Comparison of Empirical Methods and a Boussinesq-type Wave Model for Predicting Overtopping of Coastal Structures

Patrick Lynett, Texas A&M University
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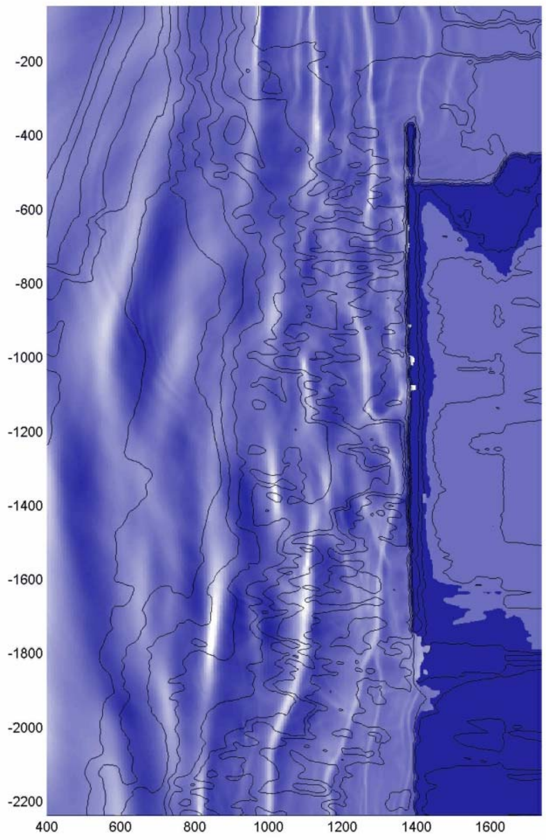
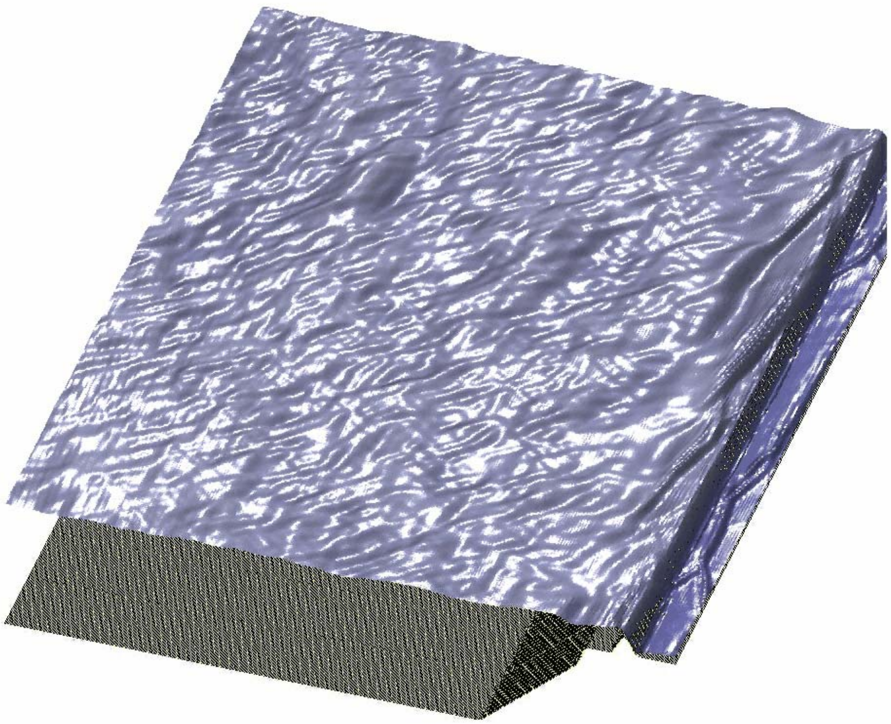
Mathijs van Ledden, Haskoning, Inc.





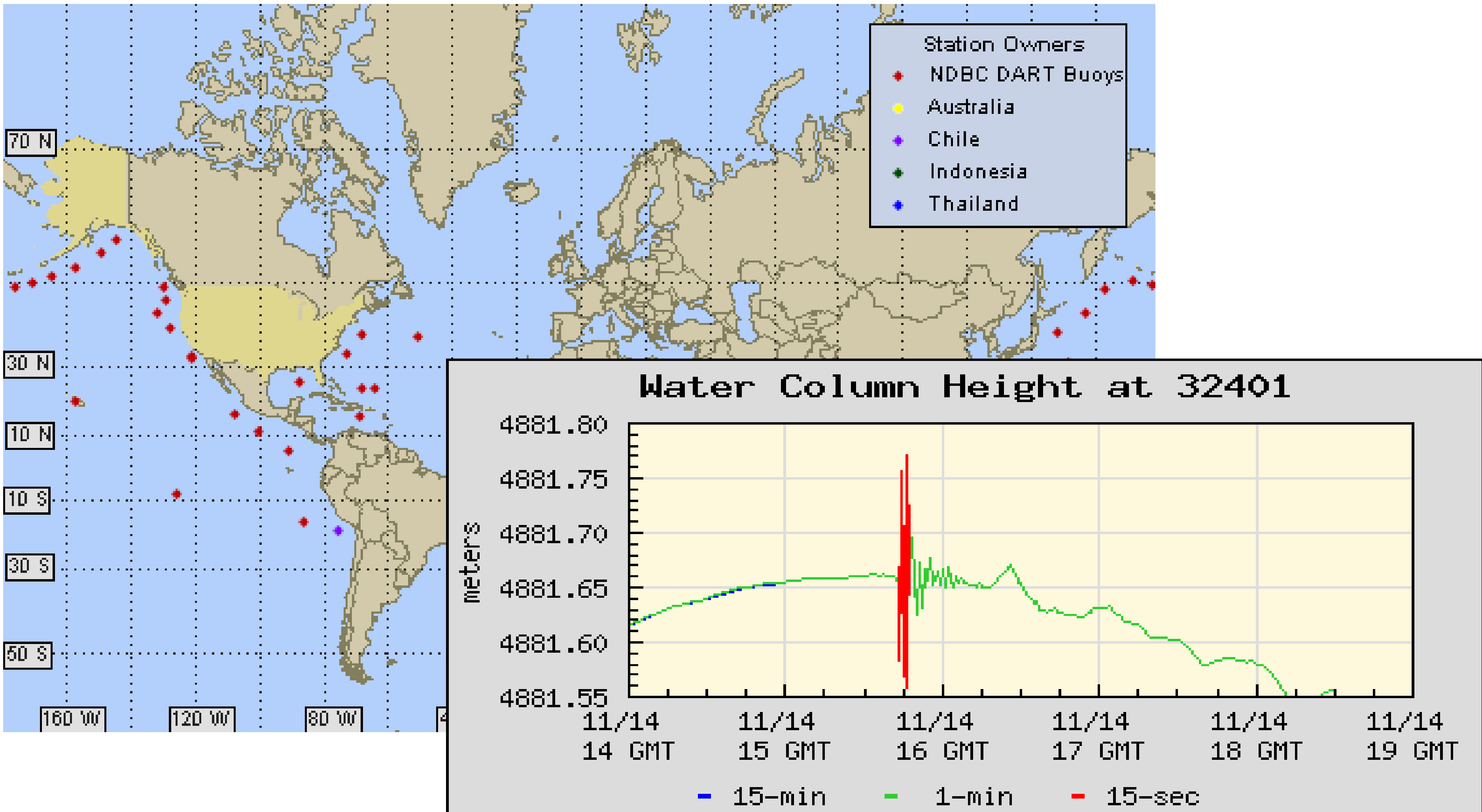
$$\frac{Q}{\sqrt{gH_{m0}^3}} = 0.2\exp\left(-2.6\frac{R_c}{H_{m0}}\frac{1}{\gamma_f\gamma_\beta}\right)$$

This slide hijacked by Tsunami

- 5:41 am HST Earthquake off Chile Coast
  - 7.7 moment magnitude
  - Depth 60 km
- 5:55 am HST Local Tsunami Warning for Chile/Peru
- 5:57 am HST Tsunami Advisory for West Coast and Hawaii
- 6:53 am HST no reports of damaging tsunami in Chile, local warning cancelled
- 6:56 am HST Advisories taken down

This slide hijacked by Tsunami

- Small tsunami was recorded by NOAA DART



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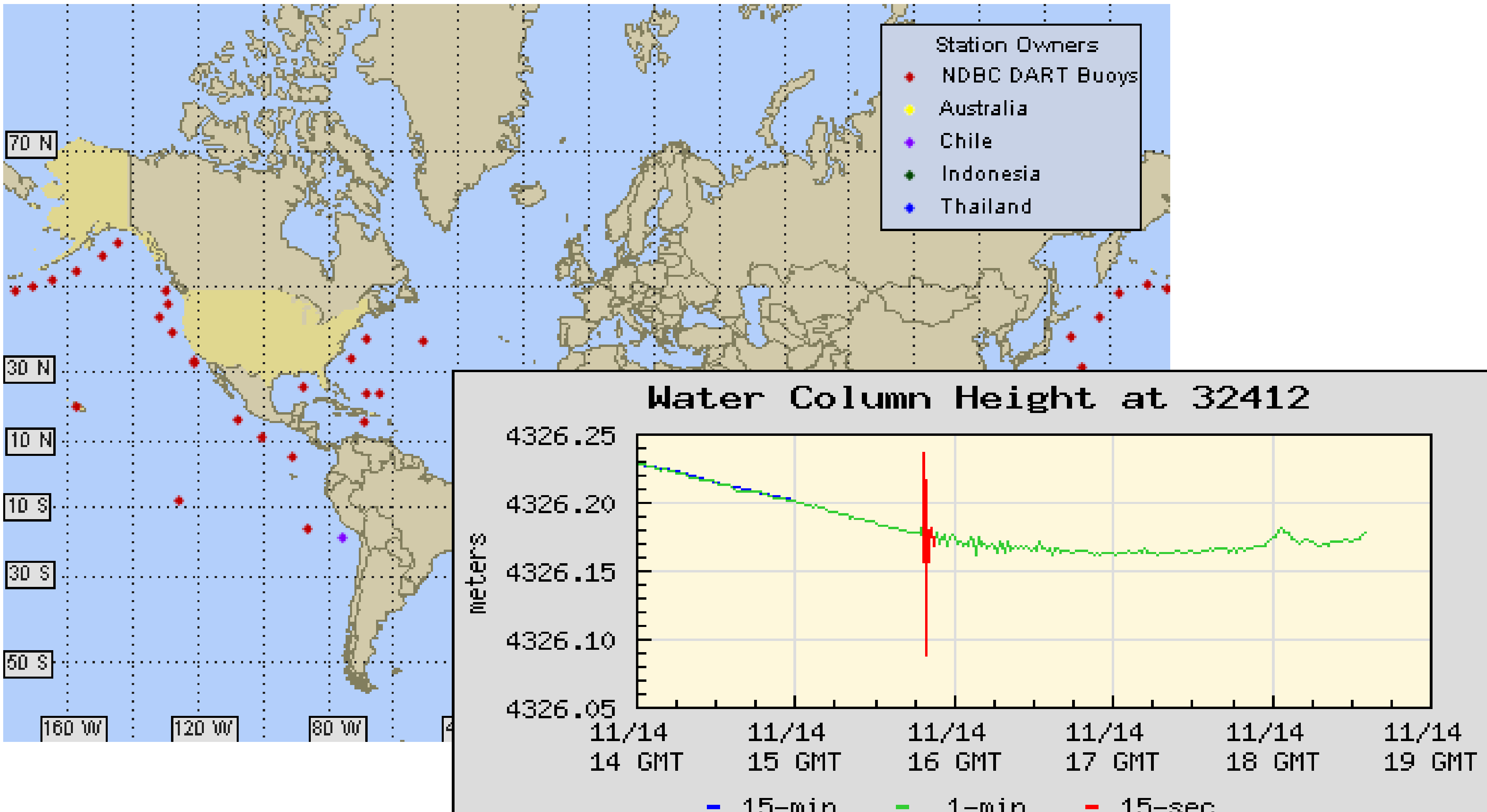

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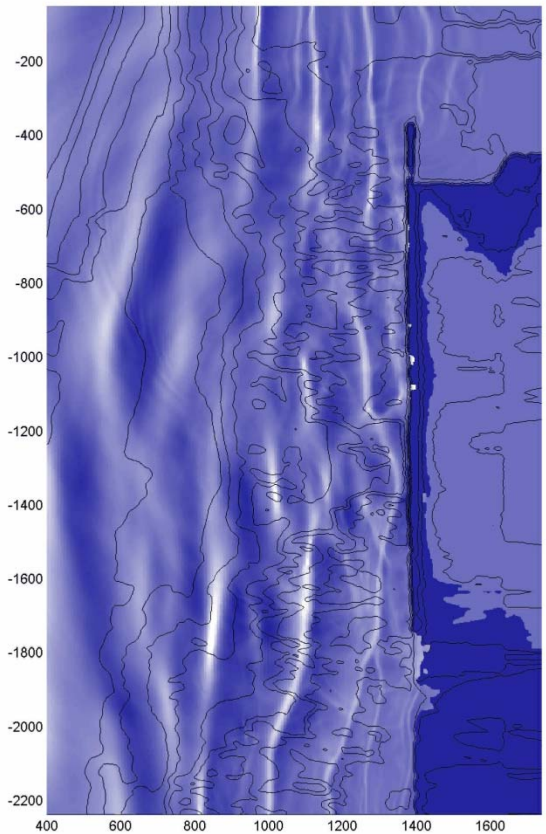
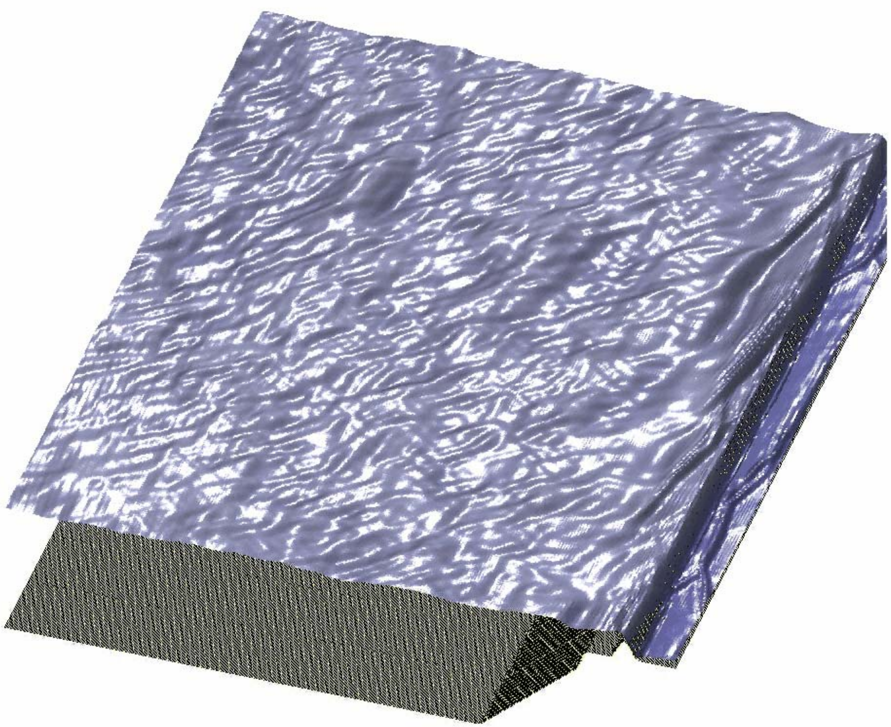
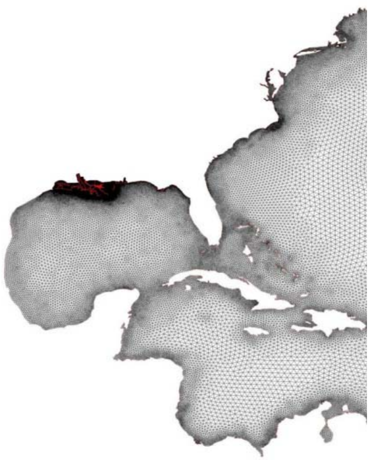


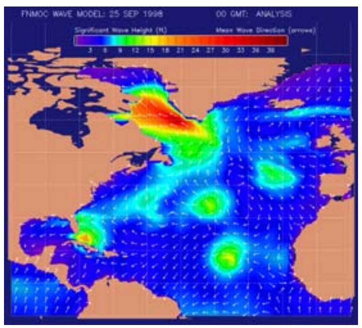


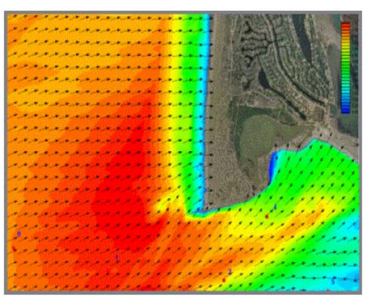
$$\frac{Q}{\sqrt{gH_{m0}^3}} = 0.2\exp\left(-2.6\frac{R_c}{H_{m0}}\frac{1}{\gamma_f\gamma_\beta}\right)$$

Motivation for Study

- There is a technical discontinuity of the hydrodynamic science/modeling/effort used in design of coastal structures





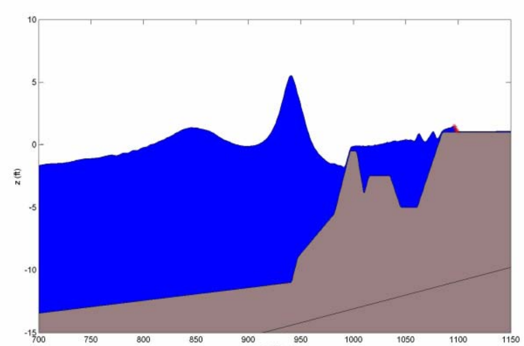


Offshore to nearshore: physics-based modeling of wave and surge (e.g. WAM, STWAVE, ADCIRC)

Models provide estimate of waves and water levels near the toe of the structures (typically a couple hundred meters seaward)

OT Rate, which controls design levee elevation, is estimated with empirical equations

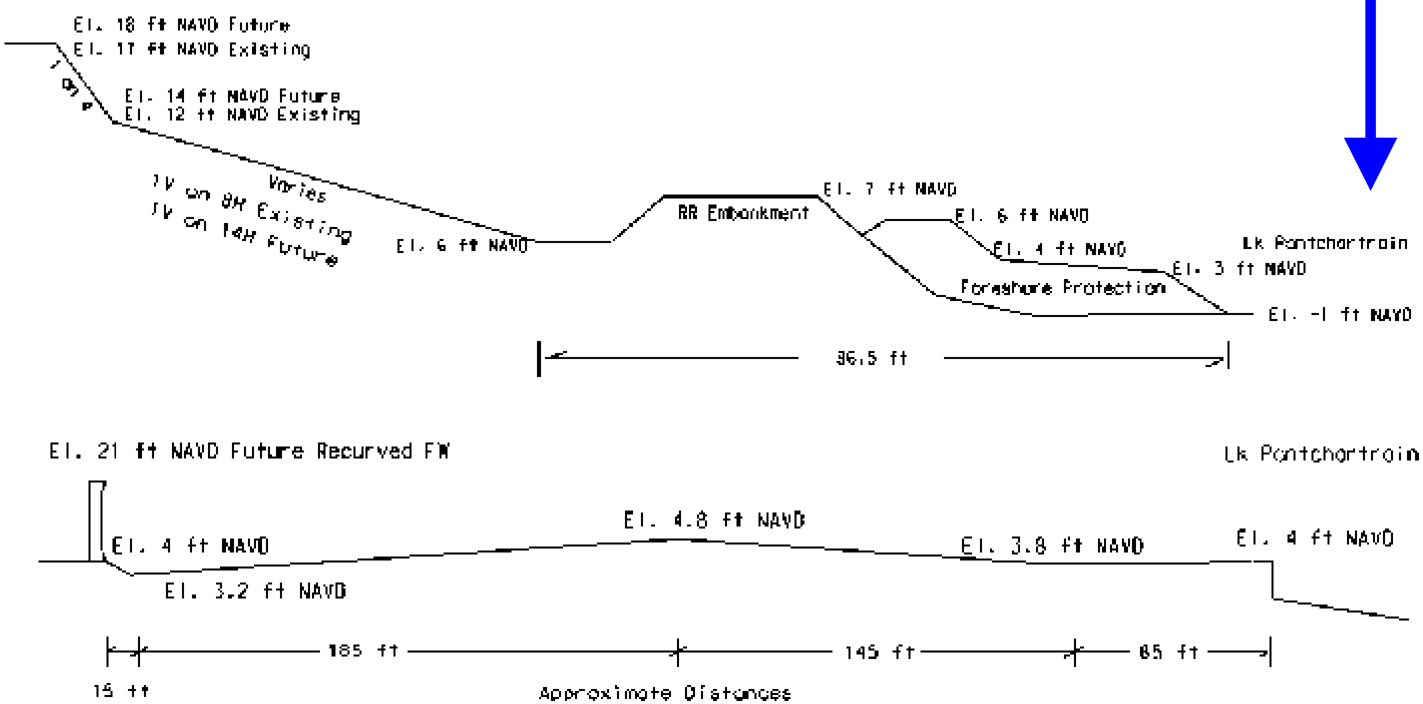
$$\frac{q}{\sqrt{gH_{m0}^3}} = \frac{0.067}{\sqrt{\tan\alpha}}\gamma_b\xi_0 \cdot \exp\left(-4.75\frac{R_c}{H_{m0}}\frac{1}{\xi_0\gamma_b\gamma_f\gamma_\beta\gamma_v}\right)$$



Motivation for Study

Structure design is based on wave height here

STWAVE output provided here



New Orleans Lakefront Airport Floodwall nr Seabrook Bridge

Methodology/Outline

- Quick empirical background
- Boussinesq review, validation, and limitations
  - Runup
  - Wave breaking
  - Overtopping
- Empirical and Boussinesq comparisons
- <u>Numbers to keep in mind:</u>
  - Critical overtopping rate used in levee design:
    - 0.1 $ft^3/s/ft$ = 0.01 $m^3/s/m$ = 10 l/s/m

Summary of Conclusions

- Boussinesq does a very good job at predicting nearshore wave evolution, including overtopping rates
- For a levee with a simple reach (single slope), there is no accuracy preference between empirical methods and Boussinesq
- If the reach is complex, empirical methods can still be used, but must be provided an estimate of the wave properties (height and setup) at the structure toe
  - Boussinesq can fill this role, represents a hybrid approach
  - Boussinesq can also do “everything”

Empirical Approach

The overtopping formulation from Van der Meer reads (see TAW 2002):

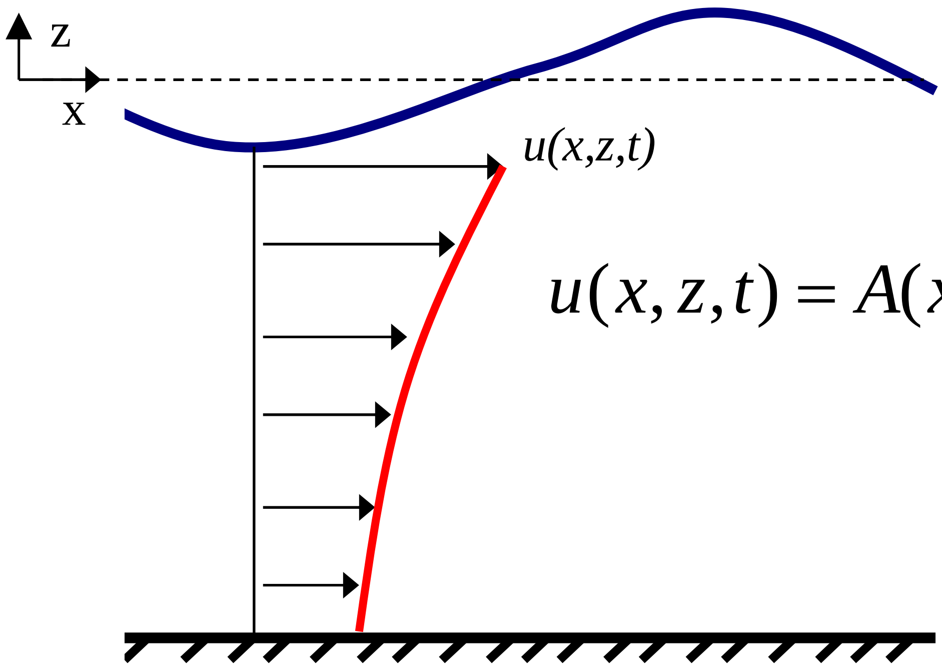
$$\frac{Q}{\sqrt{gH_{m0}^3}} = \frac{0.067}{\sqrt{\tan\alpha}}\gamma_b \xi_0 \exp\left(-4.75\frac{R_c}{H_{m0}}\frac{1}{\xi_0 \gamma_b \gamma_f \gamma_\beta \gamma_v}\right)$$

$$\text{with maximum:} \quad \frac{Q}{\sqrt{gH_{m0}^3}} = 0.2\exp\left(-2.6\frac{R_c}{H_{m0}}\frac{1}{\gamma_f \gamma_\beta}\right)$$

With:
Q : overtopping rate [cfs/ft]
g : gravitational acceleration [ft/s²]
H_{m0} : wave height at toe of the structure, *as if the structure was not there* [ft]
ξ_0: surf similarity parameter [-]
α : slope [-]
R_c : freeboard [ft]
γ : coefficient for presence of berm (b), friction (f), wave incidence (β), vertical wall (v)

Boussinesq Equations

- Boussinesq Equations (Peregrine, 1967; Ngowu, 1993):



$$u(x,z,t) = A(x,t) + \underbrace{\left[z * B(x,t) + z^2 * C(x,t) \right]}_{\text{Should be small compared to A(x,t)}}$$

- Functions *B, C* lead to 3rd order spatial derivatives in model (eqns)
- Accurate for long and intermediate depth waves, kh<~3 (wavelength > ~ 2 water depths)

Boussinesq Equations

Continuity Equation

New terms, due to the Boussinesq-type derivation

$$\boxed{\eta_t + \nabla \cdot (Hu_\alpha) -}$$

$$\boxed{\begin{gathered}\nabla \cdot \left\{ H \left(\frac{1}{6}\left(\eta^2 - \eta h + h^2\right) - \frac{1}{2} z_\alpha^2 \right) \nabla \left(\nabla \cdot u_\alpha \right) \right\} - \\ \nabla \cdot \left\{ H \left(\frac{1}{2}\left(\eta h - h\right) - z_\alpha \right) \nabla \left(\nabla \cdot \left(h u_\alpha \right) \right) \right\} = 0\end{gathered}}$$

$$H = h + \eta$$

Boussinesq Equations

Momentum Equation

New terms, due to the Boussinesq-type derivation

$$\boxed{u_{\alpha t} + u_\alpha \cdot \nabla u_\alpha + g \nabla \zeta} +$$

$$\left\{ \frac{z_\alpha^2}{2} \nabla (\nabla \cdot u_{\alpha t}) + z_\alpha \nabla Q_t + z_{\alpha t} \left[z_\alpha \nabla (\nabla \cdot u_\alpha) + z_\alpha \nabla Q \right] \right\} +$$

$$\left[Q \nabla Q - \nabla \eta Q_t + (u_\alpha \cdot \nabla z_\alpha) \nabla Q + z_\alpha \nabla (u_\alpha \cdot \nabla Q) \right] +$$

$$\left\{ z_\alpha (u_\alpha \cdot \nabla z_\alpha) \nabla (\nabla \cdot u_\alpha) + \frac{z_\alpha^2}{2} \nabla \left[u_\alpha \cdot \nabla (\nabla \cdot u_\alpha) \right] \right\} +$$

$$\nabla \left\{ \frac{\eta^2}{2} \nabla \cdot u_{\alpha t} - \eta u_\alpha \cdot \nabla Q + \eta Q \nabla \cdot u_\alpha \right\} +$$

$$\nabla \left\{ \frac{\eta^2}{2} \left[(\nabla \cdot u_\alpha)^2 - u_\alpha \cdot \nabla (\nabla \cdot u_\alpha) \right] \right\} = 0$$

$$\text{where} \ : \ Q = \nabla \cdot (h u_\alpha)$$

Why Boussinesq??

- Boussinesq model provides a practical nearshore wave processes model
  - Excellent hydrodynamic accuracy for wind waves
  - *Fundamentally irrotational and inviscid*, with empirical add-ons for approximating dissipation (more about these later)
    - Not as physically complete as Navier-Stokes models
  - Can be run in a reasonable time, ~10 seconds of desktop wall time per wave period for 1HD problems
- If you want numerous nearshore wave simulations, providing a statistical database, Boussinesq is the choice

Validation of Runup Algorithm

- Comparison with the Irribaren Law for wave runup:

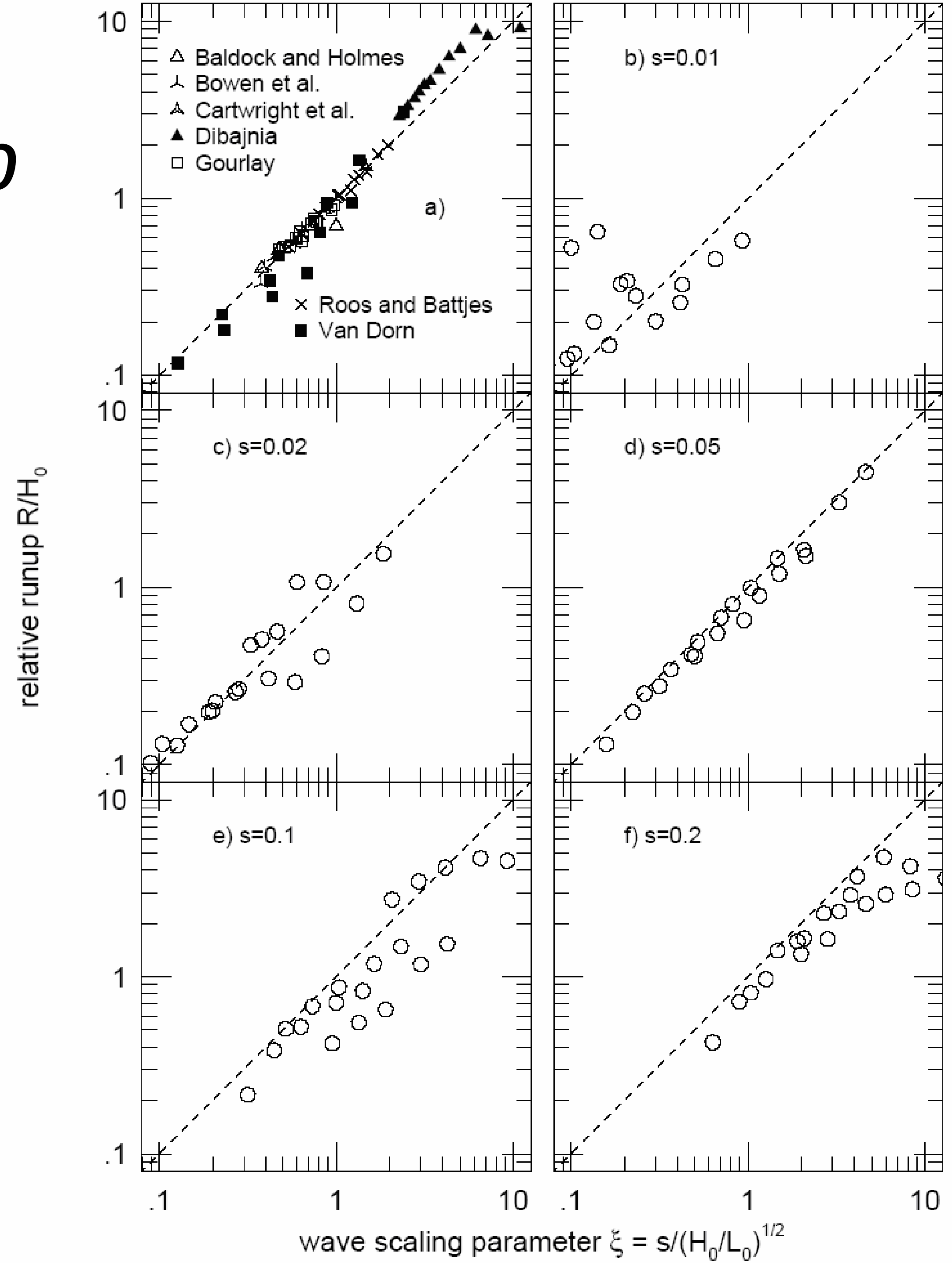
$$\frac{R}{H_0} = \xi = s(H_0/L_0)^{-1/2}$$

Lynett, P., Wu, T.-R., and Liu, P. L.-F., "Modeling Wave Runup with Depth-Integrated Equations," Coastal Engineering, 2002.

Lynett, P., and Liu, P. L.-F., "A Numerical Study of the Runup Generated by Three-Dimensional Landslides," JGR-Oceans. 2005.

Korycansky, D., Lynett, P., and Ward, S., "Runup from Impact Tsunami," GJI, 2007.

Lynett, P., "The Effect of a Shallow Water Obstruction on Long Wave Runup and Overland Flow Velocity," Journal of Waterway, Port, Coastal, and Ocean Engineering (ASCE), 2007.



Energy Dissipation Submodels (Add-ons)

$$\frac{\partial \boldsymbol{u}_1}{\partial t} + \frac{\varepsilon_o}{2}\nabla\left(\boldsymbol{u}_1 \cdot \boldsymbol{u}_1\right) + \nabla\zeta + \mu_1^2\{....\} - \boldsymbol{R}_b + \boldsymbol{R}_f = 0$$

- Bottom friction with a quadratic drag law
- Breaking dissipation, $\boldsymbol{R}_b$, following Kennedy *et al*. (2000)

$$\boldsymbol{R}_f = \frac{f}{h+\zeta}\boldsymbol{u}_b|\boldsymbol{u}_b|$$

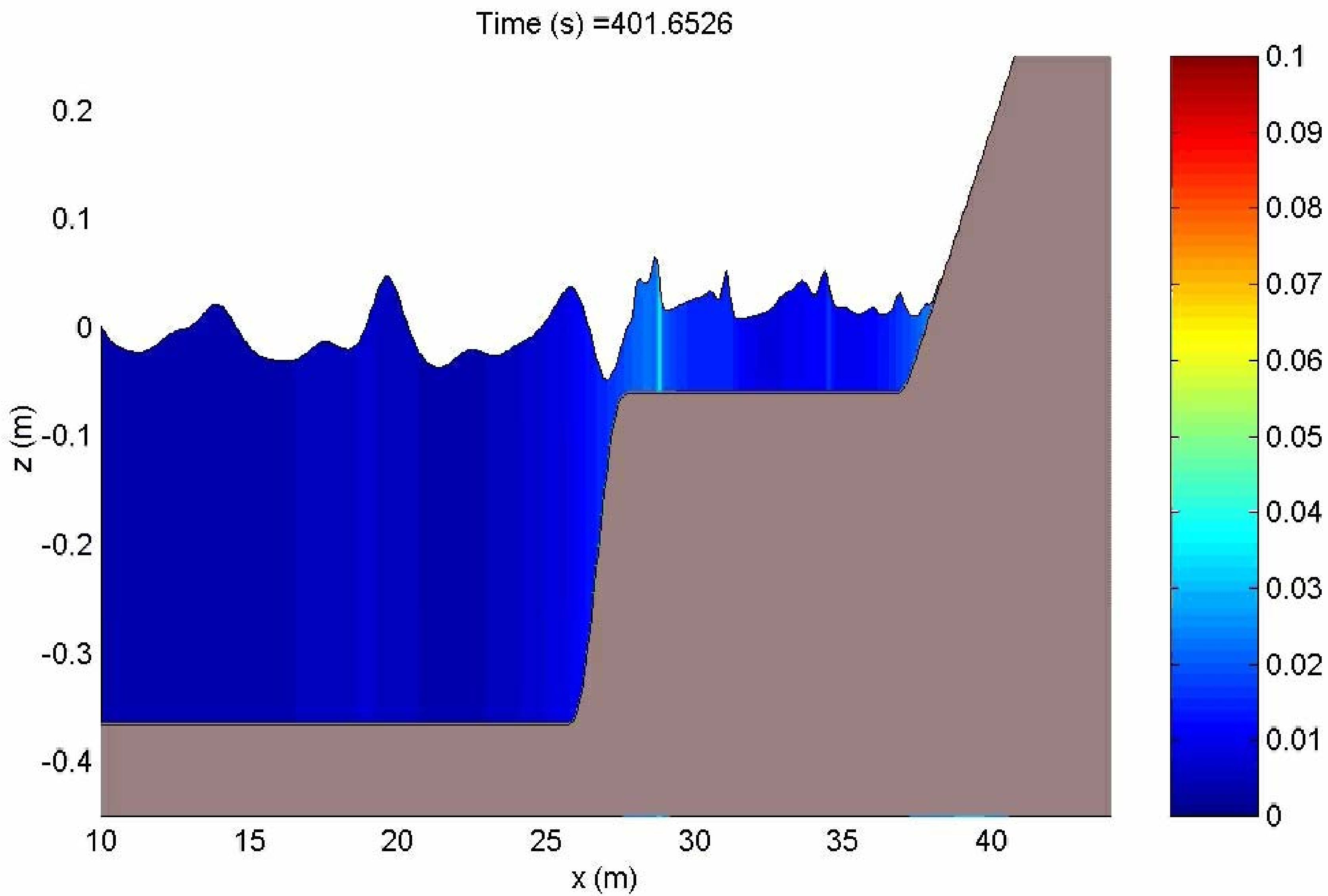
$$R_{bx} = \frac{1}{H}\left(\left[\nu\left(Hu_1\right)_x\right]_x + 0.5\left[\nu\left(Hu_1\right)_y + \nu\left(Hv_1\right)_x\right]_y\right)$$

$$R_{by} = \frac{1}{H}\left(\left[\nu\left(Hv_1\right)_y\right]_x + 0.5\left[\nu\left(Hu_1\right)_y + \nu\left(Hv_1\right)_x\right]_x\right)$$

$$\nu = B\delta^2 H\zeta_t$$

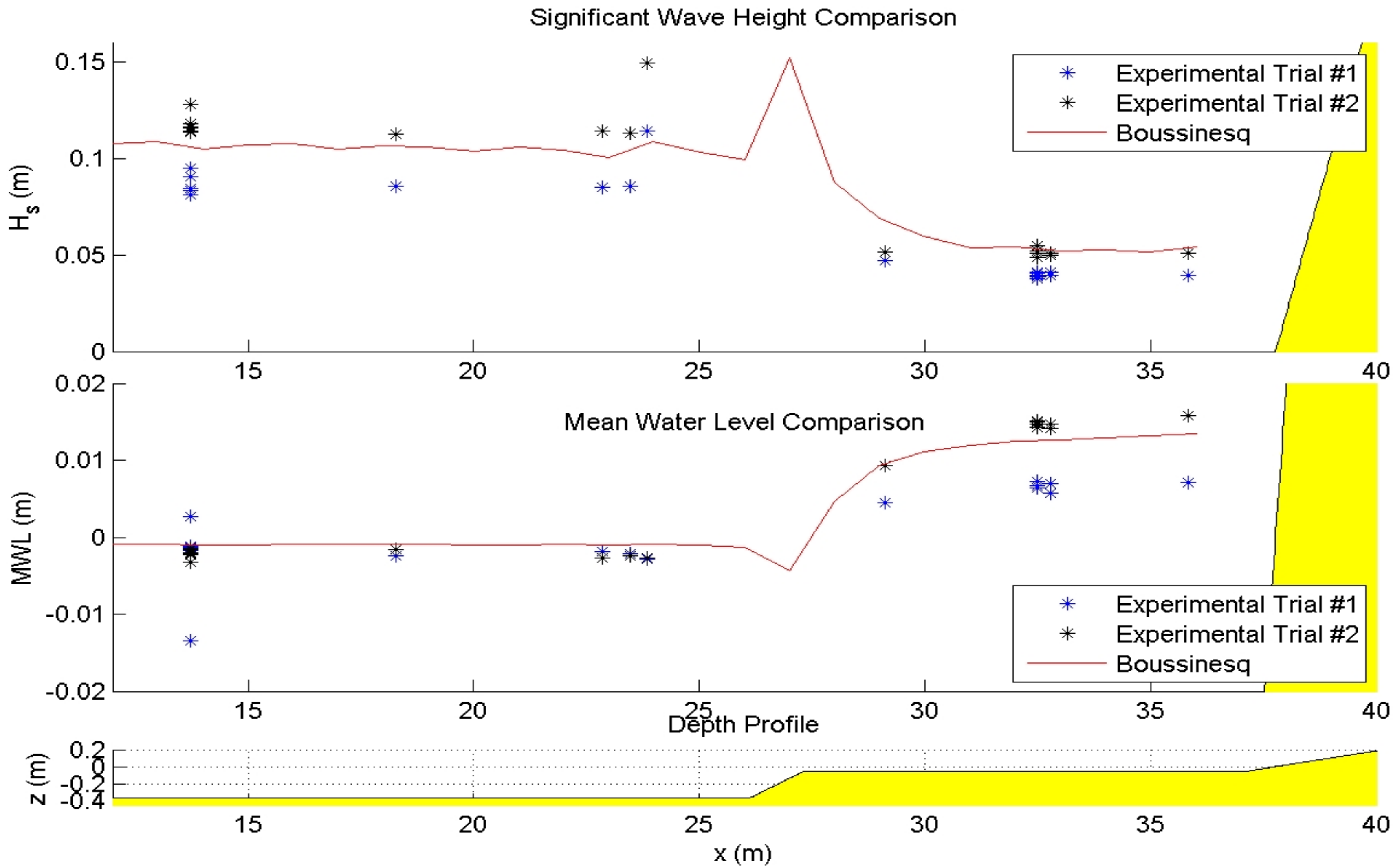
- Little verification to-date of bottom friction form and f-values in phase-resolving wave models (set to zero in validation/benchmarking)
- Breaking model is highly empirical, but has undergone a large validation exercise
  - Breaking initiates when free surface gradient, $\partial\zeta / \partial x$, exceeds some threshold
  - Turns off when dips below another threshold

Irregular Wave Breaking Onto a Shelf



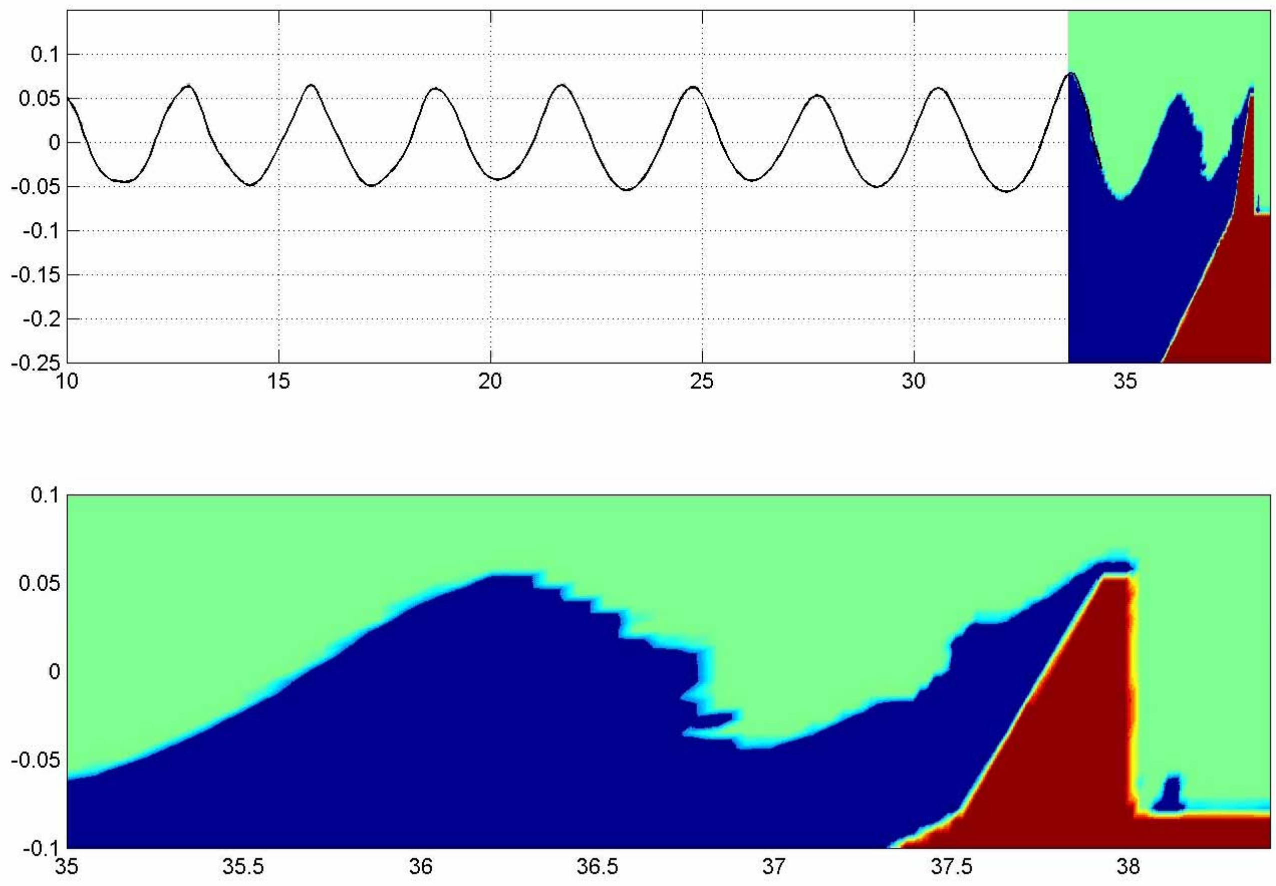
Irregular Wave Breaking Onto a Shelf

Experimental data from Don Ward et al., 2007



Wave Overtopping – Limitations with Boussinesq

- Wave overtopping is a turbulent, 3D problem
  - Strong vertical velocity and acceleration components



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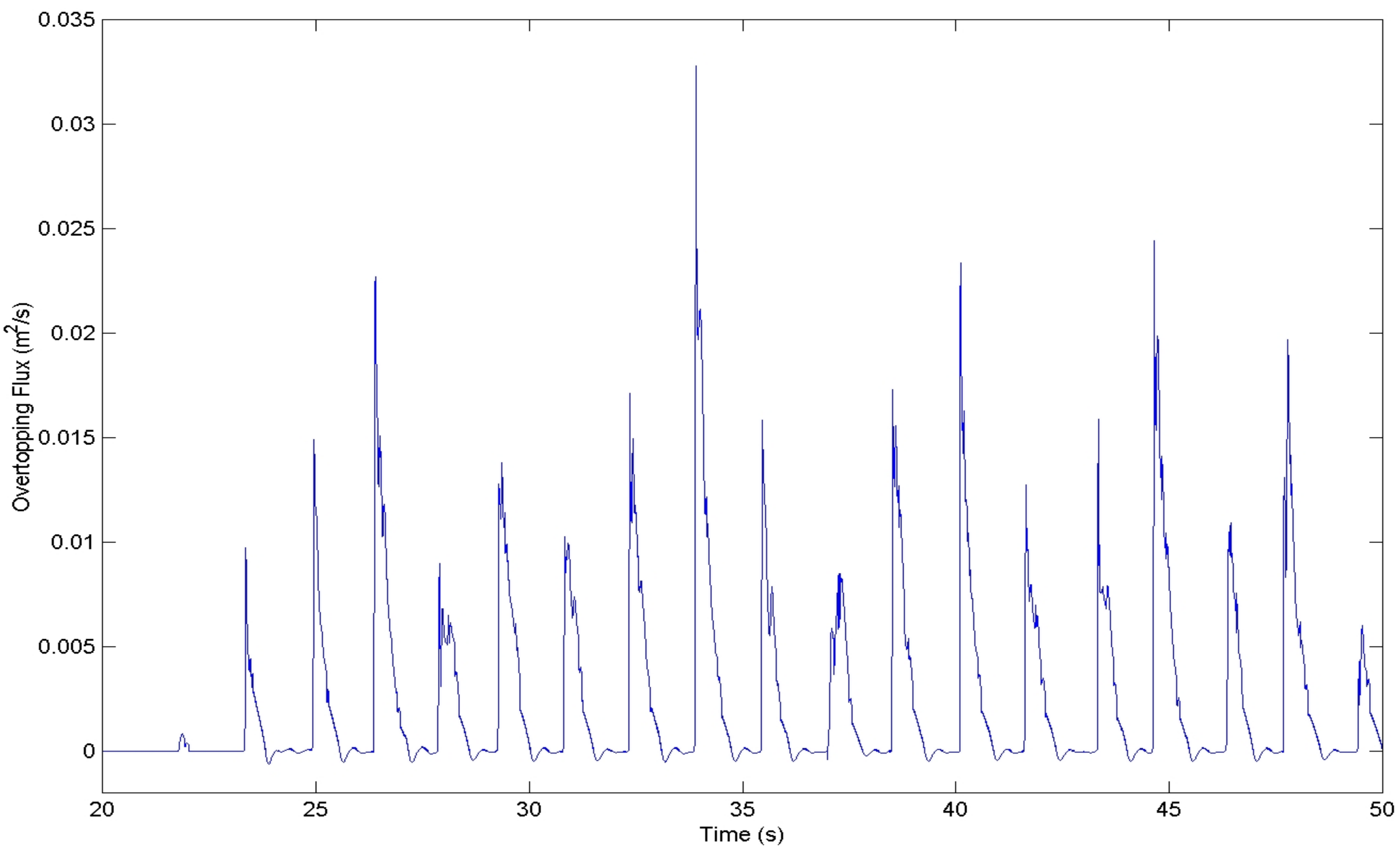
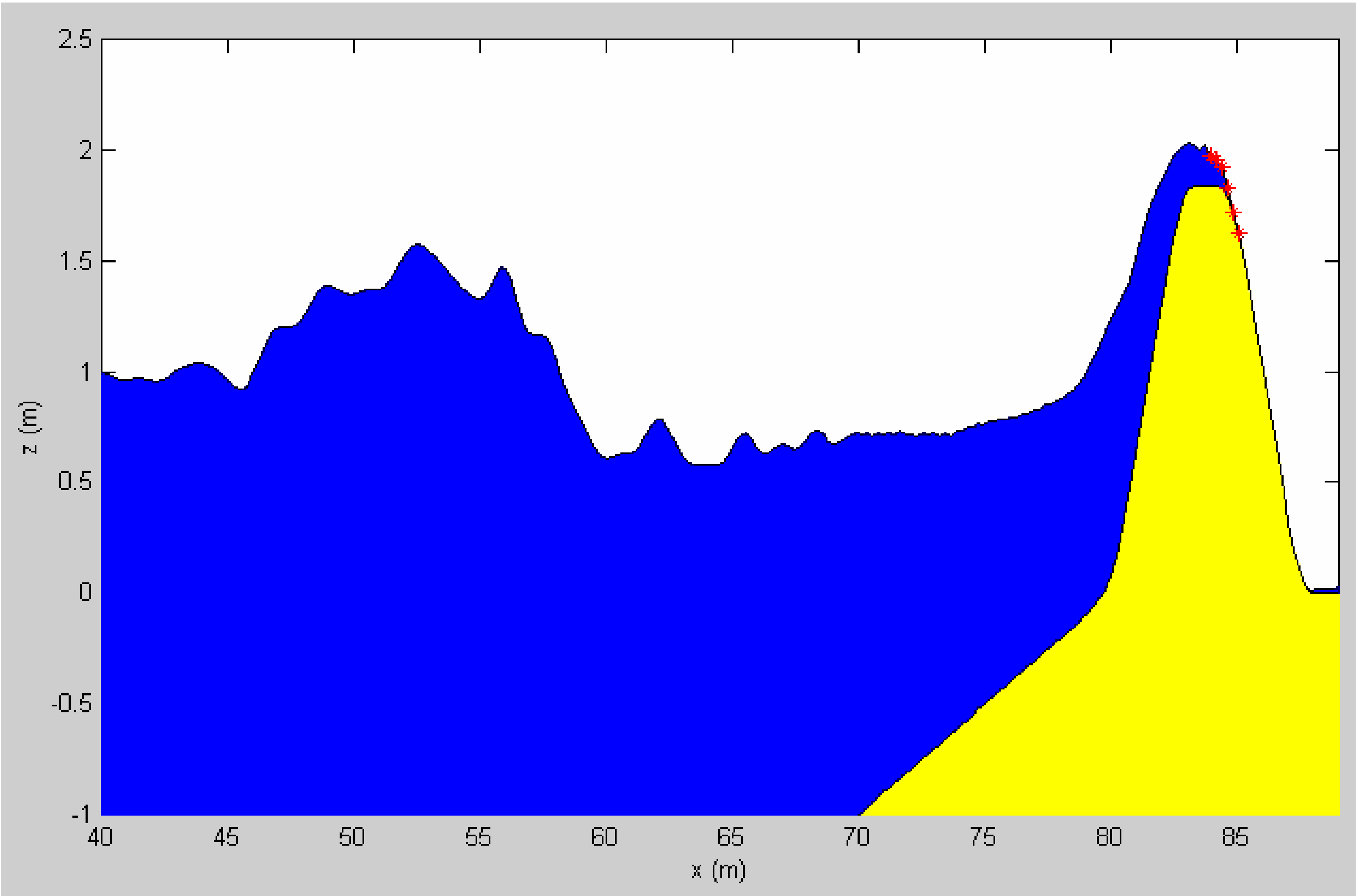
Wave Overtopping – Limitations with Boussinesq

- Turbulent interaction with reflected wave leads to a non-uniform overtopping time series, even for regular incident waves



Wave Overtopping – Limitations with Boussinesq

- Now, with the Boussinesq, we cannot model this turbulent 3D interaction
  - How important is this phenomenon to predicting overtopping???



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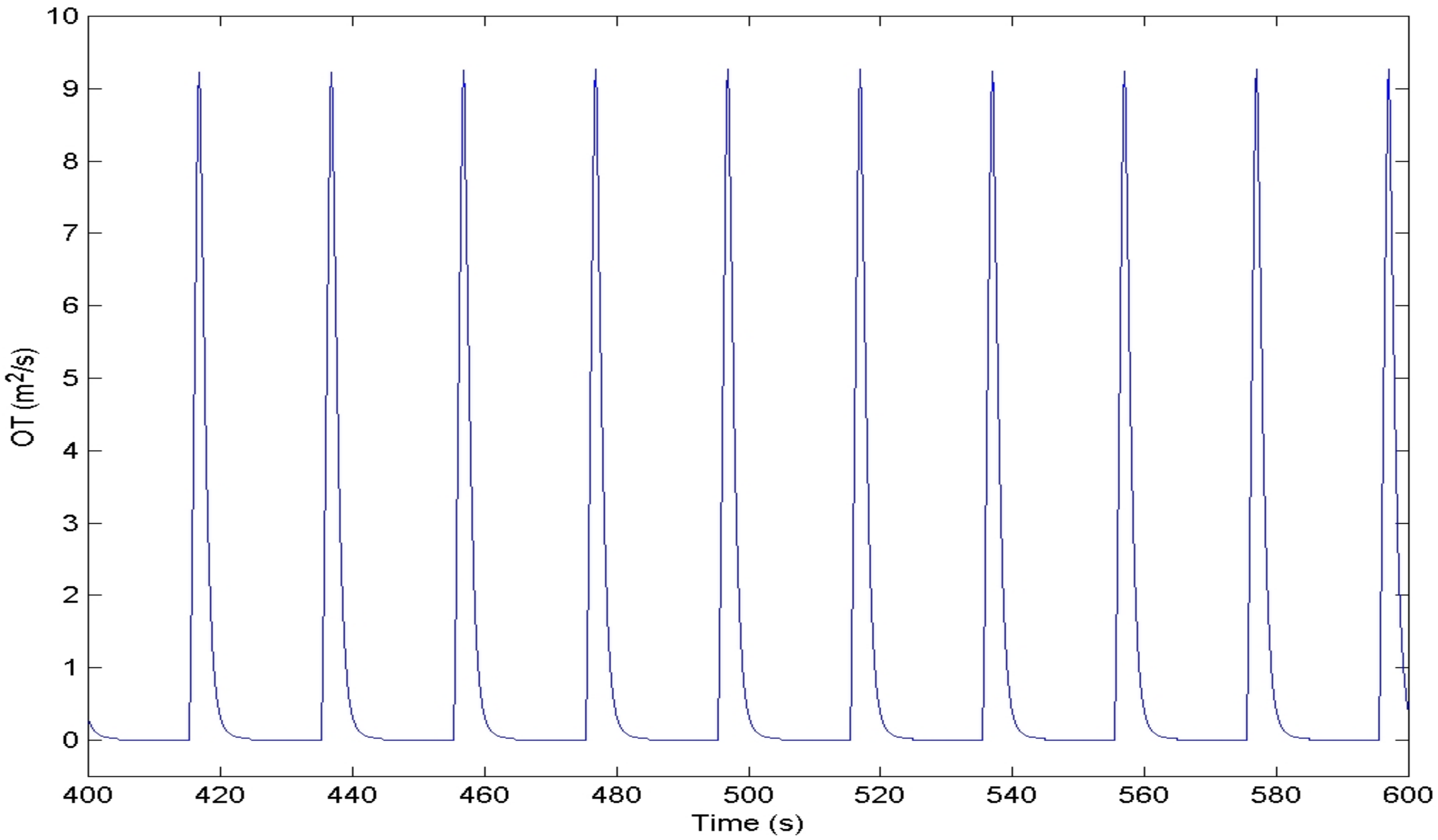
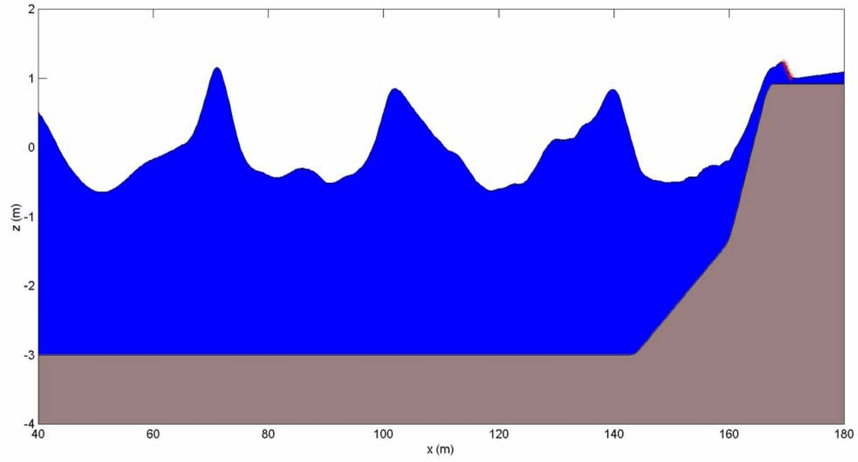


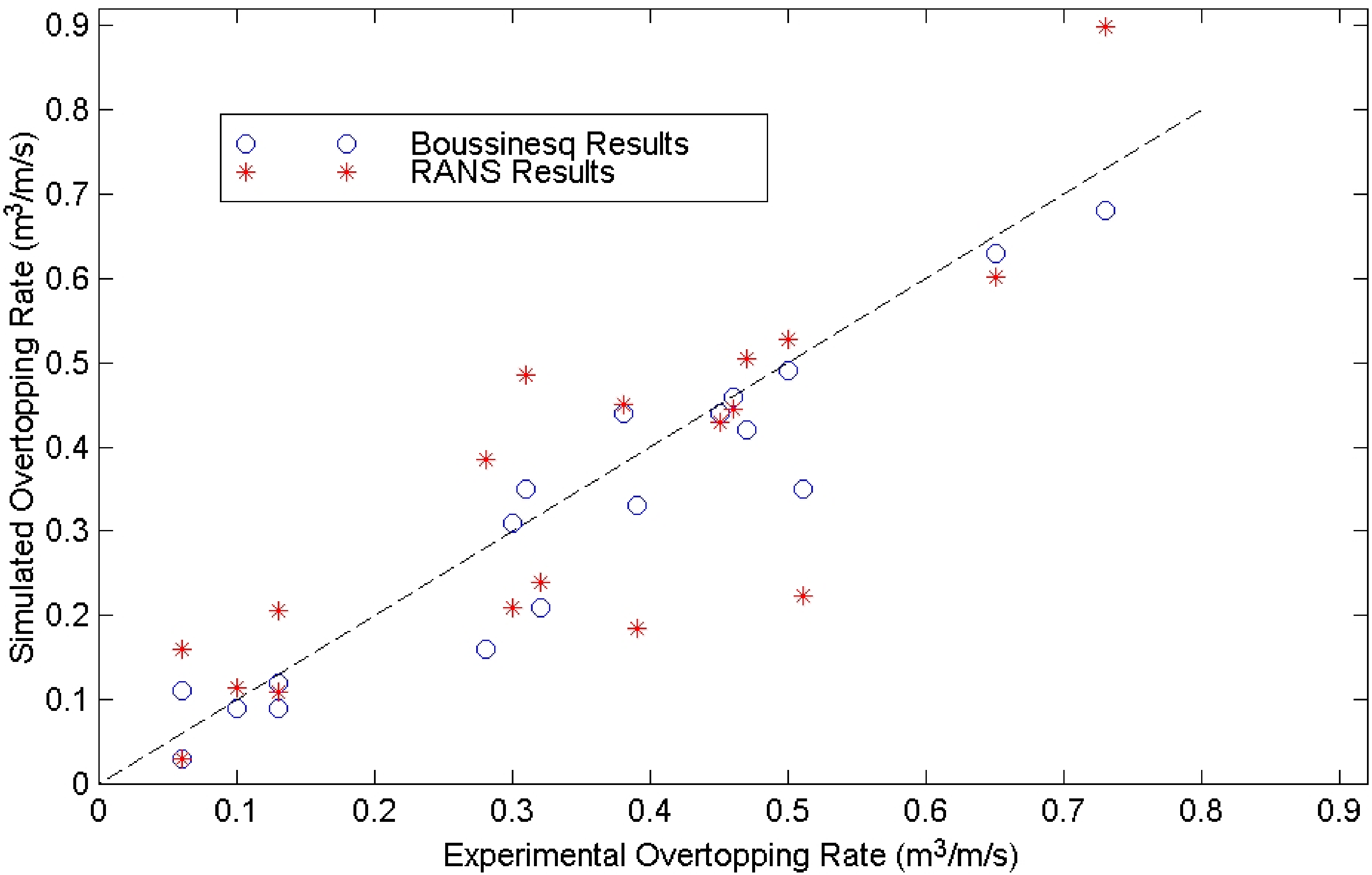
Wave Overtopping – Limitations with Boussinesq

- Now, with the Boussinesq, we cannot model this turbulent 3D interaction
  - How important is this phenomenon to predicting overtopping???
    - Experimental data comparisons indicate that, in the time-averaged sense, the Boussinesq provides reasonable results
    - Mean OT rate = OK
    - Variance statistics = not OK
      - Would need to use physical modeling or N-S modeling



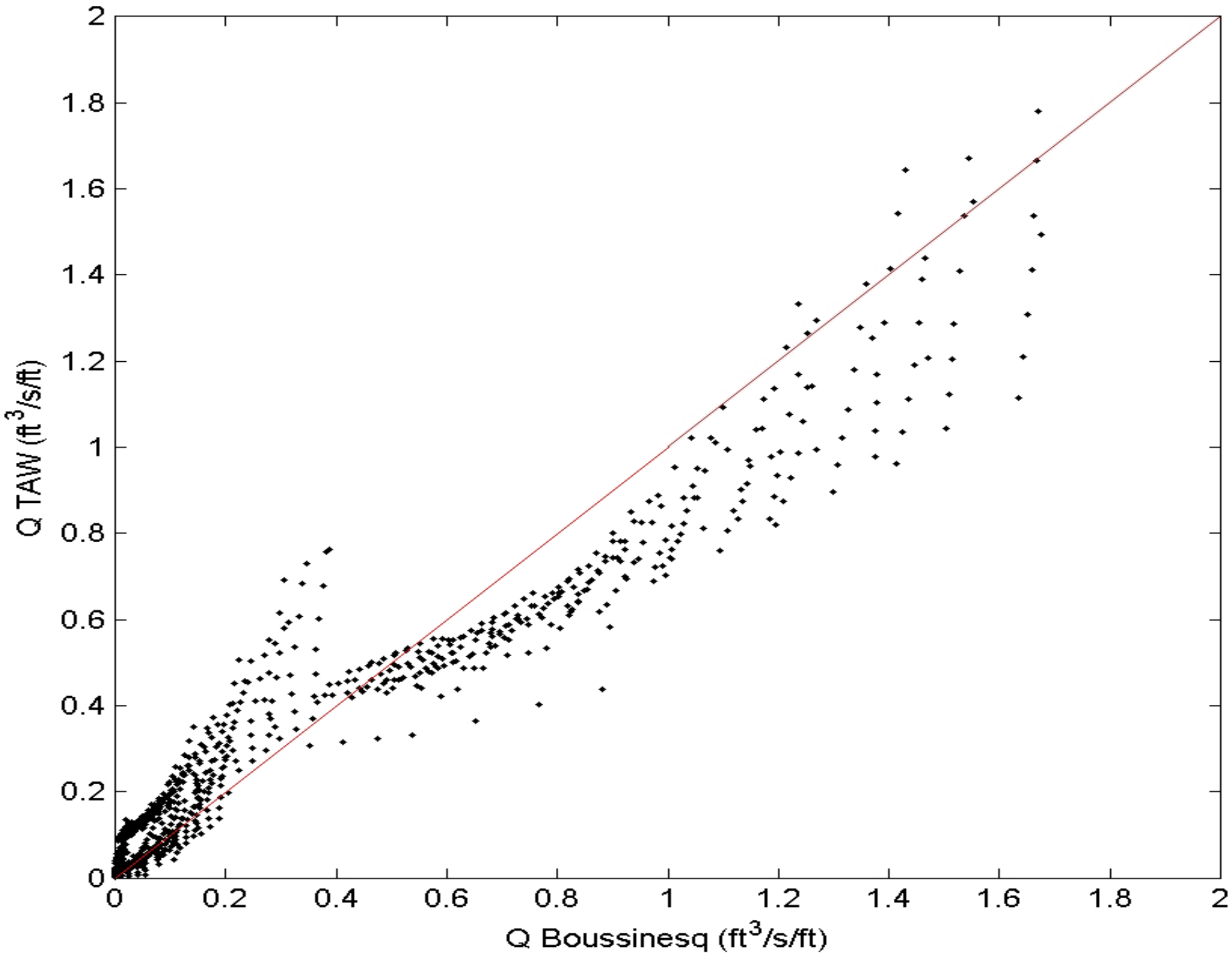
Wave Overtopping – Boussinesq Validation

- Comparison with standard benchmark data of Saville (1955)



Wave Overtopping

- Comparison with TAW formulation for Simple Levee
  - Simulation parameters:
  - Crest elevation = 17.5’
  - Toe elevation = +1’
  - 1/3<s<1/8
  - 1’<R_c<4’
  - 2’<H_s(600’)<8’
  - 8s<T_p<16s



Bous and empirical agree well for a wide range of levee configurations, **as long as H_toe (if structure was not there) is used in empirics**

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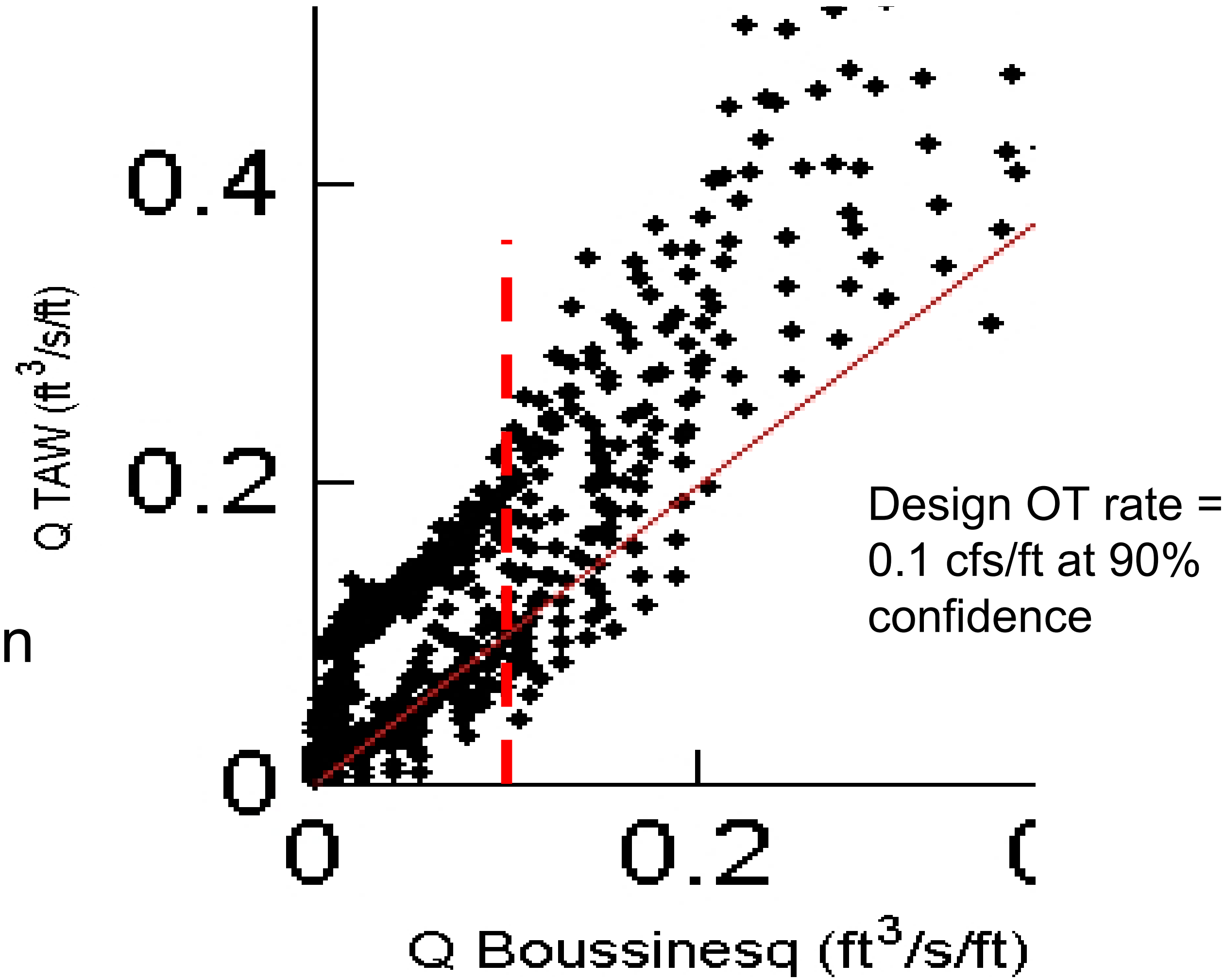

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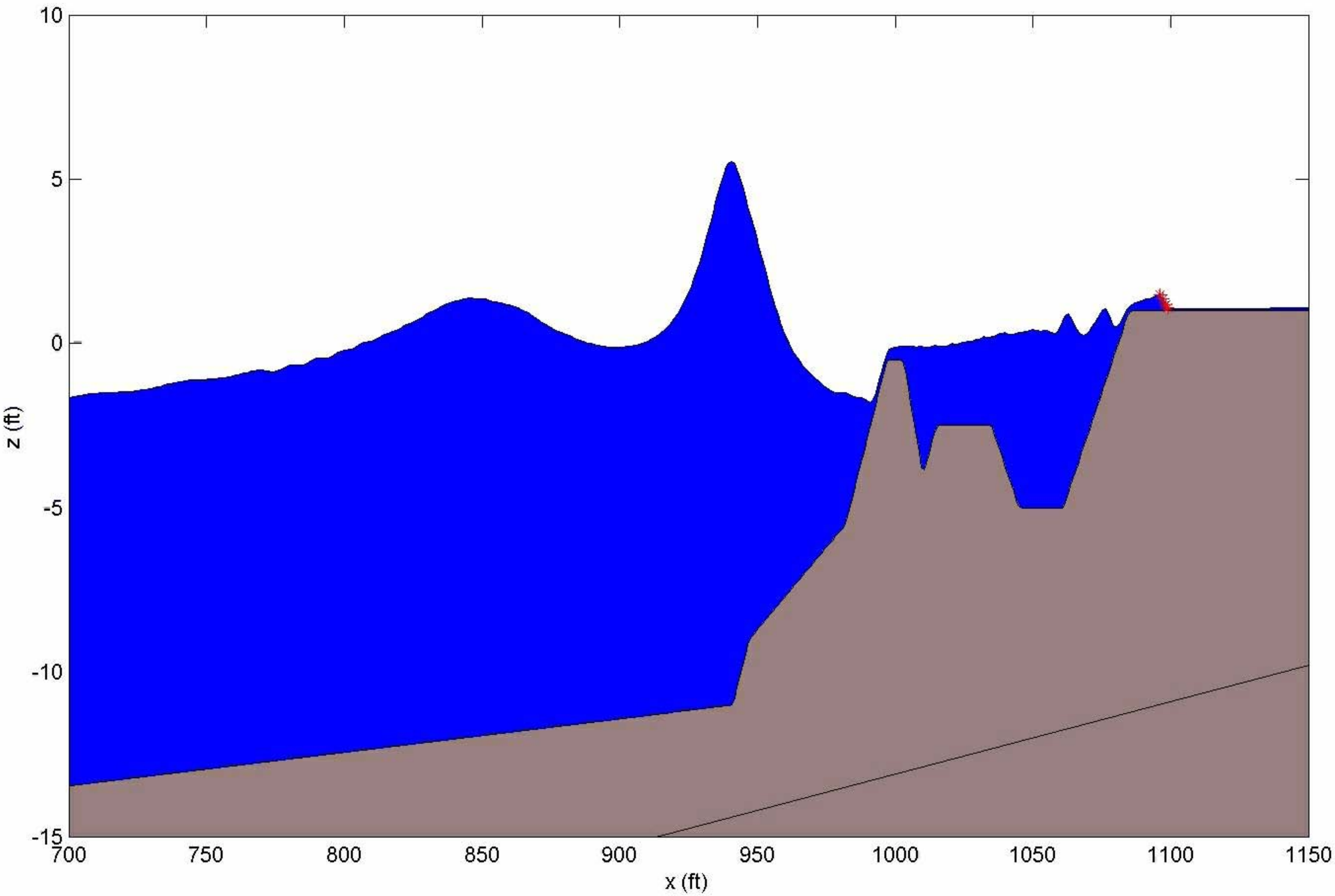
Information From Boussinesq Sims

- Example: Levee with foreshore protection



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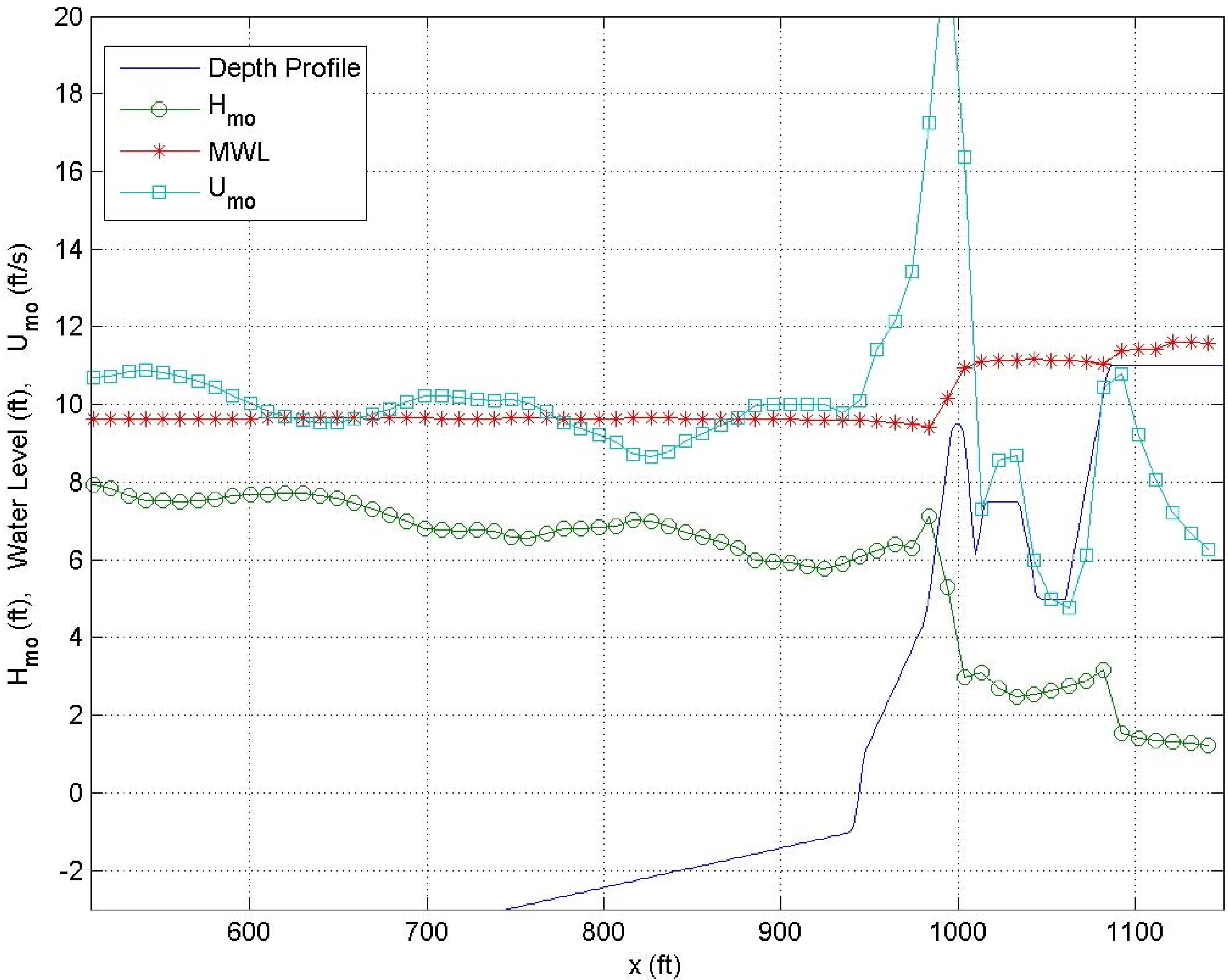

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