Freak Waves and Wave Breaking - Catastrophic Events in Ocean

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There are two types of rare catastrophic events on the ocean surface:

1. Freak waves (major catastrophic event)
2. Wave breaking (minor catastrophic event)

Freak waves are responsible for ship-wreaking, loss of boats, cargo and lives. Wave breaking is the most important mechanism of wave energy dissipation and for transport of momentum from wind to ocean.

Analytic theory is for both of these are not developed

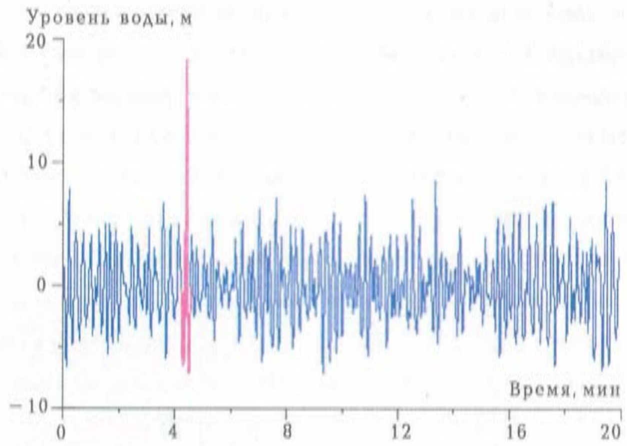


Рис. 1.9. «Новогодняя волна», зарегистрированная в Северном море 1 января 1995 г. [106]

“New Year” wave – 1995 year

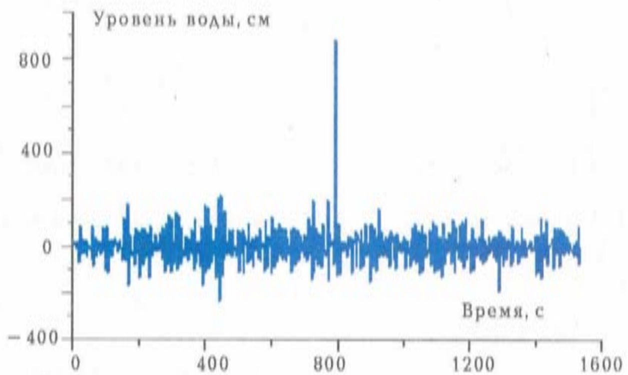
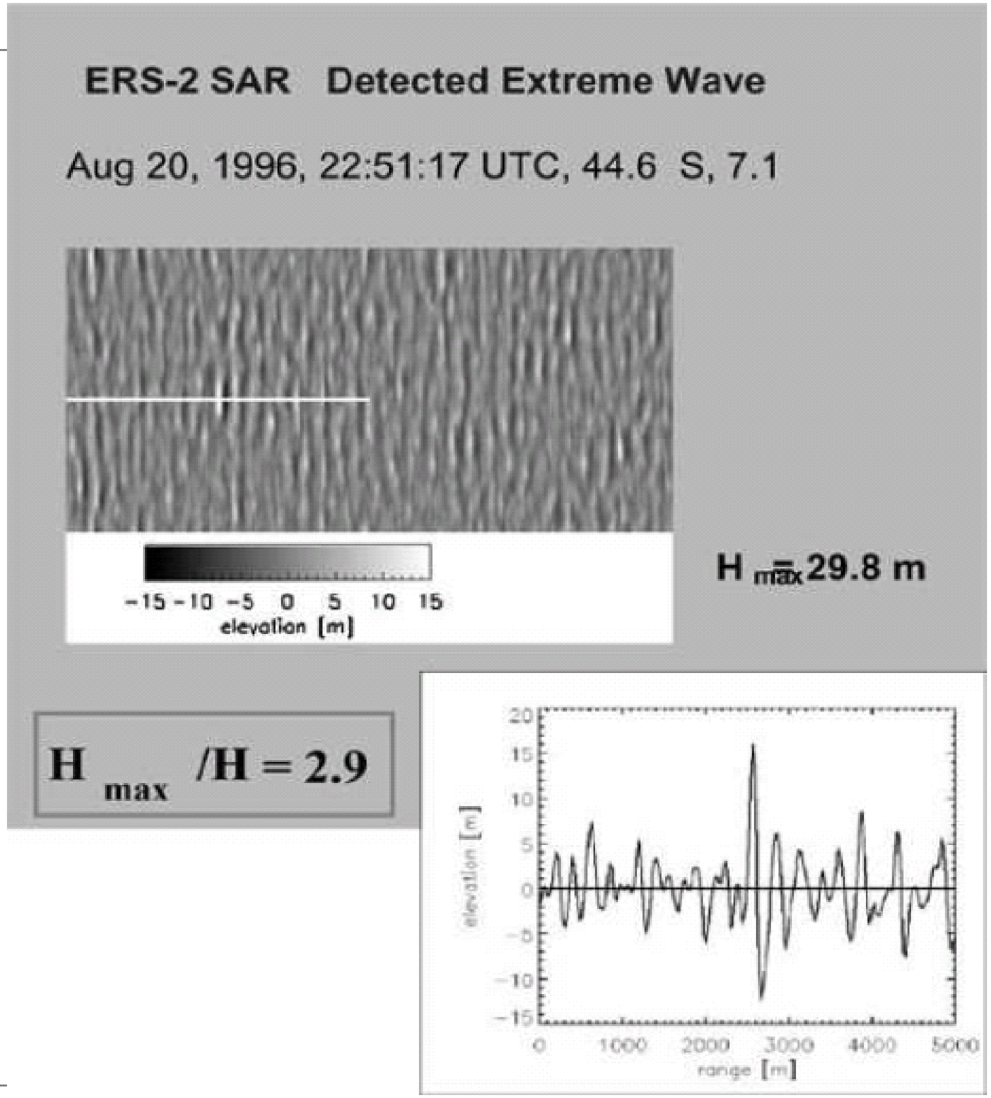
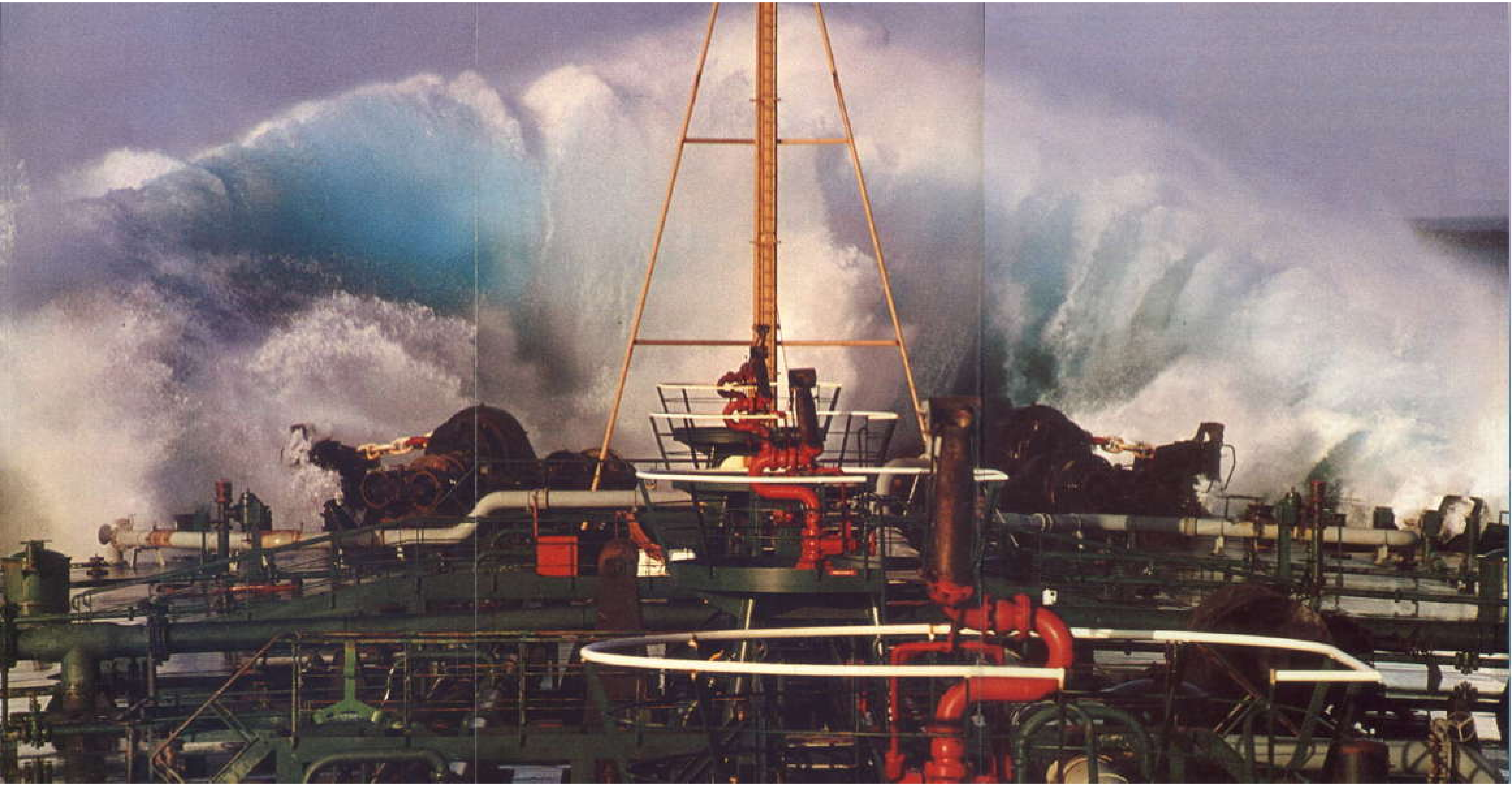


Рис. 1.10. Аномальная волна, зарегистрированная с буя в Черном море 22 ноября 2001 г. [12, 30]

Extreme wave in the Black sea – 2002 year

Satellite Radar



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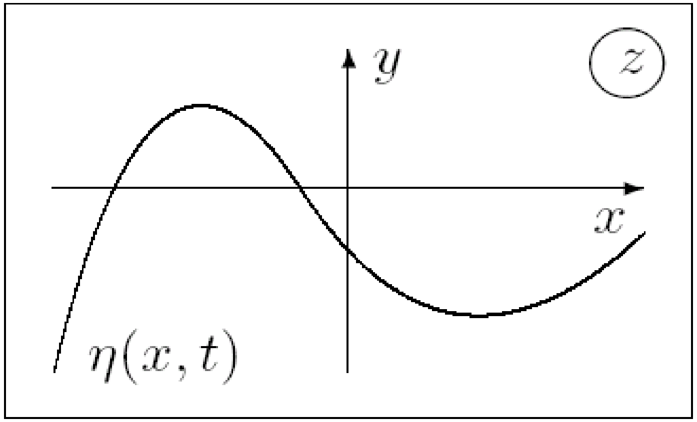
Equations



potential irrotational flow

$$\triangle\phi(x,y,t)=0$$

Boundary conditions:

$$\left[\begin{array}{l}\frac{\partial\phi}{\partial t}+\frac{1}{2}|\nabla\phi|^2+g\eta=\frac{P}{\rho},\\ \frac{\partial\eta}{\partial t}+\eta_x\phi_x=\phi_y\end{array}\right| \text{ at } y=\eta(x,t).$$

Conformal Mapping

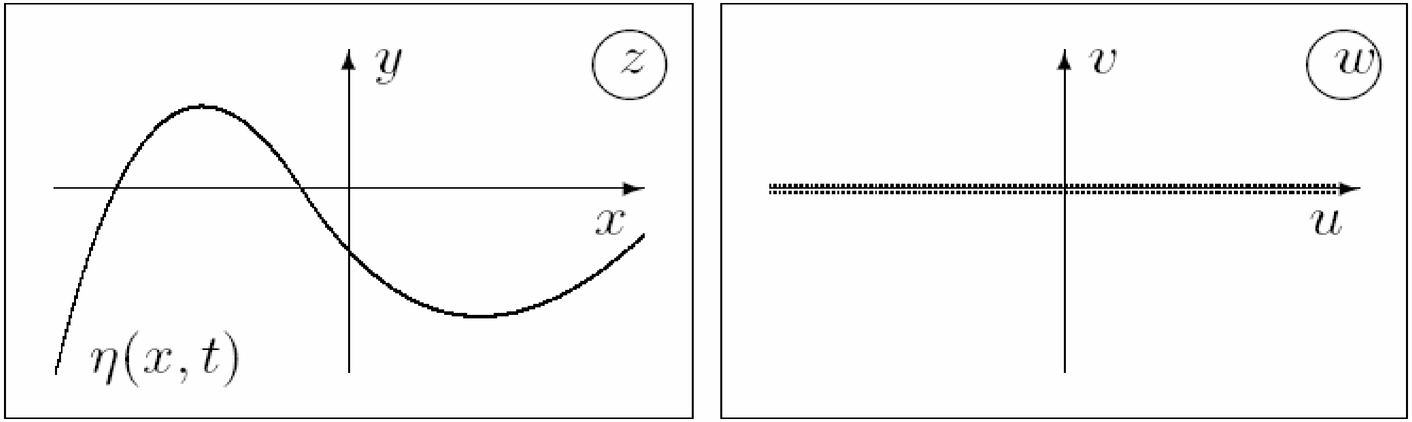
of the domain on the plane $z = x + iy$,

$$-\infty < x < \infty, \quad -\infty < y \leq \eta(x,t)\,,$$

to the lower half-plane,

$$-\infty < u < \infty, \quad -\infty < v \leq 0\,,$$

on the plane $\omega = u + iv$.



Conformal Mapping

After this mapping, the surface profile is given parametrically by

$$y = y(u,t), \quad x = u + \tilde{x}(u,t)\,.$$

Functions y and $\tilde{x}$ are coupled by the relations:

$$y = \hat{H}\tilde{x} \qquad\qquad \tilde{x} = -\hat{H}y\,.$$

Here $\hat{H}$ is the Hilbert transformation,

$$\hat{H}f(u) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{f(u')}{(u'-u)} du'\,.$$

For Fourier harmonics $y_k = i\text{sign}(k) x_k$.

Explicit Equations

If *conformal mapping* has been applied then it is naturally introduce complex analytic functions

$$Z = x + iy, \quad \text{and complex velocity potential} \quad \Phi = \Psi + \hat{H}\Psi.$$

$$\begin{aligned} Z_t &= iUZ_u, \\ \Phi_t &= iU\Phi_u - \hat{P}\left(\frac{|\Phi_u|^2}{|Z_u|^2}\right) + ig(Z - u). \end{aligned}$$

U is a complex transport velocity:

$$U = \hat{P}\left(\frac{-\hat{H}\Psi_u}{|Z_u|^2}\right). \qquad u \longrightarrow w$$

Projector operator $\hat{P}(f) = \frac{1}{2}(1 + i\hat{H})(f)$.

Cubic Equations

It turned out, that the equations can be simplified just by changing variables. Introduce instead of $Z(w,t)$ and $\Phi(w,t)$ another analytic functions $R(w,t)$ and $V(w,t)$

$$R = \frac{1}{Z_w}, \qquad \Phi_w = -iVZ_w.$$

$$\begin{aligned} R_t &= i\left[UR' - U'R\right], \\ V_t &= i\left[UV' - R\hat{P}(V\bar{V})'\right] + g(R-1). \end{aligned}$$

Complex transport velocity U is defined via $\hat{P}$

$$U = \hat{P}(V\bar{R} + \bar{V}R).$$

Setup the problem

* The shape of Stokes progressive wave is given by:

$$y = \frac{c^2}{2g}(1 - \frac{1}{|Z_u|^2}),$$

while Φ is related to the surface as

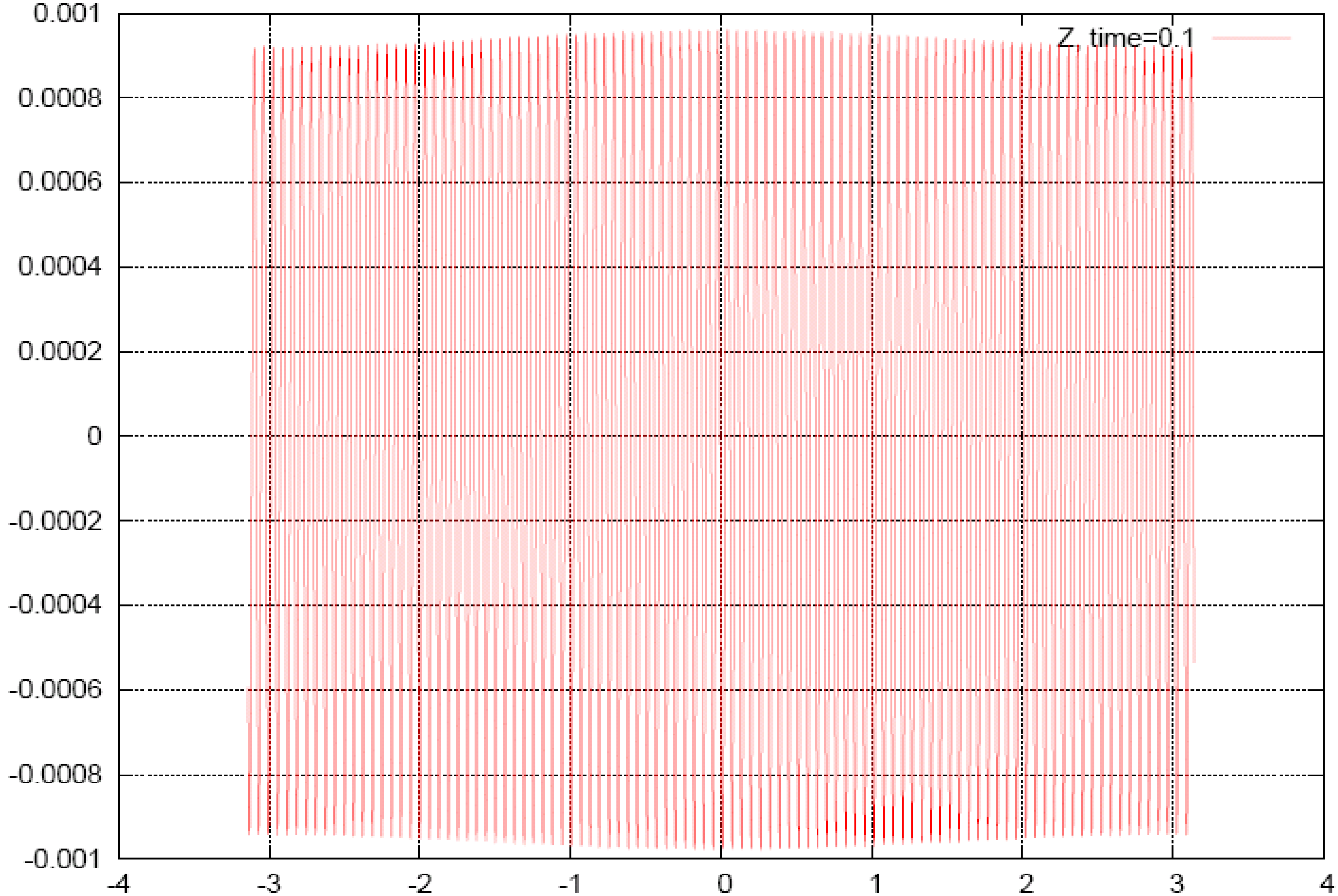
$$\Phi = -c(Z - u), \qquad\qquad V = ic(R - 1).$$

The amplitude of the wave $\frac{h}{L}$ is the parameter for I.C.,

For the sharp peaked limiting wave $\frac{h}{L} \simeq 0.141.$

100 waves $\mu = 0.095$

* Put 100 such waves with small perturbation in the periodic domain of 2π



100 waves $\mu = 0.095$

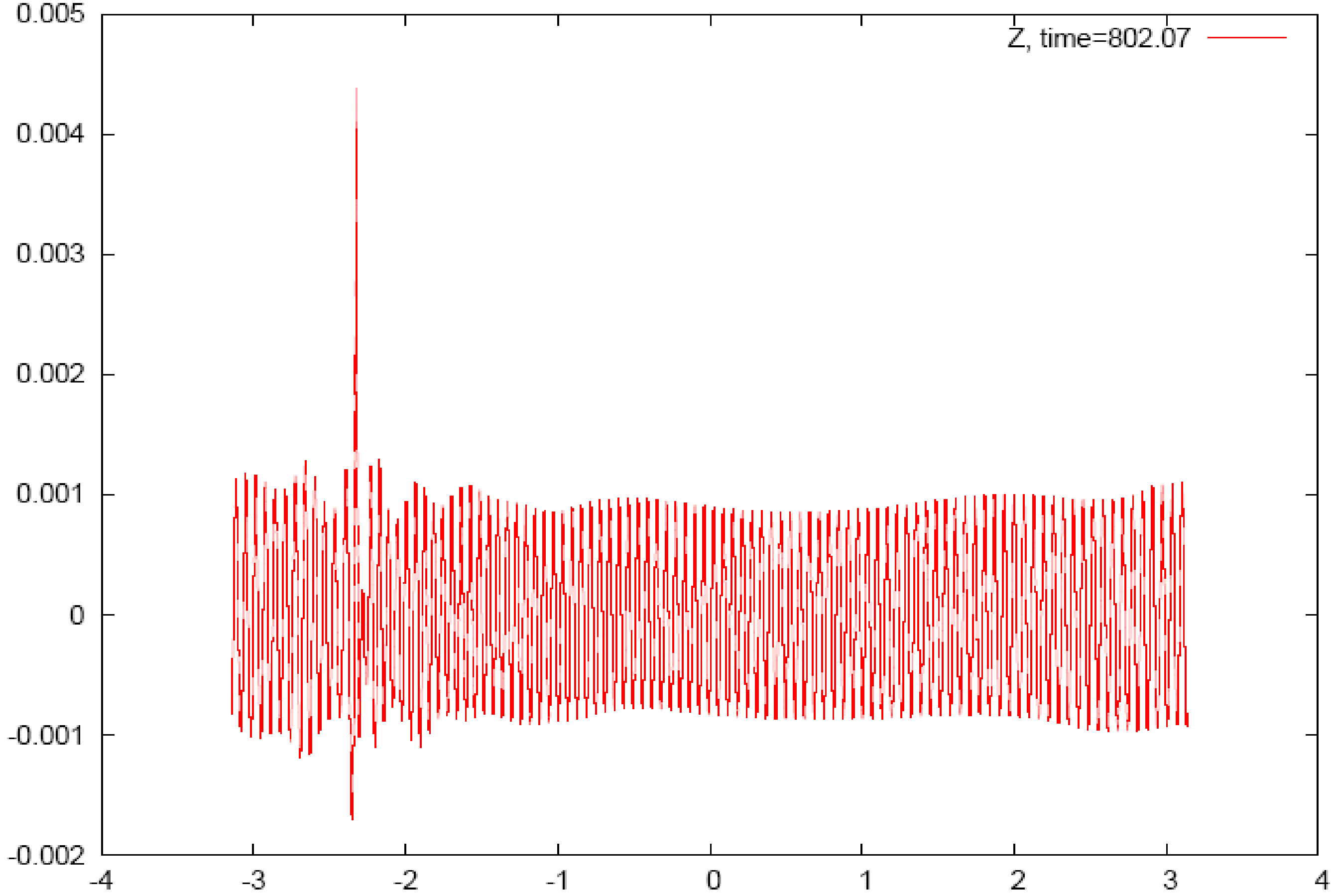


Figure 6: Surface profile

100 waves $\mu = 0.095$

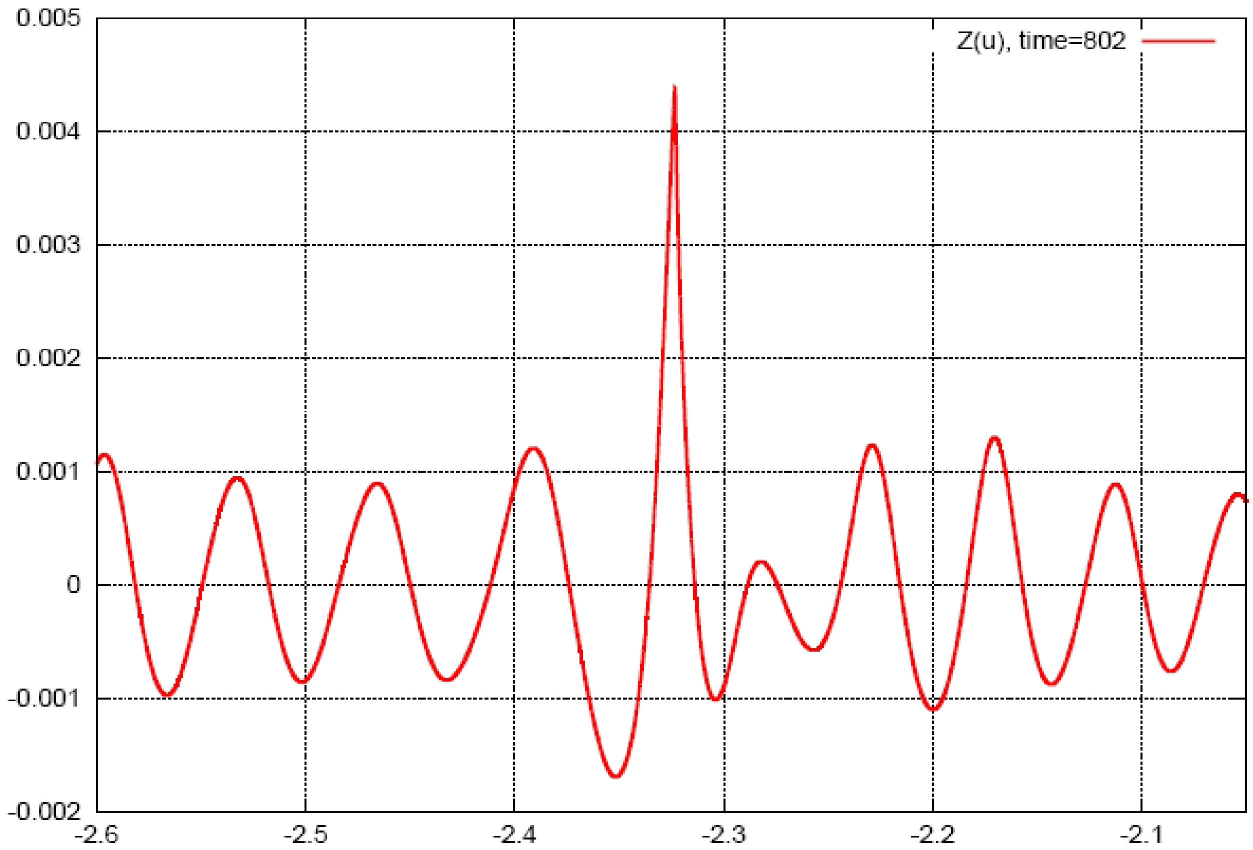


Figure 7: Zoom in surface profile

Wave breaks

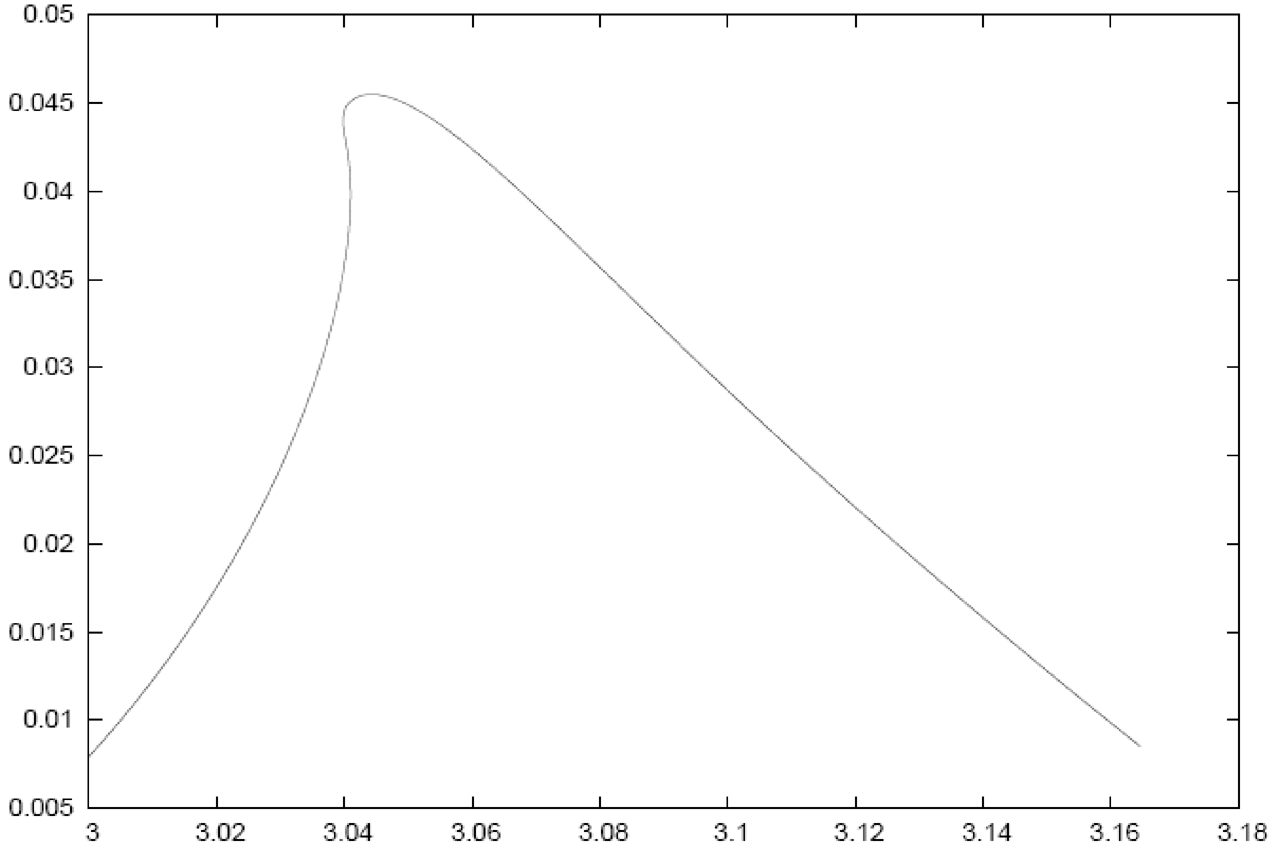
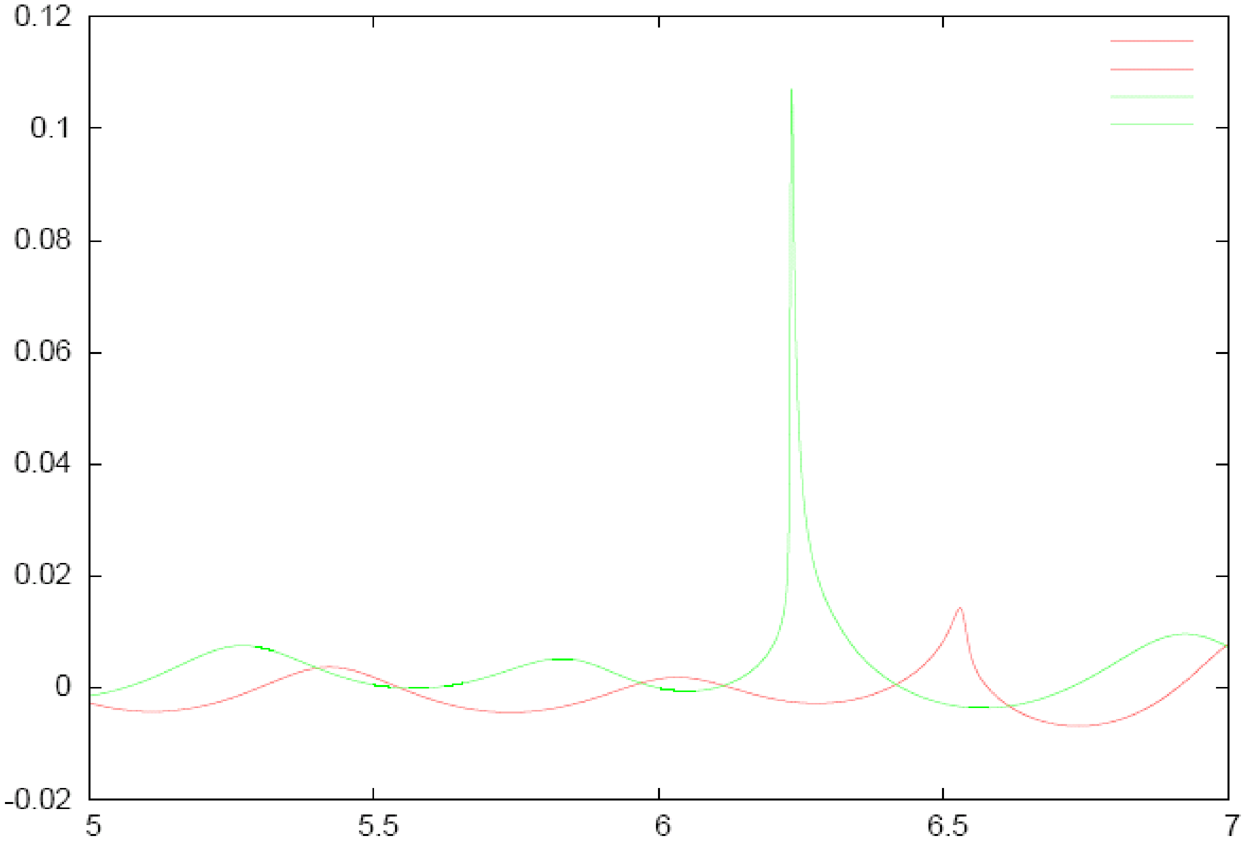


Figure 9: Overturning

Wave breaks. Momentum



NLSE Soliton $\mu = 0.1$

$$R(u) = 1 + \mu \frac{e^{-ik_0 u}}{\cosh(L_0 k_0 u)}, \qquad V(u) = -i\sqrt{\frac{g}{k_0}}\mu \frac{e^{-ik_0 u}}{\cosh(L_0 k_0 u)},$$

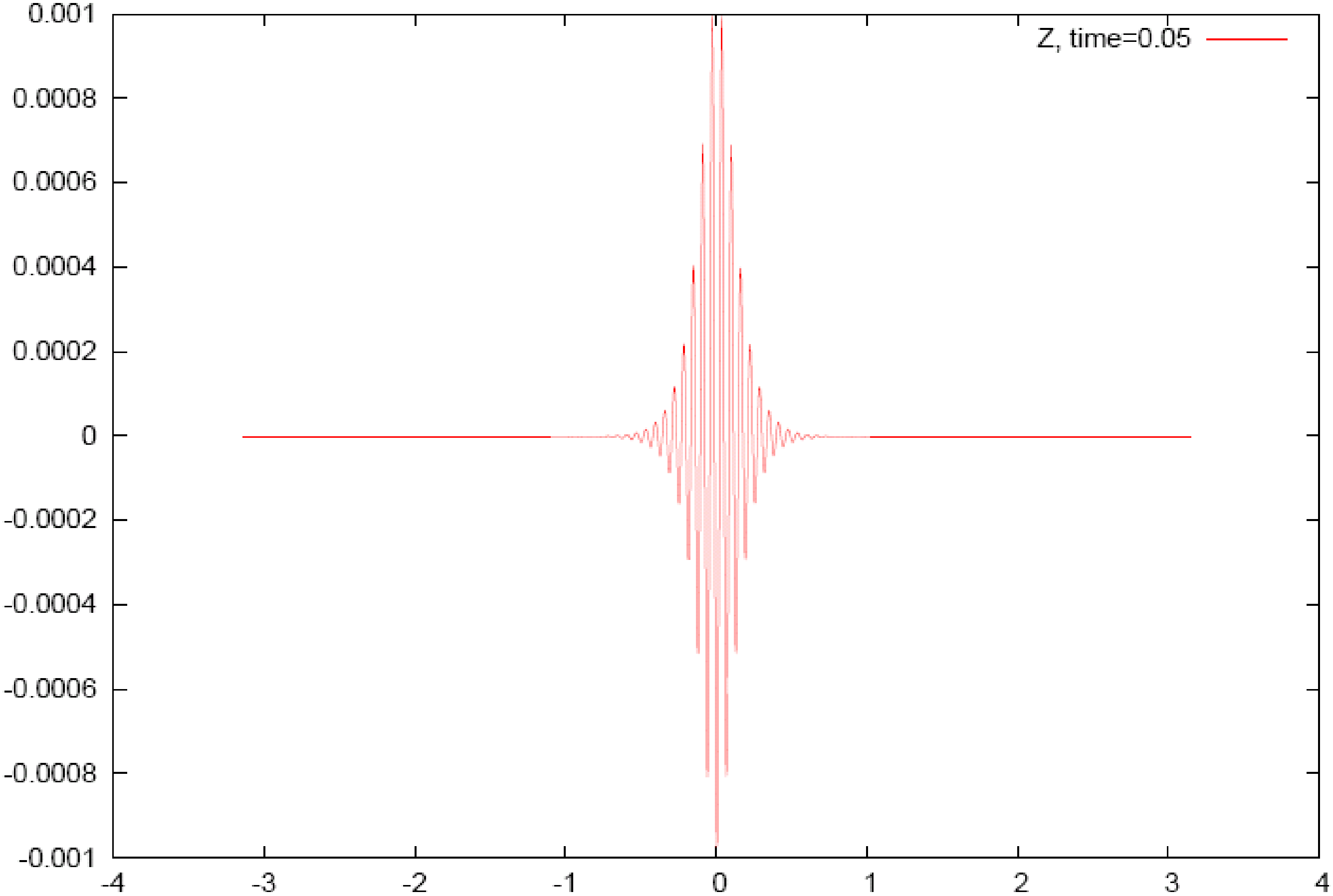


Figure 10: Initial soliton. $k_0 = 100$, $L_0 = 0.1$.

Freak waves – program for future.

We are planning to perform massive numerical simulation of different stationary wave spectra to determine dependence of probability of freak wave formation on energy spectrum.

Measuring of energy spectrum is a relatively easy problem. It will Make possible to estimate a danger of freak wave appearance in a given place in a given time…

Wave breaking

Evolution of the surface waves spectra is described by the Hasselmann kinetic equation

$$\frac{\partial N_k}{\partial t} = S_{in} + S_{nl} + S_{diss}$$

Here S_{nl} nothing but the standart quantum kinetics equation for Boson-type Quasiparticles in the limit of large occupation numbers.

S_{in} - nothing but the standart quantum kinetics equation for Boson-type quasiparticles in the limit of large occupation numbers.

S_{diss} - dissipation due to wave breaking.

To find S_{in} we should have an adequate theory of atmospheric boundary layer over ocean. This boundary layer is badly turbulent.

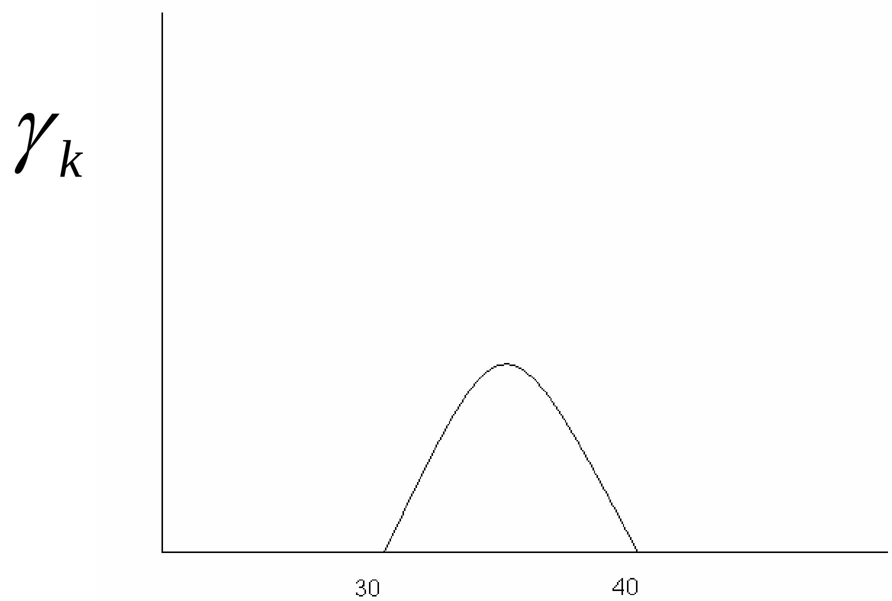
To find S_{diss} we need the theory of wave breaking. It is not developed yet.

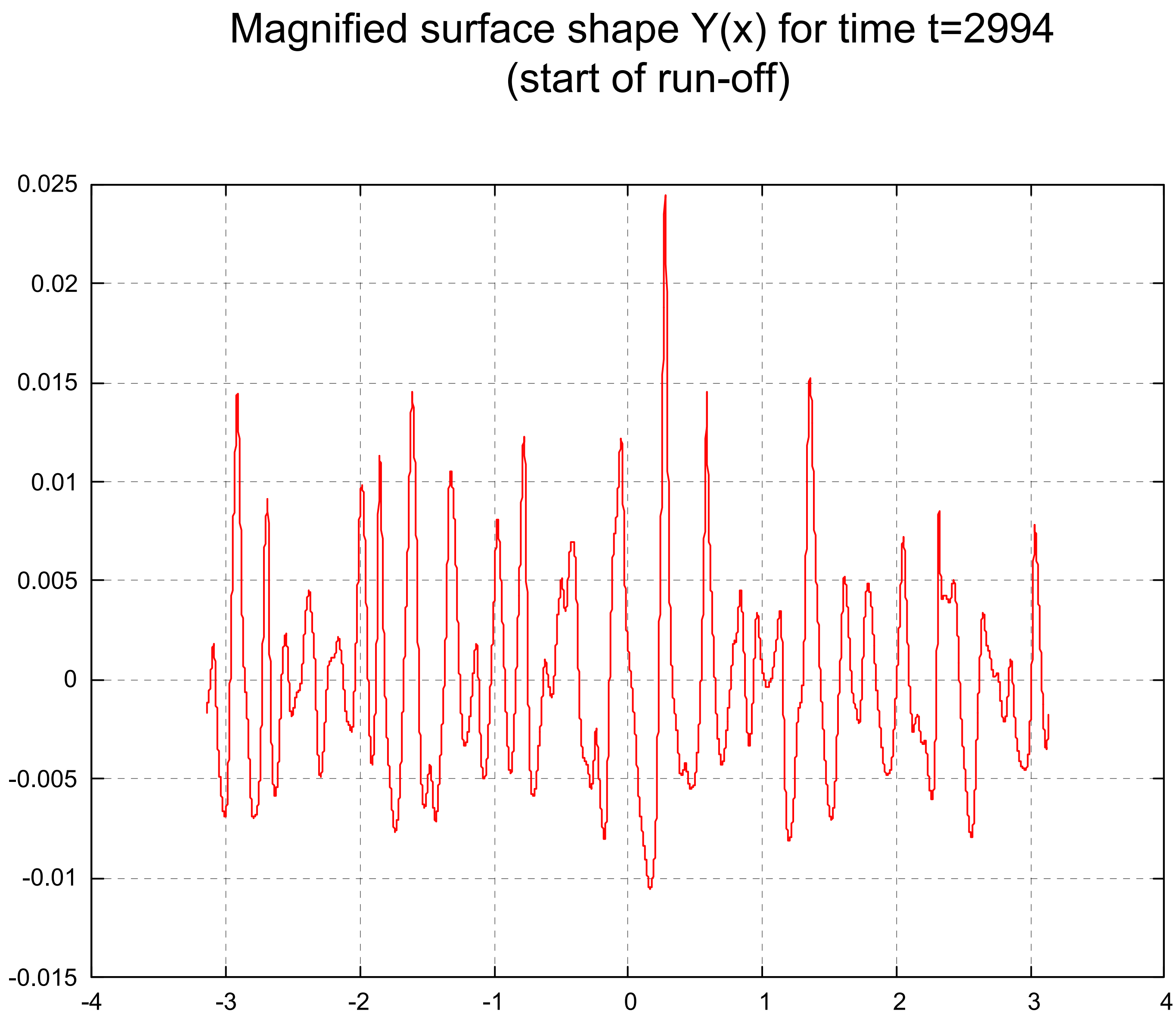
But we can perform a numerical experiment

$$\frac{\partial R}{\partial t} = i(R'U - RU') + \gamma_k R$$

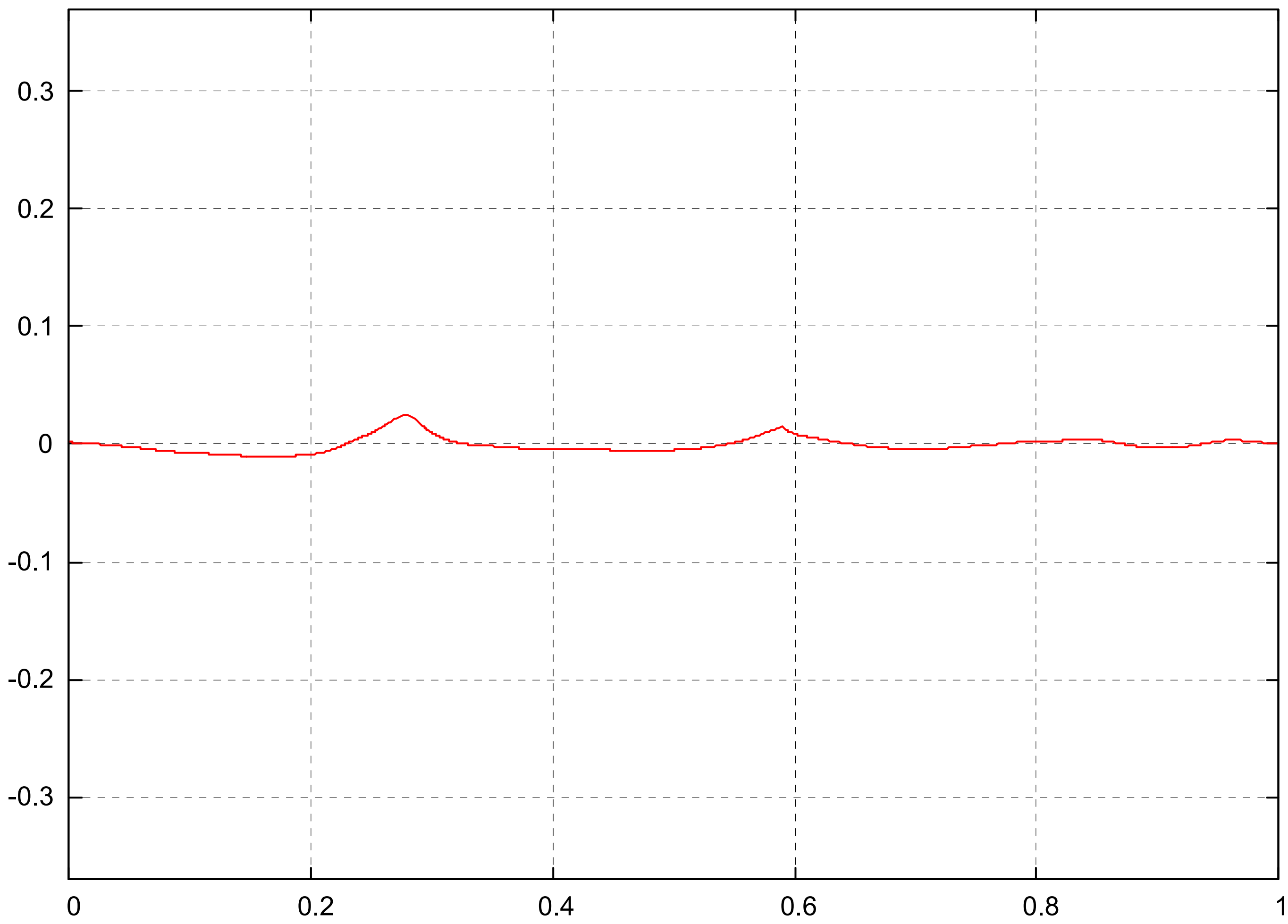
$$\frac{\partial V}{\partial t} = i(UV' - RB') + g(R-1) + \gamma_k V$$

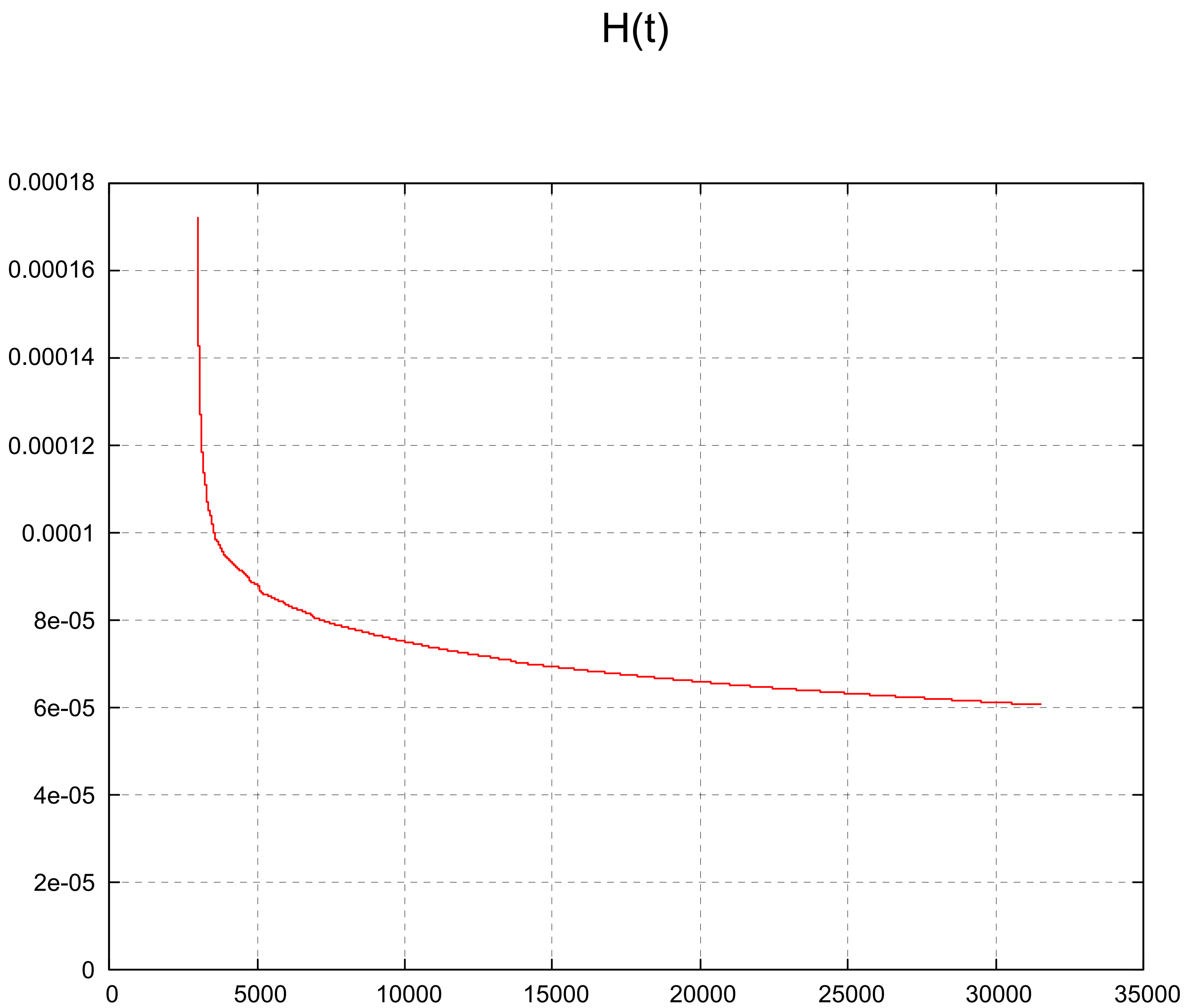
γ_k should be the same in both equations. The instability growth rate was



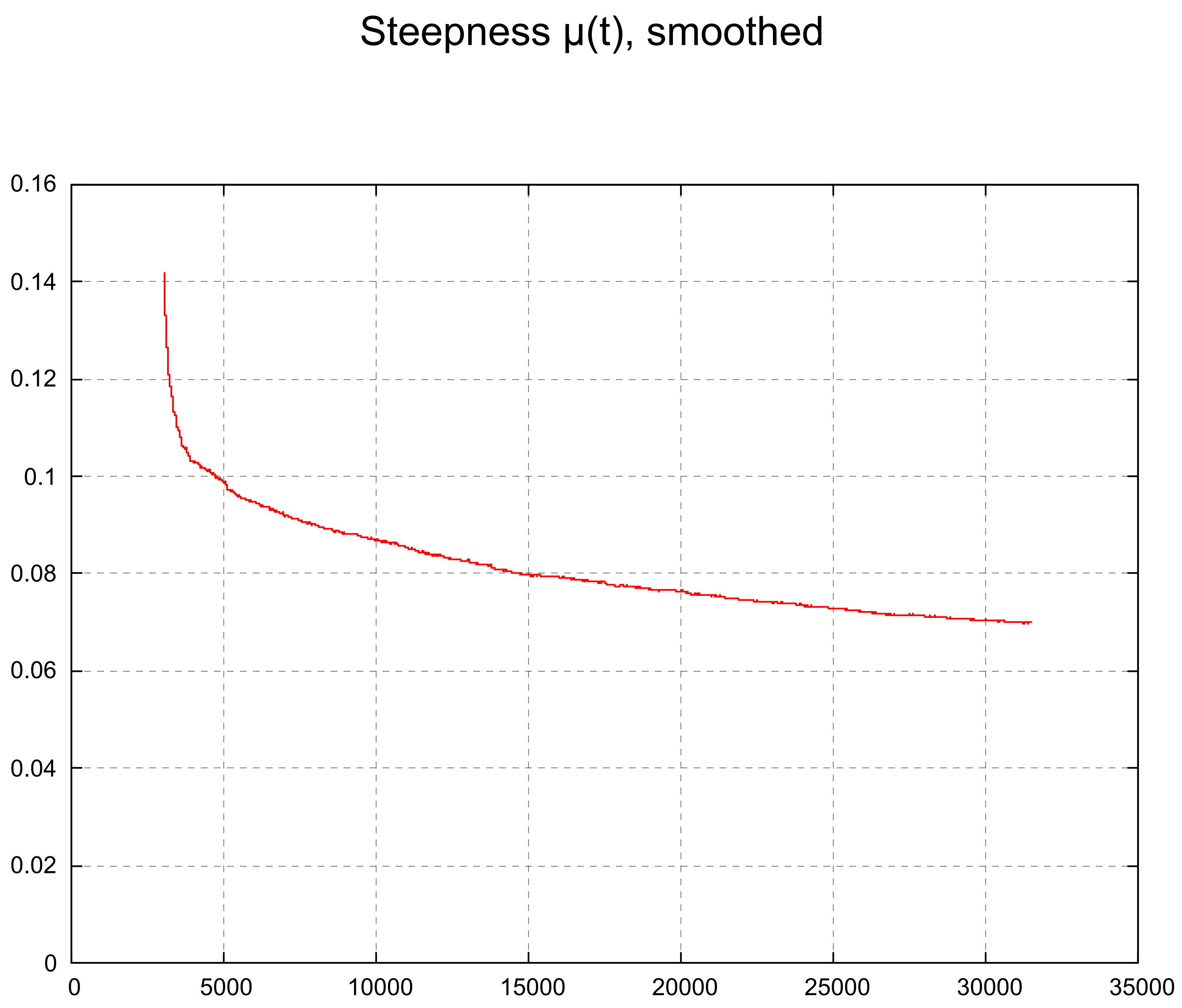

Magnified surface shape Y(x) for time t=2994
(start of run-off)
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-4
-3
-2
-1
0
1
2
3
4

Surface shape Y(x) for time t=2994
Similar scale for both axes

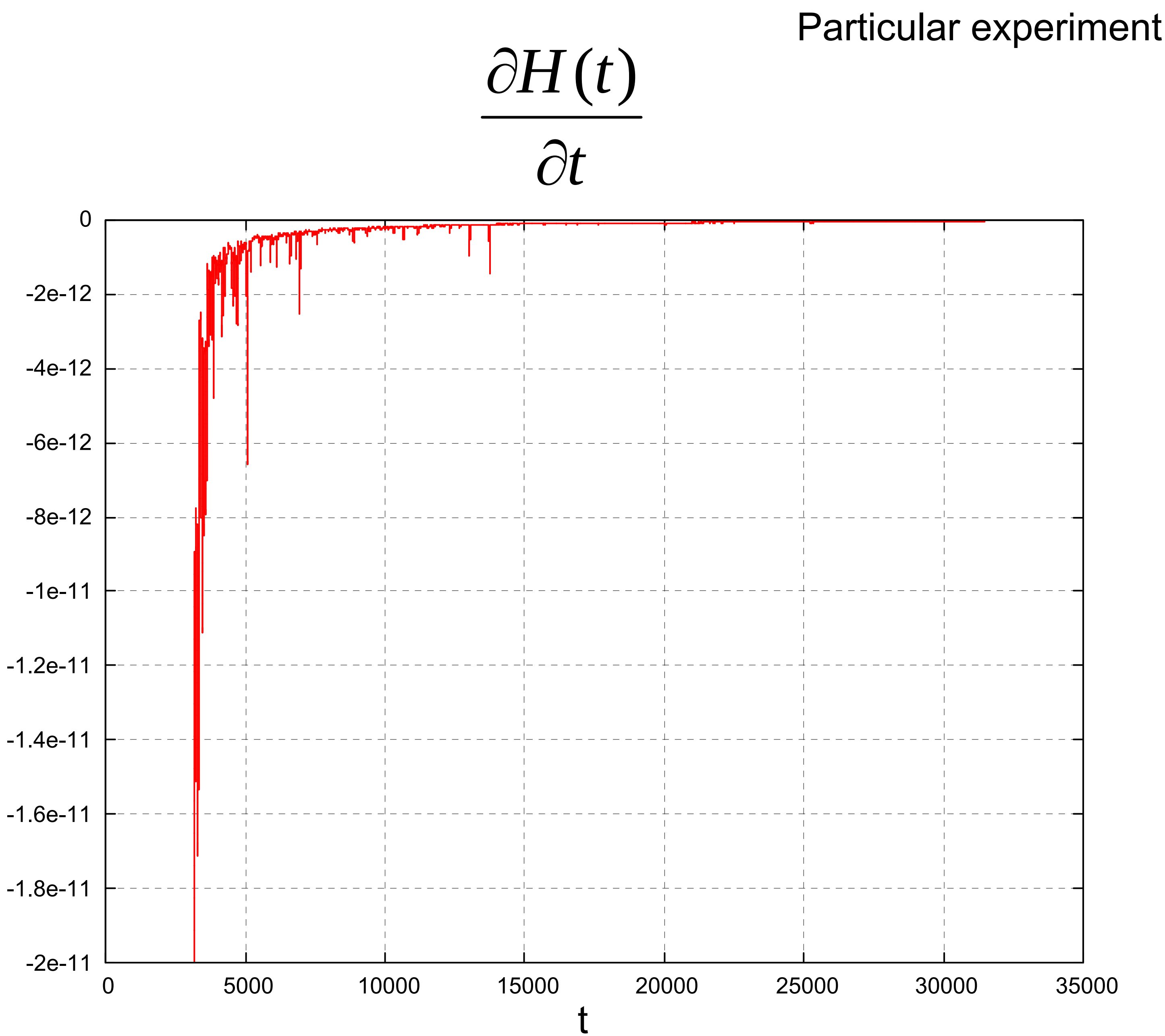




H(t)
0.00018
0.00016
0.00014
0.00012
0.0001
8e-05
6e-05
4e-05
2e-05
0
0
5000
10000
15000
20000
25000
30000
35000



Steepness μ(t), smoothed
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5000
10000
15000
20000
25000
30000
35000

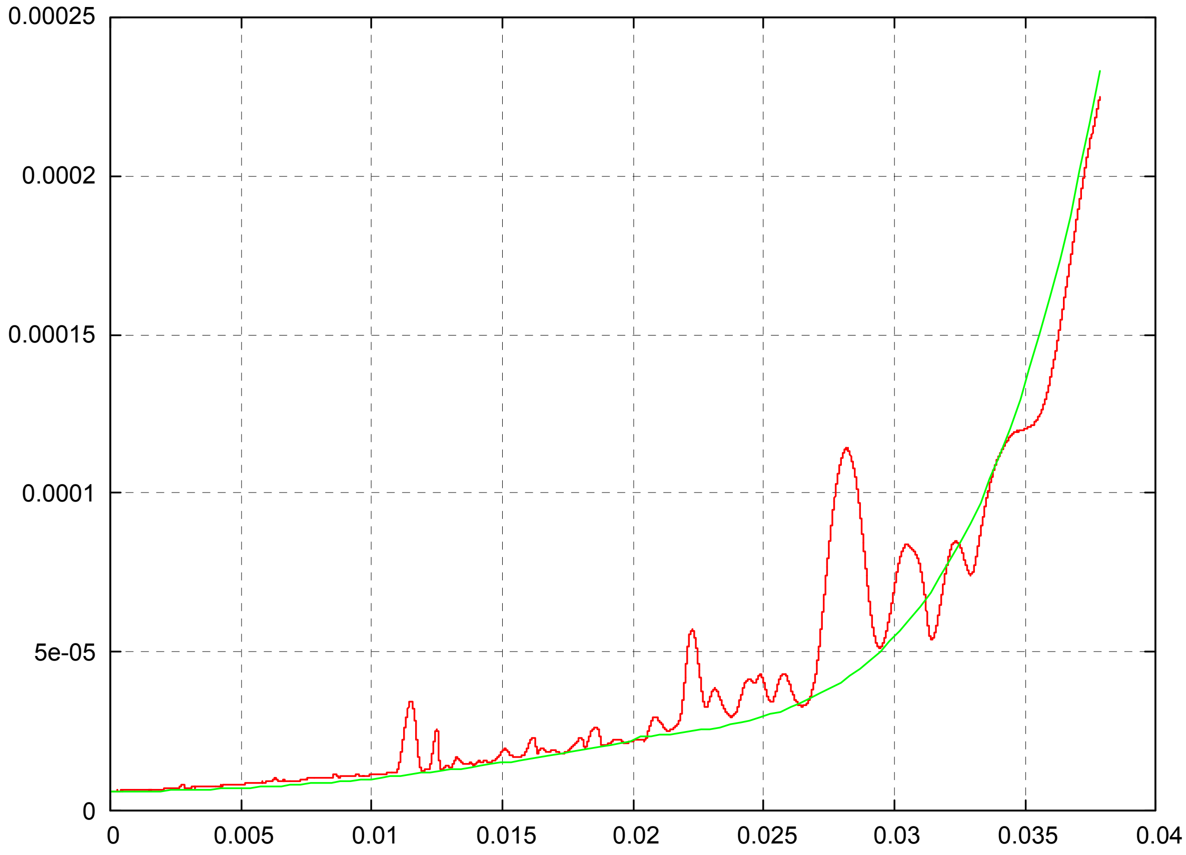

Particular experiment
$\frac{\partial H(t)}{\partial t}$
0
-2e-12
-4e-12
-6e-12
-8e-12
-1e-11
-1.2e-11
-1.4e-11
-1.6e-11
-1.8e-11
-2e-11
0
5000
10000
15000
20000
25000
30000
35000
t

Approximation of function: $\Gamma(\mu-\mu_0)=\frac{\dot{H}}{H}$

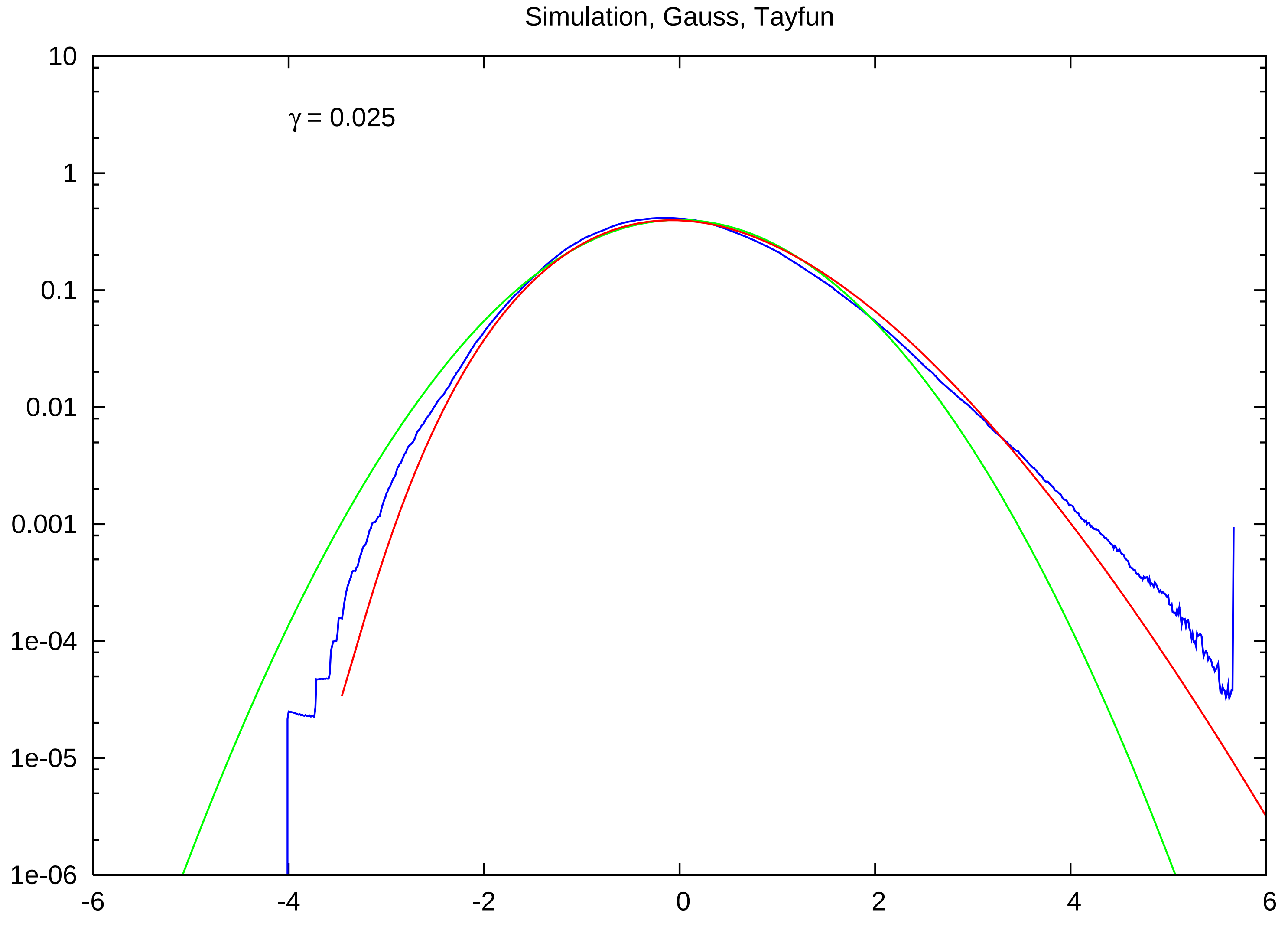
$$\mu-\mu_0<0.02: f(x)=6\cdot 10^{-6}+4\cdot 10^{-2}x^2$$

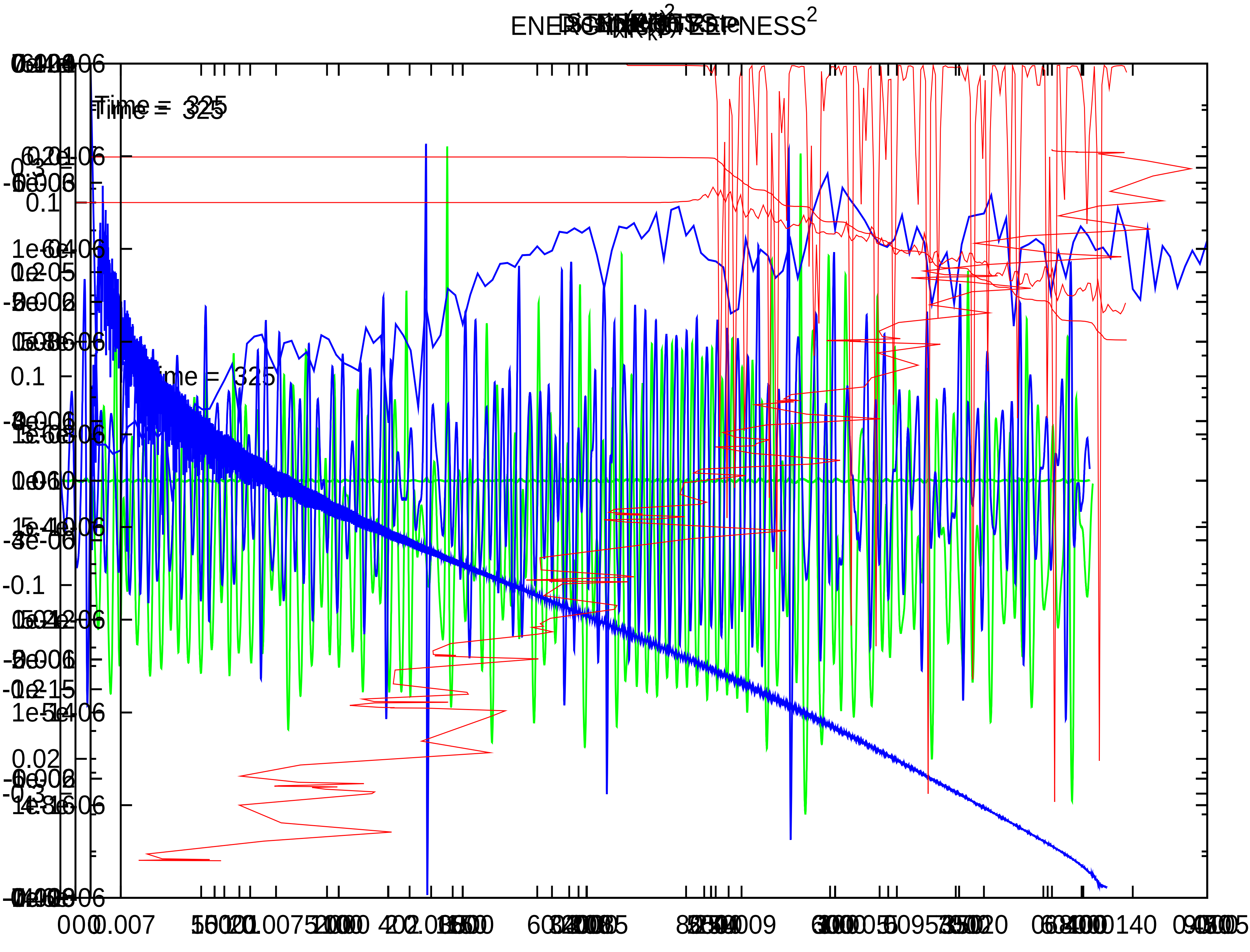
$$\mu-\mu_0>0.02: f(x)=2\cdot 10^{-5}+5\cdot 10^{7}x^8$$

(numerical values obtained by least squares method)

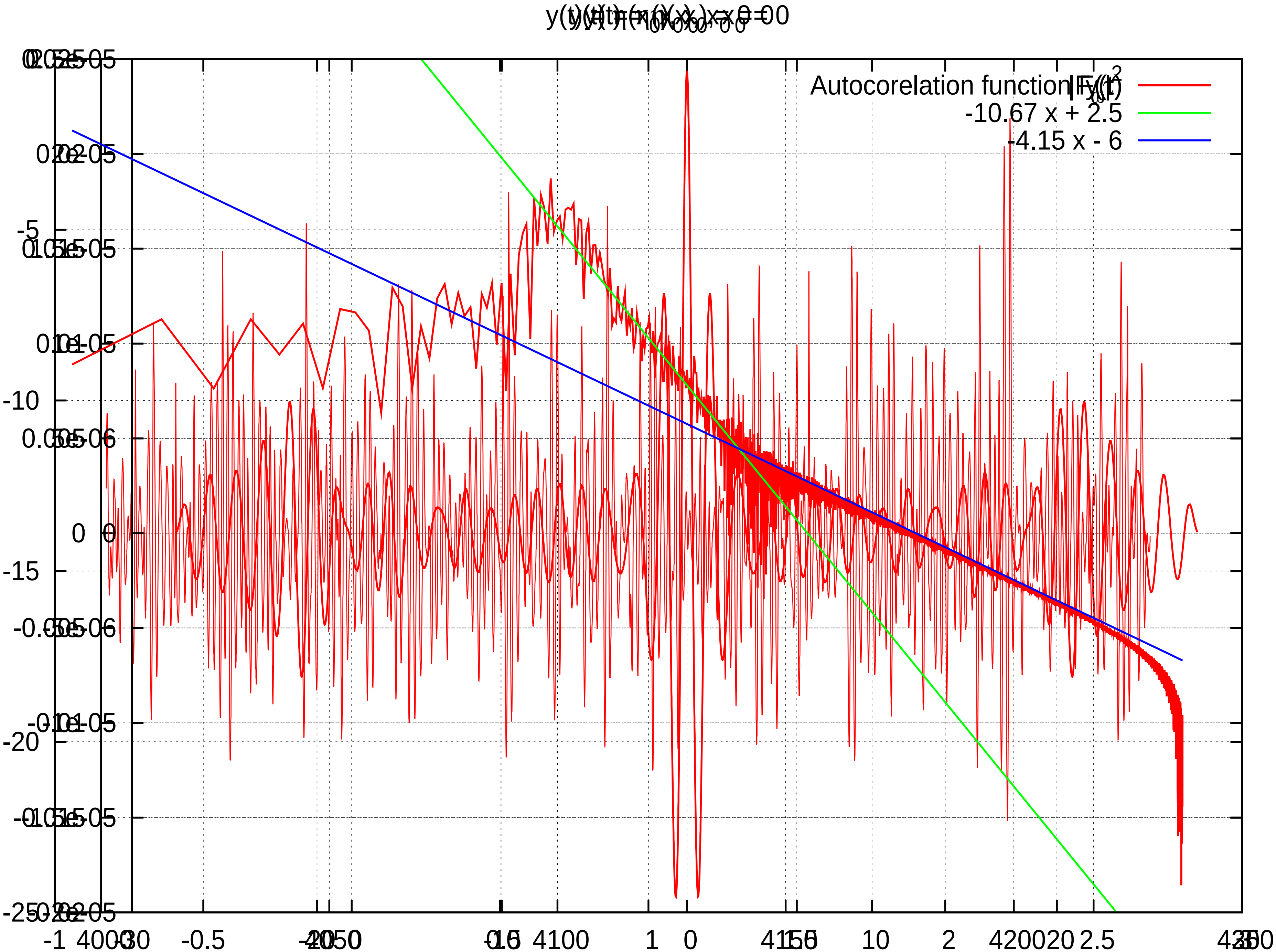


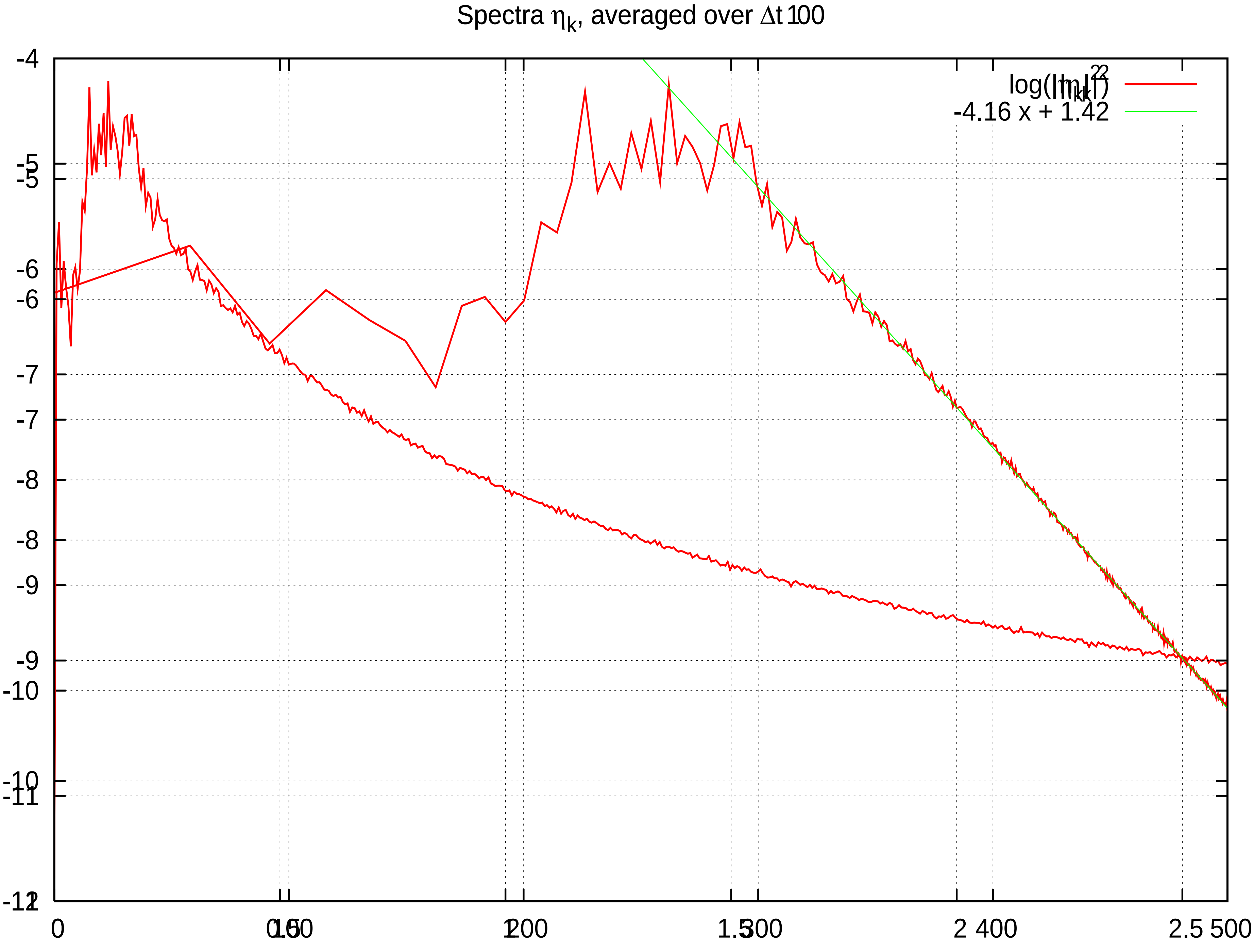
Let $\eta(r,t)$ elevation of ocean surface. In the first approximation this is quasi-stationary, quasi homogeneous random process, close to Gaussian.



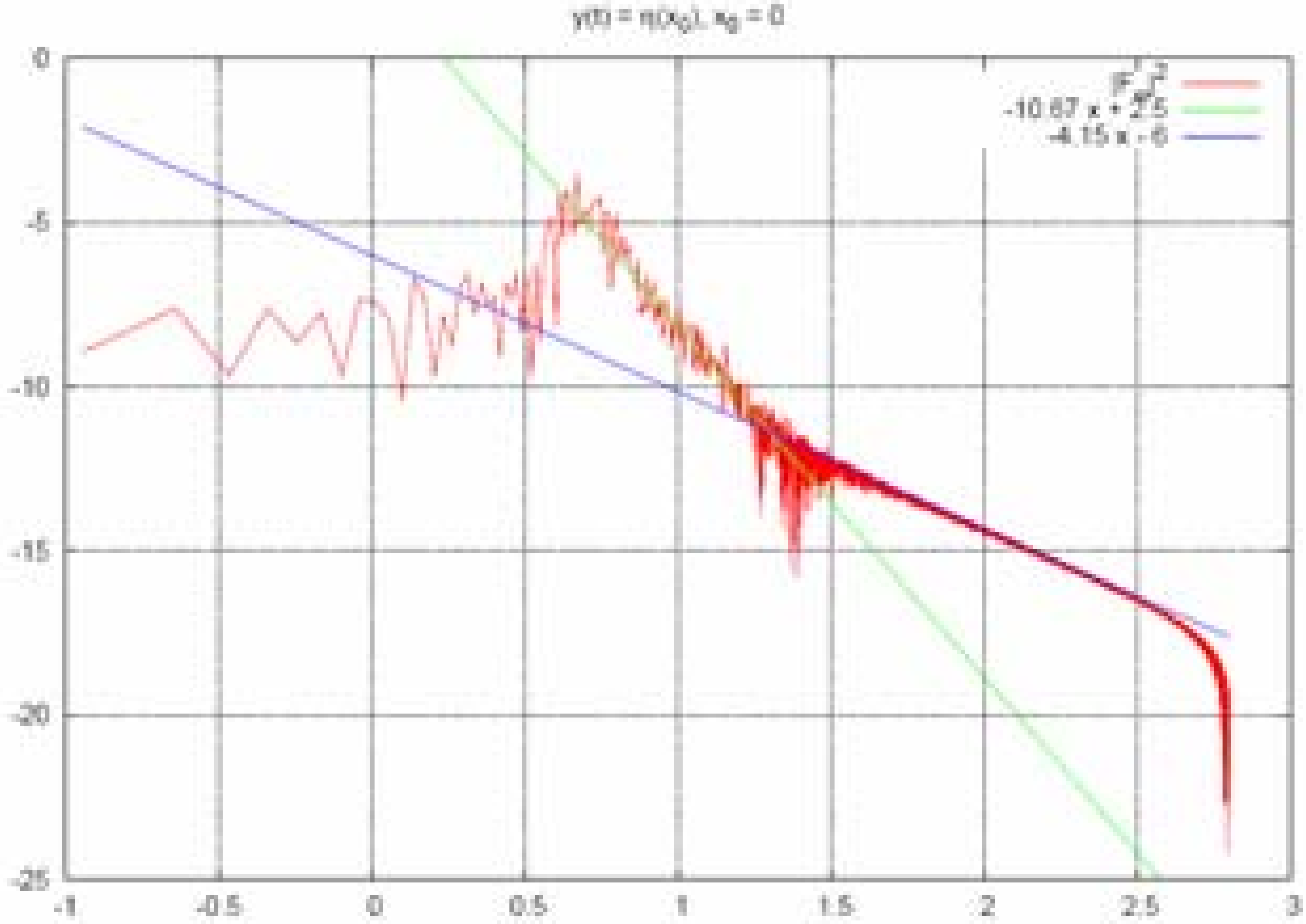


Time = 325


Autocorelation function
-10.67 x + 2.5
-4.15 x - 6



Spectra ηk, averaged over Δt 100
log(|ηk|²)
-4.16 x + 1.42
-4
-5
-6
-7
-8
-9
-10
-11
-12
0
100
200
300
400
500



y(t) = η(x_0), x_0 = 0
-10.67 x + 2.5
-4.15 x - 6
0
-5
-10
-15
-20
-25
-1
-0.5
0
0.5
1
1.5
2
2.5
3