Nonlinear Waves over Dissipative Mud

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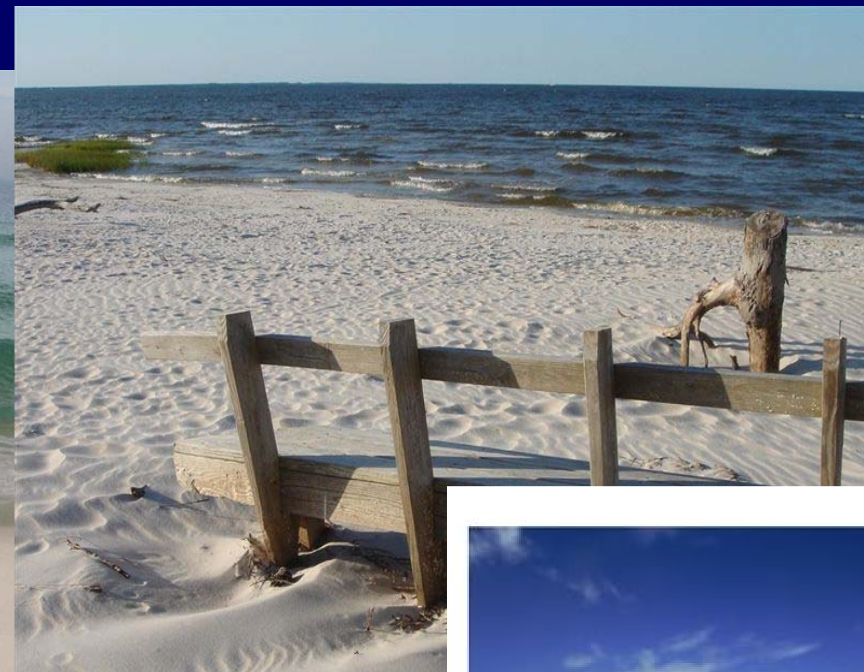
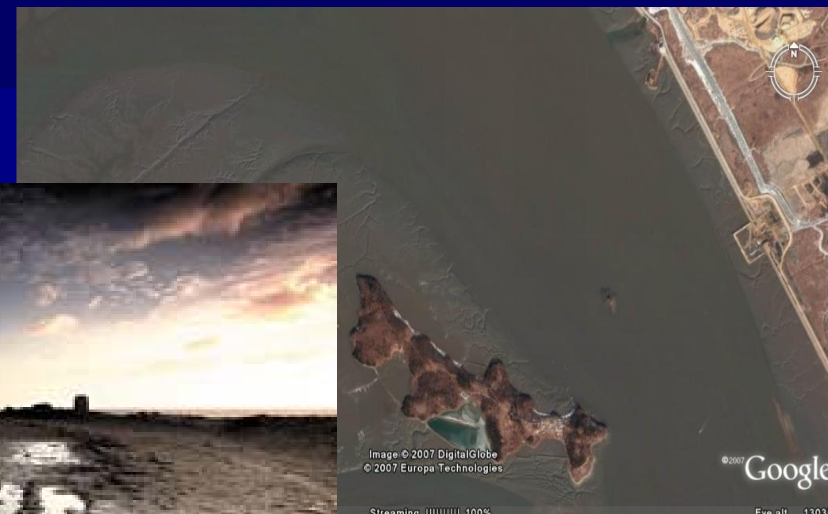
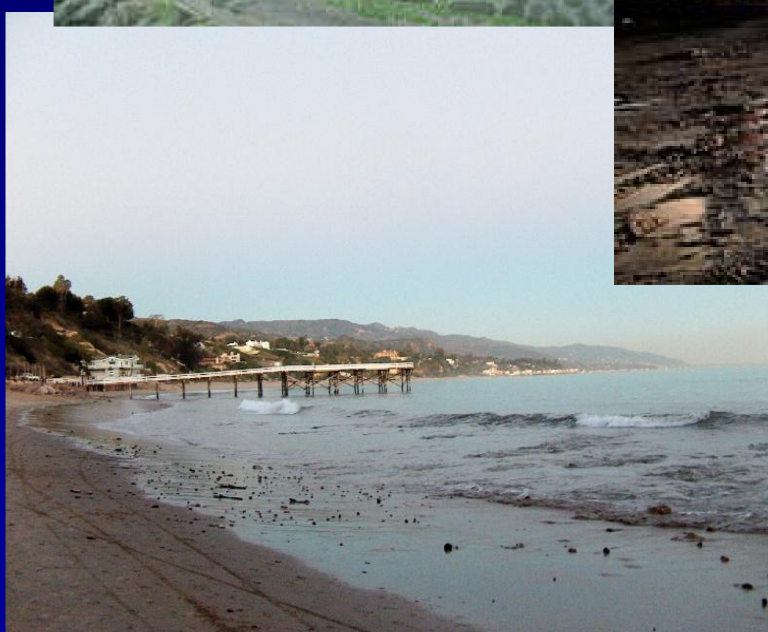
The Beach: Sand, Surf, Seashells



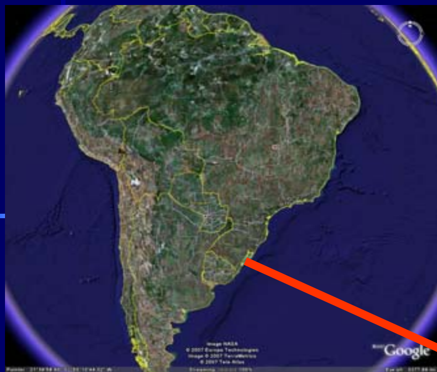
When you say “beach”, this is what you think of...

The Beach : Silts, Clays, Mangroves

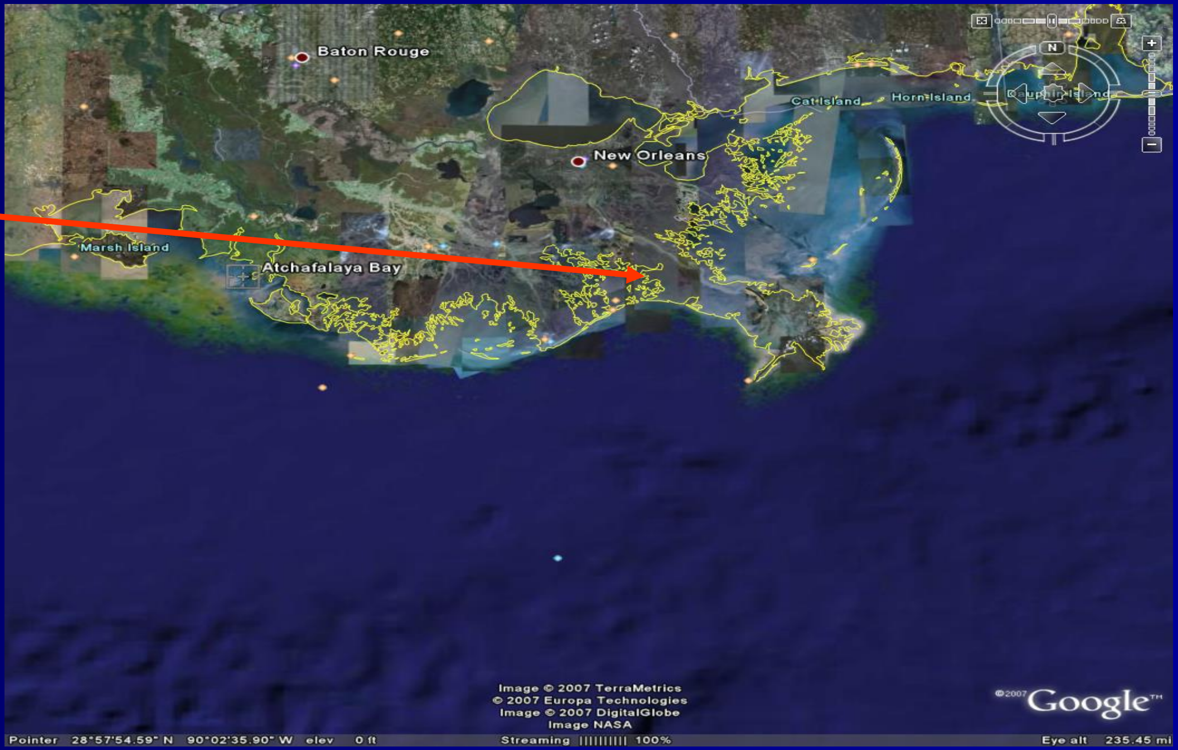




Cassino Beach, Brazil



Coastal Louisiana

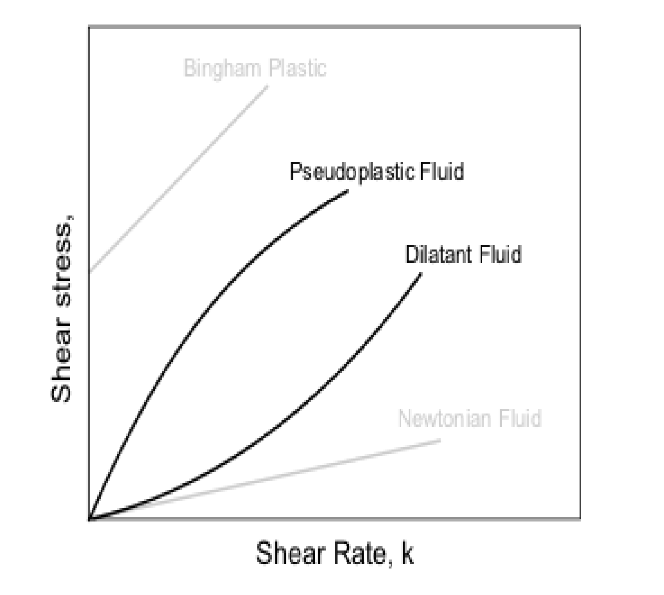


- Diversion channels proposed to build wetlands – drain off surge
- Potential for bringing large amounts of muddy sediment to coast and inner shelf
- Beneficial wave attenuation effects?

Wave Propagation Over Muds

Dissipation Mechanisms: Proxies for Mud

- *Two Layer Viscous Fluid:* waves propagating over a denser, more viscous fluid at the bottom
  - Gade (1958); Dalrymple and Liu (1978); Wen and Liu (1995); Ng (2000)
- *Visco-elastic medium:* waves propagating over a non-rigid bed (Silly Putty, chewing gum)
  - Hsaio and Shemdin (1980); Macpherson (1980); Sakakiyama and Bijker (1989); Maa and Mehta (1990); Jiang and Watanabe (1995)
- *Bingham plastic:* solid at low stresses, viscous fluid at high stresses (toothpaste, tomato ketchup)
  - Mei and Liu (1987)
- *More complex descriptions:*
  - Foda(1989): coupling viscoelastic bottom with surface sideband instabilities
  - Jiang and Mehta (1995): adjustable rheology



Wave Propagation Over Muds

Ng (2000 Coastal Engineering)

- Most theories lead to dispersion relations with complex wavenumber: many roots possible
- Ng (2000) – boundary layer reduction of Dalrymple and Liu (1978)
- Mud modeled as viscous fluid in thin layer over bottom
- Wavenumber perturbation:

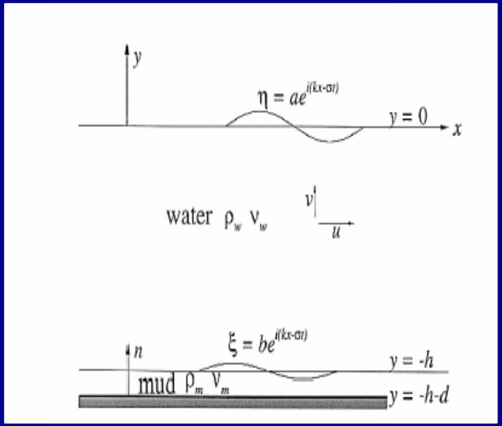
$$k = k_1 + \delta k_2 + ...$$

- Dissipative wavenumber explicit in terms of nondissipative wavenumber

Real part of wavenumber affects wavelength

$$\eta = ae^{i(k_r x - \omega t)} e^{-k_i x}$$

Imaginary part of wavenumber dampens wave



From Ng (2000)

Wave Propagation over Muds

Ng (2000 Coastal Engineering)

Linear dispersion relation for k_1 →

$$\omega^2 = g k_1 \tanh k_1 h$$

Real part of dissipative wavenumber k_2 (changes wavelength) →

$$k_{2r} = -\frac{\mathrm{Re}(B) k_1}{\sinh k_1 h \cosh k_1 h + k_1 h}$$

Imaginary part of dissipative wavenumber k_2 (dampens wave) →

$$D_n = k_{2i} = -\frac{\mathrm{Im}(B) k_1}{\sinh k_1 h \cosh k_1 h + k_1 h} = -\frac{\delta_m \left(B'_r + B'_i\right) k_1^2}{\sinh 2 k_1 h + 2 k_1 h}$$

$$B = f\left(d, \delta_m, \gamma = \frac{\rho_w}{\rho_m}, \zeta = \sqrt{\frac{\upsilon_m}{\upsilon_w}}\right)$$

d = depth of mud layer

δ_m = mud boundary layer thickness

Wave Propagation Over Muds

Nonlinear triad wave model of Kaihatu and Kirby (1995) with mud dissipation (Kaihatu et al. 2007)

Surf Zone Dissipation (*Thornton and Guza 1983*)

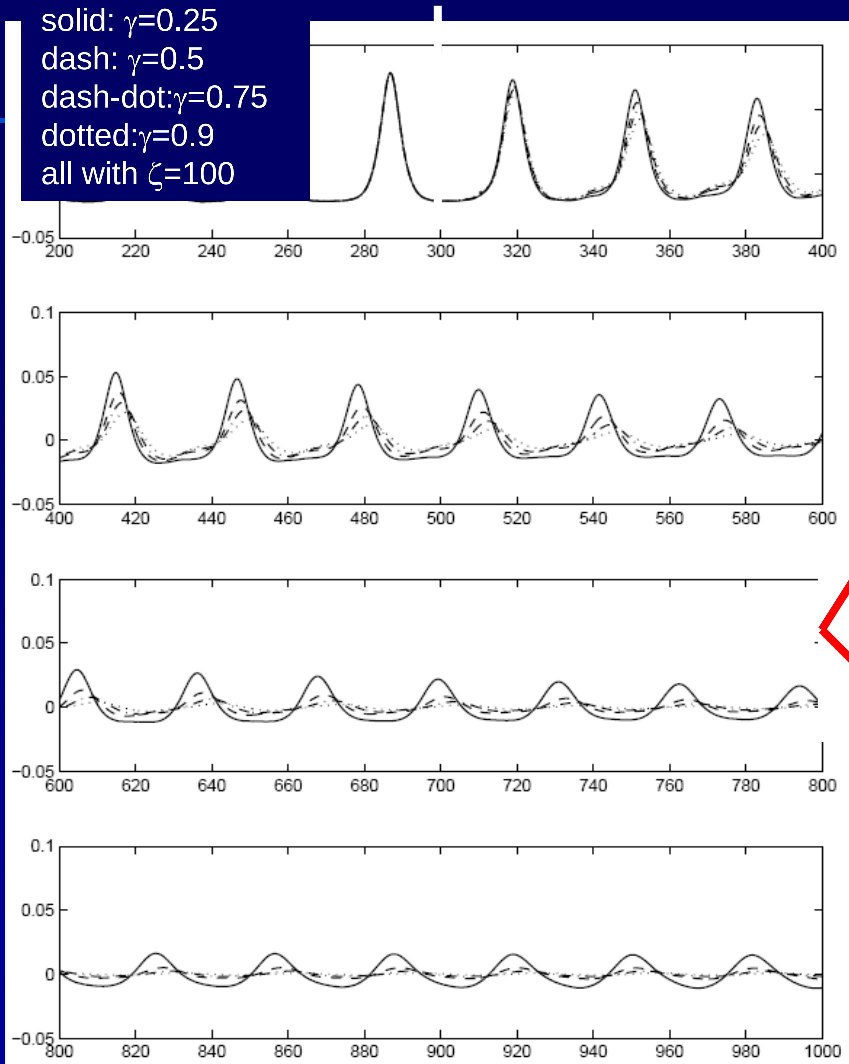
One-Dimensional

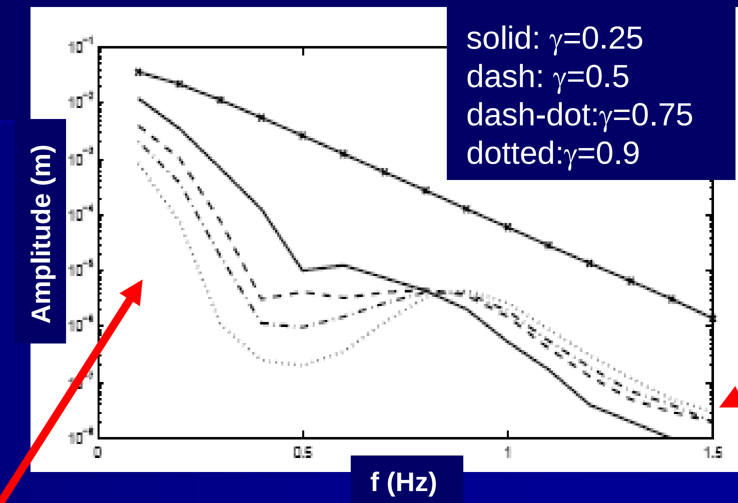
$$A_{n,x} + \frac{C_{gn,x}}{2C_{gn}} A_n + D_n A_n + \alpha_n A_n = -\frac{i}{8\omega_n C_{g,n}} \left[\sum_{l=1}^{n-1} R A_l A_{n-l} e^{i\Theta_{l,n-l}} + 2 \sum_{l=1}^{N-n} S A_l^* A_{n+l} e^{i\Theta_{n+l,-l}} \right]$$

Parabolic Two-Dimensional (**REF/DIF-S** with nearshore nonlinear interactions)

$$2i(kCC_g)_n A_{n,x} + i(kCC_g)_{nx} A_n - 2(kCC_g)_n (\bar{k}_n - k_n) A_n + \left[\left(CC_g \right)_n A_{ny} \right]_y + 2i(kCC_g)_n D_n A_n$$
$$= \frac{1}{4} \left[\sum_{l=1}^{n-1} R A_l A_{n-l} e^{i\Theta_{l,n-l}} + 2 \sum_{l=1}^{N-n} S A_l^* A_{n+l} e^{i\Theta_{n+l,-l}} \right]$$

High frequency damping – effect of subharmonic interaction





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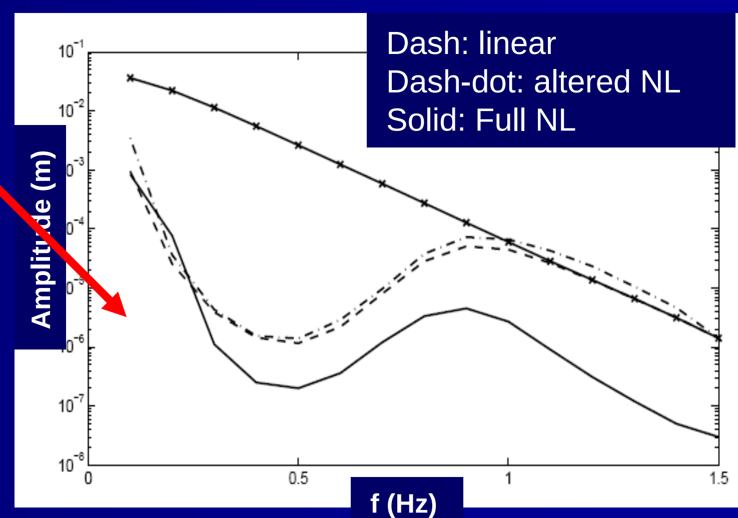
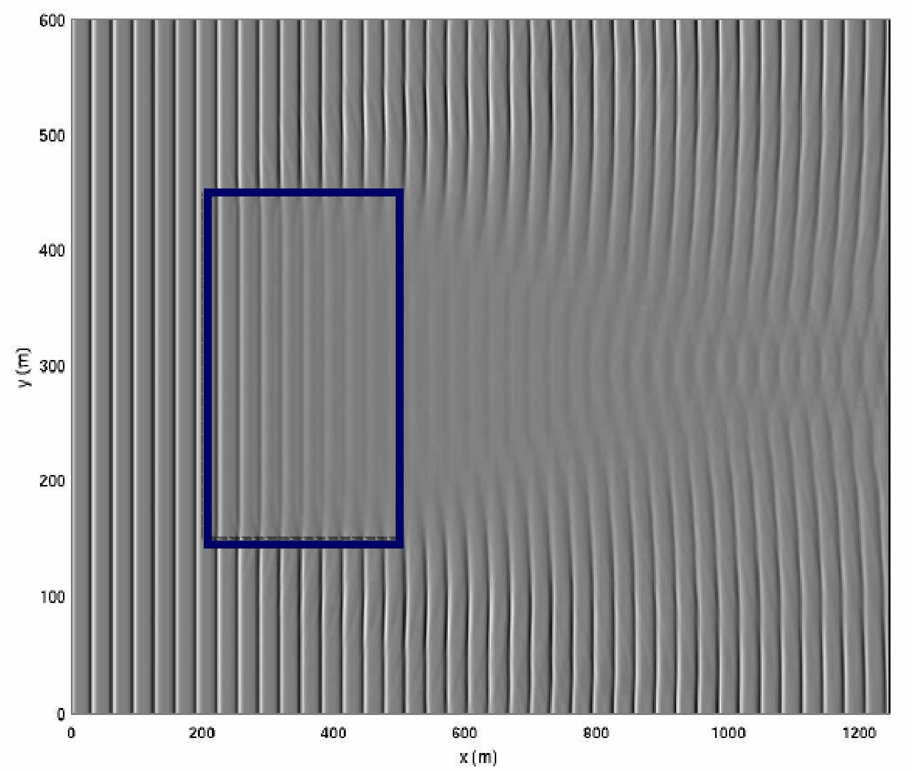


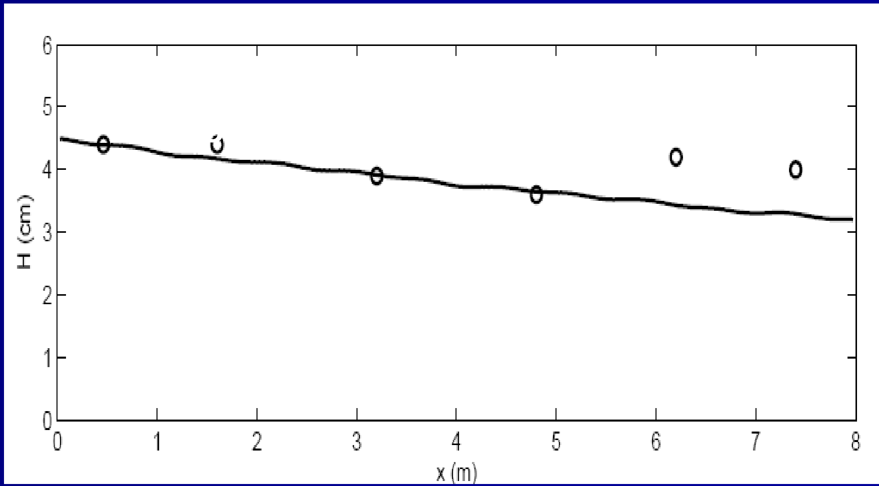
Kaihatu et al. (2007)

Two-Dimensional Wave-Mud Interaction



Two-Dimensional damping of cnoidal waves over flat bottom

Comparison to DeWit (1995)



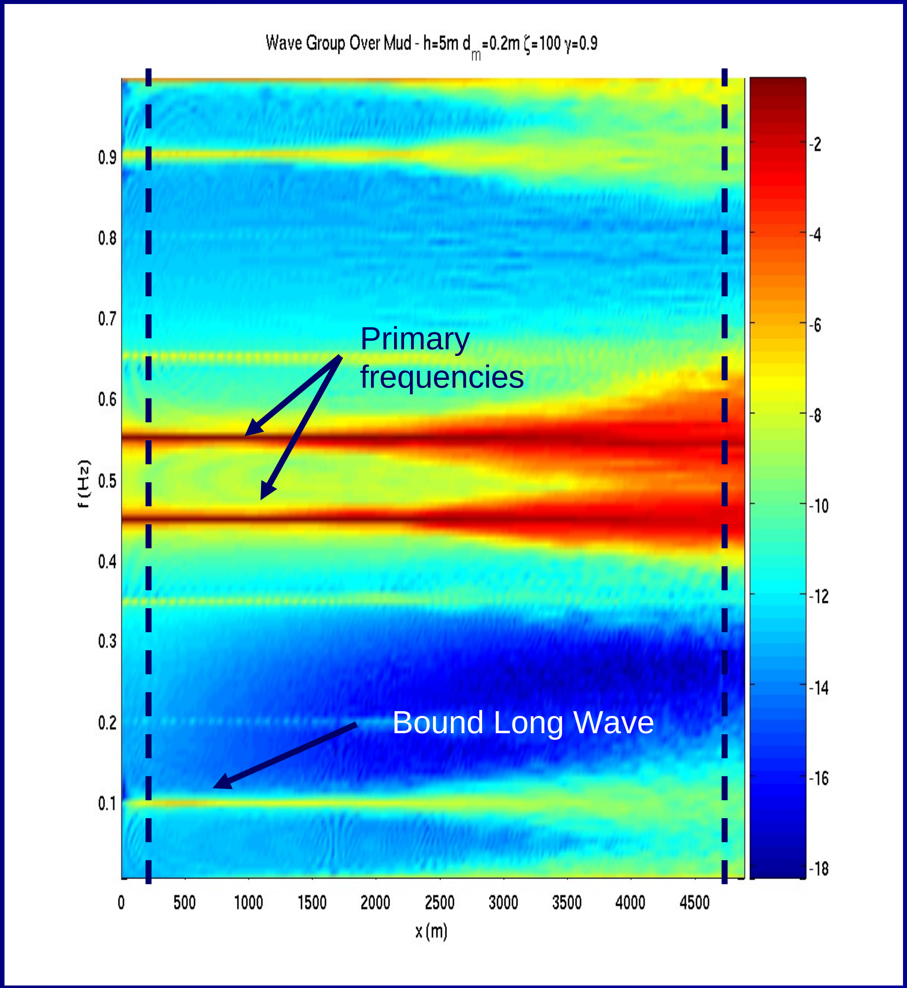
- h= 0.325 m
- ρ = 1300 kg/m^3
- d_m=0.115 m
- ν_m= 2.6x10^{-3} m^2/s (maple syrup)
- H = 0.045 m
- T = 1.5 s

γ **= 0.76** ζ **= 44.7**

Kaihatu et al (2007)

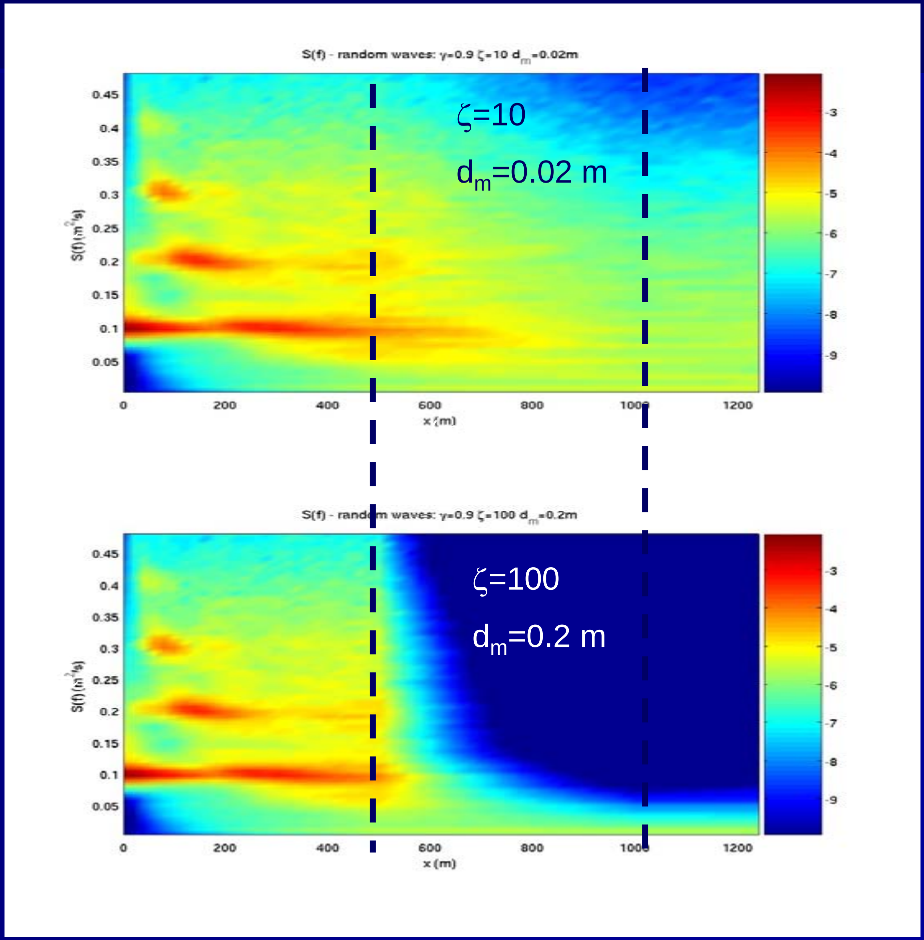
Wave Groups over Mud Patch

- Sheremet and Stone (2003): high frequency damping due to mud event
- Sheremet et al (2005): damping of high frequencies caused by dissipation of bound long wave exerting energy drain on entire system
- We run wave groups through the model; primary wave frequencies in deep water
- Bound long wave generated and dissipates slowly over 5*km*
- No significant damping of wave group frequencies, but higher frequencies do appear to damp



Random Waves Over Mud Patch

- Random waves (TMA with random phase) with peak period of *Tp*=10 s at input
- γ=0.9; ζ=(10, 100); *dm=*(0.02 m, 0.2 m)
- *h*= 1*m*
- Very strong cross-spectral damping with the high damping case (high frequencies feel bottom)

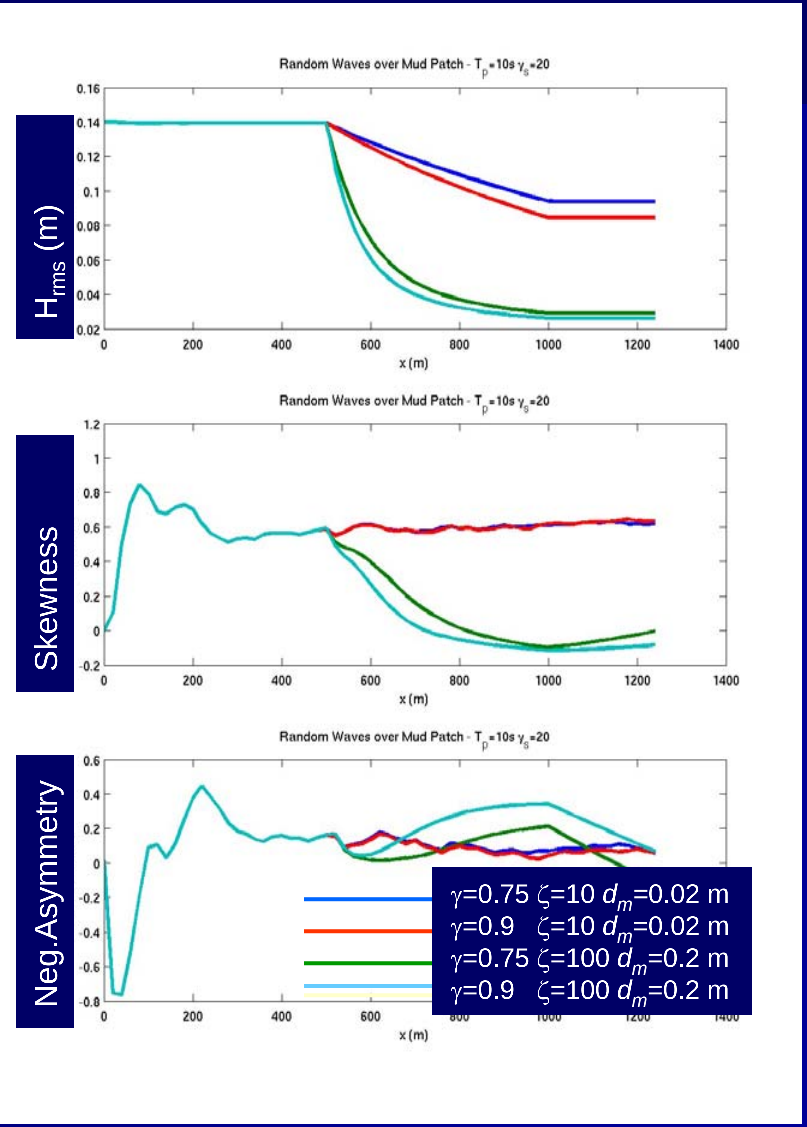


Color shade plots of log of spectral density

Random Waves Over Mud Patch

Random Waves over Mud – Effect on Wave Shape Statistics

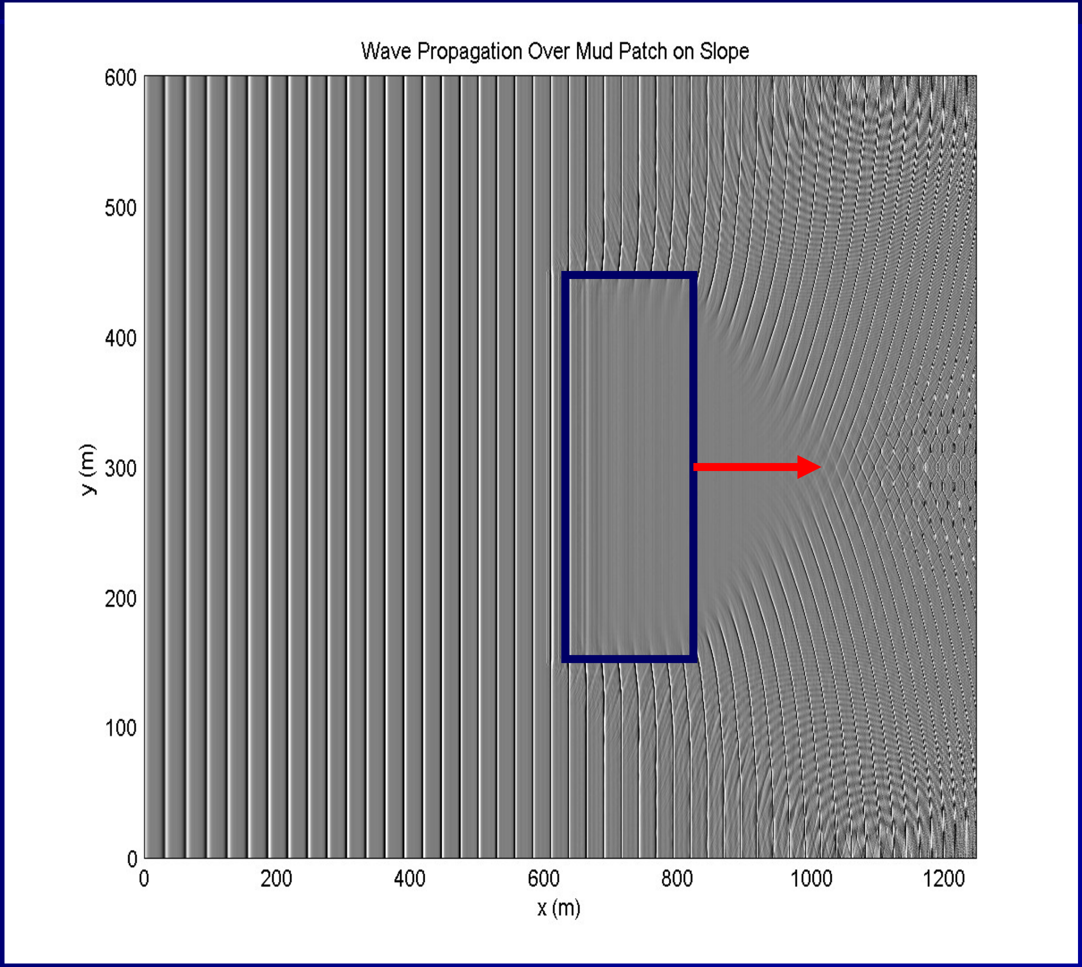
- Significant effect of mud damping on waveheight (energy)
- High-damping case strongly affects skewness
- Mud does not have a strong effect on asymmetry
    - Asymmetry not high for this situation



Breaking Wave and Mud Dissipation

Cnoidal Wave Propagation over Sloping Bottom

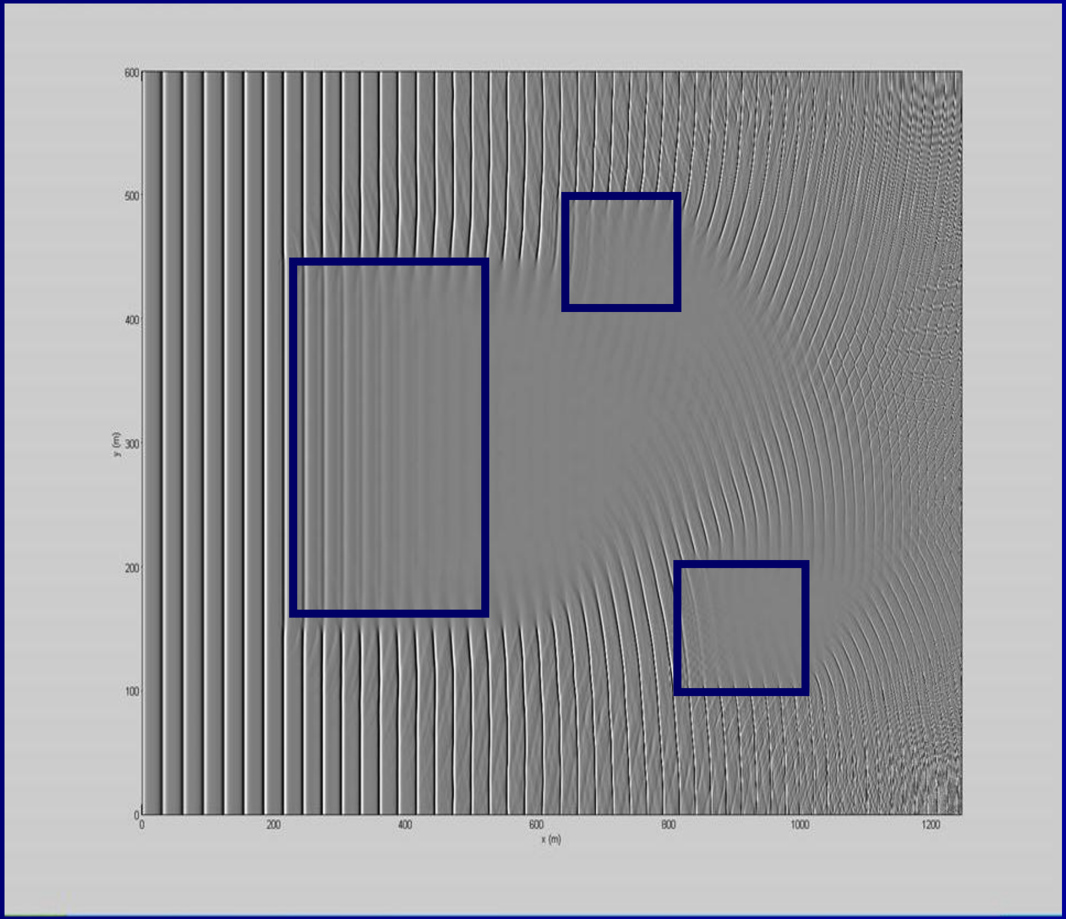
- Interest – Invert free surface for mud properties
- Strong diffraction effects behind mud patch
- Large effect on surface



Breaking Wave and Mud Dissipation

Cnoidal Wave Propagation over Sloping Bottom

- Isolated mud patches
- Interacting diffraction patterns
- Diffraction pattern will complicate inversion



Summary

- Effects of mud deposits on waves
  - Beneficial side effects?
- Nonlinear wave model with viscous mud dissipation (Kaihatu et al. 2007)
  - Mud dissipation affects subharmonic interactions
    - Indirect damping of high frequencies
  - Favorable comparison to experimental data (DeWit 1995)

Summary

- Wave groups: some indirect draining of high frequency energy
- Random waves: strong frequency dependence of damping
  - Affects third moment estimations
- Two-dimensional wave-mud interactions on slope
  - Dissipation from breaking and mud
  - Diffraction effects behind patch dependent on local depth
  - Excessive mud patchiness complicates surface image inversion