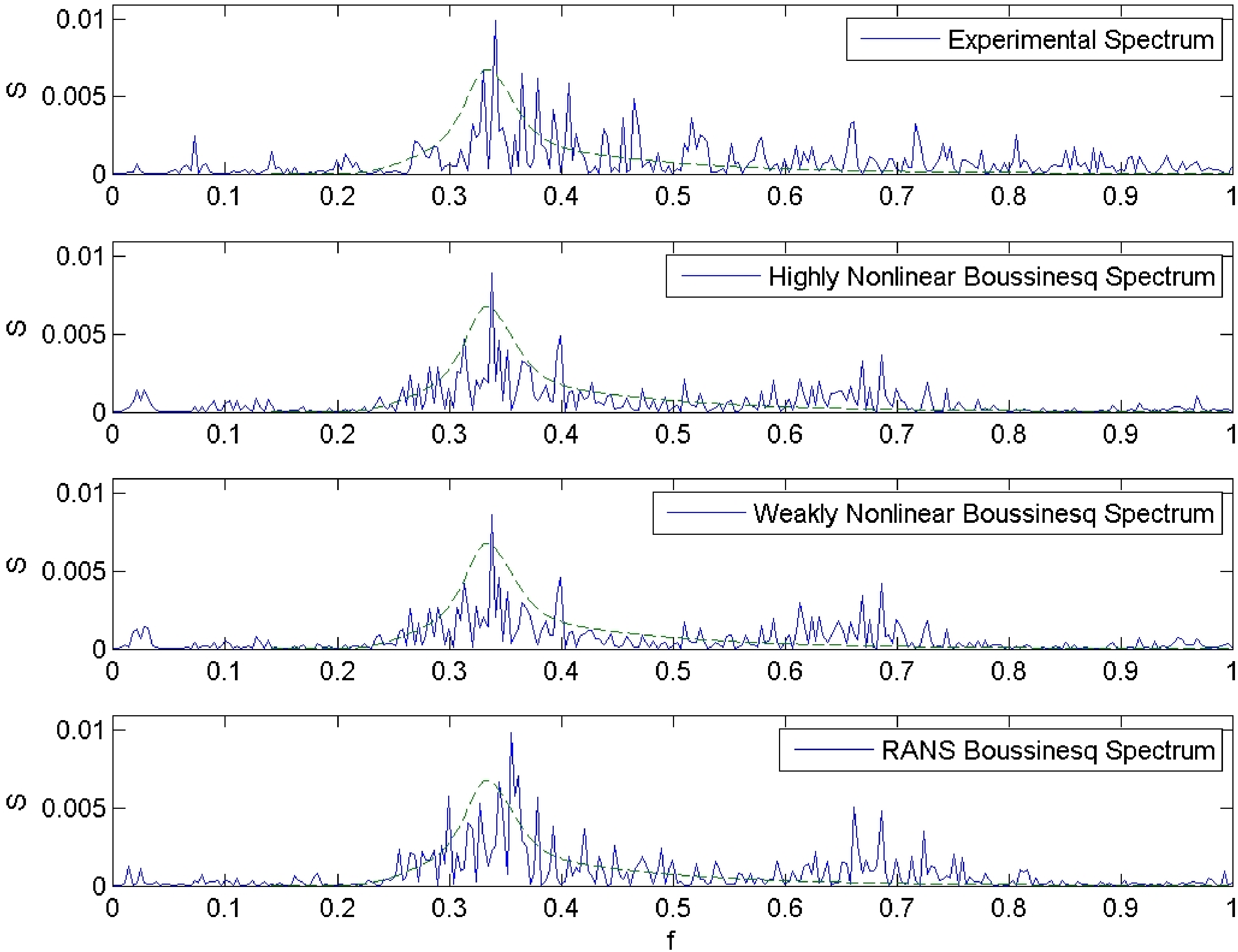


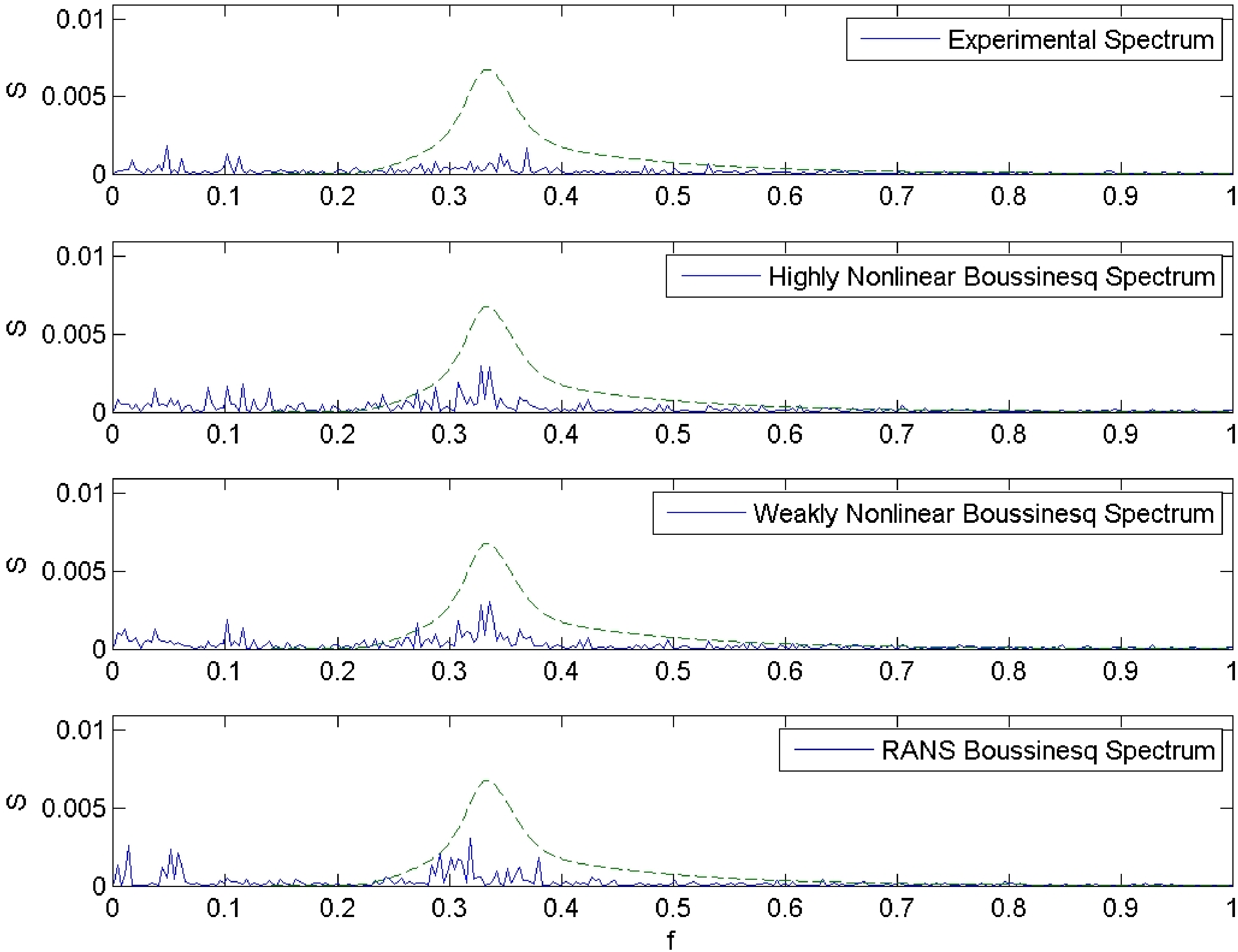


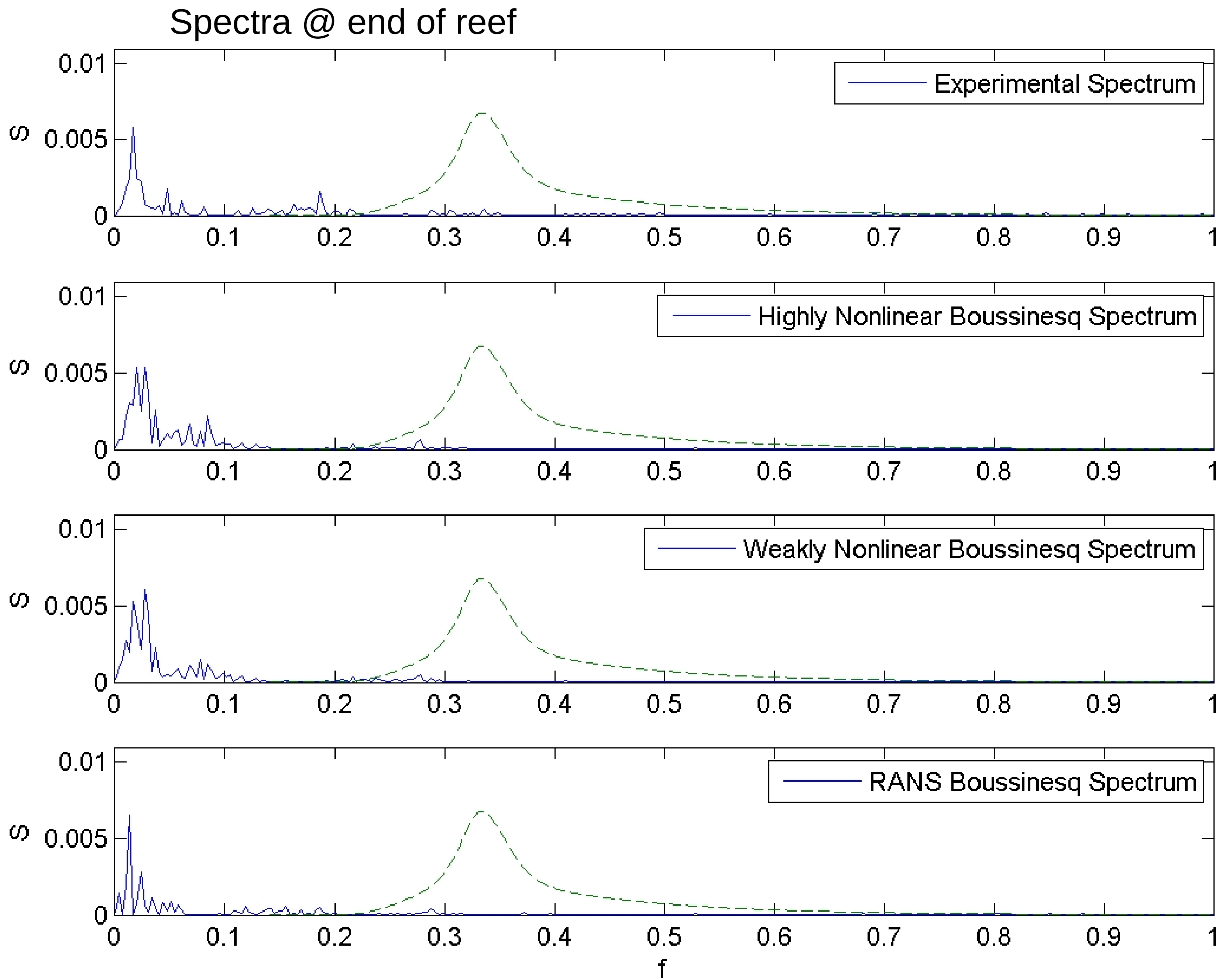
Significant Wave Height Comparison
Experimental Trial #1
Experimental Trial #2
Boussinesq Highly Nonlinear
Boussinesq Weakly Nonlinear
RANS Hybrid
H_s (m)
0.15
0.1
0.05
0
15
20
25
30
35
40
Mean Water Level Comparison
MWL (m)
0.02
0.01
0
-0.01
-0.02
z (m)
0.2
0
-0.2
-0.4
x (m)

Spectra @ 12 m (~ 2 peak wavelengths) from wavemaker

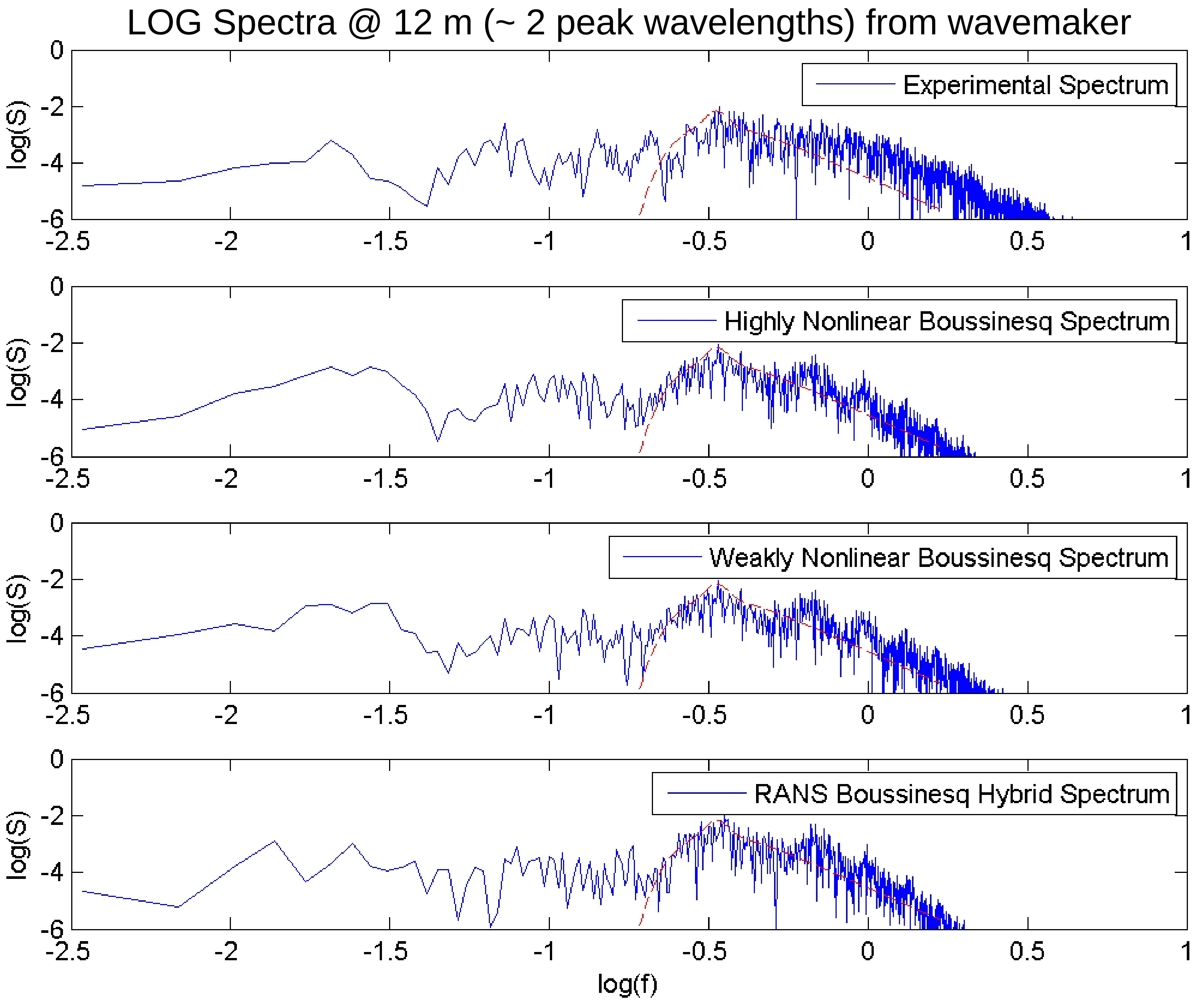


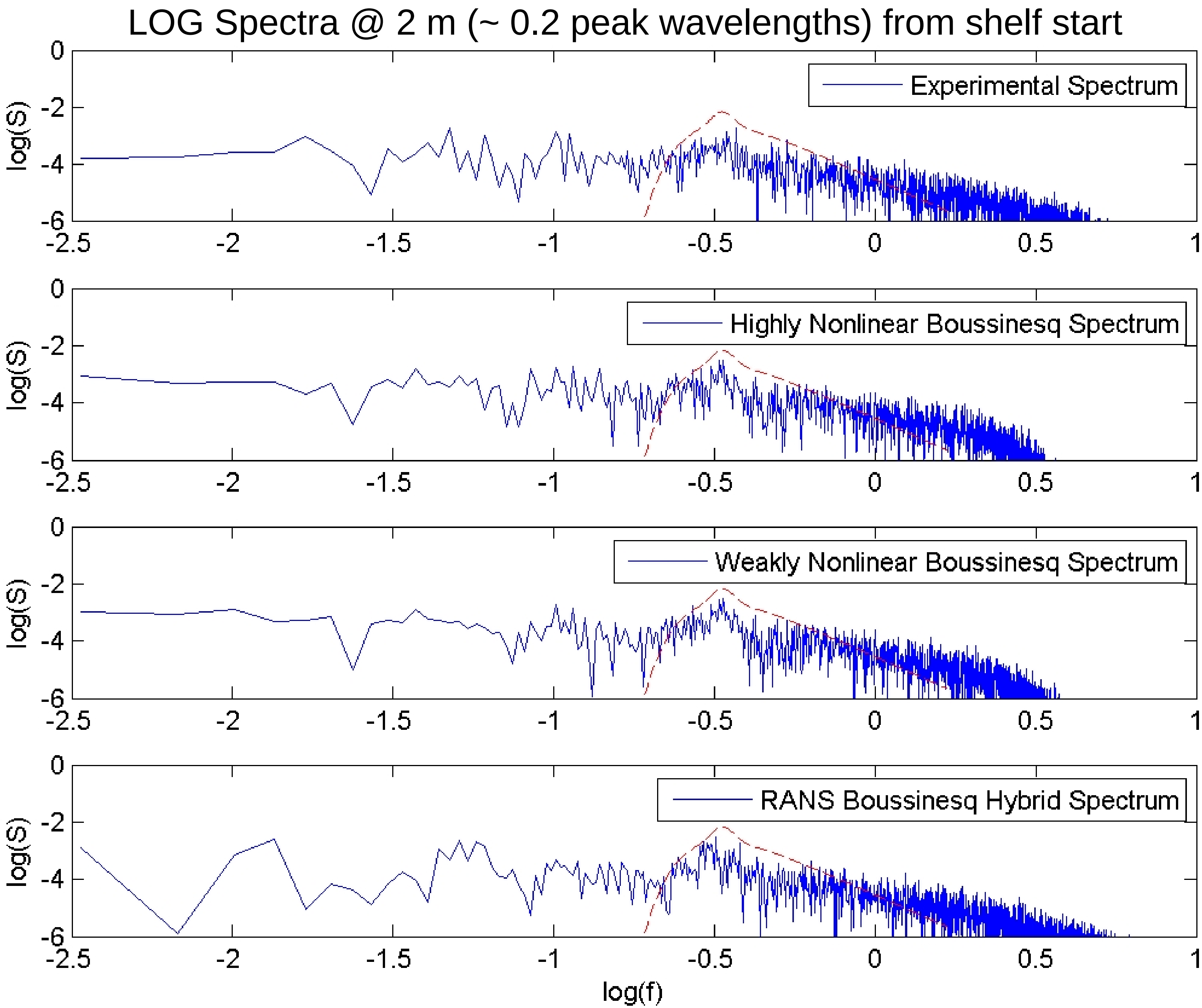
Spectra @ 2 m (~ 0.2 peak wavelengths) from shelf start





Spectra @ end of reef
Experimental Spectrum
Highly Nonlinear Boussinesq Spectrum
Weakly Nonlinear Boussinesq Spectrum
RANS Boussinesq Spectrum
S
f
0.01
0.005
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


LOG Spectra @ 12 m (~ 2 peak wavelengths) from wavemaker
Experimental Spectrum
Highly Nonlinear Boussinesq Spectrum
Weakly Nonlinear Boussinesq Spectrum
RANS Boussinesq Hybrid Spectrum
log(S)
log(f)
0
-2
-4
-6
-2.5
-2
-1.5
-1
-0.5
0
0.5
1



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0
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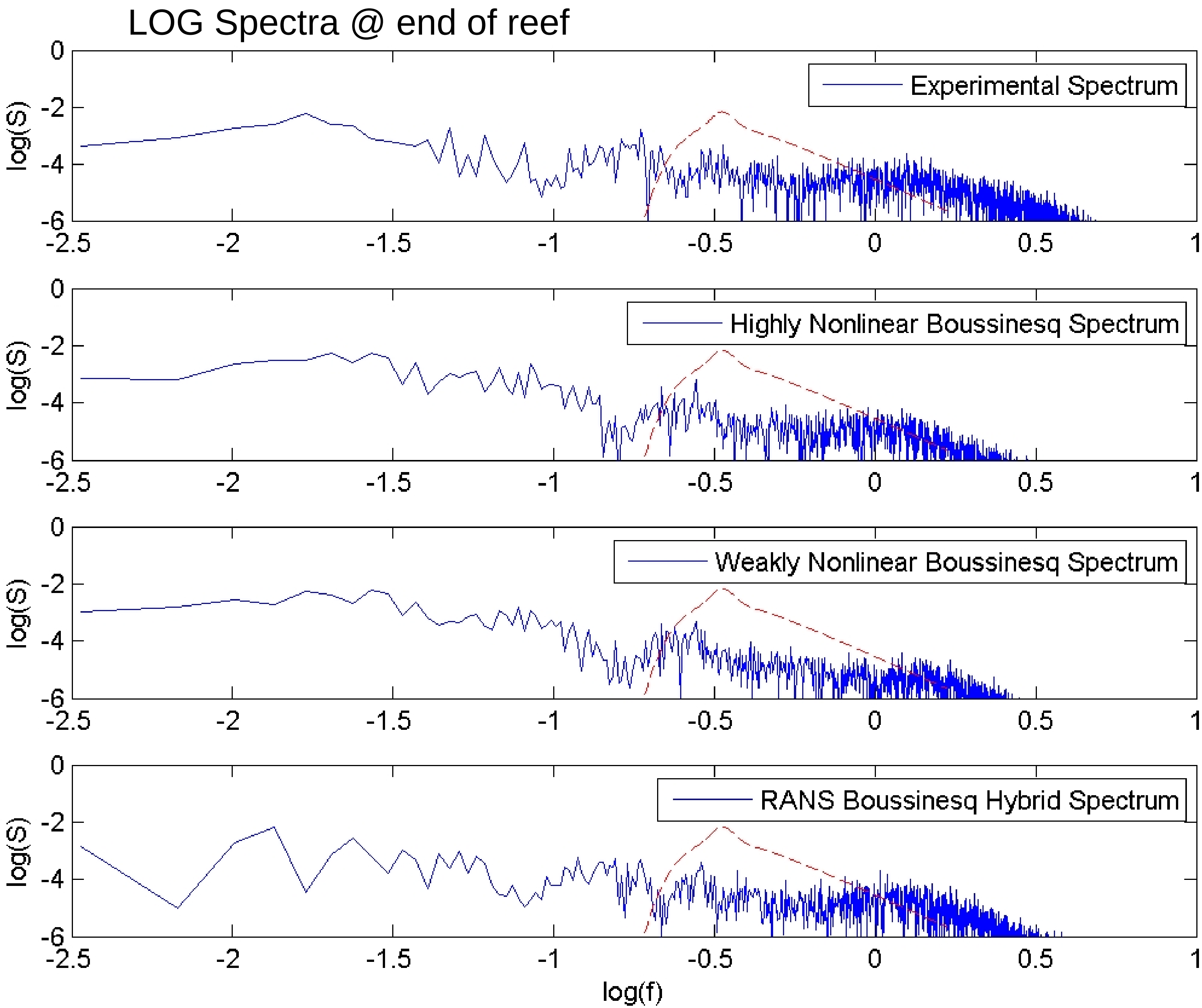
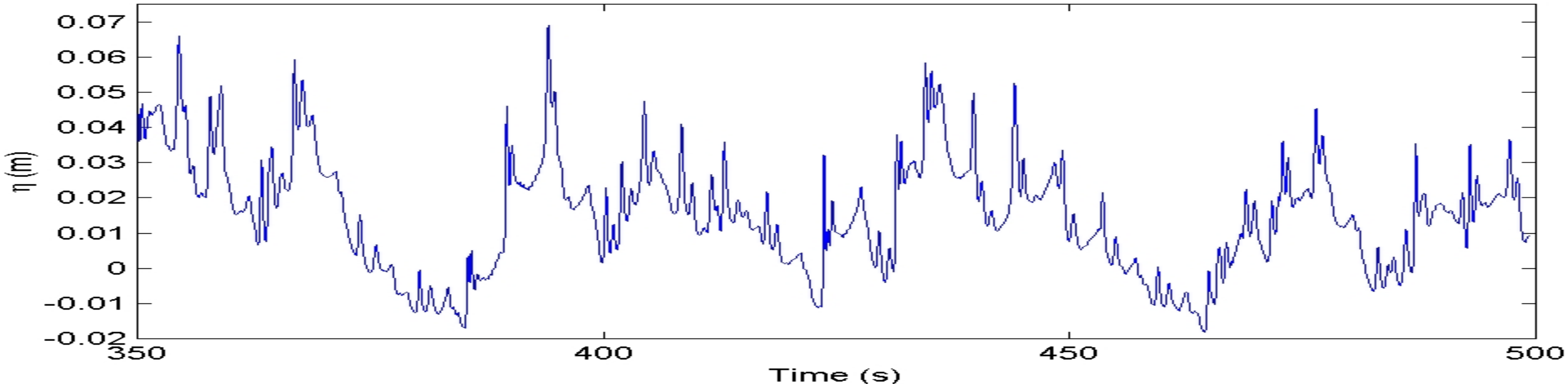
0
0.5
1



LOG Spectra @ end of reef
Experimental Spectrum
Highly Nonlinear Boussinesq Spectrum
Weakly Nonlinear Boussinesq Spectrum
RANS Boussinesq Hybrid Spectrum
log(S)
log(f)
0
-2
-4
-6
-2.5
-2
-1.5
-1
-0.5
0
0.5
1

Unidirectional Waves

- Shelf depth=0.06 m, T_p=3s
  - Spectral peak period shifts from 3s (incident) [15s prototype] to ~45s (end of reef) [3.8 min prototype]
    - Mean period from zero-crossing at end of reef =3s
    - Long period motion matches the fundamental resonance frequency of the shelf
      - Wavelength of 45s period on shelf ~38m
      - Shelf length ~10m



Unidirectional Waves

- Shelf depth=0.06 m, T_p=3s
  - H_{mo} at the end of the reef = 5.2 cm
    - H_{mo}/h=0.85
    - $H_{mo}/(h+setup)$=0.72
  - H_s from zero-crossing at end of reef = 4.1 cm
    - H_s/h=0.67
    - $H_s/(h+setup)$=0.57
  - H_{mo} at the end of reef, if beach is replaced by a 100% absorbing boundary = 2.8 cm
    - H_{mo}/h=0.47
    - $H_{mo}/(h+setup)$=0.45
    - Similar #'s from zero-crossing

Unidirectional Waves - Friction

$$\frac{\partial \boldsymbol{u}_1}{\partial t} + \frac{\varepsilon_o}{2}\nabla(\boldsymbol{u}_1 \cdot \boldsymbol{u}_1) + \nabla\zeta + \mu_1^2\{\ldots\} - \boldsymbol{R}_b + \boldsymbol{R}_f = 0$$

- Bottom friction with a quadratic drag law
- Breaking dissipation, ***R**$_b$*, following Kennedy *et al*. (2000)

$$\boldsymbol{R}_f = \frac{f}{h+\zeta}\boldsymbol{u}_b|\boldsymbol{u}_b|$$

$$R_{bx} = \frac{1}{H}\left(\left[\nu\left(Hu_1\right)_x\right]_x + 0.5\left[\nu\left(Hu_1\right)_y + \nu\left(Hv_1\right)_x\right]_y\right)$$

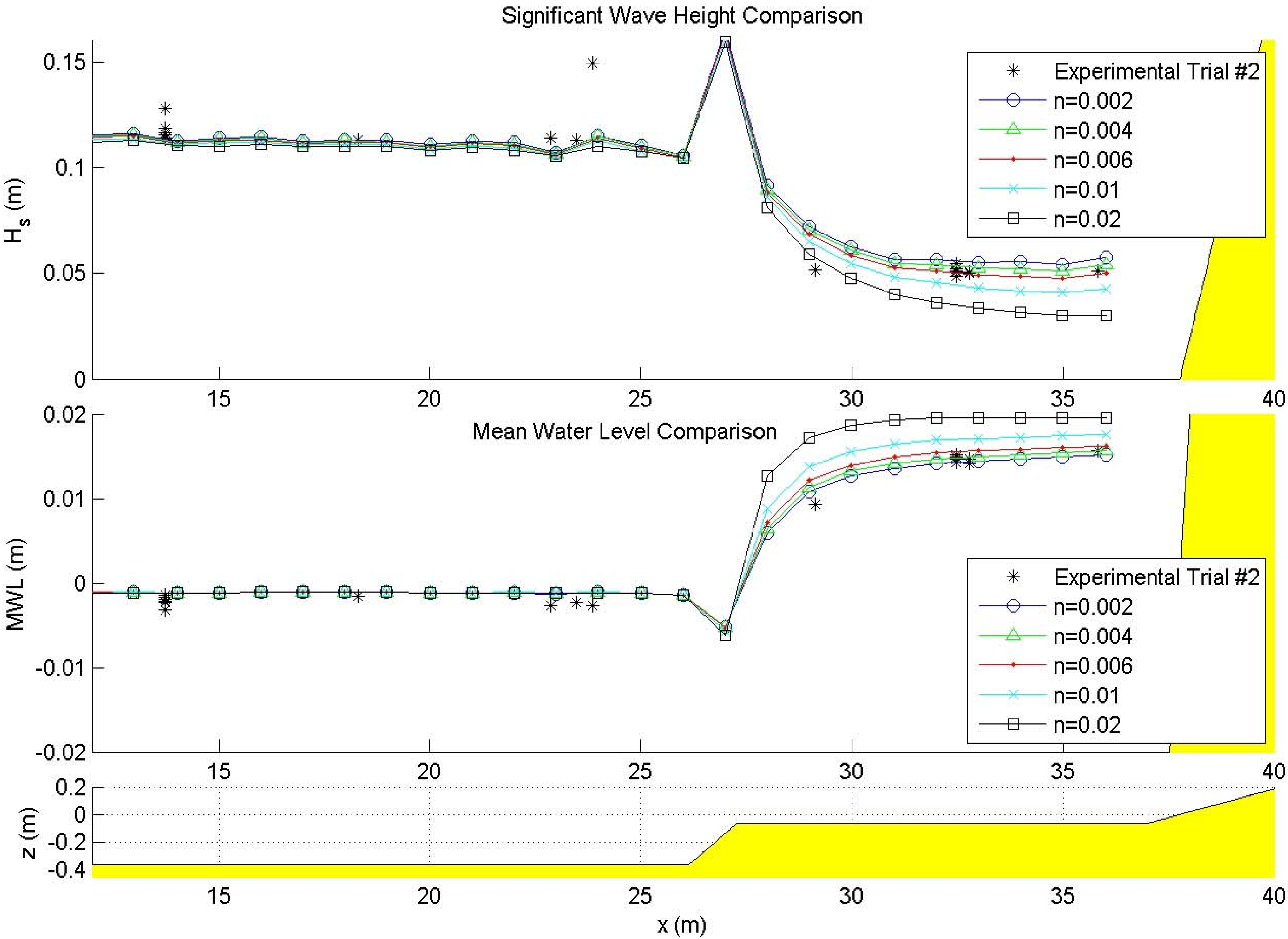
$$R_{by} = \frac{1}{H}\left(\left[\nu\left(Hv_1\right)_y\right]_x + 0.5\left[\nu\left(Hu_1\right)_y + \nu\left(Hv_1\right)_x\right]_x\right)$$

$$\nu = B\delta^2 H\zeta_t$$

- Use Mannings friction:

$$f = {8gn^2}\Big/{H_{total}^{0.333}}$$

- For reference with models using friction factors that do not carry the 8 in the numerator above, multiply the friction factors here by 2.8 for an equivalent value



Significant Wave Height Comparison
Experimental Trial #2
n=0.002
n=0.004
n=0.006
n=0.01
n=0.02
H_s (m)
0.15
0.1
0.05
0
15
20
25
30
35
40
Mean Water Level Comparison
MWL (m)
0.02
0.01
0
-0.01
-0.02
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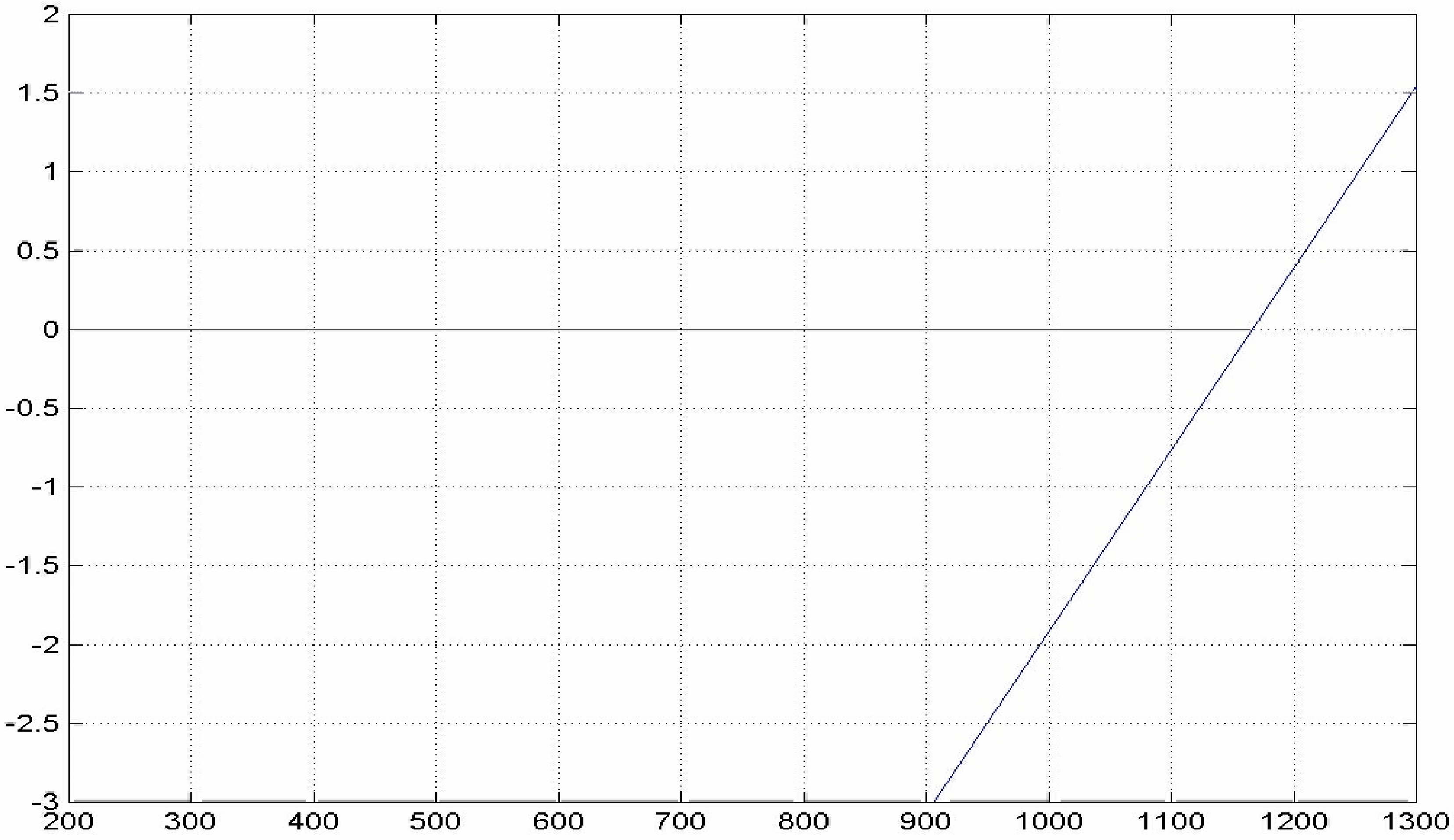
n=0.002
n=0.004
n=0.006
n=0.01
n=0.02
z (m)
0.2
0
-0.2
-0.4
x (m)

Unidirectional Waves - Friction

- Waves breaking on a constant slope



Unidirectional Waves - Friction

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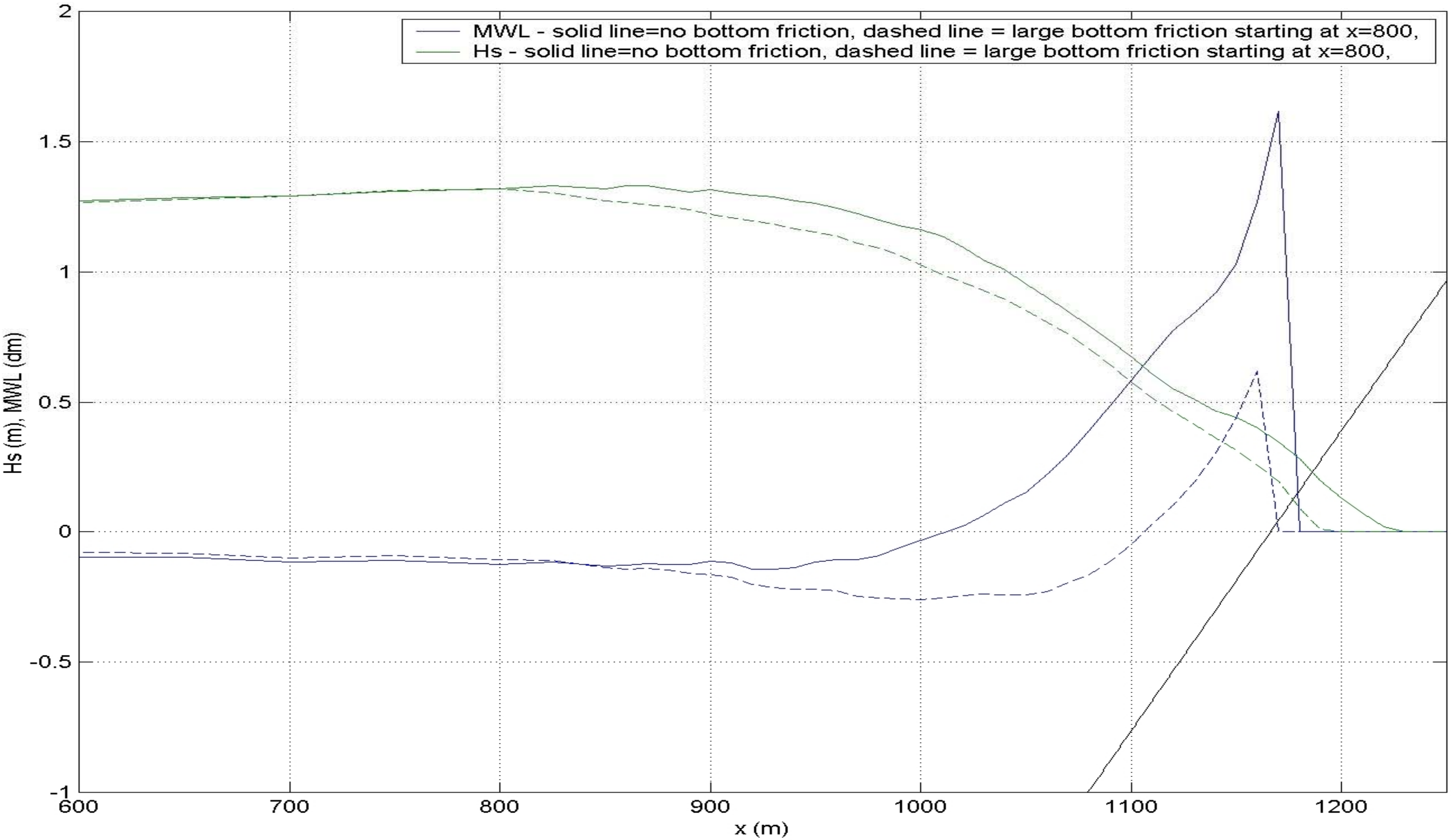
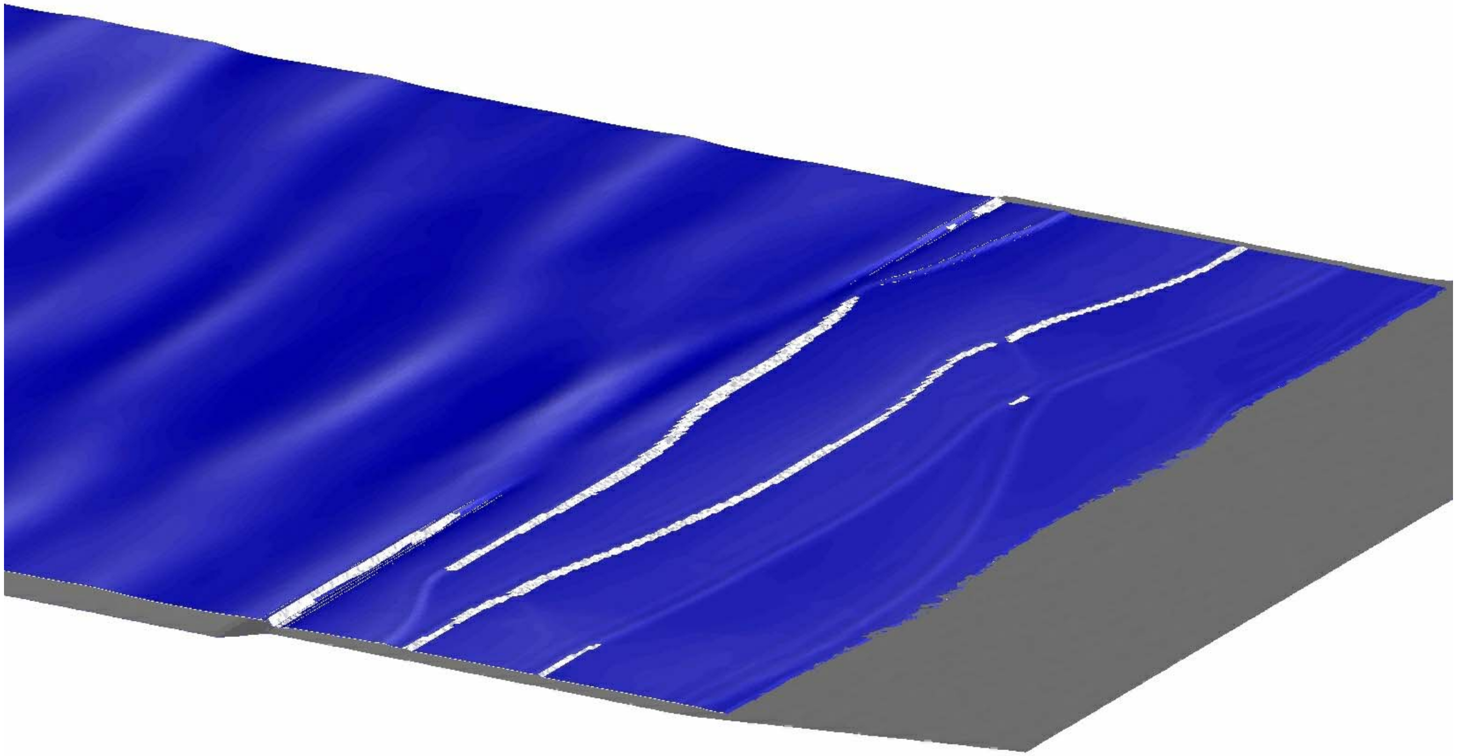
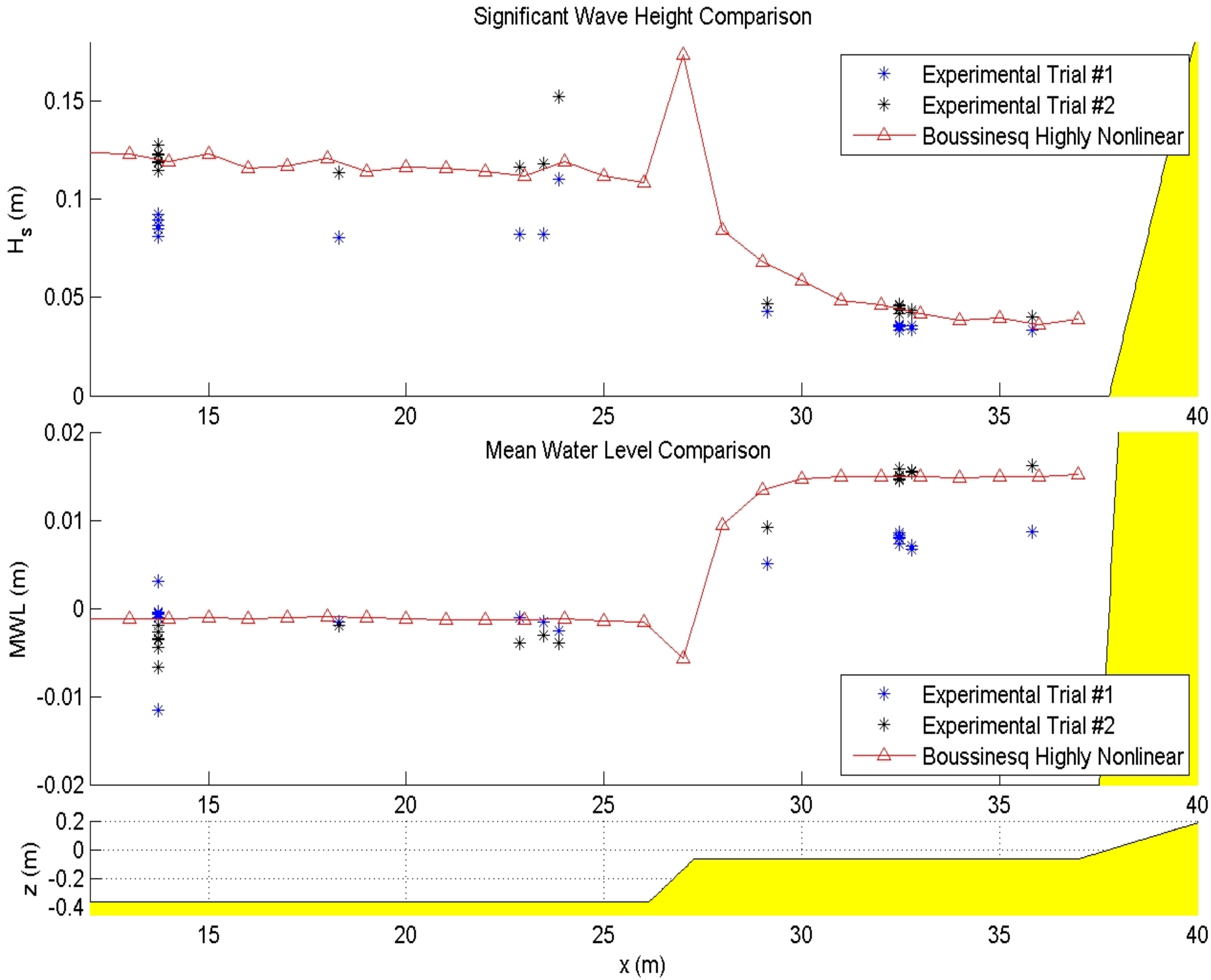


Directional Waves

- Shelf depth=0.06 m, T_p=3s





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-0.2
-0.4
x (m)

Unidirectional vs Directional

- Shelf resonance not as significant in the directional cases
  - H_{mo}'s at the end of the reef are less with directional sea, by 5%-25%
  - Setup at the end of the reef is greater with directional sea, by ~0-10%
- In the experiments (and simulations) directionality is "squished" by the side walls and the formation of mach stems

Conclusions

- If not interested in vertical and turbulent detail of the flow, Boussinesq provides equal accuracy to RANS
  - Wave height, water level, spectral transformation
- Shelf resonance is a dominant factor near the shoreline
  - Peak spectral period increases tenfold
- For the shallow reef, setup is proportional to bottom friction
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