Evolution of random directional wave and rogue/freak wave occurrence

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Motivation

- *Hypothesis*
  - Meteorological conditions and wave-current interaction preconditions the wave spectrum (dispersive and geometrical focusing)
  - Non-resonant interaction kicks in when the wave spectrum satisfies certain conditions (BFI, Kurtosis, directional spreading)
- Observations do not necessarily support the hypothesis
- Study the evolution of realistic directional waves including impact of breaking dissipation

Methodology

- Tank Experiment (comparison with weakly nonlinear theory as needed)
  - Directional wave maker (32 segments)
  - <u>Benjamin-Feir instability (uni-directional 2D)</u>
    - Initial instability
    - long-term evolution: impact of breaking dissipation
    - maximum wave height
  - <u>Random Directional Wave</u>
    - long-term statistics (~4000 wave periods)
    - steepness, spectral bandwidth, directional spreading
    - special spectral shape
    - maximum wave height, impact of breaking dissipation
- Wave hindcasting/forecasting
  - Diagnose influence of wind and current on spectral shape

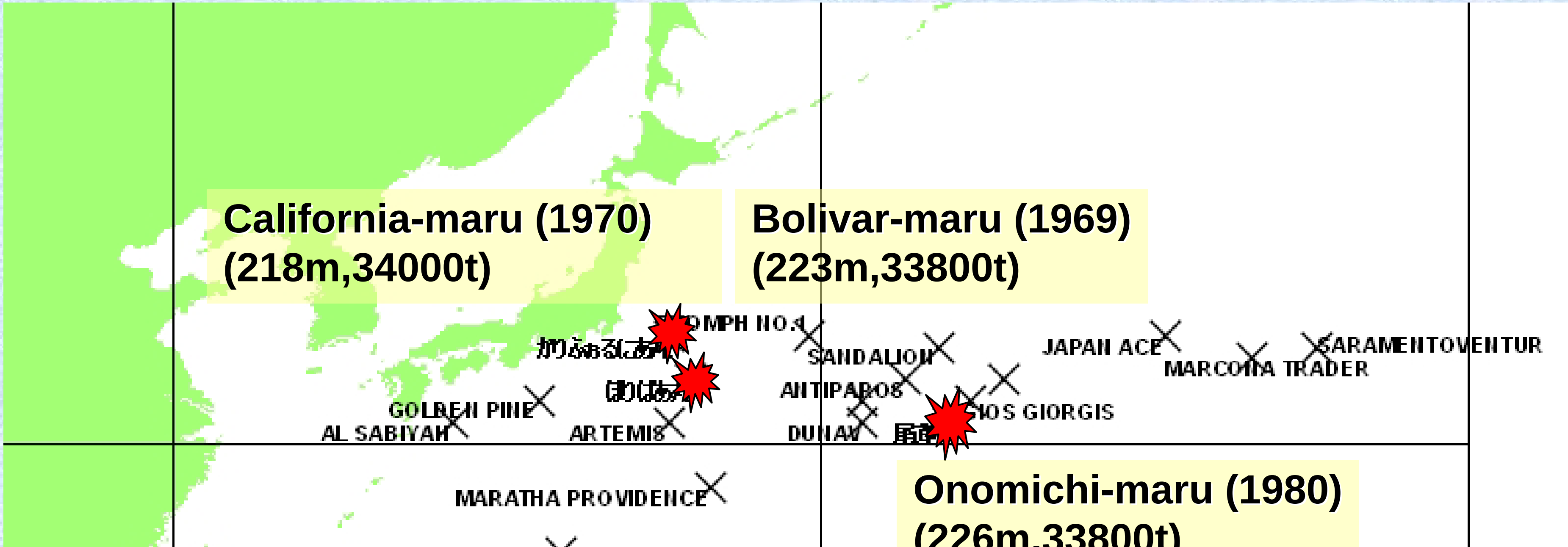
Summary of conclusion

- Maximum wave height of random directional wave
  - increases with mean steepness, similar to the unstable Stokes wave case
- Spectral downshifting due to breaking dissipation
  - spectral downshifting rate correlates well with the magnitude of energy loss, including cases with various directionality
- Extreme wave statistics
  - Kurtosis reduces as frequency bandwidth and directional spreading broadens
- Spectral parameters in real ocean
  - Broader distribution of directional spreading than frequency bandwidth

Introduction

- Safe navigation of ships near Japan
  - Group effort (U. Tokyo, NMRI, JAMSTEC) to understand generation mechanism of freak wave, its impact on ship, and prediction/avoidance
  - Tank experiment, numerical simulation, observation (radar, buoy)
- Tank experiment
  - Ship model test, radar observation
  - *Generation mechanism*

Maritime accidents near Japan



California-maru (1970)
(218m,34000t)

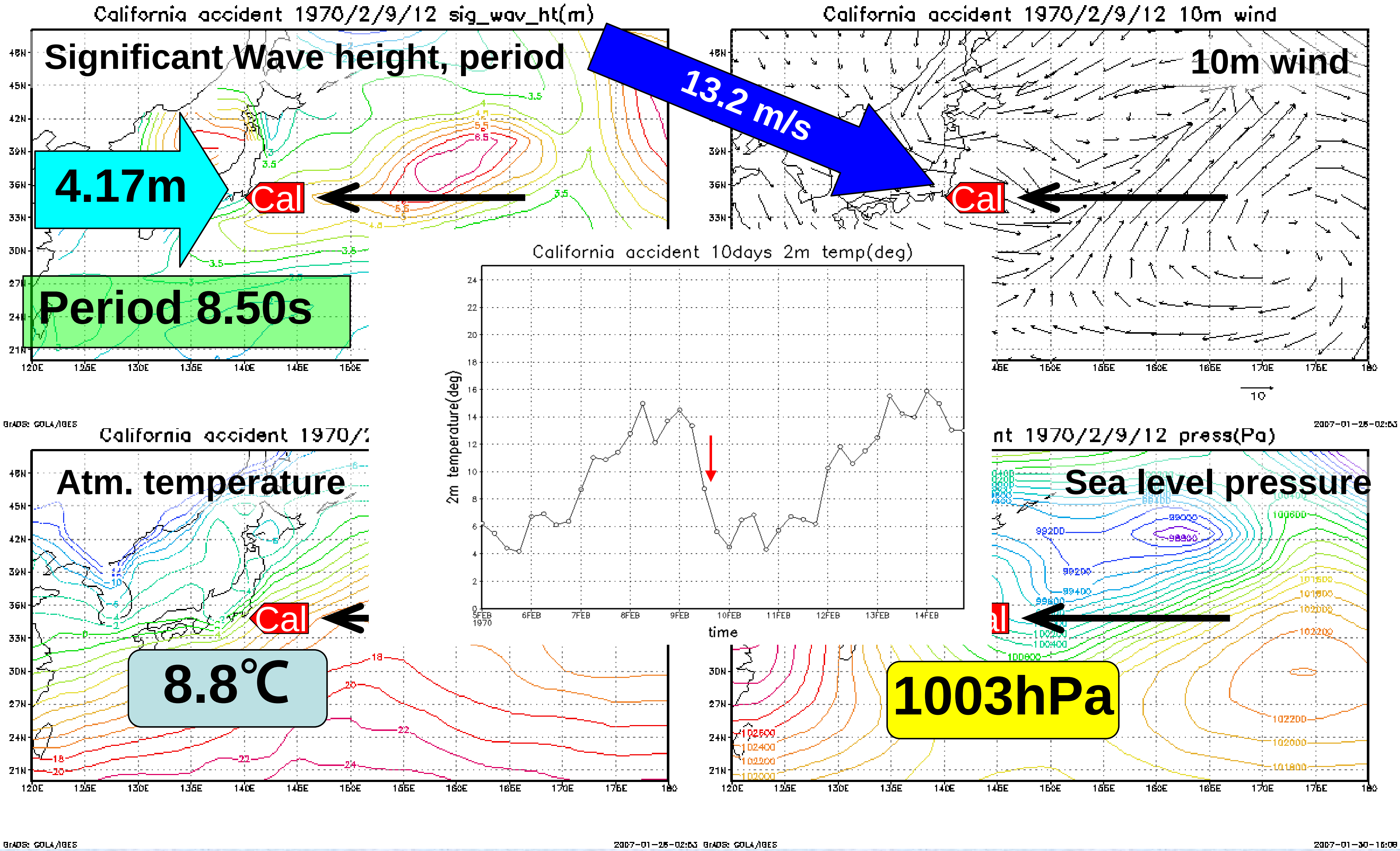
Bolivar-maru (1969)
(223m,33800t)

Onomichi-maru (1980)
(226m,33800t)

Marine Accident Inquiry Agency Report

- **Bolivar-maru**
  - **Hull damaged, lack of strength of ship**
- **California-maru**
  - **Two large waves merged, mixed sea**
- **Onomichi-maru**
  - **20m wave, slamming**

Case of California-Maru (ERA40)



February 9, 1970

Background: Relating wave spectrum and freak wave occurrence

- Evolution of Kurtosis as a result of non-resonant wave-wave interaction, Benjamin-Feir-Index (Janssen 2003, Onorato et al. 2004)
- Kurtosis as a relevant parameter to modify wave height statistics (Mori & Janssen 2006)
- Reduction of freak wave occurrence in case of directional sea (Socquet-Juglard et al. 2005, Onorato et al. 2002)

BFI=ak/(df/f)

Freak Wave occurrence and wave statistics
Janssen, Onorato, Mori, et al.

Kurtosis with higher order corrections

$$\left\langle \eta(x)^4 \right\rangle = 3\left\langle \eta(x)^2 \right\rangle^2 + 16\int M_{1,2,3,4} T_{1,2,3,4} N_1 N_2 N_3 \frac{1-Cos(\Delta\omega t)}{\Delta\omega} \delta_{1+2-3-4} dk_{1234} \qquad \text{Free Wave}$$

$$+12\int \mathrm{K}_{1,2,3,1,2,3} N_1 N_2 N_3 dk_{123} \qquad \text{Bound Wave}$$

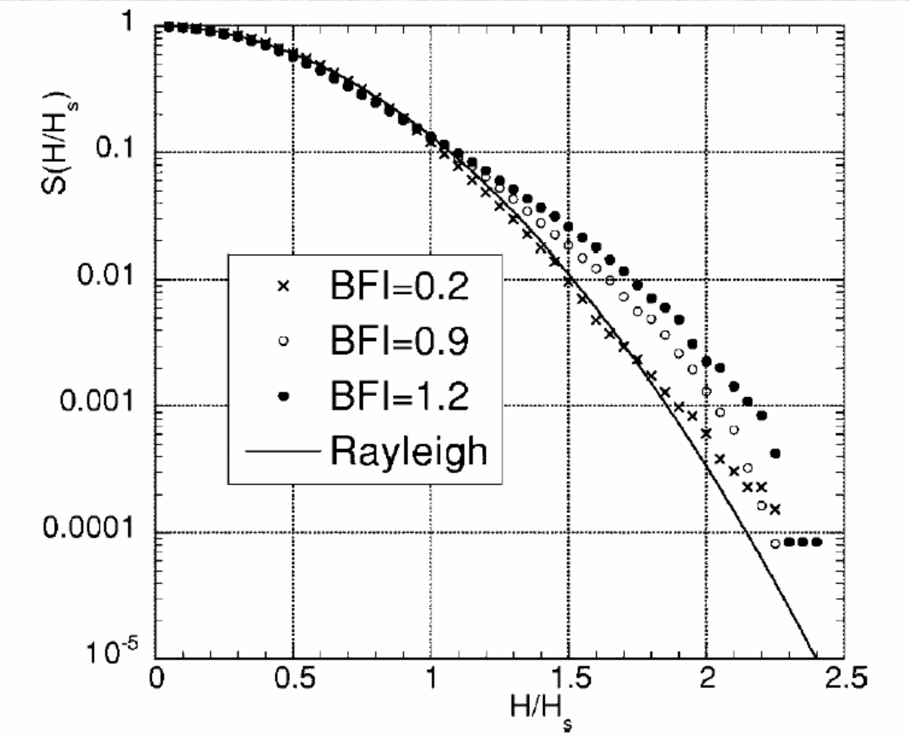
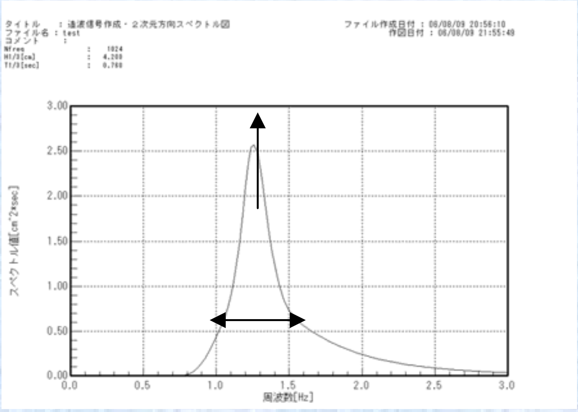


FIG. 5. Survival function at $x/L = 18.5$ for BFI0.2 (crosses), BFI0.9 (open circles), and BFI1.2 (solid circles).

Probability of freak wave occurence

$$P_{\text{freak}} = 1 - \exp[-\alpha N(1 + 8\kappa_{40})]$$

$$\kappa_{40} = \frac{\pi}{\sqrt{3}} BFI^2; \quad \boxed{BFI = \frac{\varepsilon}{\Delta\omega/\omega}\sqrt{2}}$$



Background: Instability of Stokes wave, long-term evolution and extension to a random directional wave

- Benjamin-Feir Instability
  - Stokes wave is unstable (Benjamin-Feir 1962, McLean 1982 etc.), most unstable for uni-directional
- Long-term evolution
  - Recurrence (Lake & Yuen 1972), breaking wave and permanent downshifting (Melville 1982, Tulin & Waseda 1999)
- Instability of random directional wave
  - Instability limited to narrow directional spread, <u>35.26 degree</u> (Alber & Saffman 1978)

Benjamin－Feir instability in a tank

Plunger motion

$$\eta = a_c \cos(\omega_0 t) + b_+ \cos(\omega_+ t + \varphi_+) + b_- \cos(\omega_- t + \varphi_-)$$

$$a_0^{\ 2} = a_c^{\ 2} + b_+^{\ 2} + b_-^{\ 2}$$

$$\omega_+ = \omega_0 + \Delta\omega$$

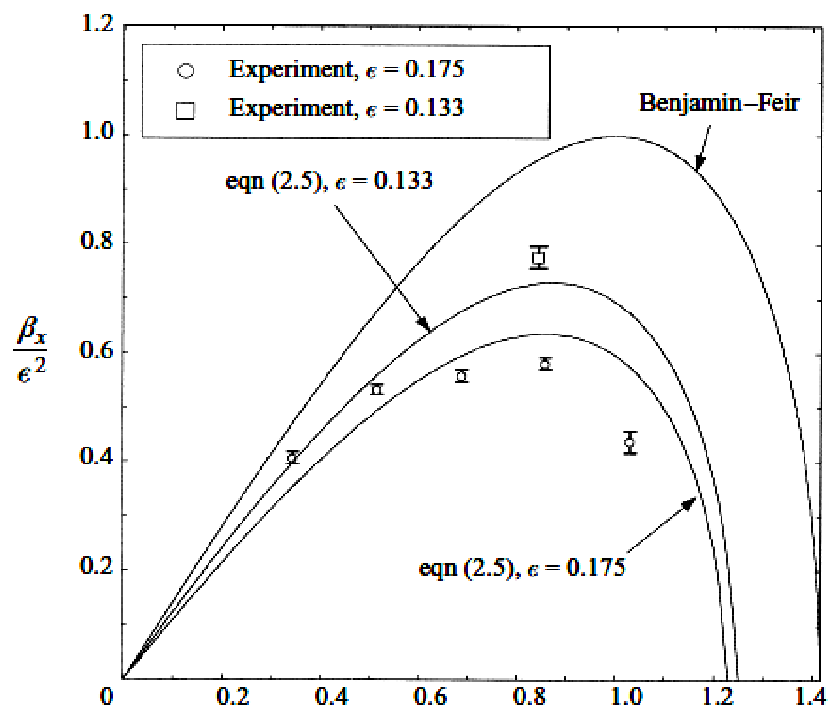
$$\omega_- = \omega_0 - \Delta\omega$$

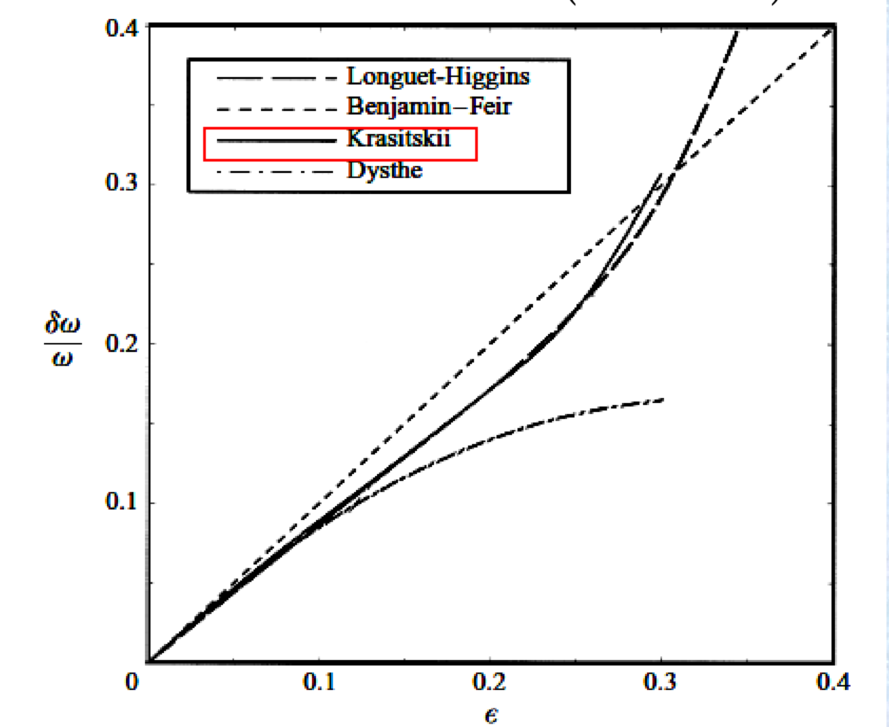
$$\varphi_+ + \varphi_- = -\frac{\pi}{2}$$

Control Parameter

$$\boxed{\begin{array}{l} a_0 k_0 \\ \delta = \dfrac{\Delta\omega/\omega_0}{a_0 k_0} = \dfrac{1}{\sqrt{2}} \dfrac{1}{\mathrm{BFI}} \end{array}}$$

Growth rate BF: $\beta = \varepsilon^2 \delta \left(2.0 - \delta^2\right)^{1/2}$

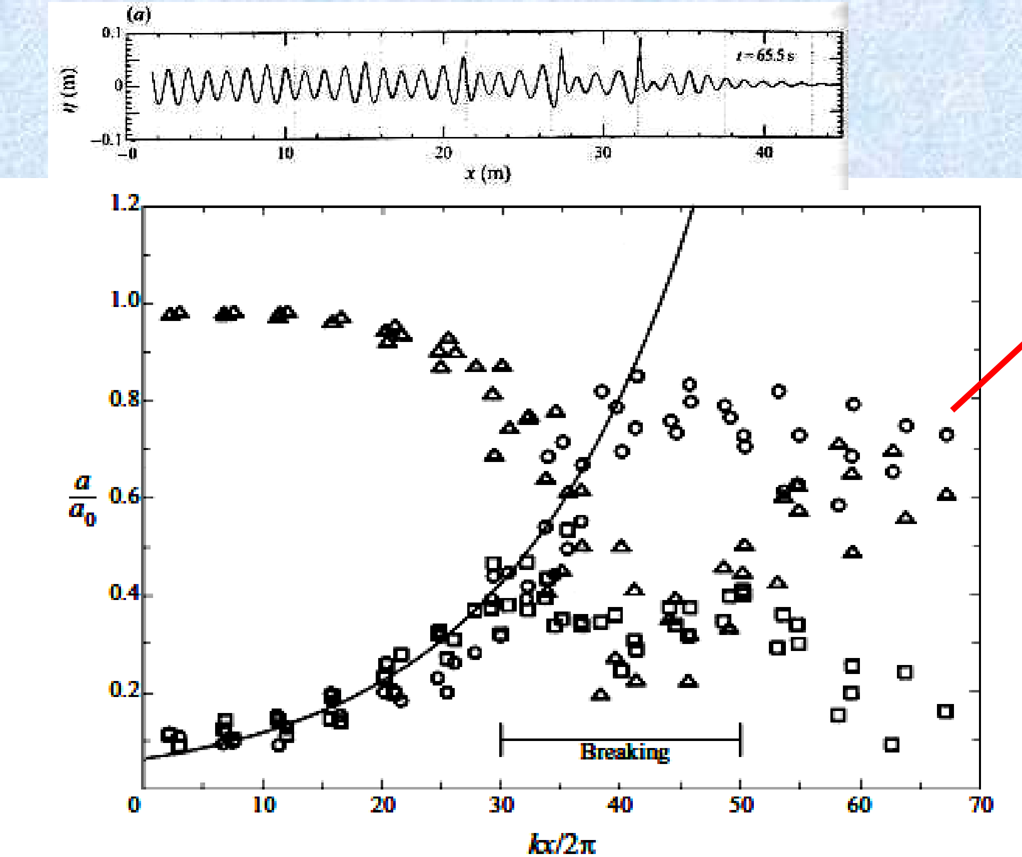




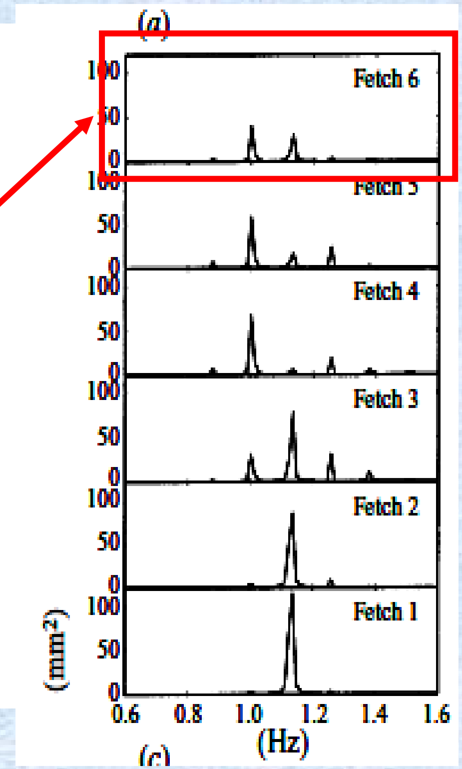
$$\boxed{\delta = \frac{\Delta\omega/\omega_0}{a_0 k_0} = \frac{1}{\sqrt{2}}\frac{1}{\mathrm{BFI}}} \longrightarrow \boxed{BFI > 0.5}$$

Waseda&Tulin JFM 1999

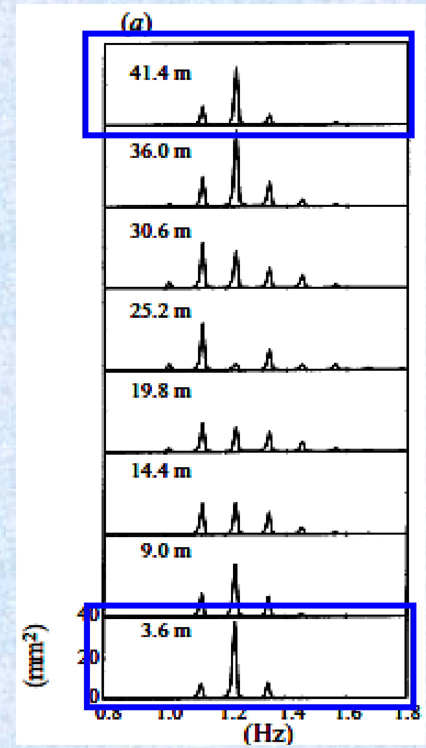
Long-term evolution of the BF Wave train – Impact of Breaking Dissipation



With breaking downshift



Without breaking recurrence



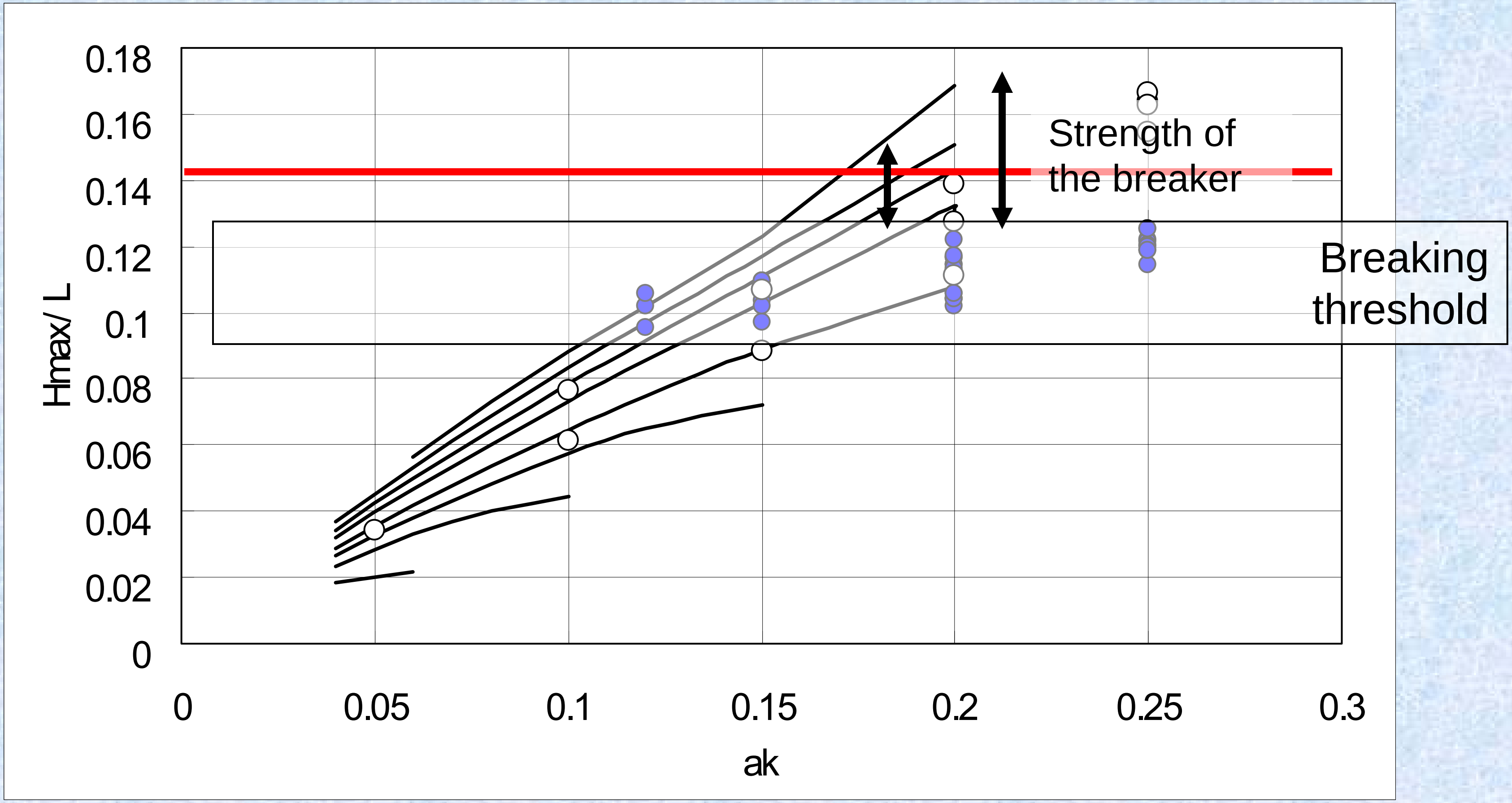
$$\eta = a_c \cos(\omega_0 t) + b_+ \cos(\omega_+ t + \varphi_+) + b_- \cos(\omega_- t + \varphi_-)$$

$$a_0^2 = a_c^2 + b_+^2 + b_-^2$$

$$\varphi_+ + \varphi_- = -\frac{\pi}{2}$$

Tulin & Waseda JFM 1999

Maximum Wave Height
Comparison of experiment and weakly nonlinear solutions



Lines: Dysthe’s eqn
Open Circles: Zakharov 4-wave reduced eqn
Blue Circles: Experiment at U. Tokyo OE Tank

Tank Experiment

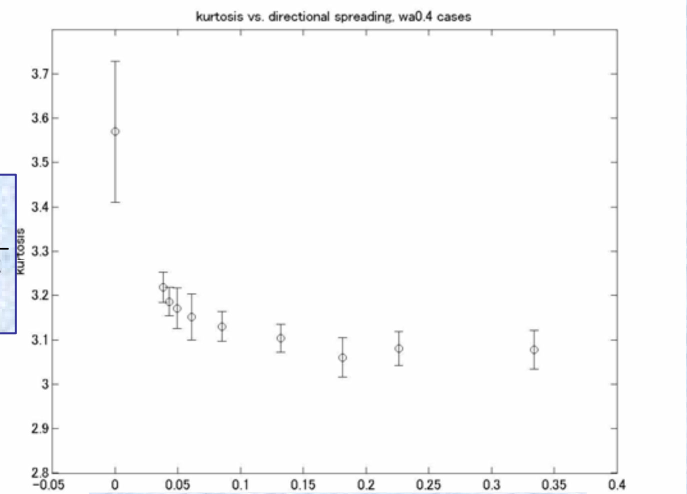
Exp01: August 2006

→result presented at the 9th wave workshop (poster)

→Reduction of Kurtosis with directionality

Strong influence of breaking dissipation

$$\frac{\left\langle \eta(x)^4 \right\rangle}{\left\langle \eta(x)^2 \right\rangle^2}$$



Directional bandwidth

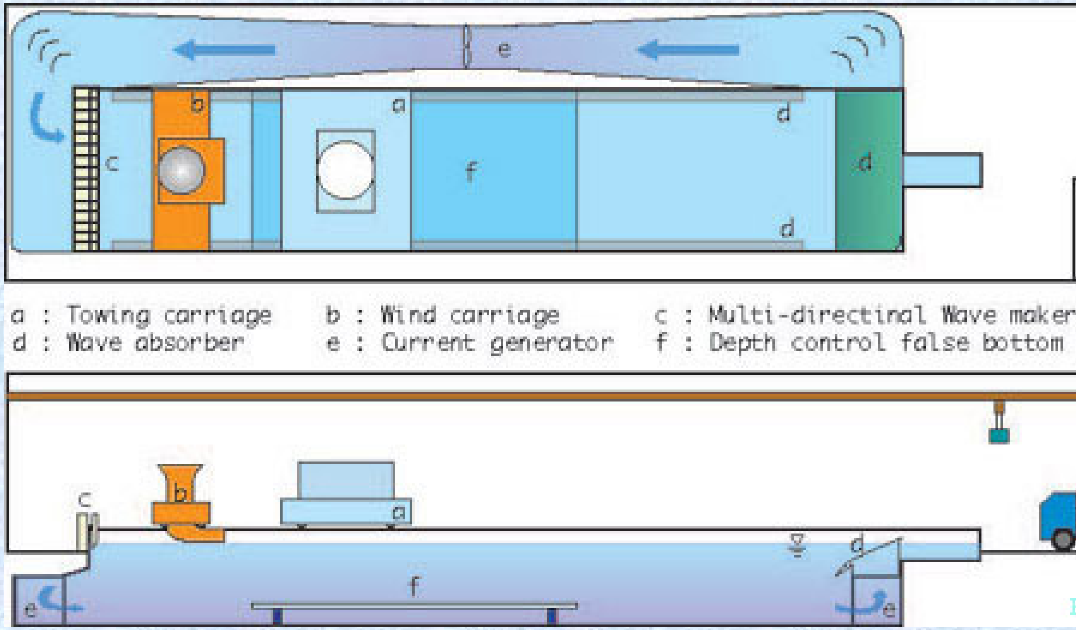
Exp02: April 2007:

wider range of spectral parameters, better control of the wave maker

Ocean Engineering Tank
Institute of Industrial Science, U. of Tokyo
Kinoshita Lab / Rheem Lab

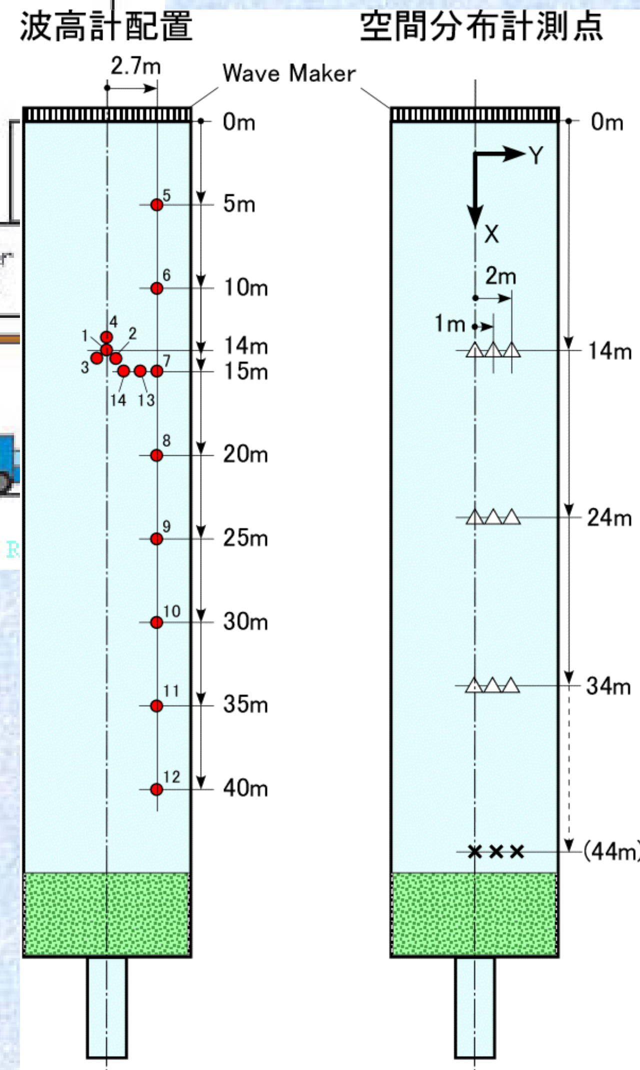
10 m

5 m



50 m

Directional wave maker
32 plungers
0.5~5 s wave period



Generation of the Directional Spectrum

- JONSWAP

$$S(f,\theta) = S(f)\cdot G(f,\theta)$$

$$S(f) = \boxed{\alpha}\, g^2 (2\pi)^{-4} \underline{f^{-5}} \exp\left\{-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right\} \boxed{\gamma}^{\exp\left\{-\frac{(f-f_p)^2}{2\sigma^2 f_p^2}\right\}}$$

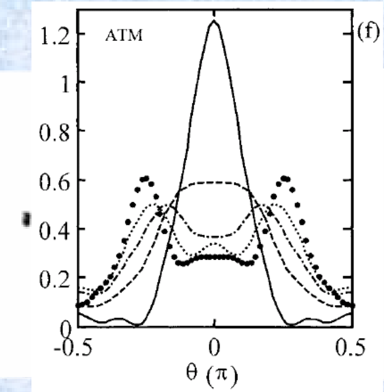
- Control Parameters

$$\alpha \Leftrightarrow H_{1/3} \quad \& \quad \gamma \Leftrightarrow \delta f / f_p$$

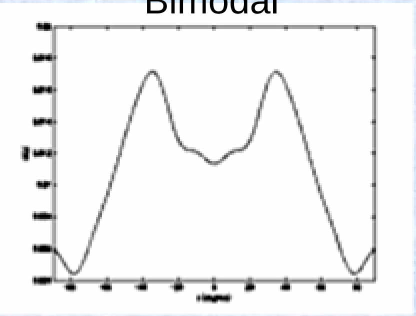
Steepness   Spectral Bandwidth

- Directional spreading $G(\theta) = C(n)\cos^{\boxed{n}}(\theta)$
  - CosN
  - Hwang
  - Bimodal

Hwang & Wang 2001



Bimodal



Generating random wave in a tank

- DS (Double Summation) method
  - Large number of waves; non-ergodic (amplitude nodes)

$$\eta(x,y,t)=\sum_{i=1}^{I}\sum_{j=1}^{J}a_{ij}\cos(\omega_i t-\mathbf{k}_{ij}\cdot\mathbf{x}+e_{ij})$$

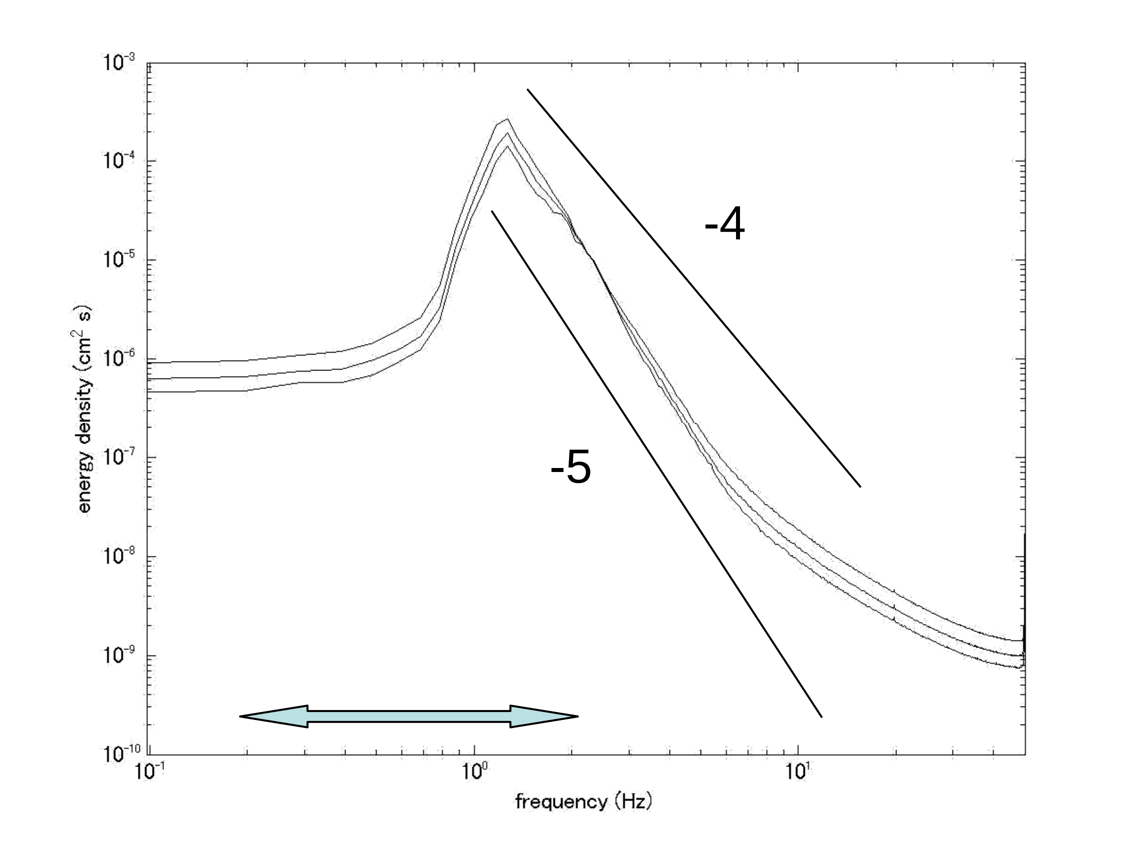
$$a_{ij}=\sqrt{2S(\omega_i,\theta_j)\Delta\omega\Delta\theta}$$

- **<u>SS (Single Summation) method</u>**
  - Ergodic (no nodes); reduced number of modes

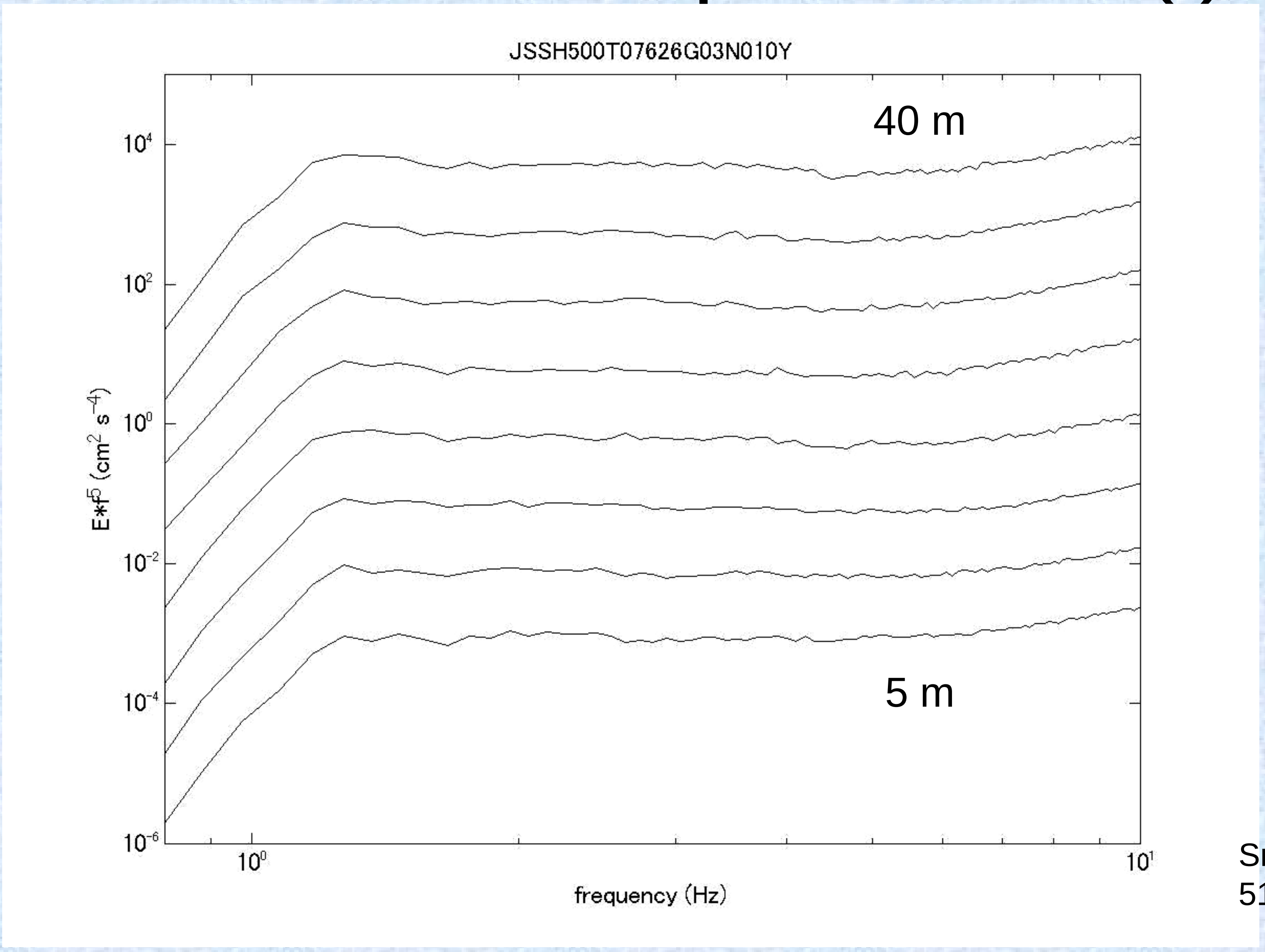
$$\eta(x,y,t)=\sum_{n=1}^{n=N}a_n\cos(\omega_n t-\mathbf{k_n}\cdot\mathbf{x}+e_n)$$

$$a_n=\sqrt{2S(\omega_n)\Delta\omega}$$

1024 waves
Discretization: conserve energy per bin
Frequency ~ 0.2 – 2.0 Hz
1 hour record (~4000 wave period)
No wind



High-frequency tail saturation spectrum: E(f)xf^5

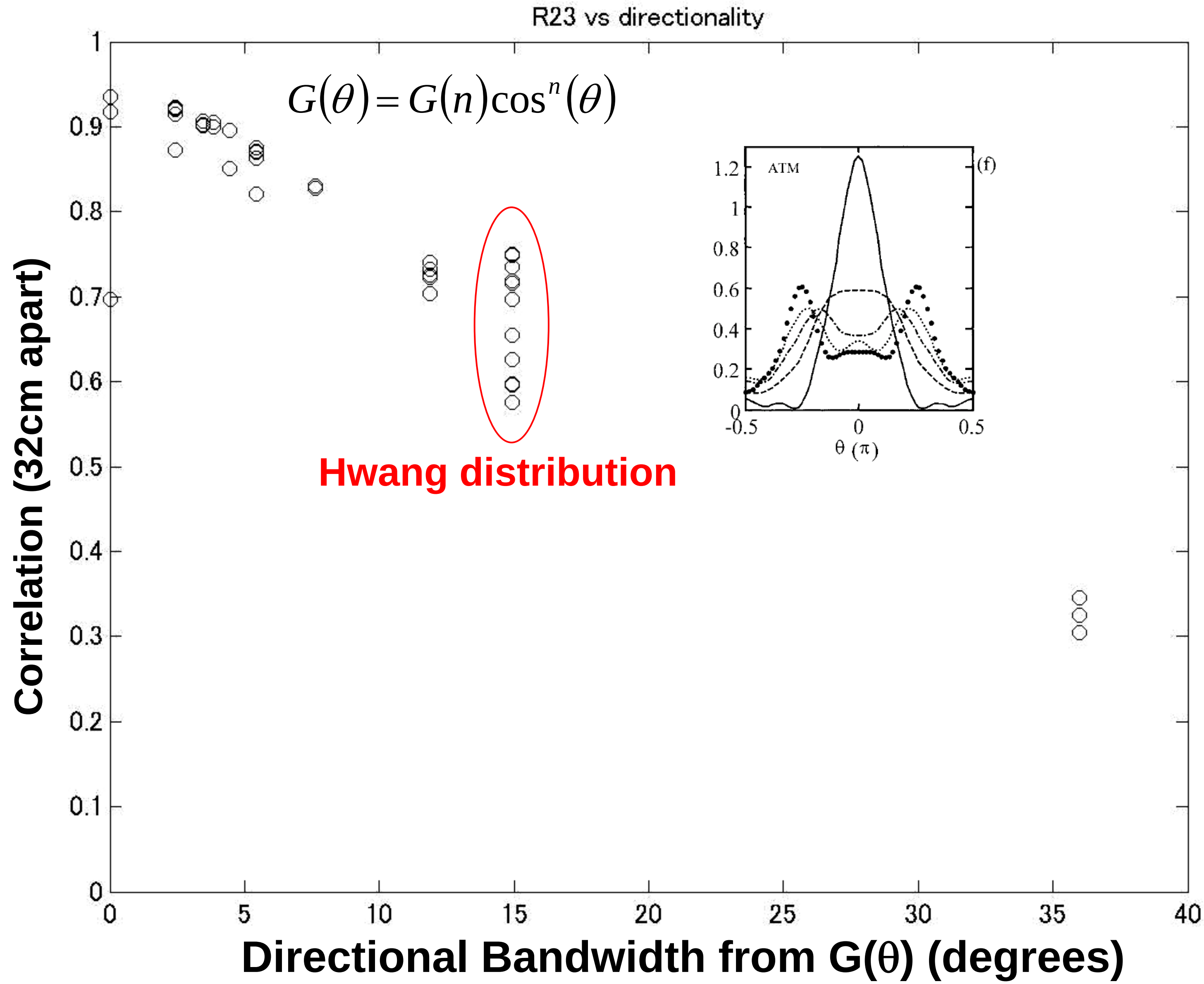


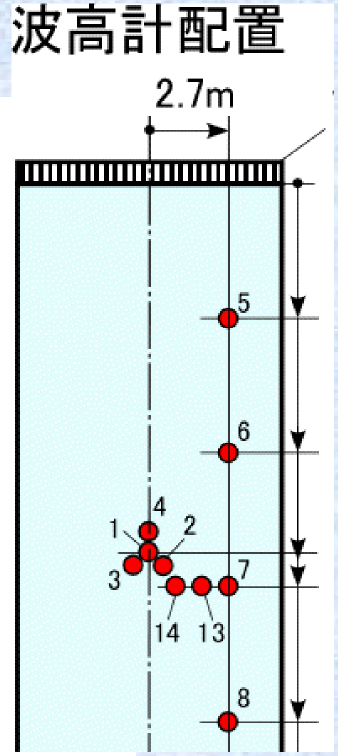
$$\gamma = 3.0$$

$$n = 10$$

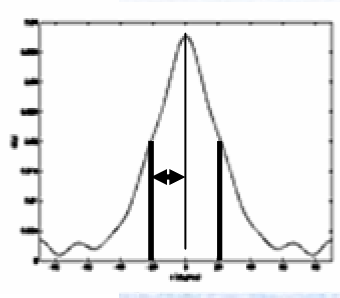
Smoothed periodogram
512 degrees of freedom

Cross-correlation



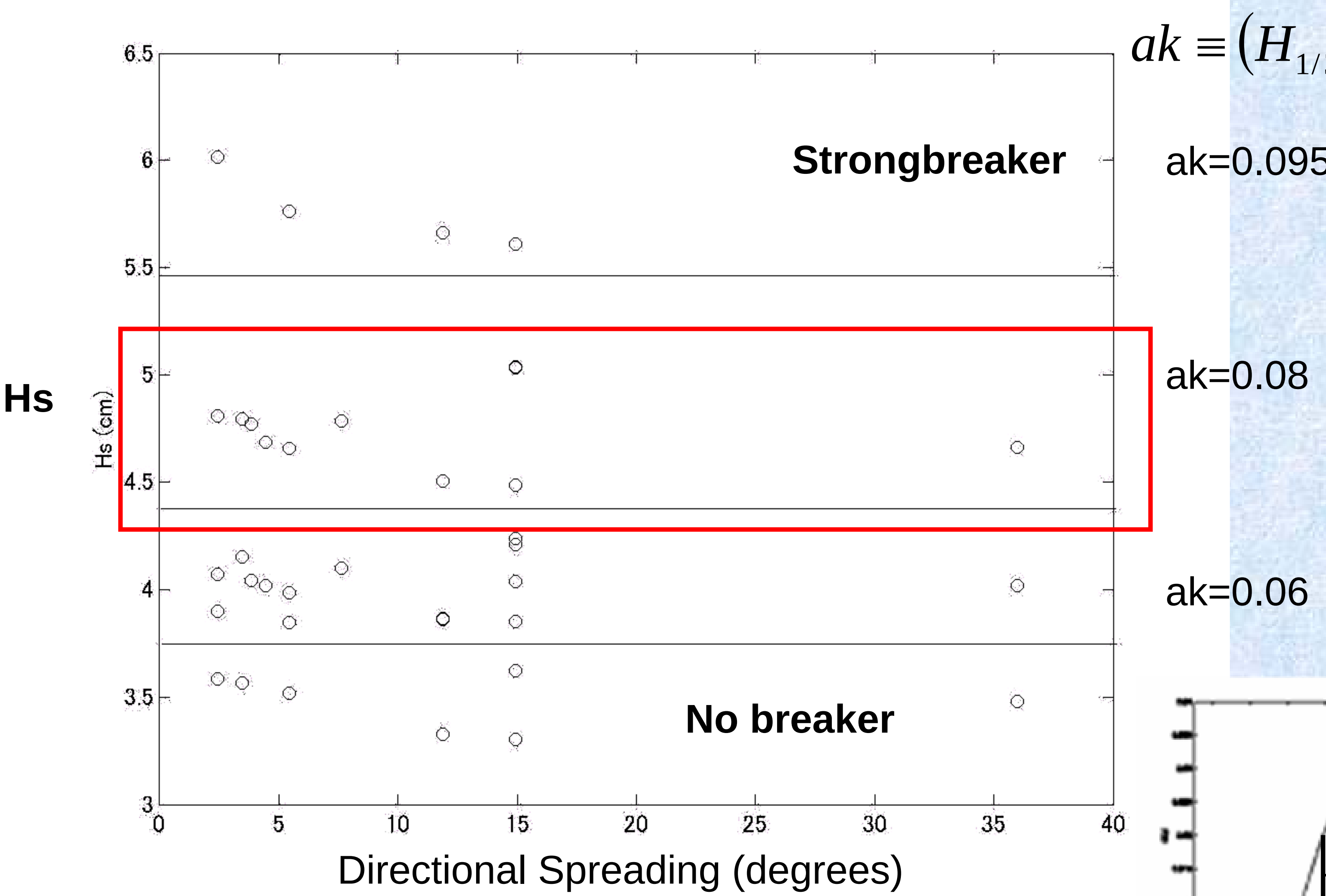


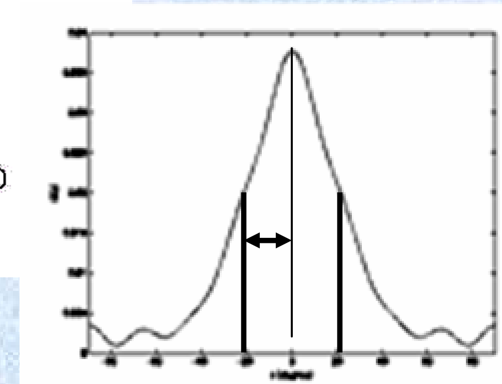
Correlation
32cm apart
across tank



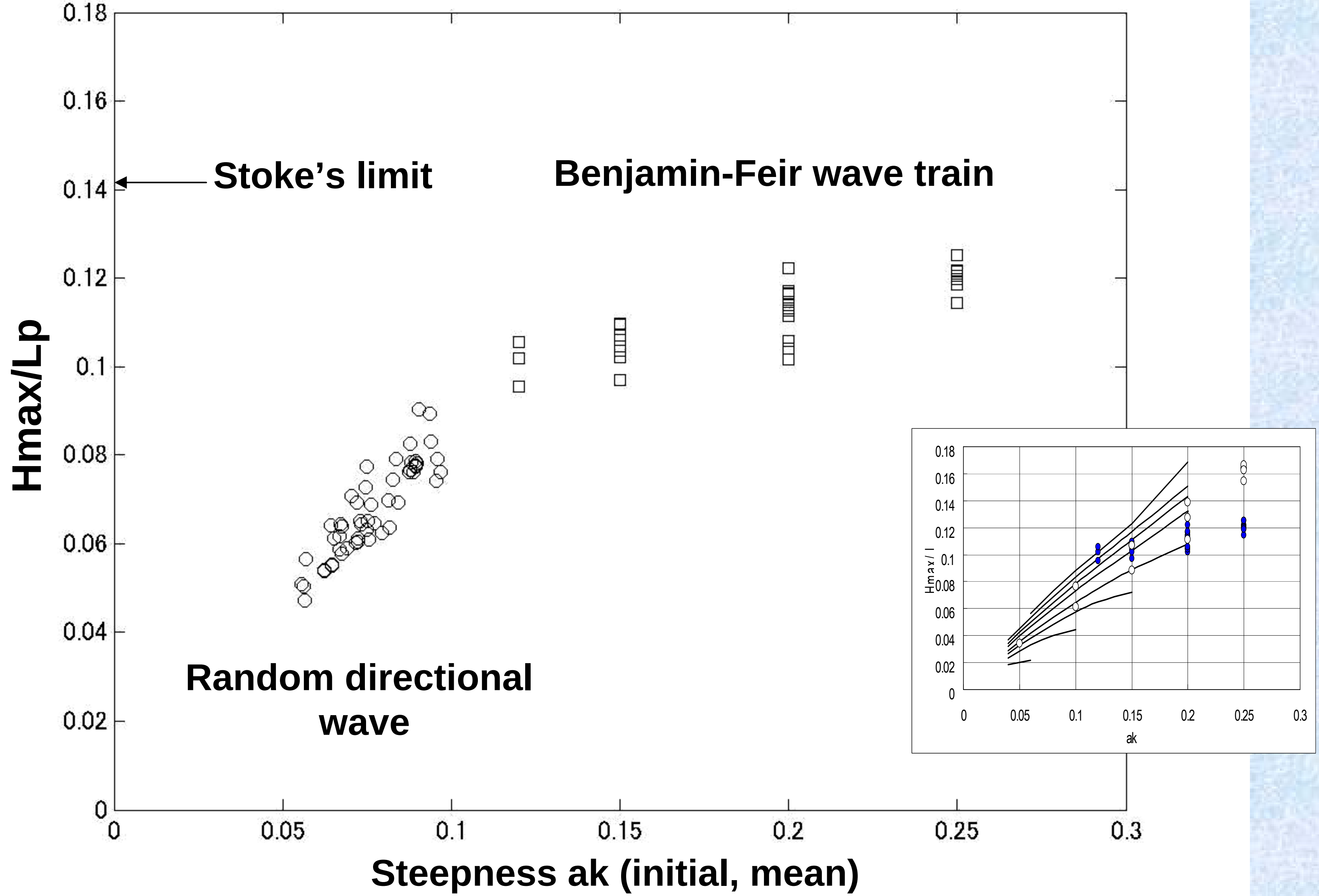
April 2007 Experimental Cases:
for a fixed frequency spectrum; Mean Wave length L=1m

$$ak \equiv \left(H_{1/3}/4\right)\left(2\pi/L\right)$$

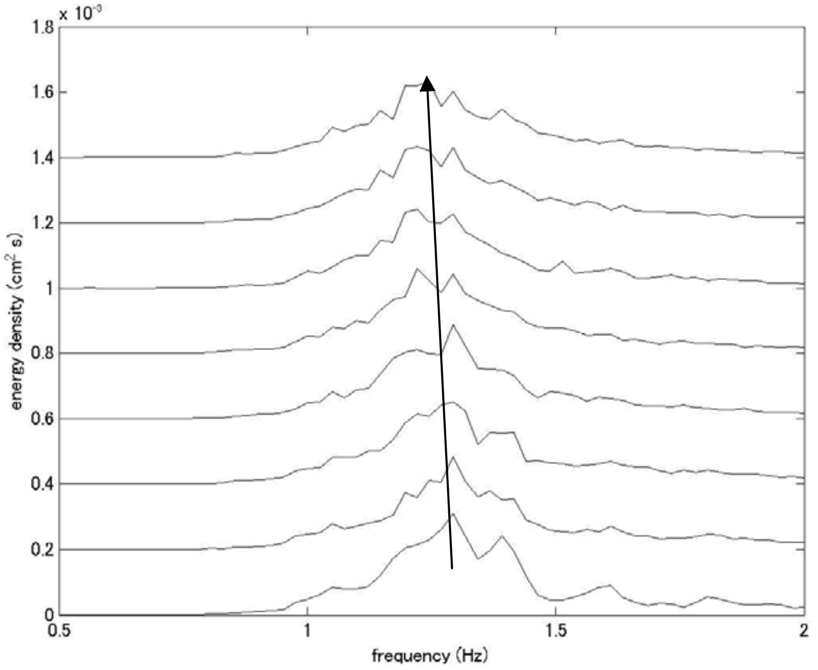




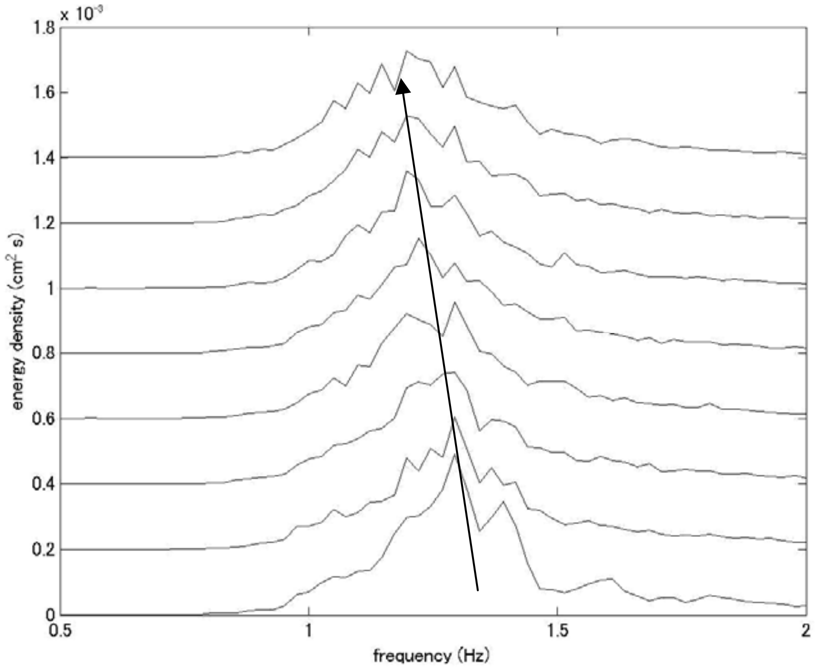
Maximum wave height



Spectral evolution (typical cases)



$\gamma = 3.0;\quad n = 125;\quad \boxed{\mathrm{Hs} = 4.15}$



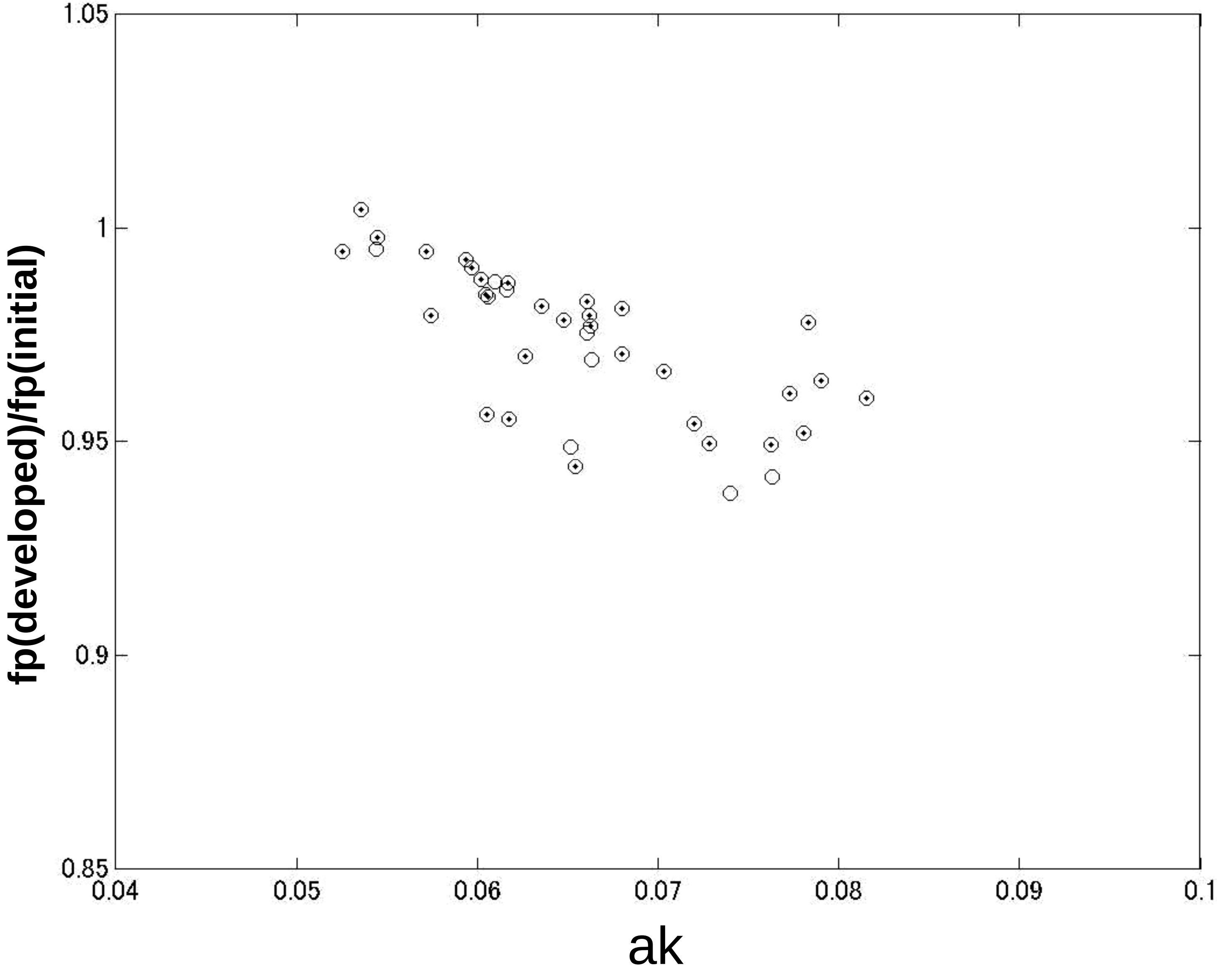
$\gamma = 3.0;\quad n = 125;\quad \boxed{\mathrm{Hs} = 5.00}$

$$S(f,\theta) = S(f)\cdot G(f,\theta)$$

$$S(f) = \boxed{\alpha}\, g^2 (2\pi)^{-4} f^{-5} \exp\left\{-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right\} \boxed{\gamma}^{\exp\left\{-\frac{(f-f_p)^2}{2\sigma^2 f_p^2}\right\}}$$

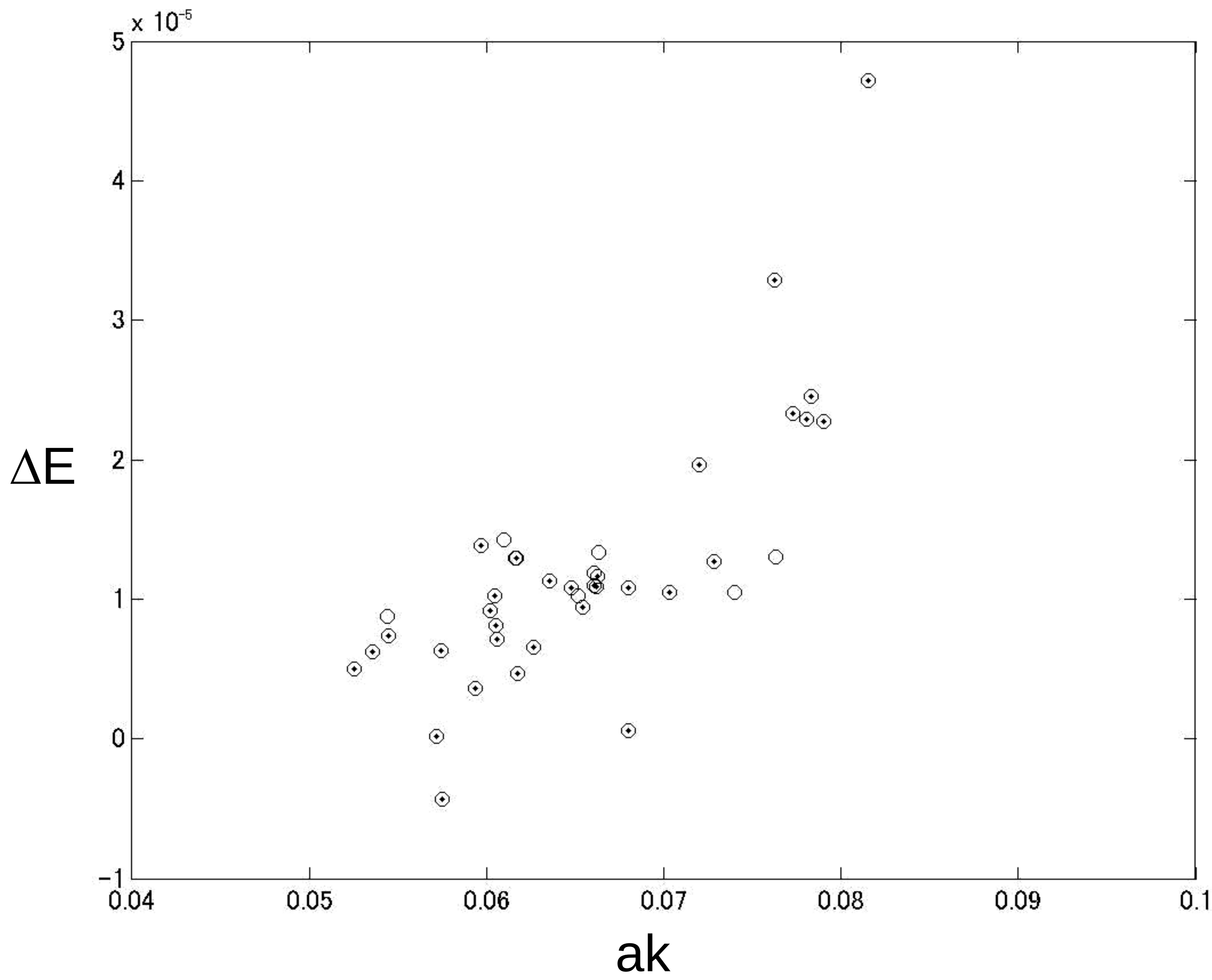
$$G(\theta) = G(n)\cos^{\boxed{n}}(\theta)$$

Spectral downshifting of random directional wave



BF

Energy Loss

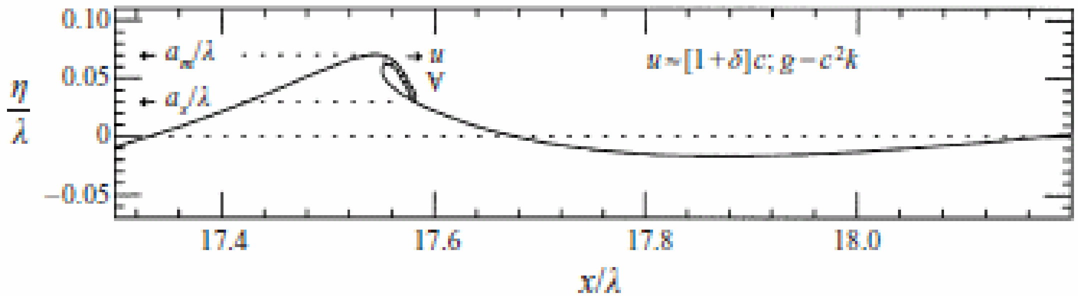


Downsh

$$E_T = \sum_j E_j$$ a

$$M_T = \sum_j M_j$$ wh

$$\omega_p = \frac{\sum_j \omega_j E_j}{\sum_j E_j}$$



$$\delta E = -\sum \left[\frac{\rho u^2}{2} + \rho g(a_m - a_s)\right]\forall = -\sum [1 + 2\delta + 2(a_m - a_s)k]\frac{\rho c^2}{2}\forall,$$

$$\delta M = -\sum \rho u \forall = -\sum \rho c(1+\delta)\forall = (1+\gamma)\frac{\delta E}{c},$$

where $$\gamma = \frac{1 - 2a_m k(1 - a_s/a_m)}{1 + 2\delta + 2a_m k(1 - a_s/a_m)} \approx 0.4 \text{ (strong) to } 0.7 \text{ (weak)},$$

$$[\delta \lesssim 0.1; \quad ka_m \approx 0.25 - 0.50; \quad a_s/a_m \approx 0.5 \text{ (strong) to } 0.9 \text{ (weak)}].$$

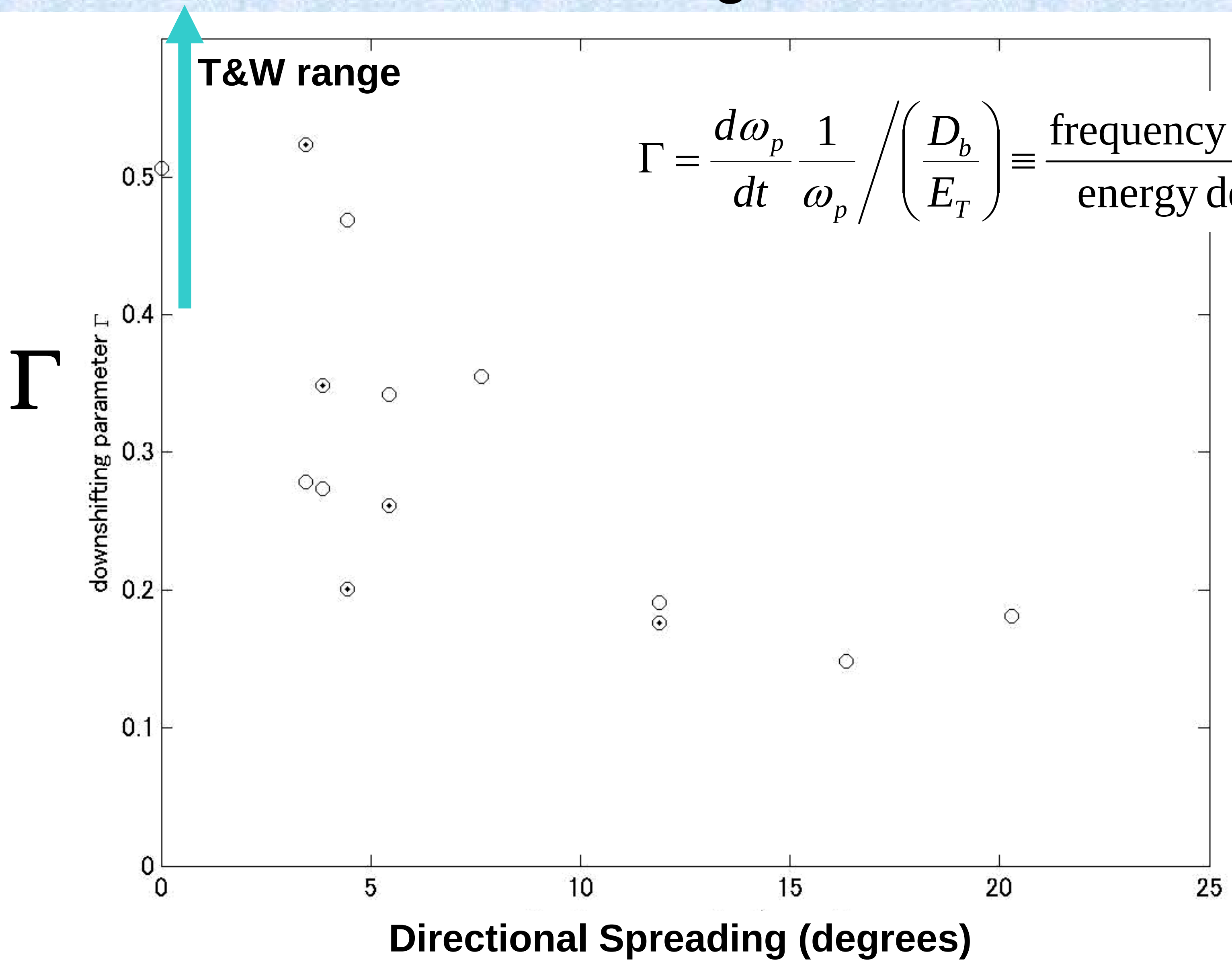
FIGURE 21. Schematic of breaking wave and jet, with quantification of energy and momentum losses.

Tulin&Waseda 1999

$$\frac{d\omega_p}{dt} = \frac{\omega_p}{E_T}\left(c_p \dot{M}_T - D_b\right) \approx -\frac{\omega_p}{E_T}\Gamma D_b$$

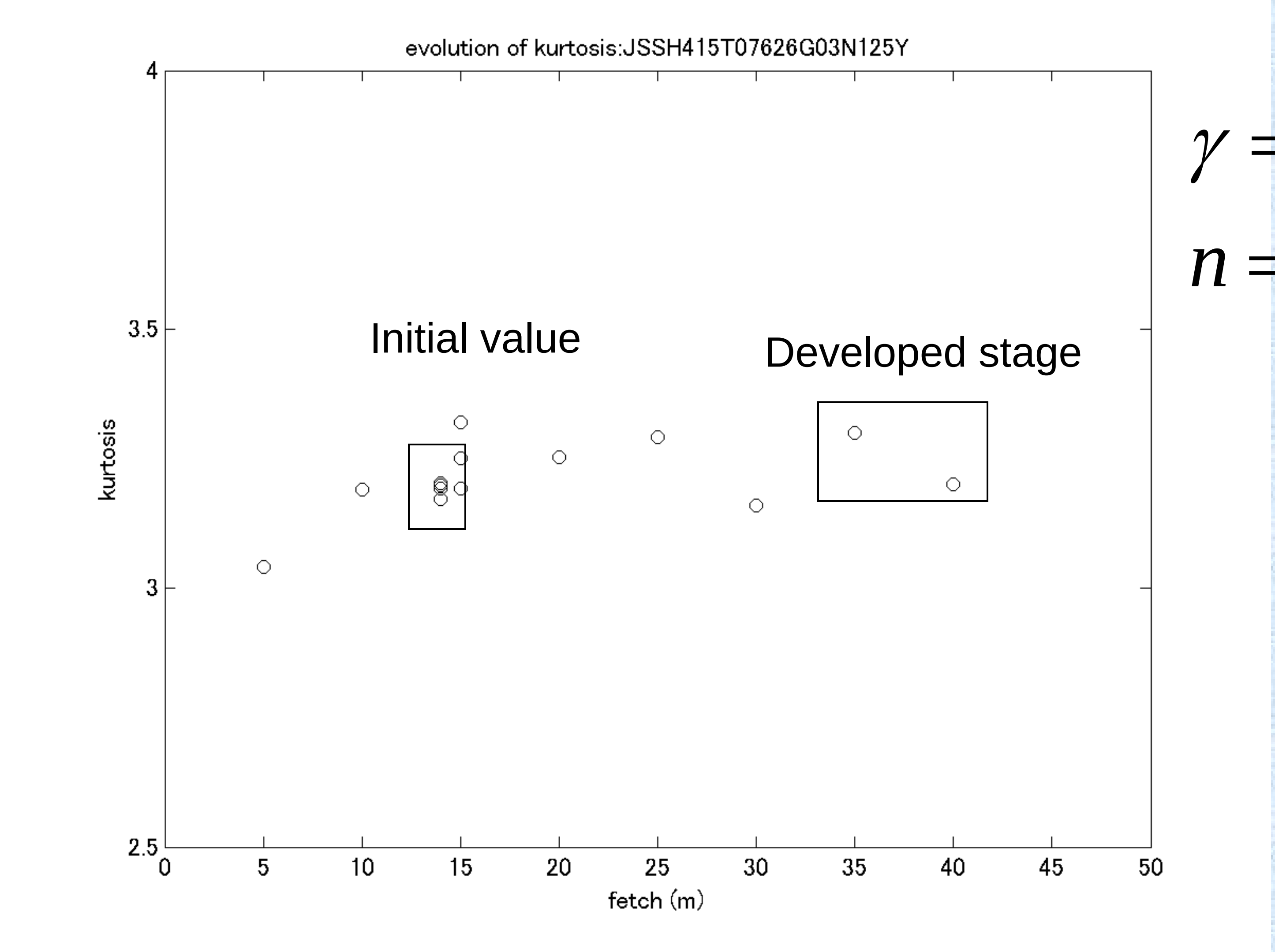
$$\boxed{\Gamma = \frac{d\omega_p}{dt}\frac{1}{\omega_p}\Bigg/\left(\frac{D_b}{E_T}\right) \equiv \frac{\text{frequency decay rate}}{\text{energy decay rate}}}$$

Downshifting parameter Γ estimated for *strong breaker* cases



$$\Gamma = \frac{d\omega_p}{dt}\frac{1}{\omega_p}\Bigg/\left(\frac{D_b}{E_T}\right) \equiv \frac{\text{frequency decay rate}}{\text{energy decay rate}}$$

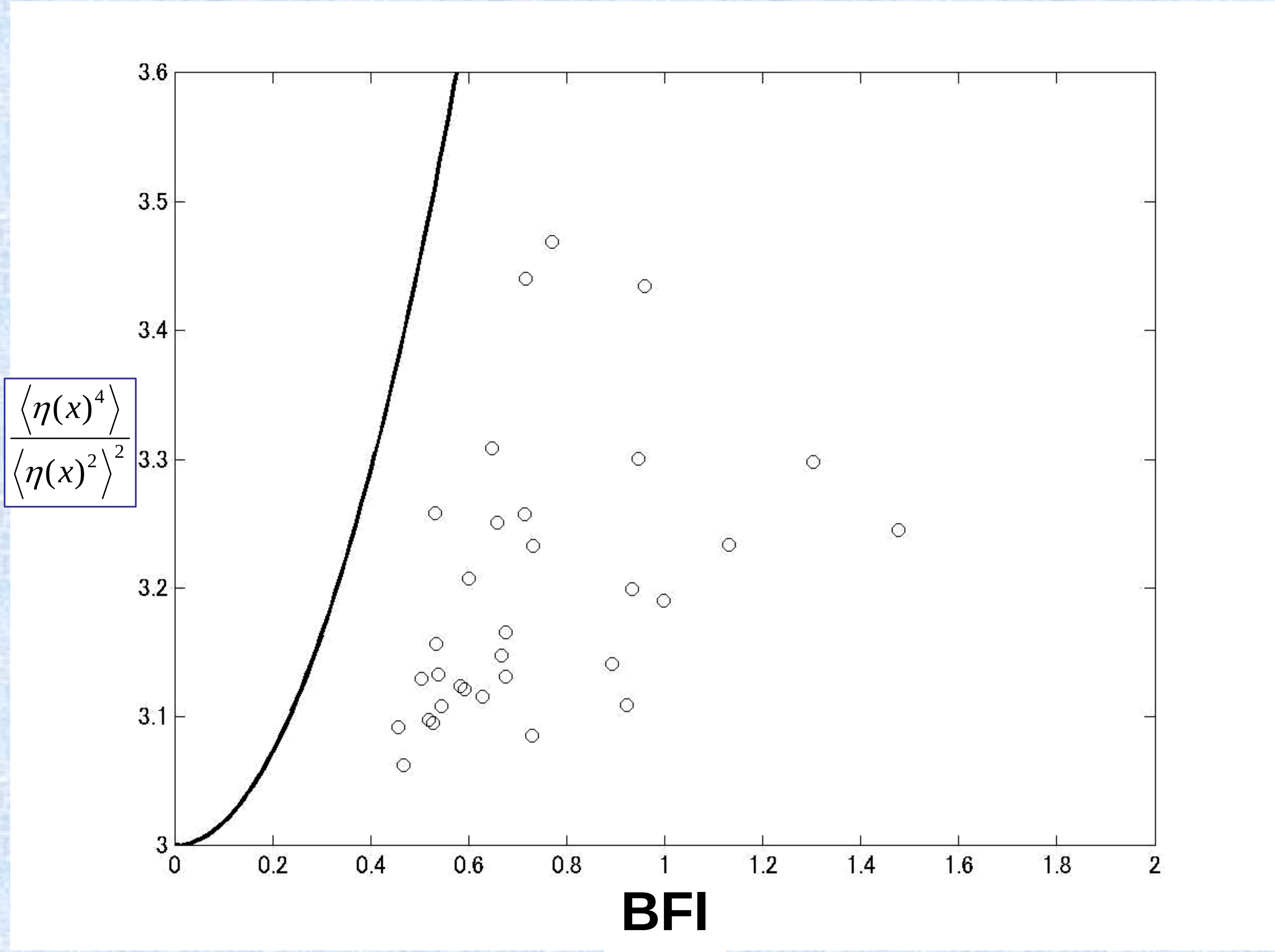
Evolution of Kurtosis with fetch



$$\gamma = 3.0$$

$$n = 125$$

BFI vs. kurtosis

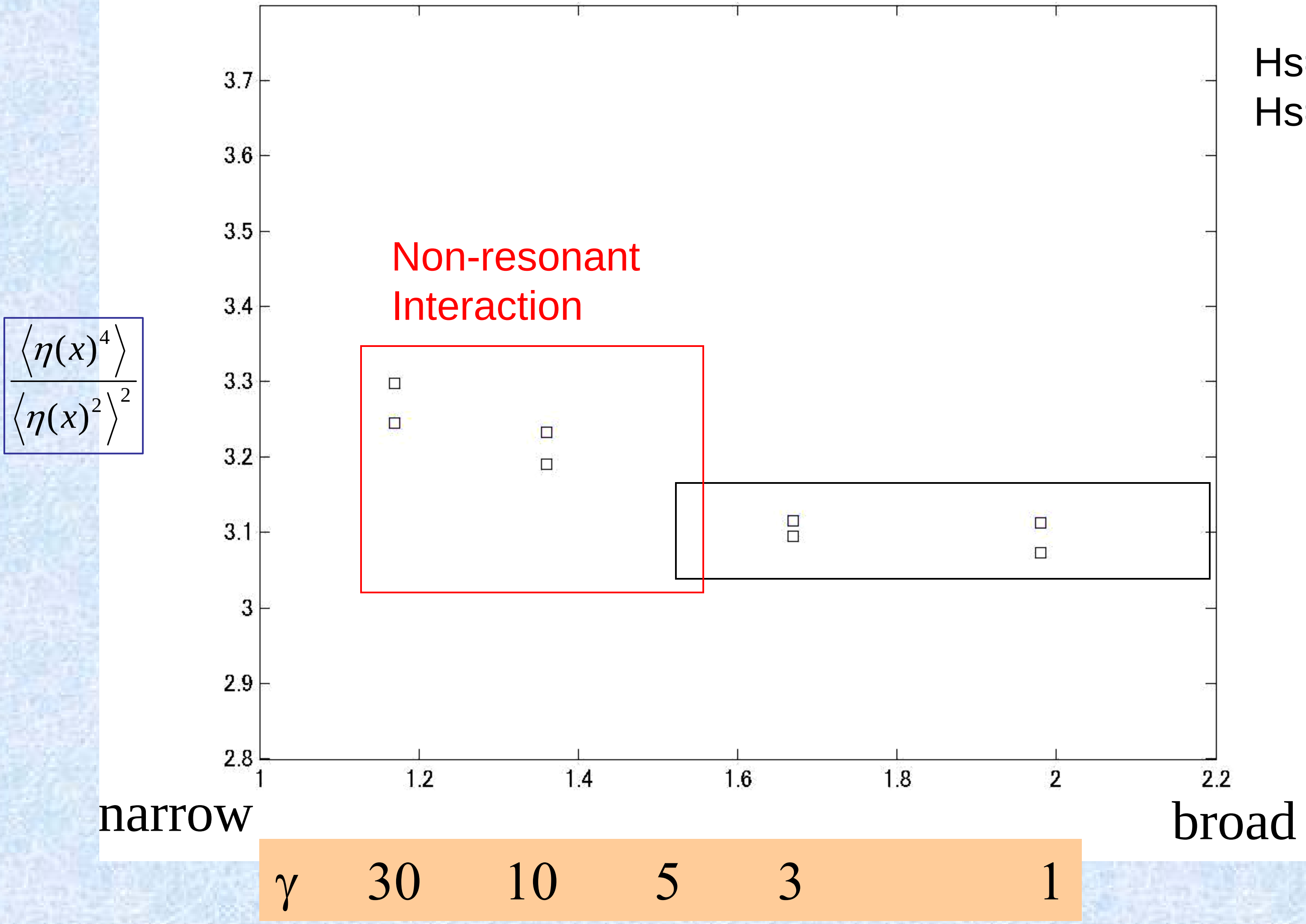


Line: $\kappa_{40} = \frac{\pi}{\sqrt{3}} BFI^2; \quad BFI = \frac{\varepsilon}{\Delta\omega / \omega}\sqrt{2}$

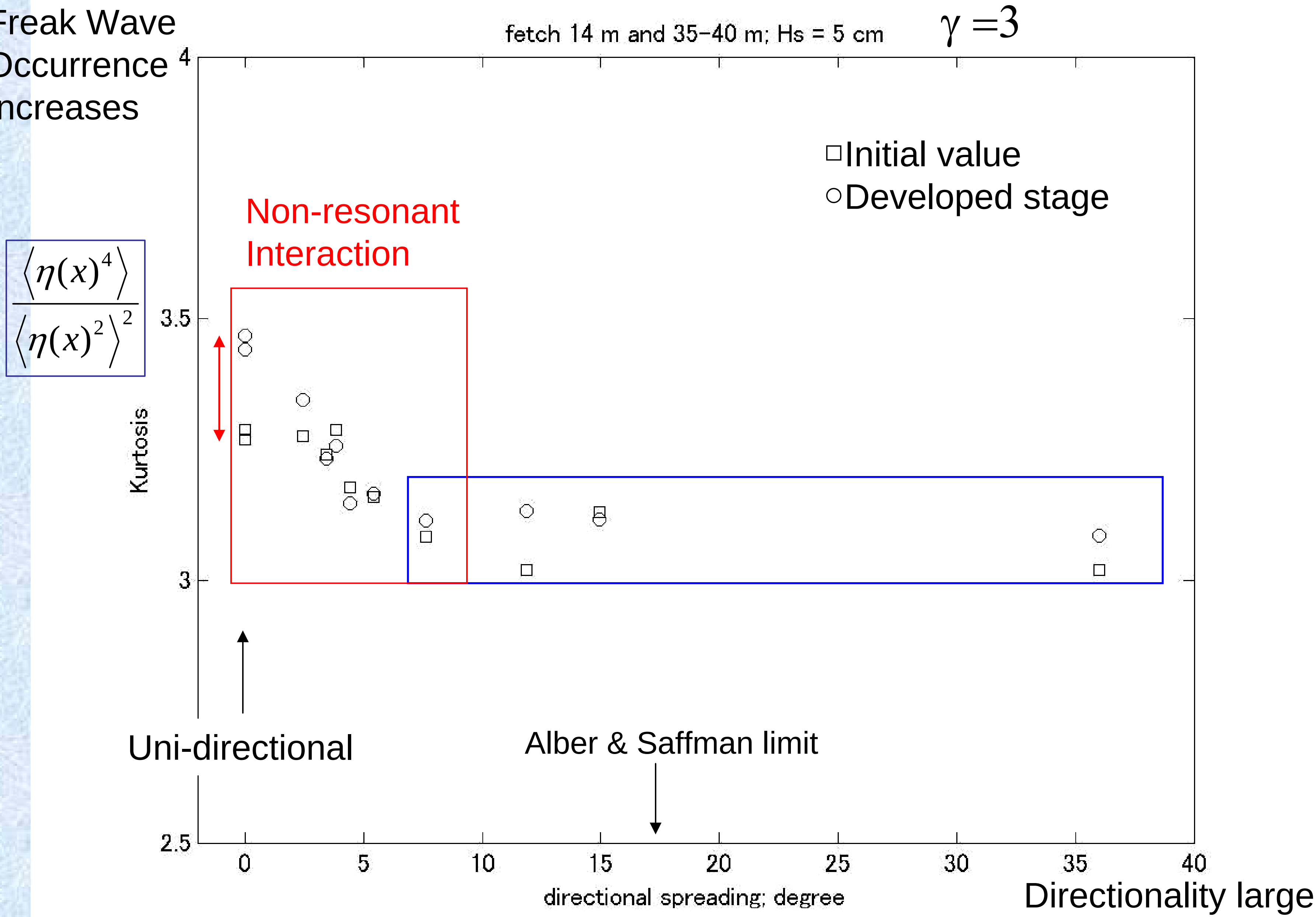
Kurtosis and spectral bandwidth (γ) Hwang directional distribution

Developed stage

Hs=4.15
Hs=5.00

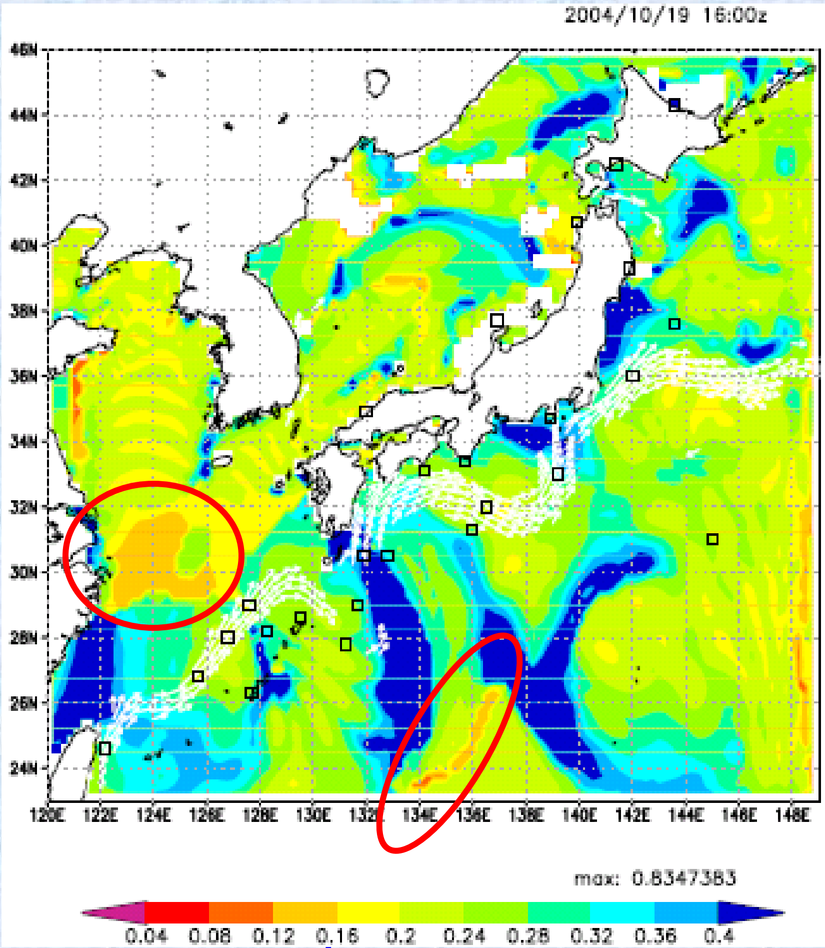


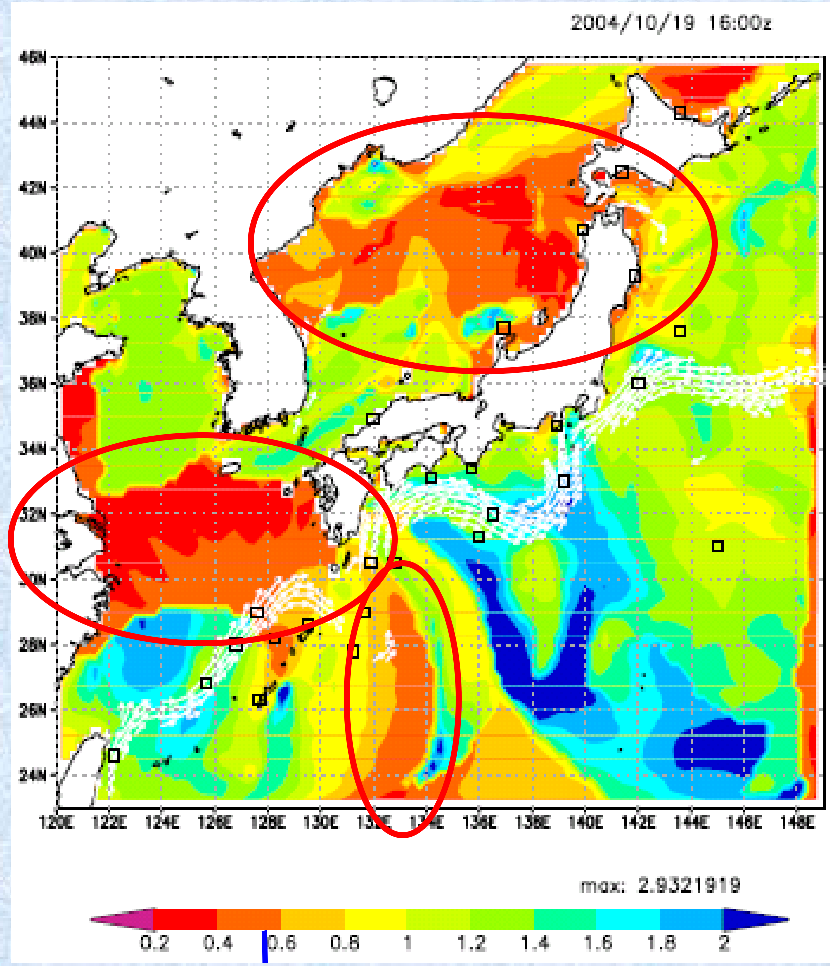
Freak Wave Occurrence Increases



n	250	50	25	10	5	3	1

High-resolution wave-current coupled model
Wave-JCOPE, Snl(SRIAM)





γ 40 5 3 2

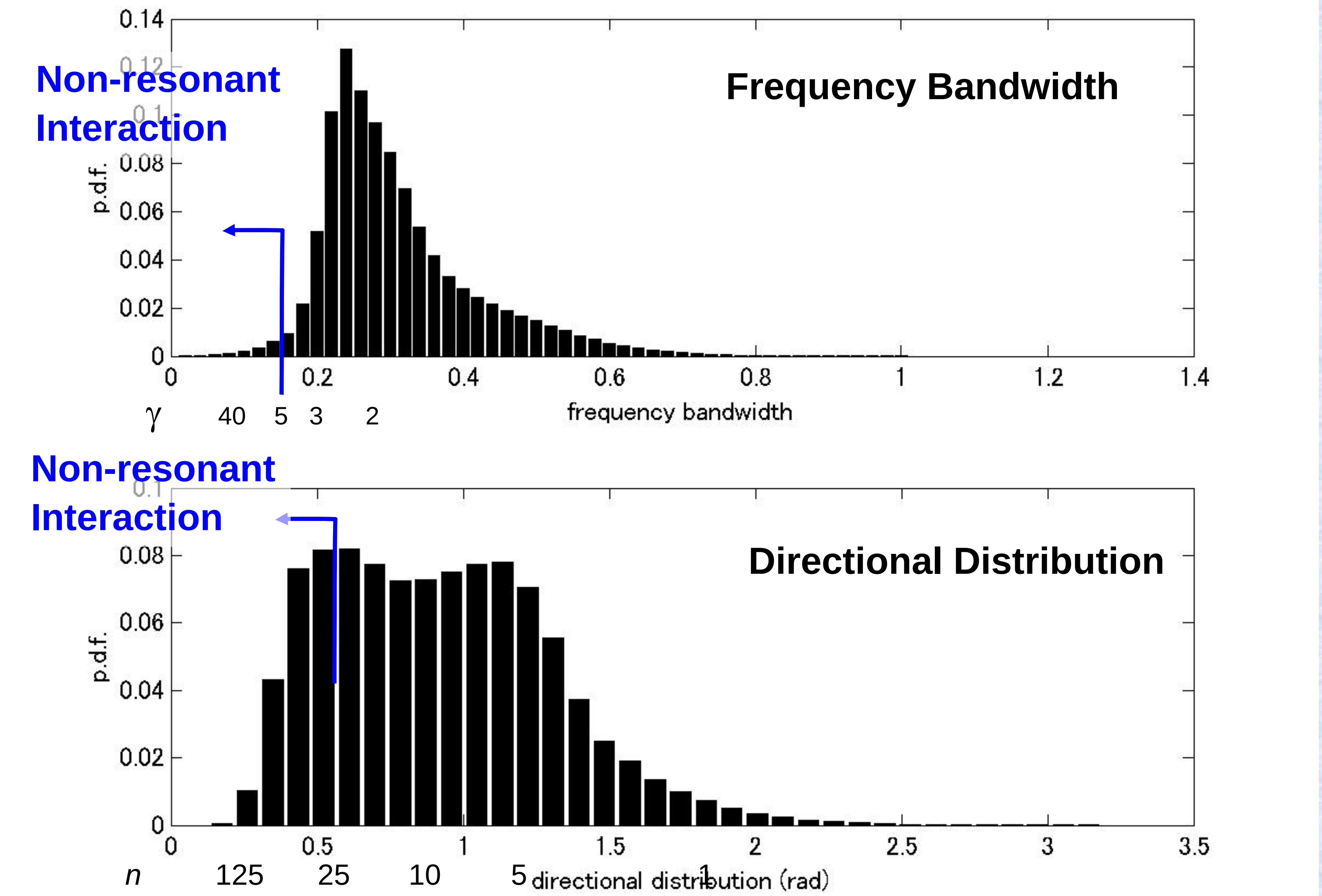
n 125 25 10 5 1

Degree 7.7 16.3 30

A&S limit

Tamura, Waseda, Miyazawa & Komatsu, 11/16 presentation

Wave parameter distribution



Concluding remarks

- Evolution of random directional wave in the laboratory tank was studied and revealed that the non-resonant interaction becomes significant as the spectrum narrows, including cases with energetic breakers
- As a result of breaking dissipation, the spectrum downshifts suggesting that the combination of non-resonant interaction and wave breaking is the relevant downshifting mechanism for narrow spectrum
- Analysis of high-resolution wind-wave coupled model suggests that favorable condition for freak wave occurrence, i.e. narrow frequency bandwidth and narrow directional spread, is rare.

Hypothesis: Freak wave is an expected realization for an abnormal wave condition when the wave spectrum is narrow