10TH INTERNATIONAL WORKSHOP ON WAVE HINDCASTING AND FORECASTING AND COASTAL HAZARD SYMPOSIUM

Freak Wave Prediction from Spectra

- Maximum Wave Height Prediction in 1D Waves -

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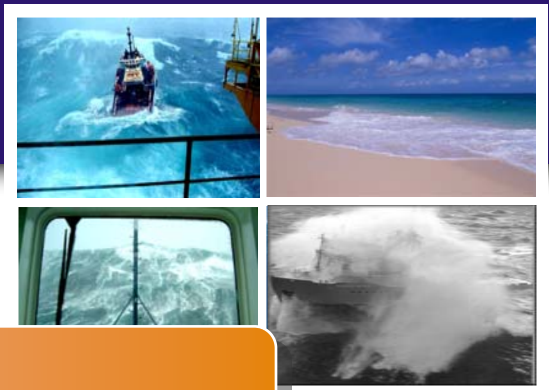
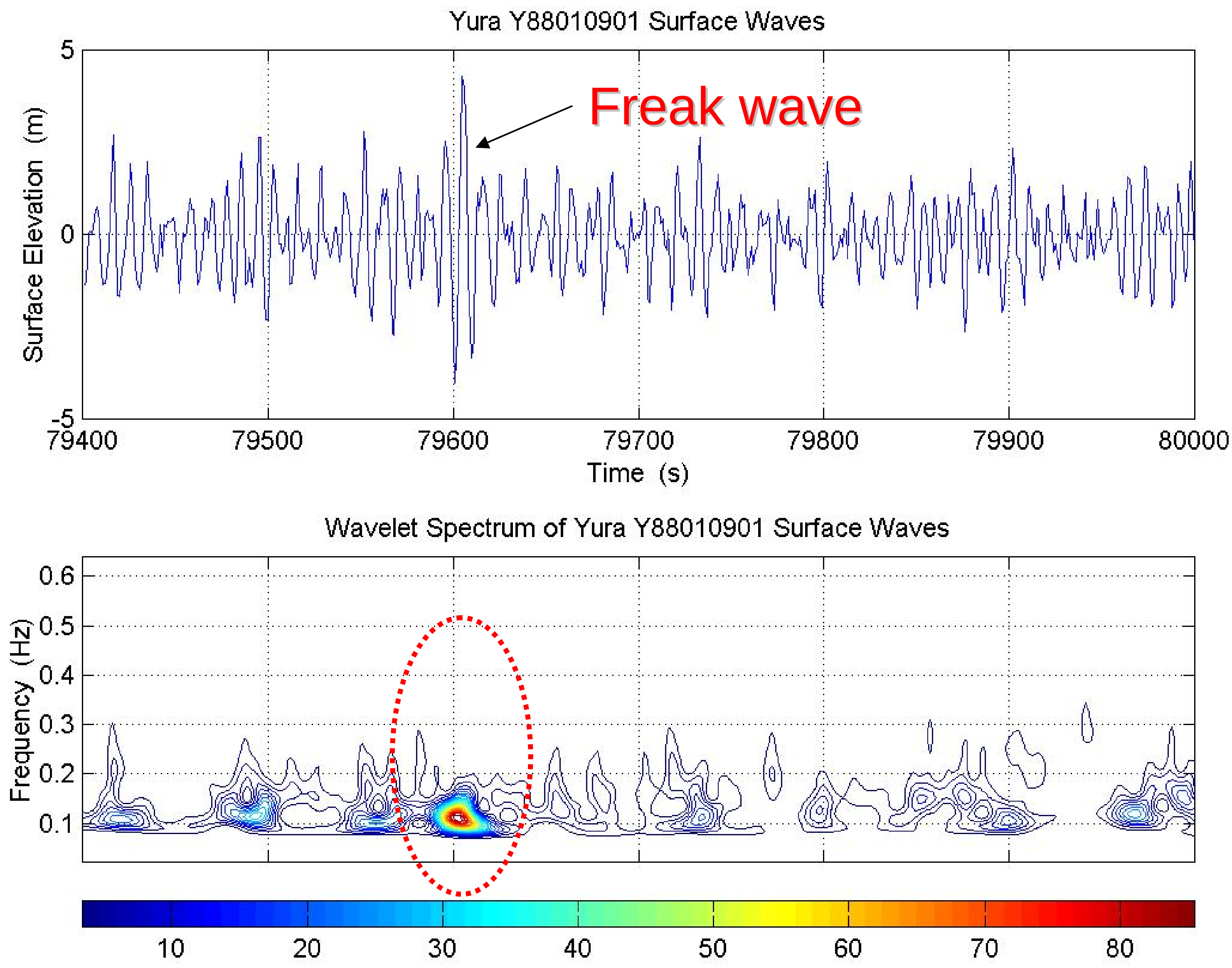
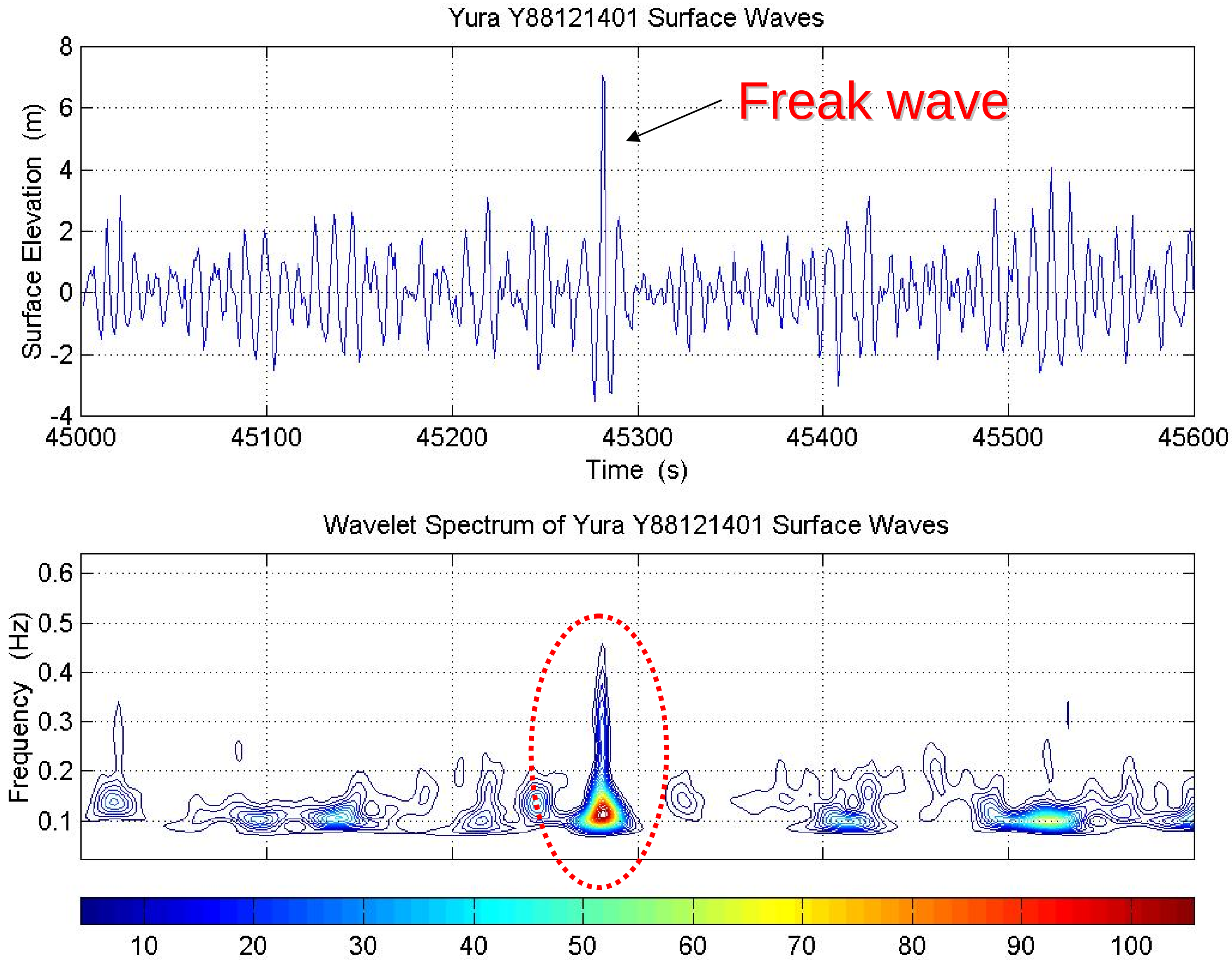
Contents



Mori and Janssen (2006) JPO
Mori, Onorato, Janssen, Osborne and Serio (2007) JGR

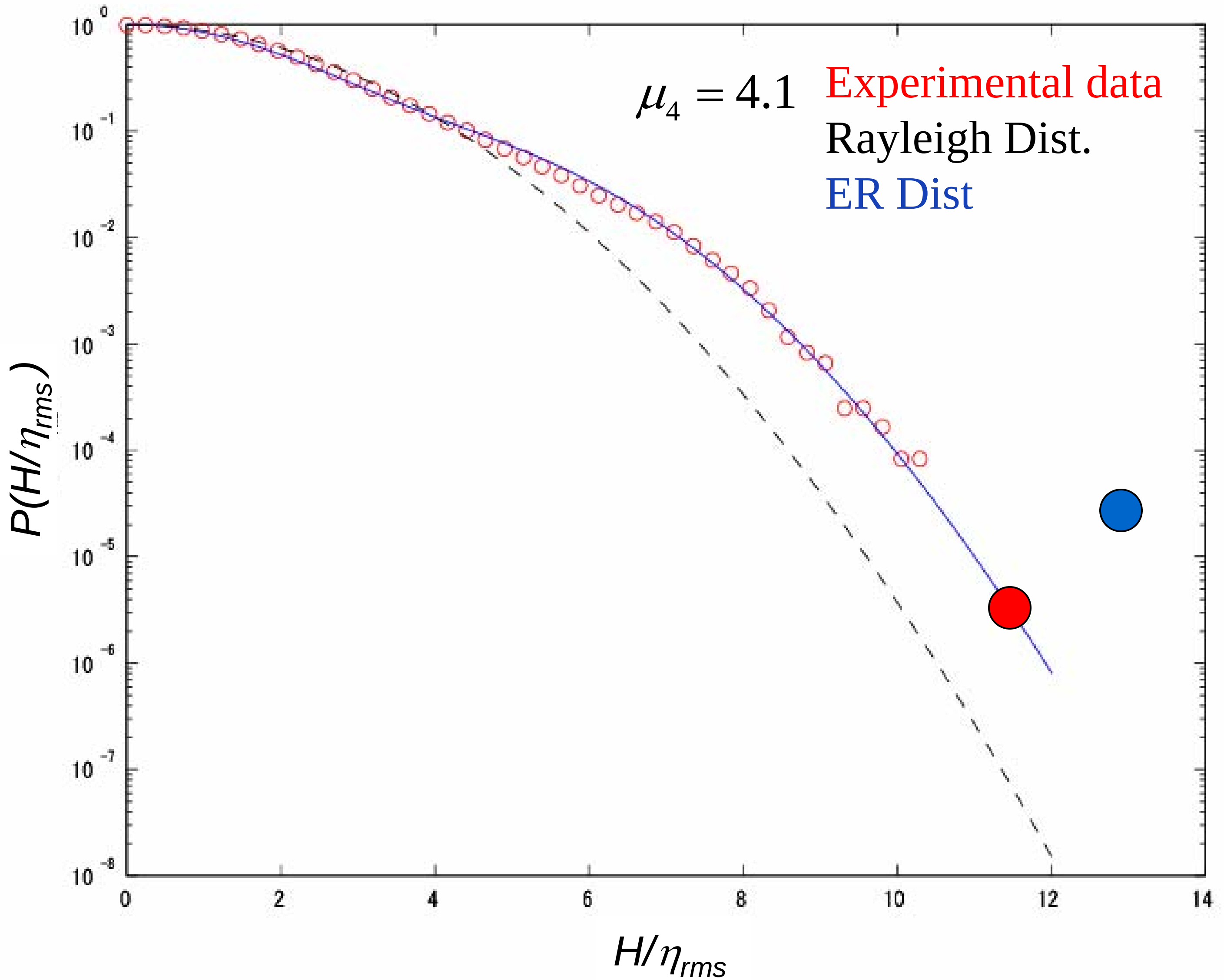


Mori, Liu and Yasuda (2002) Ocean Eng.



Mori, Liu and Yasuda (2002) Ocean Eng.

Is Freak Wave Predictable?

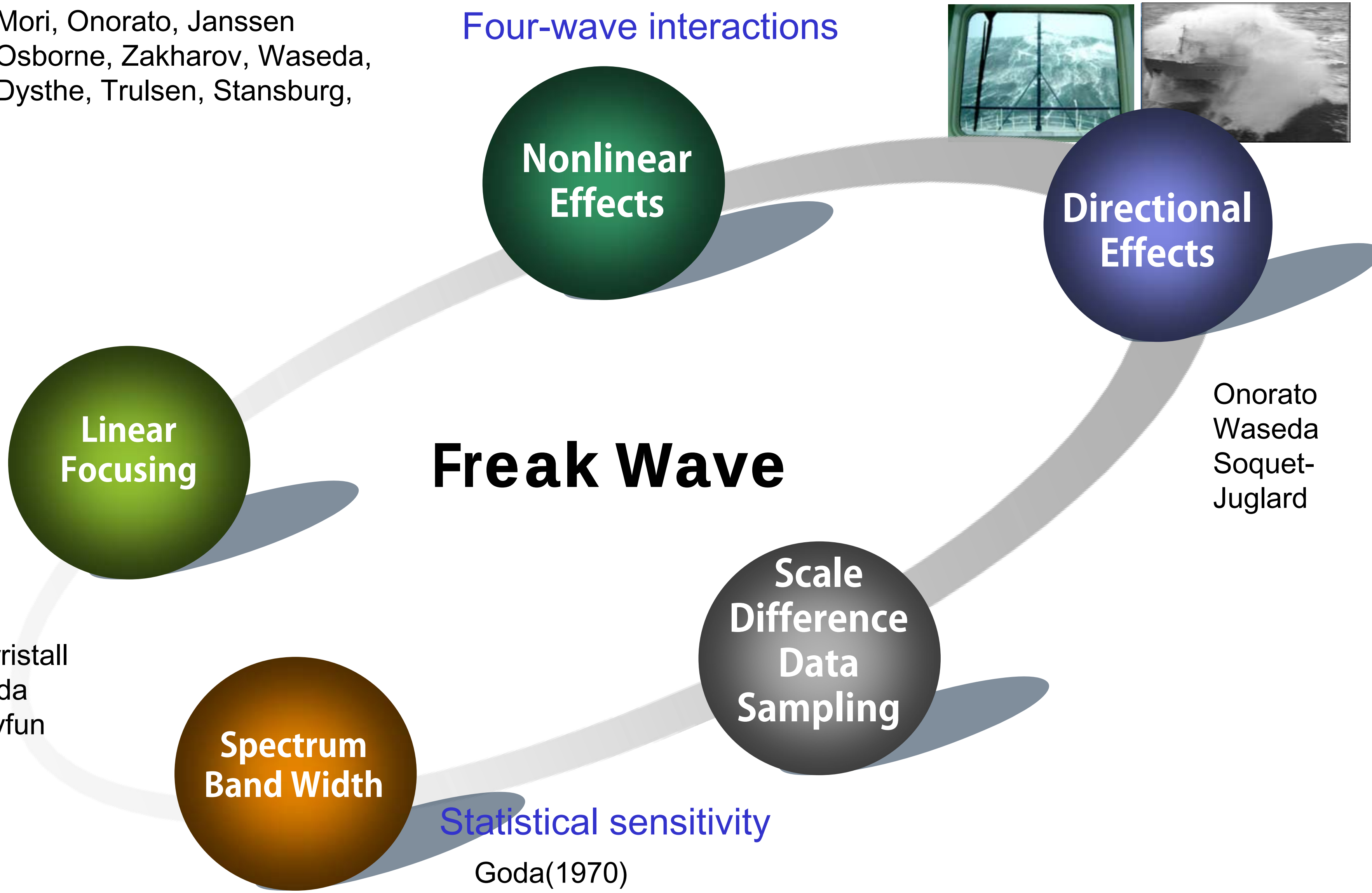


Conclusion

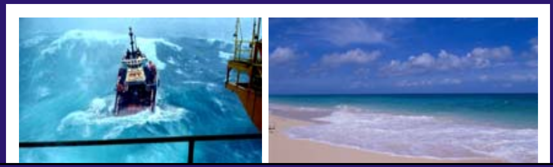
- Kurtosis depends on the shape of wave spectra and changes owing to quasi-resonant interactions.
- Freak waves/maximum wave height distributions can be predicted by a weakly non-Gaussian theory as a function of kurtosis.

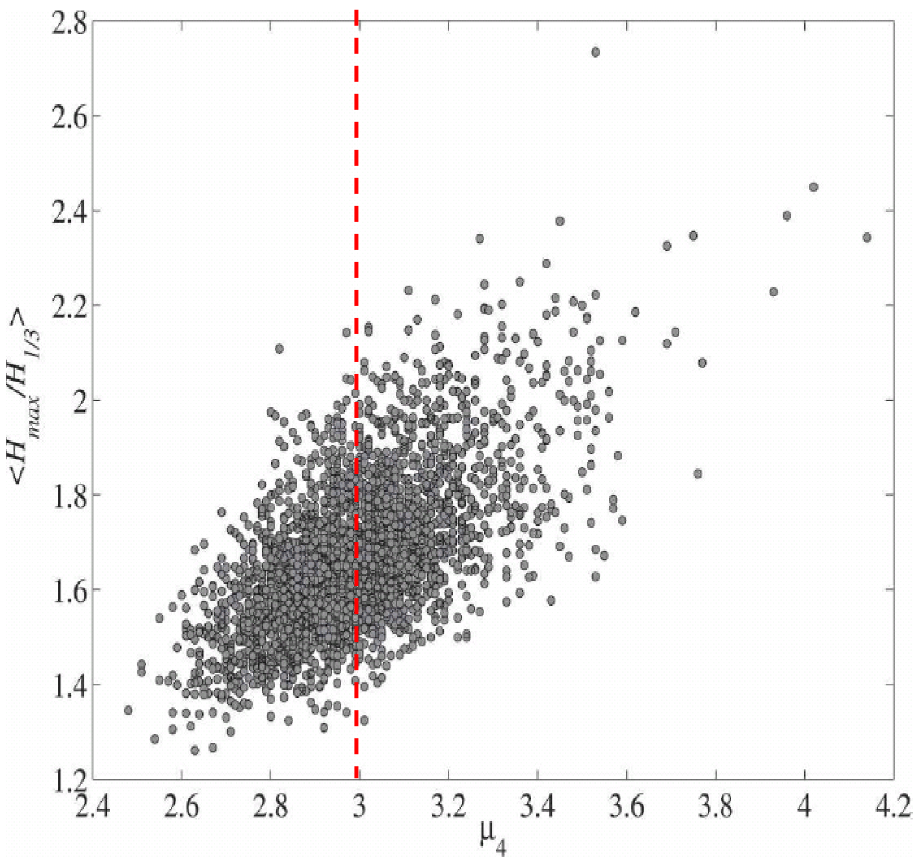
$H_{max}/H_{1/3}>2$

Mori, Onorato, Janssen
Osborne, Zakharov, Waseda,
Dysthe, Trulsen, Stansburg,

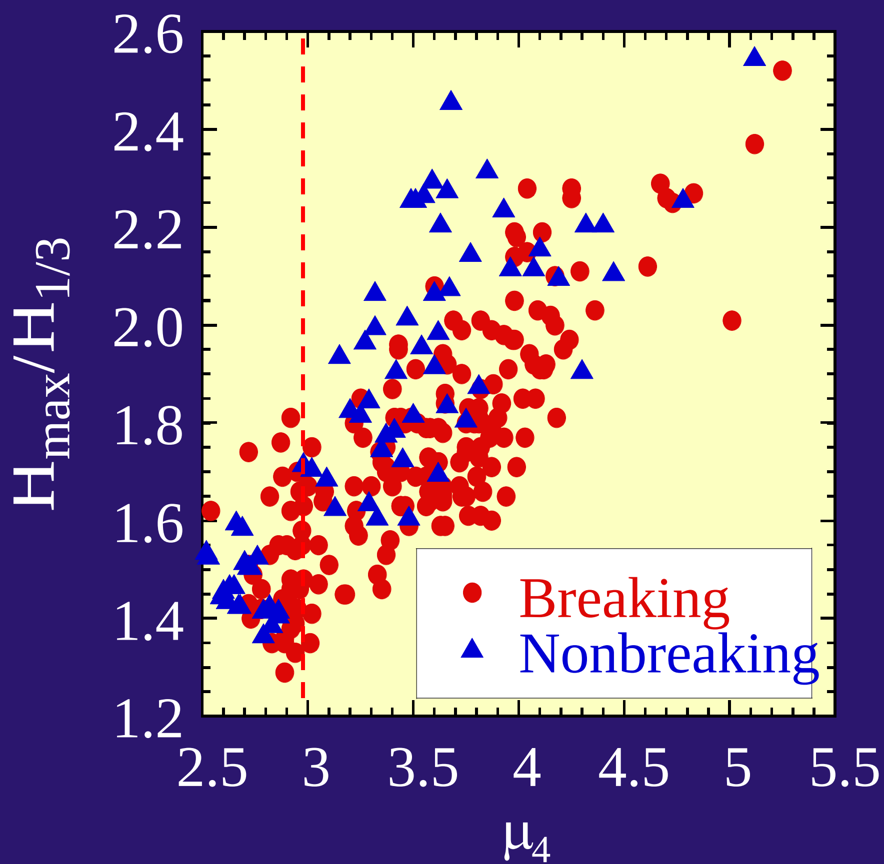


$H_{max}/H_{1/3}$ vs kurtosis





Field data (buoy)



Experimental data

Freak Wave Prediction System

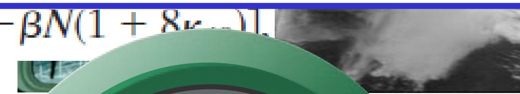
$$\kappa_{40} = \frac{\langle \eta^4 \rangle}{m_0^2} - 3$$
$$= \mu_4 - 3$$

$$\kappa_{40} = \frac{\pi}{\sqrt{3}} \mathrm{BFI}^2.$$

$$\mathrm{BFI} = \frac{\epsilon}{\Delta}\sqrt{2}.$$

$$p(H) = \frac{1}{4} He^{-(1/8}$$

$$P_{\text{freak}} = 1 - \exp[-\beta N(1 + 8\kappa_{40})],$$



Wave Spectra → **kurtosis μ_4** → **Max. Wave Height**

Assumptions
- Homogeneous
- Consider free waves
- Unidirectional
- Gaussian dist. for frequency spectra

Assumptions
- Weakly Gaussian process for η
- Narrow banded spectra (H=2A)

Mori, N. and P.A.E.M. Janssen (2006) JPO

Background

Nonlinear transfer gives rise to deviations from Gaussian

$$\kappa_{40} = \mu_4 - 3 = \frac{12}{g^2 m_0^2} \int \mathrm{d}\vec{k}_{1,2,3,4} T_{1,2,3,4} \sqrt{\omega_1 \omega_2 \omega_3 \omega_4} \, \delta_{1+2-3-4} \mathcal{R}_r(\Delta\omega, t) N_1 N_2 N_3 \quad (5)$$

where κ_{40} and μ_4 are fourth cumulant and moment of the surface elevation and

$$\mathcal{R}_r(\Delta\omega, t) = \frac{1 - \cos(\Delta\omega t)}{\Delta\omega} \quad (6)$$

The kurtosis is determined by both resonant and non-resonant interactions.

Simplified Equation for Kurtosis

Eq.(5) is can be simplified for unidirectional case.

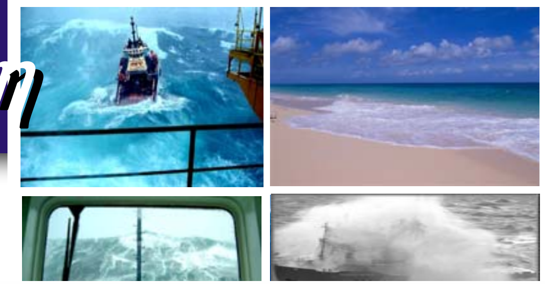
$$\kappa_{40} = \quad = \quad \frac{\pi}{\sqrt{3}} BFI^2 \tag{10}$$

$$BFI \quad = \quad \frac{\sqrt{2}\epsilon}{\delta_\omega} k_0 \sqrt{m_0} Q_p^2 \tag{11}$$

$$Q_p \quad = \quad \frac{2}{m_0} \int d\omega \, \omega E^2(\omega) = \frac{1}{\sqrt{\pi}} \frac{\omega_0}{\sigma_\omega} = \frac{1}{\sqrt{\pi} \Delta_\sigma} \tag{12}$$

where σ_ω is the width of freaquency spectrum and $\Delta = \sigma_\omega / \omega_0$ the relative frequency spectrum band width. The equation indicates that the kurtosis of unidirectional wave train depends on wave steepness and spectrum band width. This result agrees with the Alber and Saffman (1978).

Weakly Non-Gaussian PDF of η

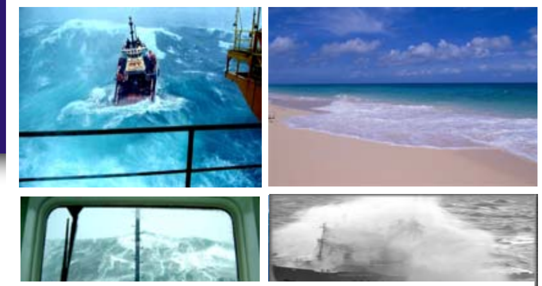


Assuming nonlinear contribution to the surface elevation considering upto fourth-order cumulant, the PDF of the surface elevation can be described by the Edgeworth distribution. The joint probability density function of surface elevation $\eta(t)$ and its envelope $\zeta(t)$ becomes

$$
\begin{aligned}
p(\eta,\zeta) &= \frac{1}{2\pi}\exp\left[-\frac{1}{2}(\eta^2+\zeta^2)\right] \\
&\times\left[1+\frac{1}{3!}\sum_{n=0}^{3}\frac{3!}{(3-n)!n!}\kappa_{(3-n)n}H_{3-n}(\eta)H_n(\zeta)\right. \\
&\left.+\frac{1}{4!}\sum_{n=0}^{4}\frac{4!}{(4-n)!n!}\kappa_{(4-n)n}H_{4-n}(\eta)H_n(\zeta)\right] \qquad (18)
\end{aligned}
$$

Edgeworth Expansion of Gaussian Process (1907)

Wave Height Distribution



Mori and Janssen (2006) found the second-order joint cumulant of η and ζ, κ_{22}, as

$$\kappa_{22} = \frac{\langle \eta^2 \zeta^2 \rangle}{m_0^2} - 1 = \frac{1}{3}\kappa_{40} \tag{19}$$

and proposed modified Edgeworth-Rayleigh (MER) distribution for wave heights.

$$p(H) = \frac{1}{4} H e^{-\frac{1}{8}H^2} \left[1 + \kappa_{40} A_H(H)\right] \tag{20}$$

$$A_H(H) = \frac{1}{384}\left(H^4 - 32H^2 + 128\right) \tag{21}$$

where H is normalized wave height by rms value of η and $p(H)$ is the PDF of wave heights.

Maximum Wave Height Distribution

The PDF of maximum wave height p_m in wave trains can be obtained once the PDF of wave height is known (Goda, 1986).

$$p_m(H_{max})dH_{max} = \frac{N}{4}H_{max}\, e^{-\frac{H_{max}^2}{8}}\left[1+\kappa_{40}A_H(H_{max})\right] \times \exp\left\{-Ne^{-\frac{H_{max}^2}{8}}\left[1+\kappa_{40}B_H(H_{max})\right]\right\}dH_{max} \quad (22)$$

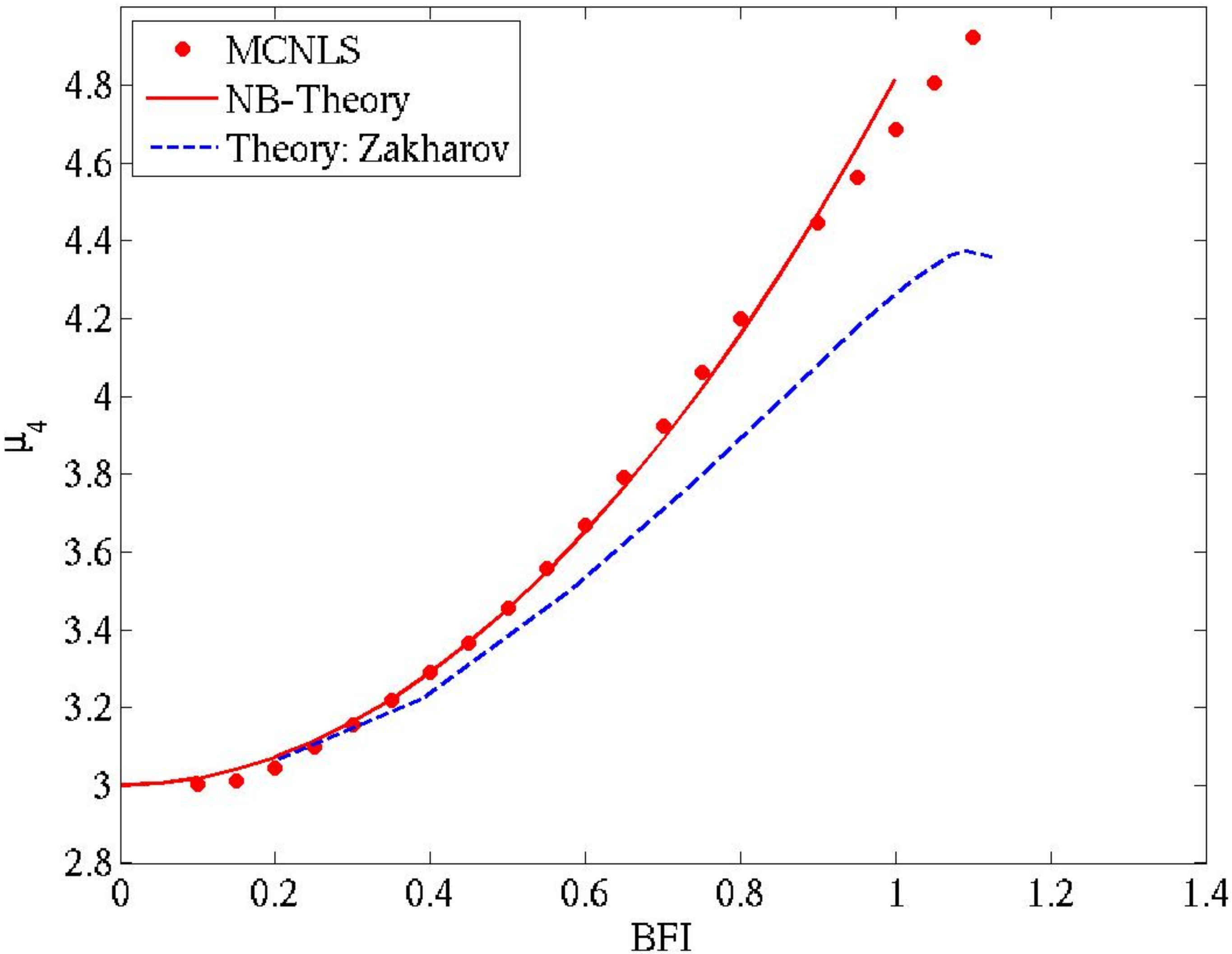
This with $\kappa_{40}=0$ corresponds to the PDF of H_{max} based on the Rayleigh distribution.

The occurrence probability of freak wave can be derived assuming $H_{freak} \geq 8m_0^{1/2}$ as a function of N and κ_{40},

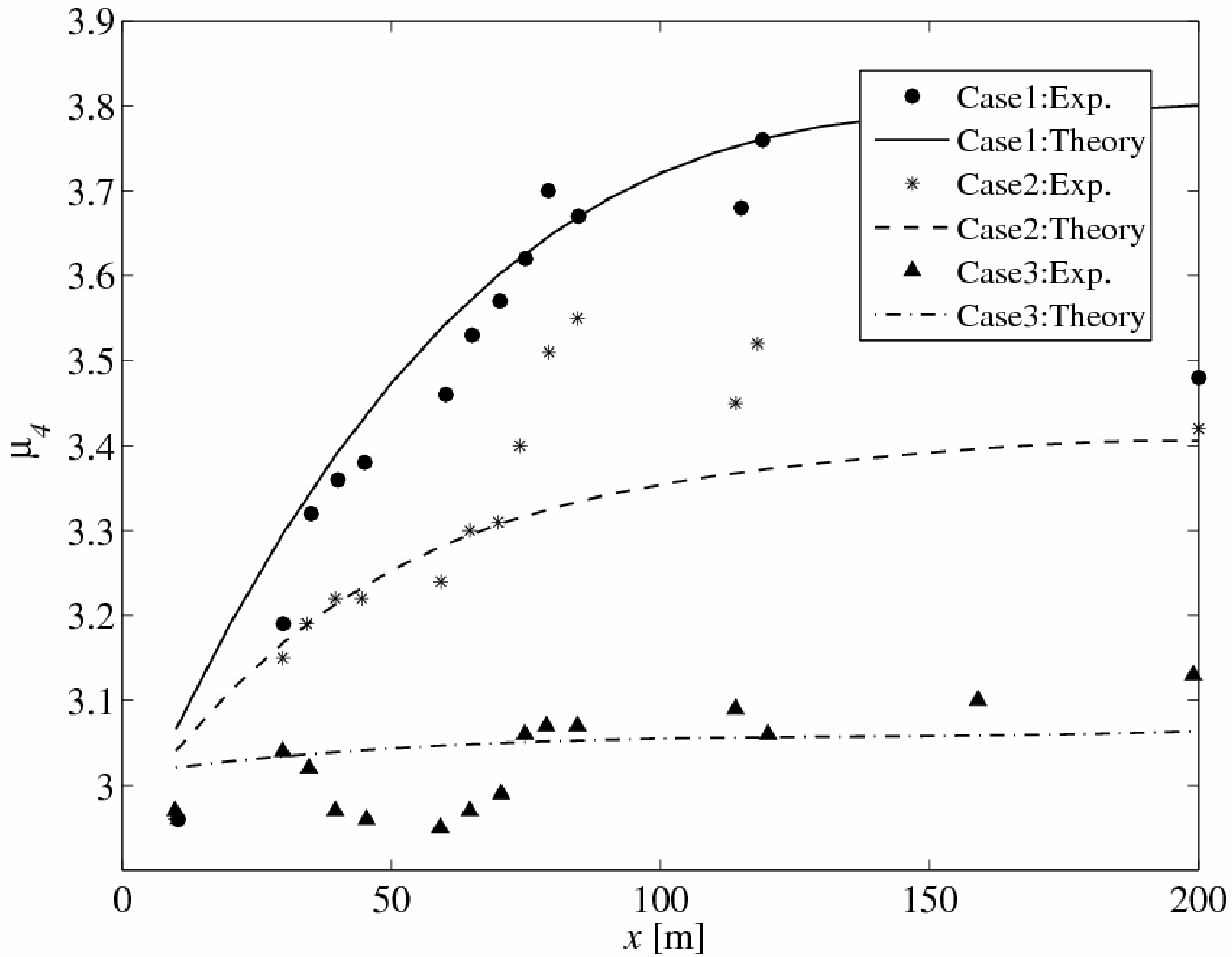
$$P_{freak} = 1 - \exp\left[-\mathrm{e}^{-8}N(1+8\kappa_{40})\right] \quad (23)$$

No ad-hoc parameters are included!

Kurtosis and BFI: 1D Case

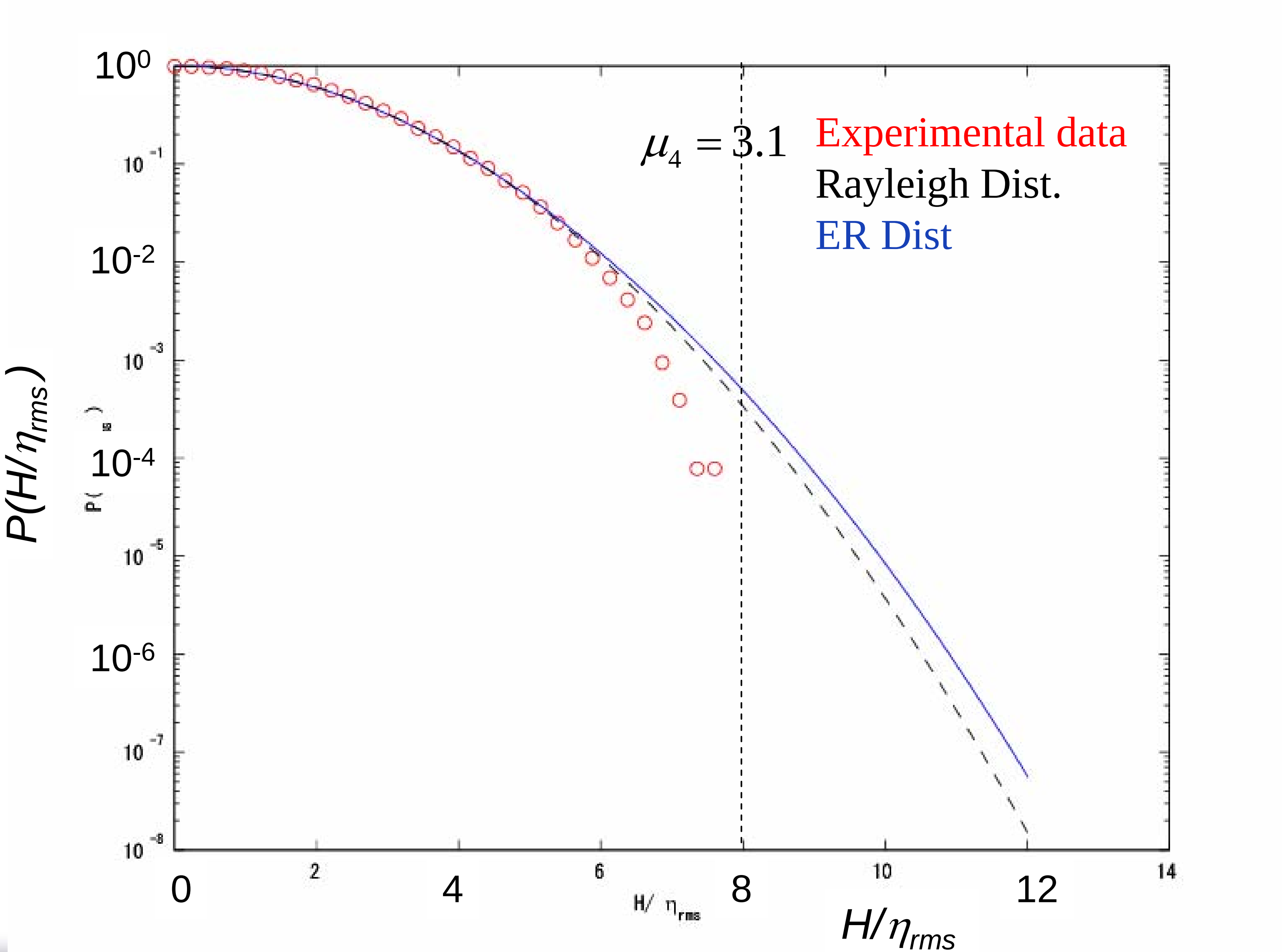


Kurtosis Evolution in Tank

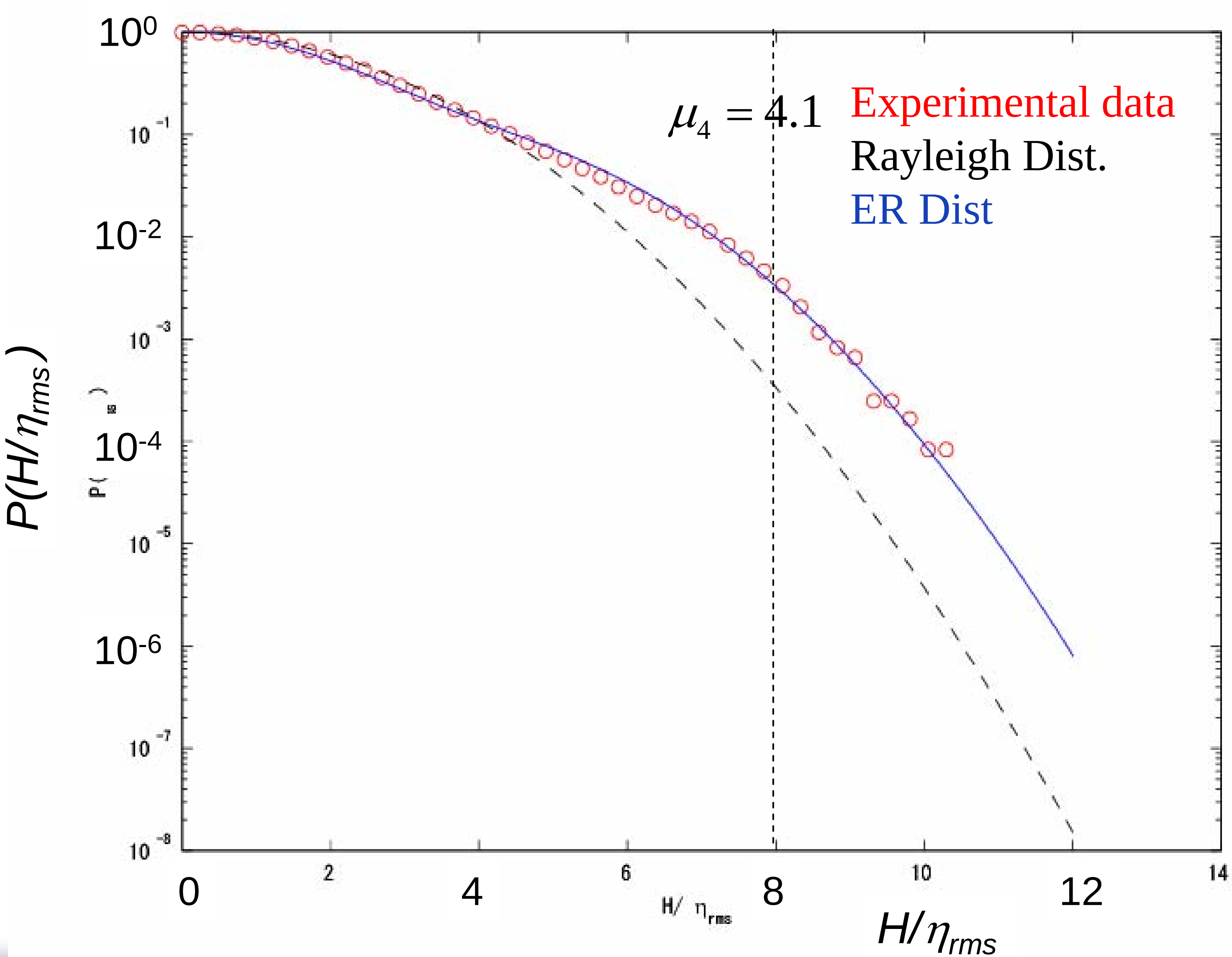


Onorato et al.(2006) Eupo. J. Fluid Mech.

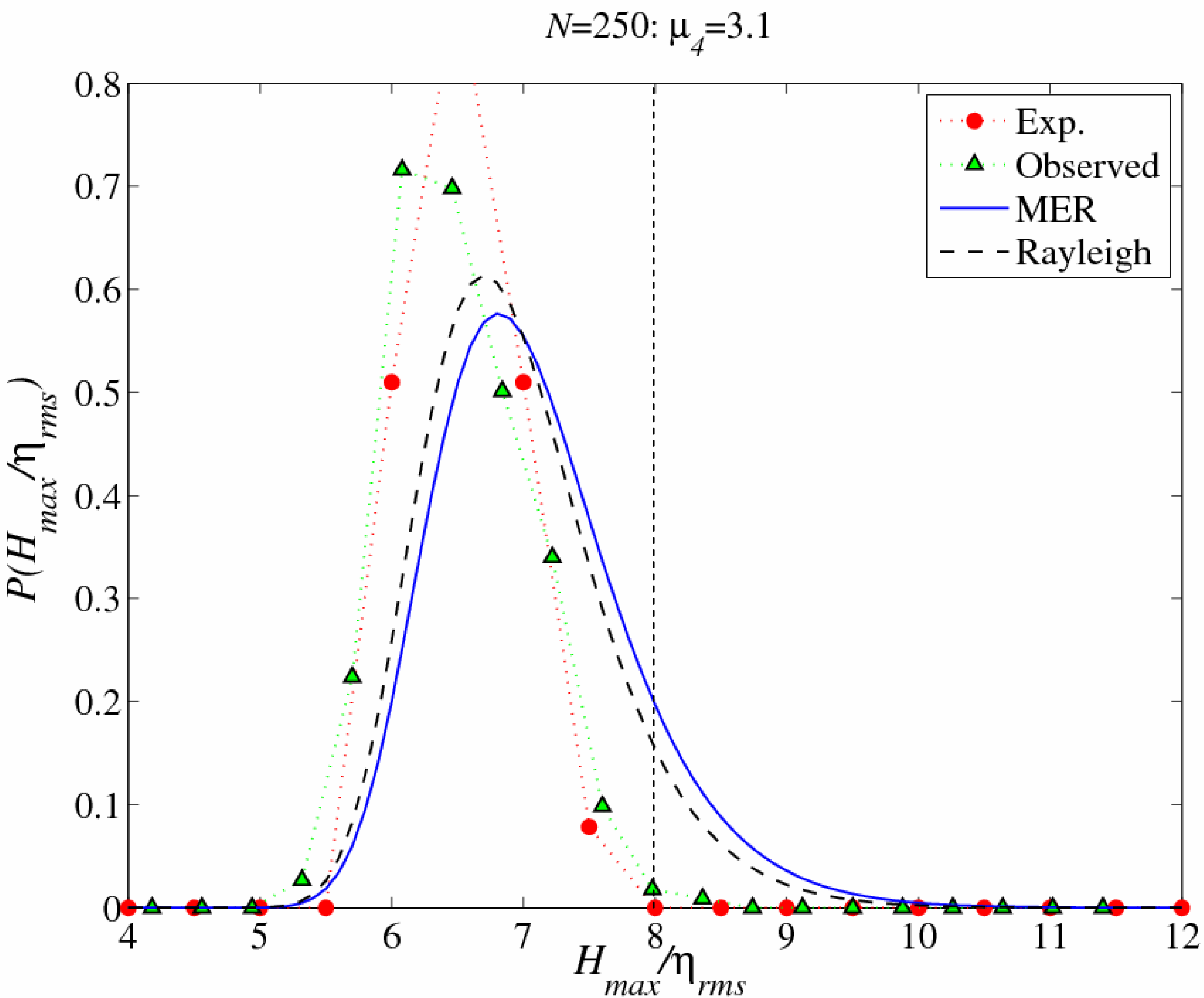
Wave Height Distribution



Wave Height Distribution

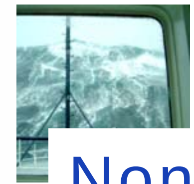


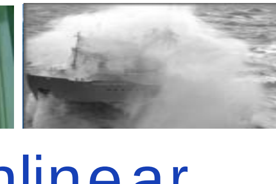
Maximum Wave Height Dist.

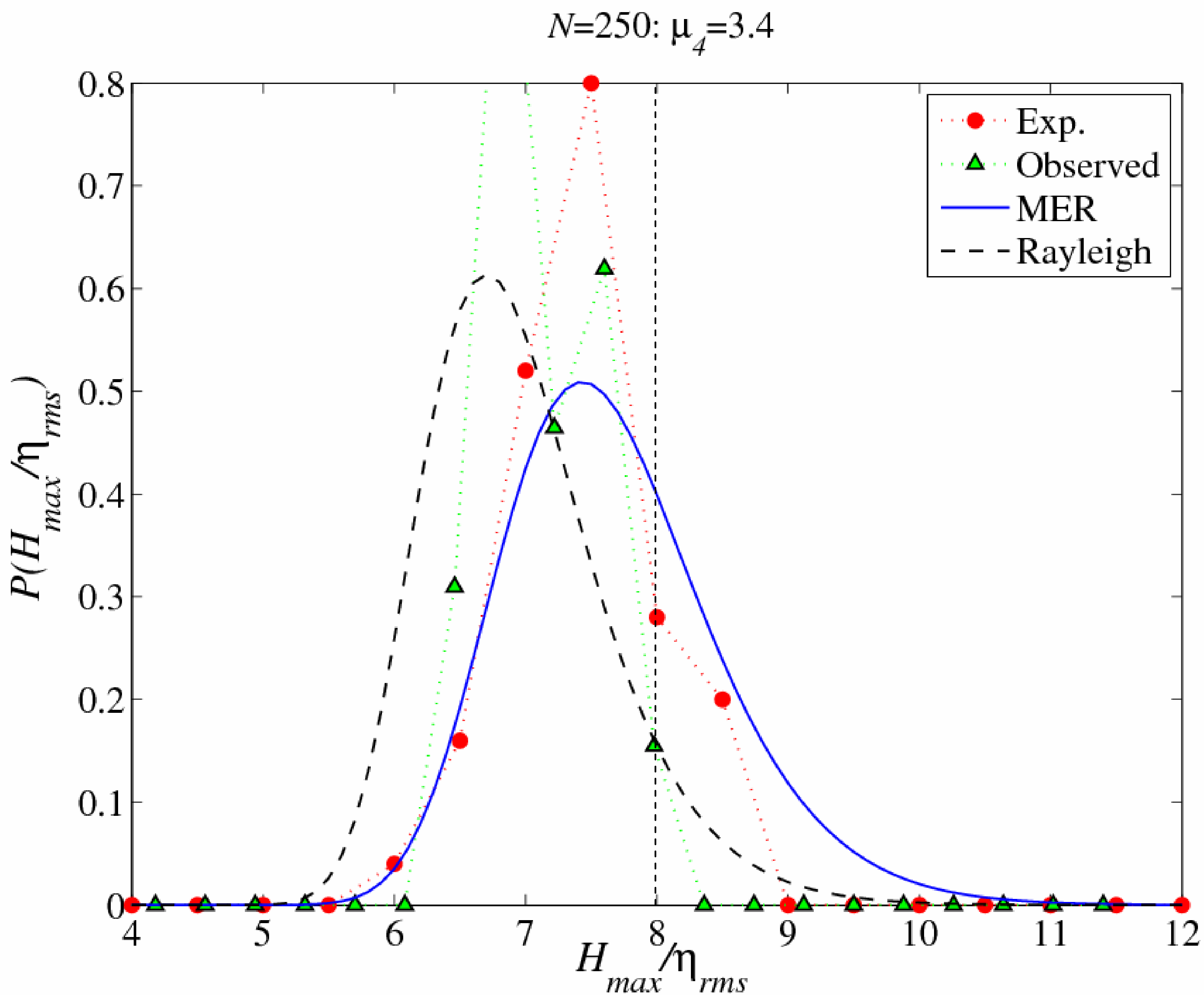


Nonlinear
Linear
Observed
Experiments
N=250
μ_4=3.1

Maximum Wave Height Dist.







Nonlinear

Linear

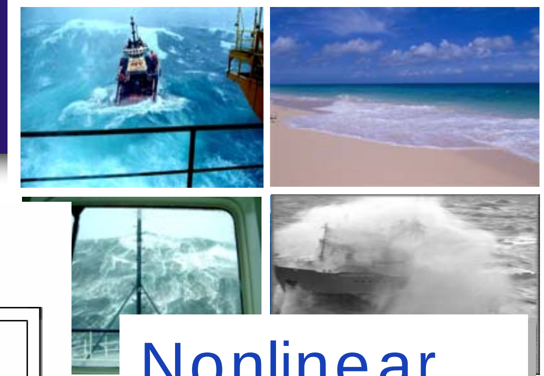
Observed

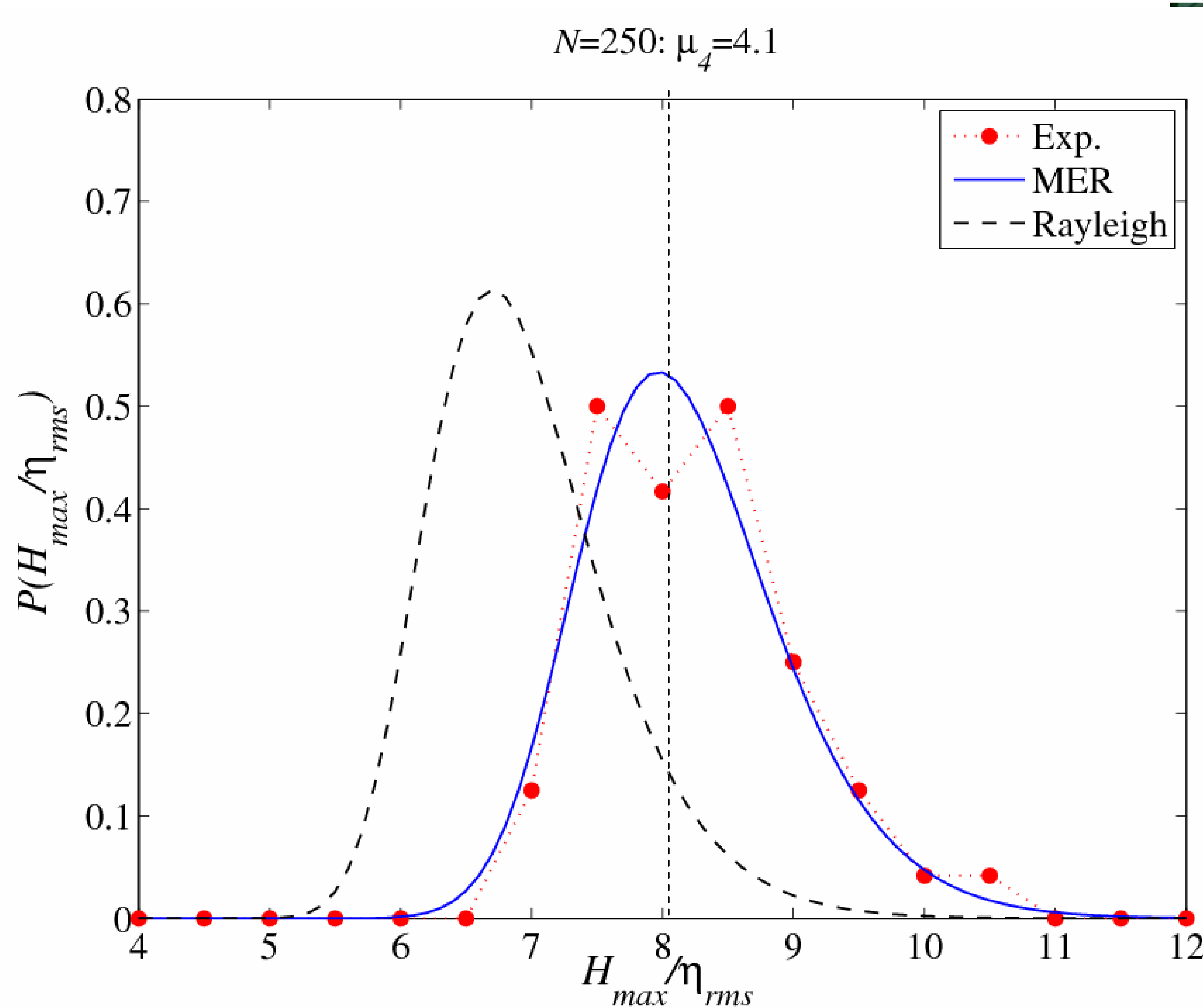
Experiments

N=250

μ_4=3.4

Maximum Wave Height Dist.

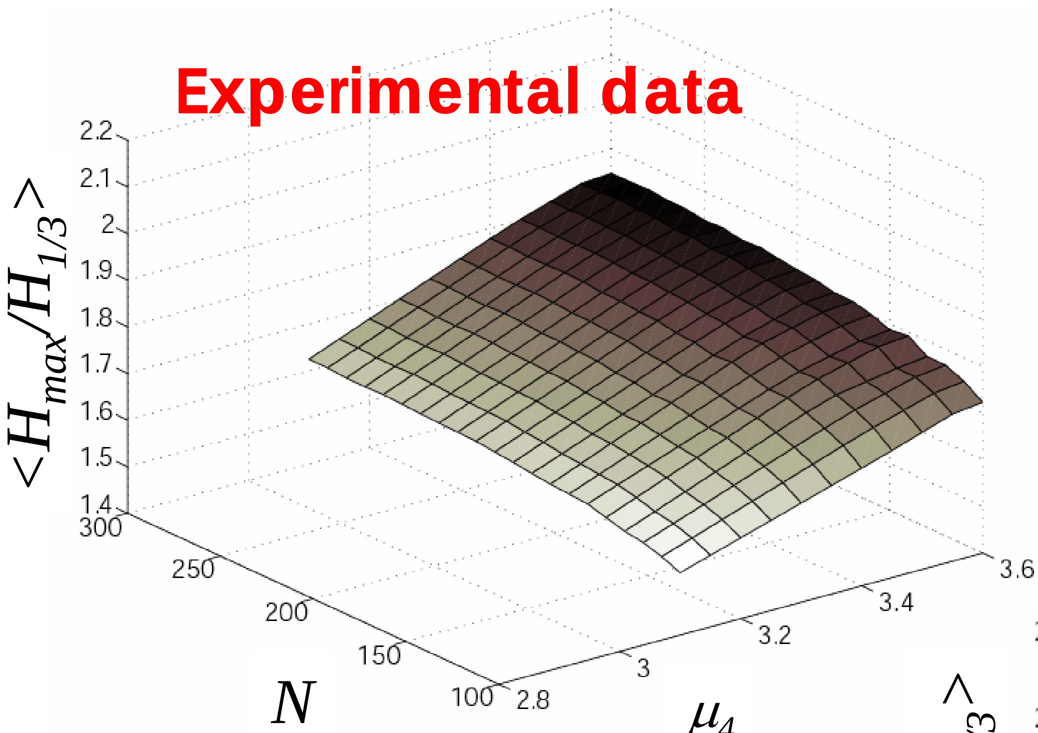




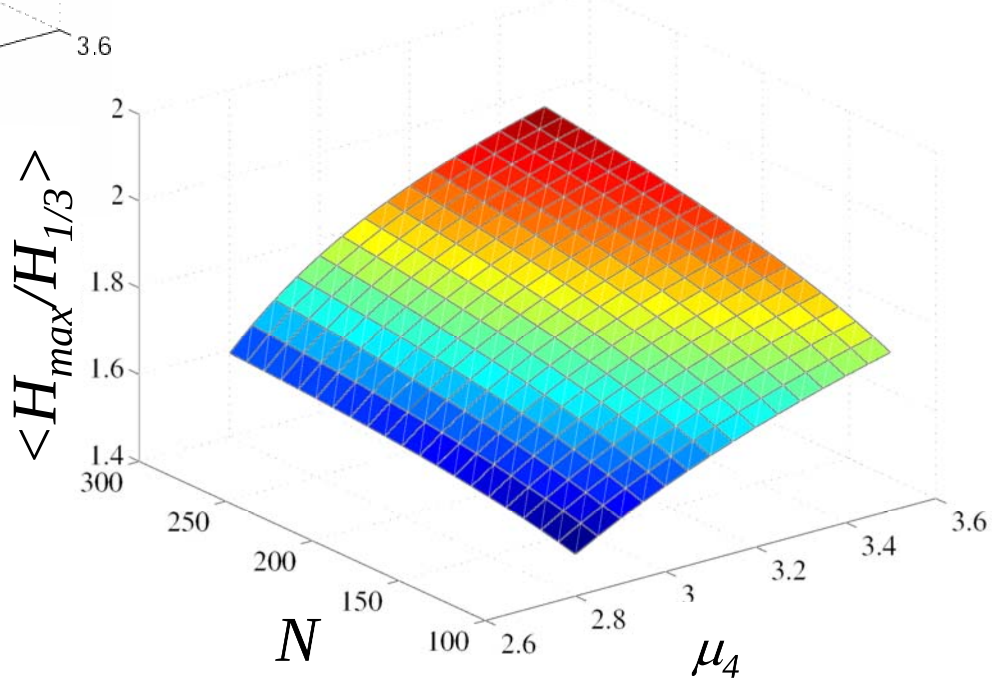
Nonlinear
Linear
Observed
Experiments
N=250
μ_4=4.1

Expected Value of H_{max}

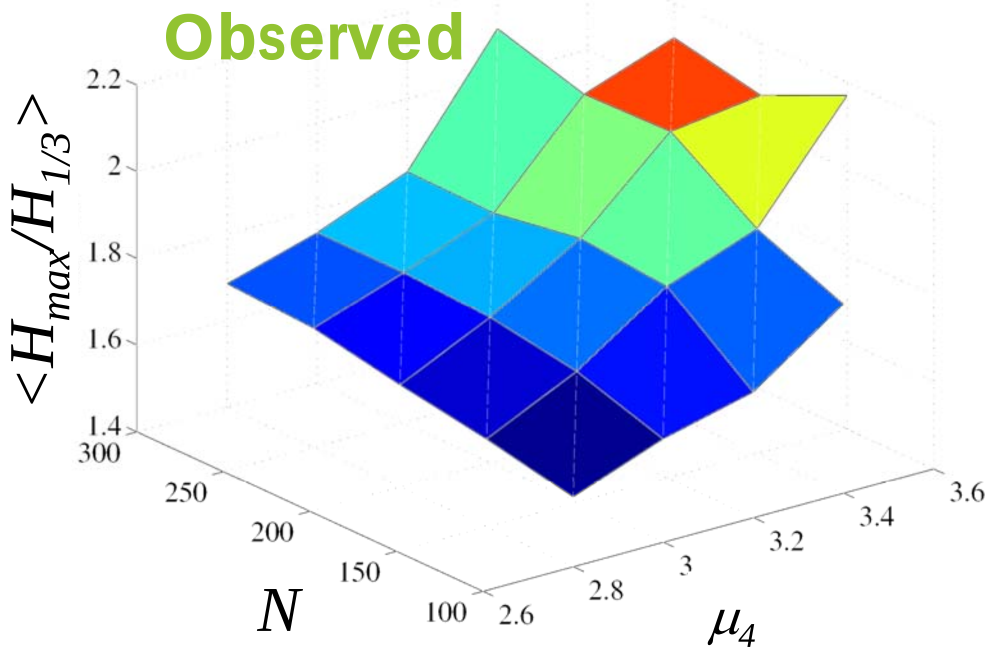
Experimental data

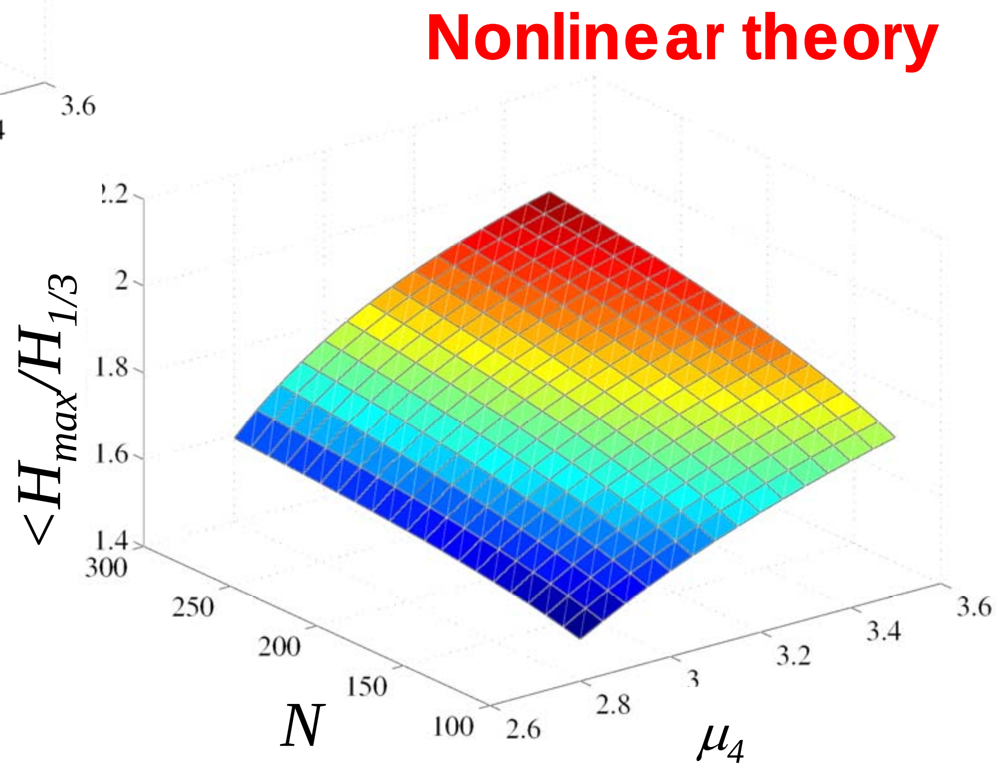


Nonlinear theory

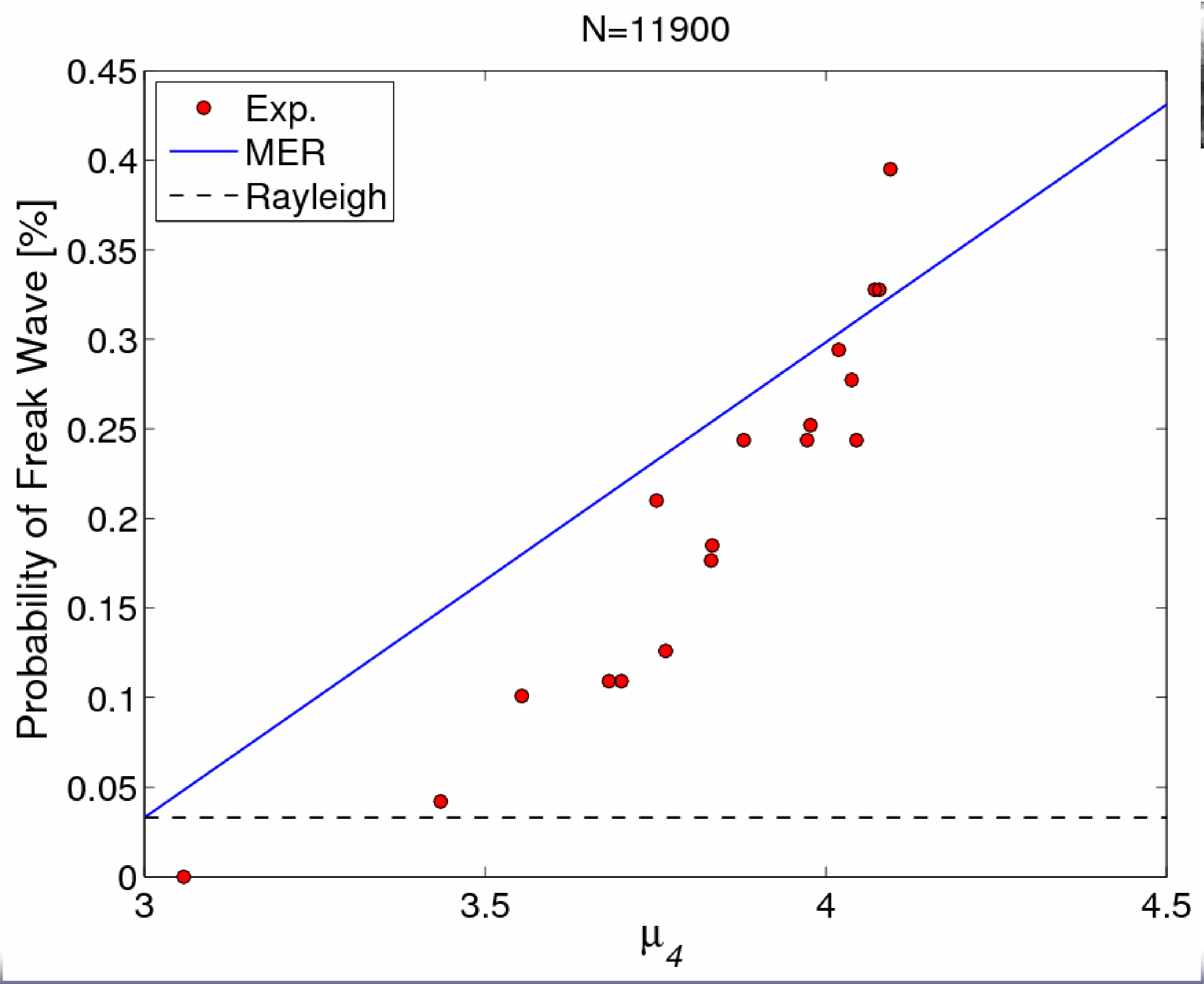


Expected Value of H_{max}

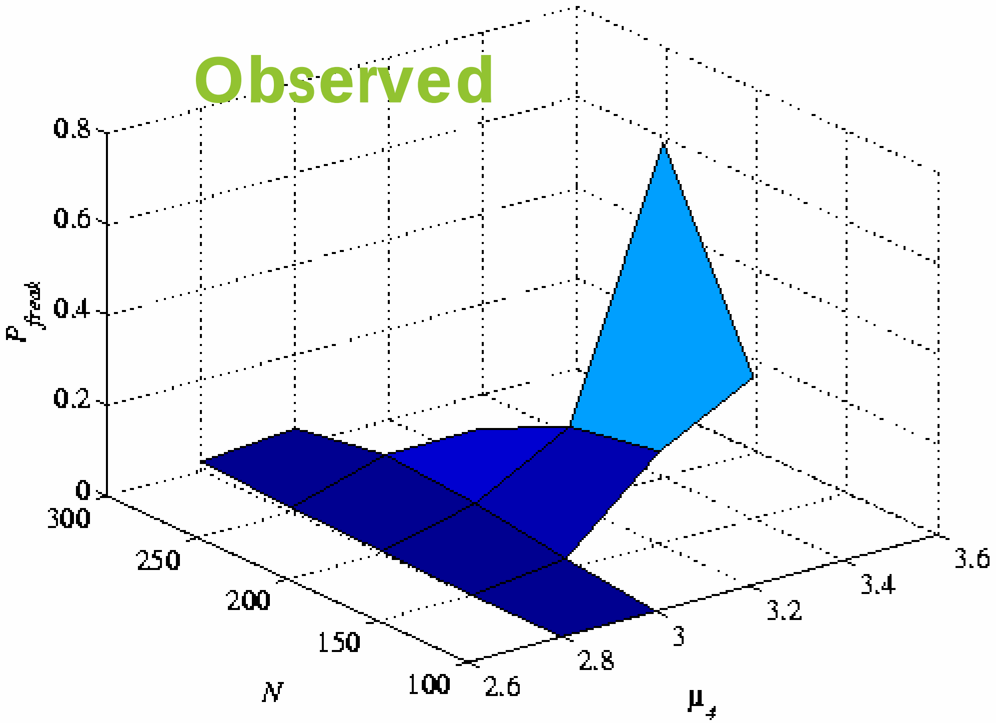


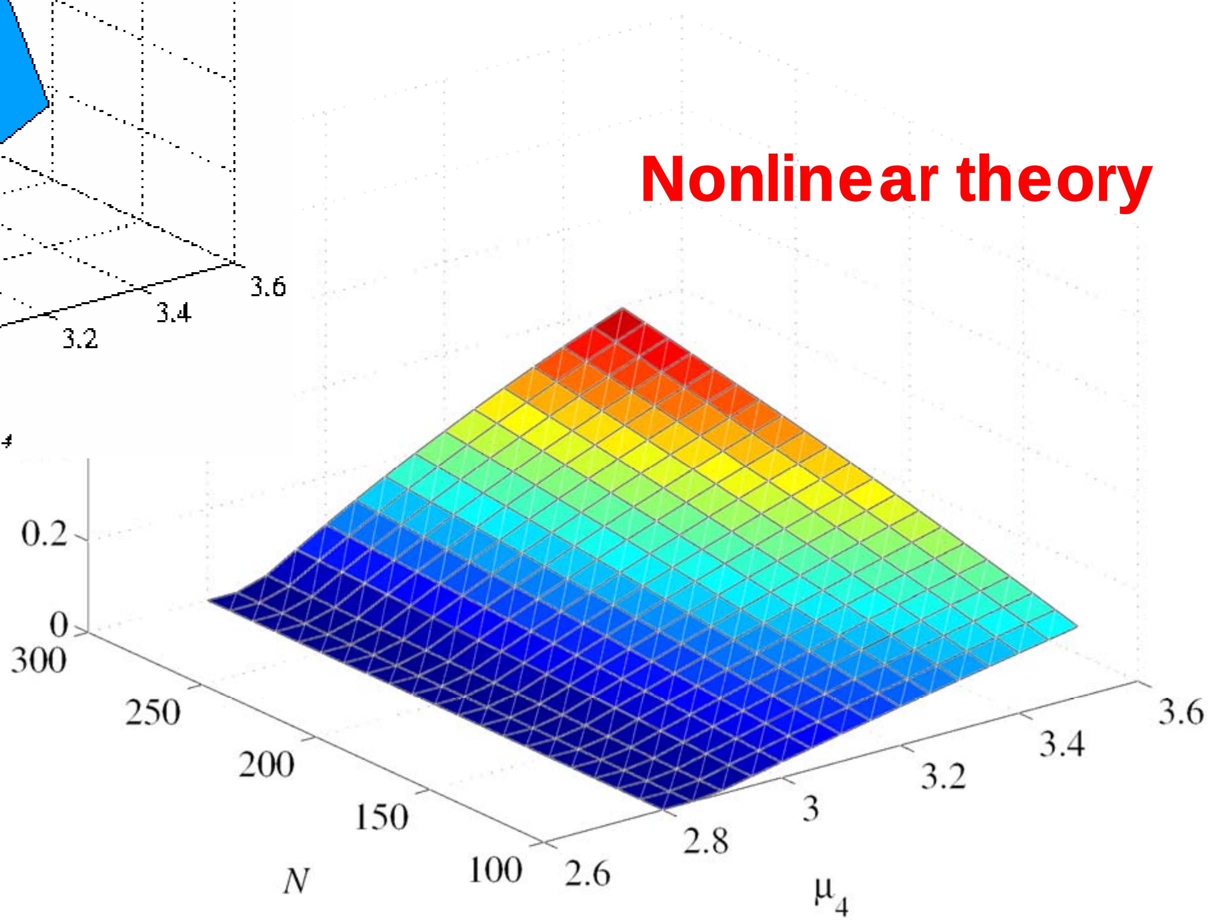


Probability of Freak Wave in Tank



Occurrence Probability of Freak Wave





Conclusion and Outlook

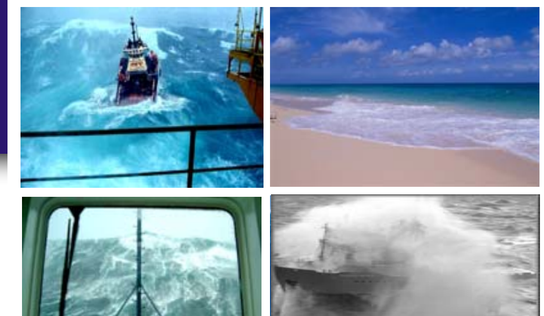
❖ Summary

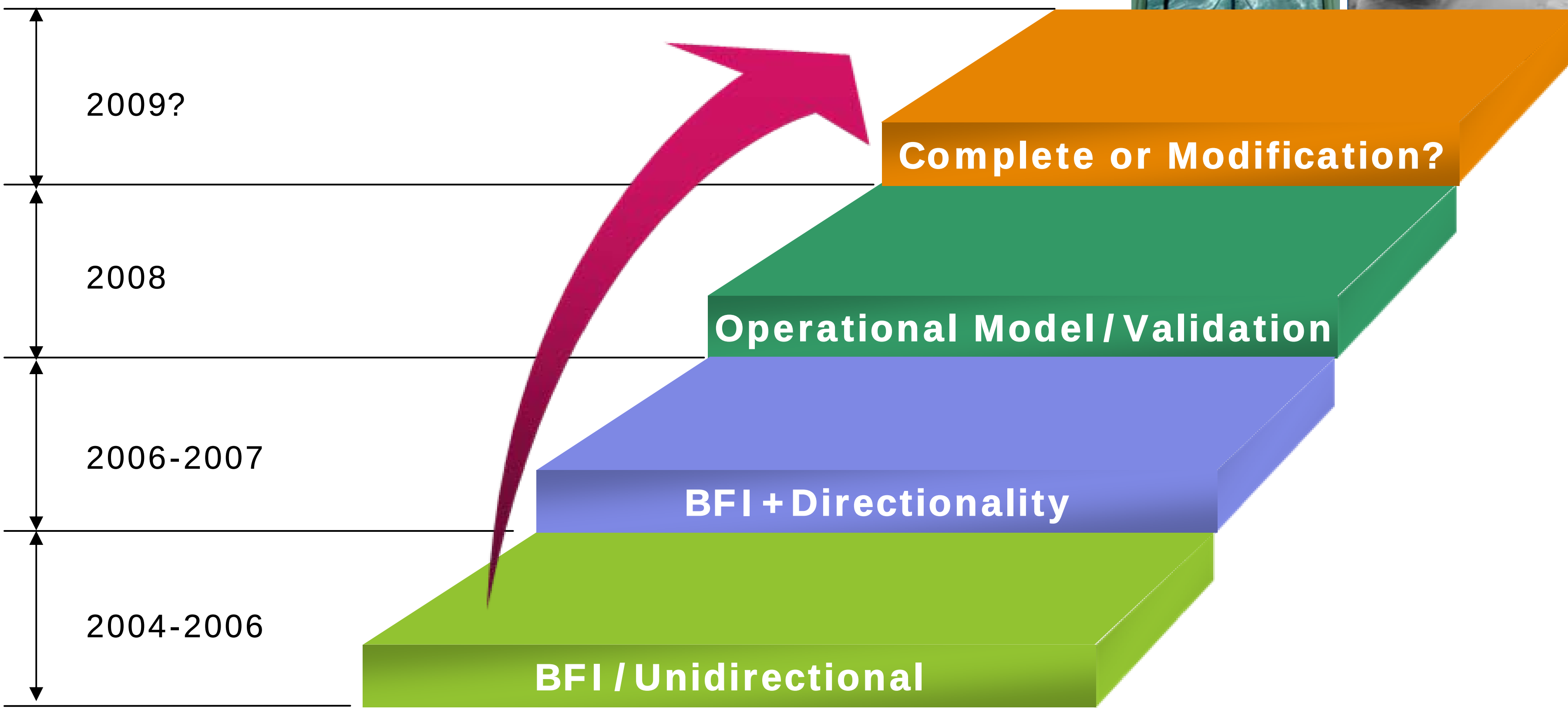
- Kurtosis depends on the shape of wave spectra and changes owing to quasi-resonant interactions.
- The maximum wave height distribution and occurrence probability of freak wave can be predicted by weakly non-Gaussian theory.

❖ Further Study

- Expansion of the theory including directional effects
- Broad banded spectrum effects
- Application to operational wave forecasts
- Reliable measurements and experiments, careful statistical analysis, and numerical simluations

Freak Wave Prediction System





Thank You

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